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Note—Commonality Strategies: Value Drivers and Equivalence with Flexible Capacity and Inventory Substitution

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Commonality strategies assemble different products from at least one common component and one other product-specific component. The distinguishing feature of commonality, i.e., the presence of dedicated components to be assembled with a common component, is shown to be mathematically inconsequential in the sense that the unified commonality problem for two products can be reduced to an equivalent substitution flexibility problem without those dedicated components. This significant simplification provides the first general, closed-form condition for commonality adoption and identifies its value drivers. Commonality is optimal even for perfectly correlated demands if products have sufficiently different margins. This introduces the "revenue-maximization option" of commonality as a second benefit that is independent of the traditional risk-pooling benefit. "Pure commonality" strategies are never optimal unless complexity costs are introduced. Dual sourcing, externalities, and operational hedging features of commonality are discussed.

Key words: component commonality; assemble to order; risk pooling; flexibility; substitution; dual sourcing
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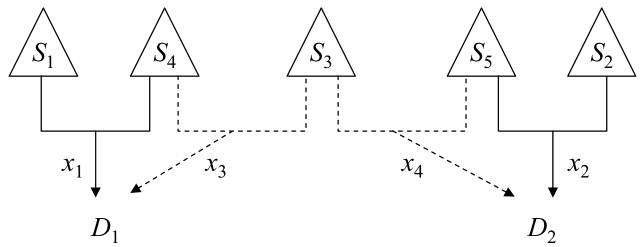
1. Summary and Literature Review

Commonality strategies assemble different products from at least one common component and one other product-specific component. Analytic studies of commonality traditionally compare two distinct models: a *no-commonality model*, where each product requires two product-dedicated components, and a *pure commonality model*, where each product requires one dedicated and one common component. The value of commonality is then explained in terms of the risk-pooling benefit in the pure commonality model. Clearly, this benefit comes at a cost, as the common component is more expensive than the dedicated ones that it replaces. The traditional requirement to adopt commonality is that the optimal value of the commonality model exceeds that of the no-commonality model, yet no simple conditions have been available.

This note analyzes a single unified model with five inputs and two products, as shown in Figure 1, that captures these two models as special cases. Under the no-commonality strategy, product $i \in \{1, 2\}$ uses dedicated components i and $i+3$, while under the pure commonality strategy it uses common component 3 and its other input $i+3$. In general, however, the model allows for stocking both dedicated and common components and for alternate assembly allocations by substituting components 1 or 2 for the common component 3. Hence, we will refer to inputs 1 and 2 as *substitutable inputs*, while inputs 4 and 5 may be called *always-dedicated*, or simply "*other,*" *inputs*. The required presence of other inputs in addition to a common input is the defining feature of commonality that distinguishes it from substitution flexibility. At their core, capacity flexibility, inventory substitution, and dual sourcing refer to having a second option to fill a product demand; they typically do not require the assembly of multiple inputs. In other words, those strategies only have inputs 1, 2, and 3 in a two-product setting. (Among many, Bassok et al. 1999, Tibben-Lembke and Bassok 2002, and Van Mieghem 1998 (hereafter abbreviated as VM98), analyze substitution flexibility.)

This note shows that the distinguishing feature of commonality, i.e., the required presence of the other components 4 and 5, is mathematically inconsequential in the sense that the unified commonality problem is equivalent to the substitution flexibility problem of VM98. This equivalence is first established in a single-period model and then extended to a multiperiod setting with i.i.d. demand and negligible leadtimes. The equivalence between a five- and three-variables problem is mathematically surprising and significantly simplifies the analysis of commonality. It also establishes a precise relationship between different literatures that intuitively are related, yet have been developed in isolation. The key advantage of the unified

Figure 1 A Unified Commonality Model: Product i Always Requires Dedicated Component $i+3$ and Either Dedicated Component i or Common Component 3



model is in allowing the optimal strategy, service levels, and commonality adoption conditions to emerge through direct optimization. This note presents the first[1] general, closed-form condition for when commonality should be adopted, and identifies its driving factors. In addition, contrary to earlier statements in the literature,[2] commonality can be valuable even with perfect correlation when risk pooling is impossible. The explanation is found in the concept of a *revenue-maximization option* that is introduced as another benefit of commonality, independent of risk pooling. The note ends by describing the dual sourcing, externalities, and operational hedging features of optimal commonality strategies.

Commonality falls within the broader supply chain operations umbrella of assemble-to-order systems, which are reviewed by Song and Zipkin (2003) and combine elements of assembly and distribution systems. Commonality research comes in three forms: parsimonious analytic studies, detailed mathematical programming formulations, and product-design studies. Early examples of analytic studies are Collier (1982), Baker et al. (1986) and follow-up work by McClain et al. (1984) and Gerchak et al. (1988). All analyze the total inventory reduction due to risk pooling inherent in commonality. Eynan (1996), Eynan and Rosenblatt (1996, 1997), and Hillier (1999) add cost considerations but restrict attention, as do all predecessors, to uniform demand distributions. Gerchak and Henig (1989), Hillier (2000), and Rudi (2000b) continue comparing the two distinct models, but under general demand distributions. Eynan (1996) and Groenevelt and Rudi (2000) are the only papers, to our knowledge, that consider correlation in the specific settings of bivariate uniform and four-state discrete demand, respectively. Song (2002) and Lu and Song (2003) analyze significantly more general commonality problems with leadtimes; however, as with all predecessors, complexity defied obtaining adoption conditions.

The second strand of commonality research, starting with Dogramaci (1979), investigates detailed mathematical programming formulations and captures more reality, including setup costs and design complexity costs. Thomas (1991) proposes clustering heuristics to trade off production and design complexity costs. Swaminathan and Tayur (1998, 1999) use simulation-based optimization to determine the appropriate amount and type of product differentiation using component commonality. Thonemann and Brandeau (2000) formulate all costs as a function of the component design and use branch-and-bound and simulated annealing algorithms.

Commonality is also studied in the product design literature, which is largely separate from the two strands above. Desai et al. (2001) review this literature and provide an analytic model that trades off the cost benefits[3] with the revenue loss that can result from the perceived quality deterioration when a high-end product shares a component with a low-end product. Groenevelt and Rudi (2000) also investigate risk pooling and product-design choice using commonality.

Finally, a dual-sourcing approach is pursued in Hale et al. (2000) and Rudi (2000a), whose setup is closest to this paper: The dedicated strategy involves long-leadtime make-to-stock operations and commonality enables short-leadtime assemble-to-order, while dual sourcing involves a mixed strategy.

2. Model and Equivalence Result

Model. The unified commonality model of Figure 1 is a newsvendor network as defined in Van Mieghem and Rudi (2002) with three data sets: (1) Demand data: The probabilistic forecast for demand vector $\mathbf{D}$ is represented here by a bivariate continuous probability measure P. (2) Financial data: Activity vector $\mathbf{x}$ yields gross margin $\mathbf{m}'\mathbf{x}$, where $\mathbf{m}$ equals price minus any marginal assembly and transportation cost; inventory incurs unit purchasing and holding costs $\mathbf{c}_S$ and $\mathbf{c}_H$, and unmet demand incurs shortage cost $\mathbf{c}_P$. (3) Network data: The input-output matrices $\mathbf{R}_S$ and $\mathbf{R}_D$, where $R_{S,ij}$ denotes the amount of input stock i consumed per unit of activity j, and $R_{D,ij}$ is the amount of output i per unit of activity j. Without loss of generality, we assume the bill of material requires one

[1] This is the first such condition to our knowledge, as corroborated by the statement in Hillier's (2000, p. 756) literature review that "it is not possible to easily determine conditions under which employing a common component would be beneficial" and by the lack of mentioning any condition in the survey by Song and Zipkin (2003).

[2] For example, Eynan (1996, p. 1591) states that with perfect correlation "no savings can be realized as we cannot pool the risks."

[3] Using mostly deterministic models, the product design literature often abstracts from risk-pooling benefits.

unit of each input per unit output, so that

$$\mathbf{R}_S = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & 1 \\ 1 & & 1 & \\ & 1 & & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{R}_D = \begin{bmatrix} 1 & & 1 & \\ & 1 & & 1 \end{bmatrix}.$$

The objective is to maximize expected value by choosing inventory vector $\mathbf{S}$ before demand is observed, and allocation or assembly activity $\mathbf{x}$ afterwards. Let $\mathbf{v} = \mathbf{m} + \mathbf{R}'_D\mathbf{c}_P + \mathbf{R}'_S\mathbf{c}_H$ denote the *net value vector* associated with the various processing activities, and let $\mathbf{c} = \mathbf{c}_S + \mathbf{c}_H$. Denote an ex post optimal recourse allocation vector by $\mathbf{x}^*(\mathbf{S}, \mathbf{D}) = \arg\max\{\mathbf{v}'\mathbf{x}\colon\ \mathbf{x} \geq 0, \mathbf{R}_S\mathbf{x} \leq \mathbf{S}, \mathbf{R}_D\mathbf{x} \leq \mathbf{D}\}$. The expected firm value to be maximized then is $V(\mathbf{S}) = \mathbb{E}[\mathbf{v}'\mathbf{x}^*(\mathbf{S}, \mathbf{D}) - \mathbf{c}'_P\mathbf{D}] - \mathbf{c}'\mathbf{S}$, which is concave for any newsvendor network and strictly concave with a continuous probability measure P. Thus, the optimal stocking strategy $\mathbf{S}^*$ is unique.

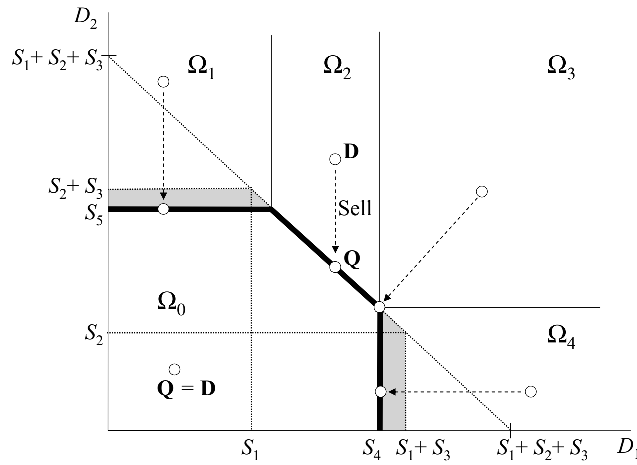
Natural assumptions are that: (1) The common input is more expensive but a viable alternative: $\max(c_1, c_2) < c_3 < c_1 + c_2$; (2) it is economically profitable to produce either product: $c_i + c_{i+3} < v_i$; (3) $v_1 = v_3 \geq v_2 = v_4 > 0$. Assumption 3 labels product 1 as the higher-margin product and follows tradition in commonality research by assuming that ex post usage of the common component versus the substitutable component does not affect value. (The analysis carries through with value penalty for assembling a product using the common component.)

Analysis. Under Assumption 3, a greedy rule that prioritizes product 1 in the ex post allocation of the common component is optimal. For the ex ante stocking decisions, it obviously is suboptimal to stock more dedicated inputs for product i than maximal possible product i output so that $S_i^* \leq S_{i+3}^* \leq S_i^* + S_3^*$. It also is suboptimal to stock more of the common component than ever can be used, so that $S_3^* \leq (S_4^* - S_1^*) + (S_5^* - S_2^*)$. These intuitive constraints extend those in Gerchak et al. (1988) for the case of pure commonality (i.e., $S_1 = S_2 = 0$), as first noted by Rudi (2000a). With these constraints, the optimal allocation decisions can be represented as in Figure 2. The thick-lined area Ω_0 is the capacity of the firm where all demand can be met. However, the commonality problem can be reduced further:

Property 1 (Reduction). The optimal commonality strategy sets $S_4^* = S_1^* + S_3^*$ and $S_5^* = S_2^* + S_3^*$.

Proof. Assume that $S_{i+3}^* < S_i^* + S_3^*$ for product $i \in \{1, 2\}$. This would imply $S_3^* > 0$. Let $j \neq i$ denote the other product. Reducing S_3^* by $\epsilon > 0$ and increasing S_j^* by ϵ would not change the effective operating profit $\mathbf{v}'\mathbf{x}$ (the Ω partition in Figure 2 is unchanged, as is the total product output[4]) but would reduce stocking costs by $(c_{S_3} - c_{S_j})\epsilon > 0$, contradicting assumed optimality. □

Figure 2 **Total Product Sales Quantities $\mathbf{Q} = (x_1 + x_3, x_2 + x_4)$ Depend on the Ω-Area in Which the Demand Vector D Falls**



Note. The optimal commonality strategy has no shaded areas.

(This generalizes a similar property established by Eynan and Rosenblatt 1997 in a simpler model that minimizes cost subject to an exogenous service-level constraint and uniform demand uncertainty.) With only three independent variables, S_1^*, S_2^*, and S_3^*, the reduced commonality problem is identical to the problem in VM98:

Corollary 1 (Equivalence). *The optimal commonality strategy sets $\mathbf{S}_{1:3}^* = (S_1^*, S_2^*, S_3^*)$ according to the optimal flexible capacity strategy of VM98 with dedicated capacity costs $\tilde{c}_i = c_i + c_{i+3}$ and flexible capacity cost $\tilde{c}_3 = c_3 + c_4 + c_5$ with necessary and sufficient conditions: There exists a $\mu \in \mathbb{R}_+^3$ such that $\mu'\mathbf{S}_{1:3}^* = 0$ and*

$$\begin{pmatrix} 0 \\ v_2 \\ v_2 \end{pmatrix} P(\Omega_1(\mathbf{S}_{1:3}^*)) + \begin{pmatrix} v_2 \\ v_2 \\ v_2 \end{pmatrix} P(\Omega_2(\mathbf{S}_{1:3}^*)) + \begin{pmatrix} v_1 \\ v_2 \\ v_1 \end{pmatrix} P(\Omega_3(\mathbf{S}_{1:3}^*)) + \begin{pmatrix} v_1 \\ 0 \\ v_1 \end{pmatrix} P(\Omega_4(\mathbf{S}_{1:3}^*)) = \tilde{\mathbf{c}} - \mu. \quad (1)$$

The optimal assemble-to-order commonality system dominates a pure assemble-to-stock (ATS) system, which is a simple and useful benchmark. Clearly, the optimal ATS system does not use commonality

[4] This substitution is even more profitable in the more general case when $v_1 \geq v_3 \geq v_2 \geq v_4$. The substitution of common input 3 for input j yields a substitution of activity x_{j+2} for higher-value activity x_j and increases the expected operating profit by $(v_j - v_{j+2})\epsilon \Pr(D_j \in [S_j, S_j + \epsilon])$.

and is denoted by $\overline{\mathbf{S}} = (\overline{S}_1, \overline{S}_2, 0, \overline{S}_1, \overline{S}_2)$, which is defined by two separate newsvendor solutions:

$$v_i P(D_i > \overline{S}_i) = \tilde{c}_i \Leftrightarrow \overline{S}_i = \overline{F}_i^{-1}\left(\frac{c_i + c_{i+3}}{v_i}\right), \qquad (2)$$

which are a special case of (1) given that $P(\Omega_2(\overline{\mathbf{S}})) = 0$ and where $\overline{F}_i$ is the tail distribution of D_i.

3. Commonality Adoption Criteria and Value Drivers

Property 2 (Adoption). Commonality should be adopted ($S_3^* > 0$) if and only if $c_3 < \bar{c}_3$, where

$$\bar{c}_3 = c_1 + c_2 - v_2 P\big(D_1 > \overline{F}_1^{-1}((c_1 + c_4)/v_1),\; D_2 > \overline{F}_2^{-1}((c_2 + c_5)/v_2)\big) \qquad (3)$$

$$= c_1 + c_2 - \frac{(c_1 + c_4)(c_2 + c_5)}{v_1} \quad \text{(iff } D_1 \text{ and } D_2 \text{ independent).} \qquad (4)$$

(Expression (4) is new, while VM98 presents similar expressions for perfectly positive and negative demands.) A higher-margin product increases the potential value of the ex post allocation option of a common component and increases the parameter domain of c_3, for which commonality is valuable. Similarly, higher costs for the substitutable inputs 1 and 2 allow for higher commonality cost. More interestingly, adoption also depends on the presence of the "other" inputs 4 and 5, but *only* through their costs. This is due to an externality: While a common component substitutes inputs 1 and 2, it requires a complementary increase in the other inputs, as shown by Property 1. Therefore, the higher the cost of the other components, the higher the effective cost—and the lower the likelihood—of adopting a commonality strategy.

In general, the threshold cost (3) for commonality adoption depends on the financial terms and on the demand distribution. When demands are independent or perfectly correlated, however, the criterion is *entirely independent* of the distributions of D_1 and D_2. This strongly suggests that the impact of demand uncertainty on the commonality adoption decision is predominantly driven by the correlation coefficient ρ, and not by the mean or variance of either product demand. While that conjecture has not been proved in general, it is true for correlated normally distributed demand: Its threshold cost $\bar{c}_3$ depends only on uncertainty through ρ and is decreasing in ρ. While commonality adoption thus becomes less likely as correlation increases, it can remain optimal—and thus valuable—with perfect correlation:

Property 3 (Revenue-Maximization Option). Commonality remains optimal with perfectly positively correlated demand if $\Delta v = v_1 - v_2 > 0$, $(c_1 + c_4)/v_1 < (\min(c_1, c_2))/v_2$, and $c_3 < \bar{c}_3$, where

$$\bar{c}_3 = c_1 + c_2 - \frac{v_2(c_1 + c_4)}{v_1} = \frac{\Delta v}{v_1} c_1 + c_2 - \frac{v_2}{v_1} c_4 > \max(c_1, c_2). \qquad (5)$$

(This extends Proposition 6 in VM98 by imposing the more stringent condition $\max(c_1, c_2) < \bar{c}_3$.) Thus, if product 1 has a higher margin and a higher markup, commonality is optimal if $c_3 \in [\max(c_1, c_2), \bar{c}_3)$. The property highlights a benefit that has not been identified in the commonality literature: Given that product demands move in lockstep if $\rho = 1$, the reason for commonality cannot be risk pooling. Rather, it is the ex post *revenue-maximization option* inherent in commonality: Stocking the common component creates the option to produce more (compared to stocking only dedicated components $\overline{S}$) of the higher-margin product at the expense of the other product when demand exceeds capacity. (VM98 shows the associated revenue gain by detailed comparison to the dedicated strategy.)

In summary, commonality adoption depends *only* on the financials and the correlation coefficient; it is less likely with more costly "other" components and higher correlation, but is independent of variances and the magnitude of risk pooling. The discussion above has focused on whether or not commonality should be adopted. It is straightforward to show that the value $V(\mathbf{S}^*)$ of the optimal commonality strategy decreases in any of the cost parameters, increases in any of the margins and expected demand, but decreases in variances and covariances (or correlation),[5] reflecting the decreasing value of risk pooling. The "option value of commonality," $V(\mathbf{S}^*) - V(\overline{\mathbf{S}})$, also decreases in correlation but, as most options, is expected to increase in variance terms.[6] The revenue-maximization option, however, can remain valuable even with perfect positive correlation, so that commonality adoption remains optimal.

4. Commonality Strategy Implementation Features

The nature of the optimal strategy gives insights into how commonality should be implemented in terms of product and process design, as well as in terms of tactical inventory-stocking quantities. Proposition 2 in VM98 shows that the optimal strategy takes on at most three forms, none of which completely

[5] The operating profit function is submodular so the result follows directly from Proposition 3 in Van Mieghem and Rudi (2002).

[6] Aside from supporting numerical evidence, no general proof is available to our knowledge. Such a proof must establish that $V(\mathbf{S}^*)$ decreases less in σ_{D_i} than $V(\overline{\mathbf{S}})$.

substitutes all components 1 and 2 by the common component. Hence:

PROPERTY 4 (DUAL SOURCING). "Pure commonality" ($S_1 = S_2 = 0$) is never optimal in this model.

One should follow a dual-sourcing strategy for the higher-margin product even if commonality is very inexpensive, in which case $S_1^*, S_3^* > 0$ while $S_2^* = 0$. Anupindi and Akella (1993) also find that dual sourcing is preferred in a different setting where supply is uncertain. Obviously, pure commonality would be optimal if it were free ($c_3 \leq \max(c_1, c_2)$) or if we would include a large fixed cost for product and process redesign or "cost of complexity" for sourcing and handling dual inputs.

PROPERTY 5 (EXTERNALITY). Adopting commonality should be accompanied by an increase in the stocking level of the "other," nonsubstitutable inputs (4 and 5 in our model): $S_i^* < \overline{S}_i = \overline{S}_{i+3} < S_i^* + S_3^* = S_{i+3}^*$.

Property 5 generalizes similar existing relationships between the dedicated strategy and the (suboptimal) pure commonality strategy. Commonality decreases the level of substitutable inputs as expected, but also introduces an externality by increasing that of the other components. The effect of commonality adoption on total stock (and thus safety stock) is, however, parameter specific, as is typical in newsvendor systems where pooling can increase or decrease inventory depending on financial data (e.g., for a symmetric distribution depending on whether the critical fractile is below 0.5 or above).

Commonality allows for alternate processing modes and can be viewed as a form of operational hedging that shares the "imbalance" feature and the "insurance" interpretation as discussed in Van Mieghem (2003):

PROPERTY 6 (HEDGING). Commonality always overstocks the "other" components, *regardless of their cost*.

One will never use all the "other" components, given that total output $\sum_i x_i^* \leq S_1^* + S_2^* + S_3^* < S_1^* + S_2^* + 2S_3^* = S_4^* + S_5^*$. While either S_4 or S_5 can be fully used, never will both be, so that it is optimal to invest in insurance or overstock in $S_4 + S_5$.

5. Extension to the Multiperiod Model

Van Mieghem and Rudi (2002) show that a base-stock policy with the level $\mathbf{S}^*$ of the single-period newsvendor network remains optimal for the dynamic version of that newsvendor network with i.i.d. demand and negligible replenishment leadtimes. Therefore, our results directly extend to a dynamic setting with i.i.d. demand and negligible replenishment leadtimes, both under lost sales and backlogging.[7] The open question is whether the analysis also extends to leadtimes and a general number of products and inputs. The conjecture is that it does when procurement leadtimes are identical, yet it may fail otherwise. Song (2002) and Lu and Song (2003) consider commonality with leadtimes, while Rudi (2000a) presents analytic expressions of the optimality conditions for n products and one common component with zero leadtimes, yet complexity defied crisp analysis on commonality adoption criteria and value drivers. It is hoped that those papers form a stepping stone to generalize reduction Property 1, and at the same time simplify their analysis.

[7] This always holds when unmet demand results in lost sales. For our commonality problem it also holds under backlogging by Proposition 6 in Van Mieghem and Rudi (2002) given that the discretionary activities x_3 and x_4 are "strong nonbasic."

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