Online Appendix to: Multi-market Facility Network Design with Offshoring Applications

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I. Extended Literature Review

A. Operations strategy. Configuring the right multiplant facility network plays an important role in a firm's operations strategy. Hayes & Wheelwright (1984) illustrate four approaches for formulating a multiplant facility strategy: physical facilities analysis, geographical network analysis, functional needs and corporate philosophy analysis, and product-process focus analysis. They argue that these approaches represent different perspectives and should be used in a proper combination. Our analysis combines the geographical network and product-process focus approaches. The geographical network analysis is often observed when transportation costs constitute a significant portion of total production cost. This paper illustrates the pivoting role of transportation cost in choosing centralized versus localized commonality strategy. The basis of the product-process focus approach is the concept of operational focus, proposed by Skinner (2006) and recently modeled by Van Mieghem (2008). Firms may choose to focus their facilities according to volumes, product, process, or service. Similar in spirit, our model incorporates two products with different demand characteristics, two geographically separated markets with distinct economic characteristics, and two processes with different purposes: common component manufacturing versus dedicated assembly. We illustrate how these elements interact and drive the optimal network decisions.

B. Facility location & network design. Our research falls within the vast literature on facility location and supply chain network design. Snyder (2006) presents a recent and broad review of facility location research. Our paper follows the stream that deals with facility decisions in a global context. We categorize and summarize the related literature into four groups of papers according to their research methodologies:

B1. Mathematical programming. One of the seminal papers that formulate global manufacturing strategic planning as a mathematical programming problem is Cohen & Lee (1989). Their model is capable of capturing a large number of factors affecting resource deployment decisions in a multi-country model, such as regional demand requirements, sourcing constraints, interplant transshipments, taxation and tariffs. In a global setting, firms' manufacturing decisions are significantly affected by international trade barriers and regulations. Arntzen et al. (1995) construct and implement a mixed integer programming model for global supply chain design at Digital Equipment Corporation. Their integrated manufacturing and distribution decision framework enables the company to restructure its existing global supply chain and save over $100 million. Munson & Rosenblatt (1997) focus on the impact of local content rules on global sourcing decisions. They incorporate local content rules into the classical plant location problem and provide an efficient solution procedure. Kouvelis et al. (2004) present a mixed integer programming model that incorporates government trade policies, such as financing subsidies, tariffs, and taxation. The solution of their model confirms and quantifies expected determinants of the structure of global facility networks. (For example, expensive transportation of subassemblies leads to centralized manufacturing and distribution networks while increased trade tariffs makes decentralized distribution networks more attractive.) A common feature of these mathematical programming formulations is that the decision framework is deterministic, i.e., no demand, financial, production, or regulatory uncertainties.

B2. Stochastic programming. Some papers explicitly model uncertainty in the global manufacturing environment using a stochastic programming approach. Santoso et al. (2005) propose a model and a solution algorithm for solving large-scale stochastic supply chain design problems. Others specifically evaluate the benefit of operational flexibility embodied in owning international operations. Kogut & Kulatilaka (1994) treat a multinational operating network as a real option whose value depends on exchange rates. They use stochastic dynamic programming to solve the option valuation problem and conclude that high variance of exchange rates increases the value of multinational networks. Similar to Kogut & Kulatilaka (1994), Huchzermeier & Cohen (1996) develop a stochastic dynamic programming formulation for valuation of global manufacturing strategy options in the face of switching costs and correlated exchange rate processes. Kouvelis et al. (2001) study the effects of real exchange rates on the ownership structure of global production facilities. They identify a hysteresis phenomenon that characterizes switching behavior between three ownership strategies: exporting, joint ventures with local partners, and wholly owned production facilities in the foreign country. A more recent paper by Kazaz et al. (2005) characterizes the value of production hedging and allocation hedging in global production planning in the presence of exchange-rate uncertainty.

B3. Newsvendor networks. This stream of papers use parsimonious newsvendor models to generate managerial insights pertaining to demand risk in general and specifically the value of transshipment. Robinson (1990) shows that transshipment can not only reduce costs considerably but also affect the optimal ordering policy. Rudi et al. (2001) take a new approach to the transshipment problem by extending it to an interfirm setting and studying the impact of local vs. centralized decision making on joint profits.

Van Mieghem & Rudi (2002) present a systematic approach to study network design in a newsvendor setting, which was adopted by Kulkarni et al. (2004) and Kulkarni et al. (2005). The latter is closely related to our model and numerically determines the better of two predetermined network configurations for a multiplant network with commonality: process plant (corresponding to our U.S.-centralization) and product plant (corresponding to our market-focused configuration). Four major distinctions separate our work from Kulkarni et al. (2005). First, we enlarge the feasible configuration set and endogenize the location of common component by incorporating Asia-centralization and the most general configuration, i.e., the hybrid. Second, we let the optimal network configuration emerge from optimization and provide analytical optimality conditions. Third, instead of using numerical sensitivity analysis, we analytically demonstrate how economic and demand characteristics impact optimal network configurations. Last, we explain how differences in transportation costs and revenue maximization benefits determine optimal network design under demand uncertainty and demonstrate how this can even lead U.S.-centralization to be optimal.

B4. Conceptual and empirical approaches. A group of papers draw on extensive interviews and case studies to examine the strategies and trends in facility location selections (Schmenner (1979), Bartmess & Cerny (1993), Bartmess (1994), MacCormack et al. (1994)). Bartmess & Cerny (1993) emphasize the strategic impact of plant location decisions and champion capability building in the objective of facility location decisions. MacCormack et al. (1994) document the new trend of locating global manufacturing sites location based on changes in production technologies, workforce sophistication, and organizational philosophies. Other papers conduct empirical studies on facility strategies of large manufacturing firms. Key characteristics that affect the attractiveness of four prevailing multiplant strategies (i.e., product, market area, process, and general purpose) are identified for the Fortune 500 firms (Schmenner (1982)). Similarly, Brush et al. (1999) empirically investigate the determinants of multinational manufacturing firms' choices between integrated and independent plants, and between domestic and foreign plants. The novelty of their approach is to combine perspectives from international business and manufacturing, and examine the interplay of the two perspectives in shaping managers' facility decisions.

C. The commonality, dual sourcing, and offshoring literature is also relevant to our work. Commonality is about assembling multiple products from common components and product-specific components, according to Van Mieghem (2004). Multiproduct firms often use commonality to add flexibility to their existing production networks. Kulkarni et al. (2005) examine the trade-offs between risk pooling and logistics cost for two extreme configurations (process vs. product) of commonality in a multiplant network. In contrast to Kulkarni et al. (2005), we endogenize the location decision of commonality and allow different centralization configurations to arise from optimization.

Our work also studies the choice between single sourcing (centralization) and dual sourcing (hybrid) strategies for commonality. Anupindi & Akella (1993) study how to optimally allocate quantities between two suppliers with yield uncertainty and its effects on the buyer's inventory policies. Yazlali & Erhun (2004) examine the trade-off between responsiveness and cost in global sourcing strategies using an imbedded multi-period inventory model. Tomlin & Wang (2005) study unreliable supply chains with risk averse firms that trade-off the level of mix flexibility against risk diversification through dual sourcing. Tomlin (2006) also studies sourcing mitigation strategies in the presence of different reliable suppliers as compared to adopting inventory mitigation and contingent rerouting for managing supply chain disruptions.

Finally, our research belongs to the growing literature on offshoring. The related literature is mostly found in the economics, international business, and popular management journals. Ferdows (1997) categorizes the strategic roles of foreign plants in a manufacturing firm's facility network and suggests that firms upgrade the roles of their foreign plants over time in order to gain competitive advantage in manufacturing capability. In contrast, Markides & Berg (1988) argue that offshore manufacturing does not build long-term competitive advantages, but is rather a "short-term tactical move." Farrell (2004) and Farrell (2005) lay out a conceptual framework for firms considering offshoring and shows (based on a recent study by the McKinsey Global Institute) that firms can significantly lower their costs by moving their production to low-wage locations. She also argues that cost savings from offshoring in turn enables firms to reduce prices and attract new customers, and therefore offshoring creates enormous value for both firms and the global economy. The related economics literature focuses on the impact of offshoring on domestic labor markets (Feenstra & Hanson (1996), Baily & Lawrence (2004)).

II. Proofs

Proposition 1 Proof. Since demand is deterministic, choosing the most profitable processing activity for each product yields the optimal configuration and capacities. ■

Lemma 1 Proof. The ordering follows immediately from the definition of the $v_i's$ and the assumptions. ■

Proposition 2 Proof. If $c_{T,1} \leq -\Delta c_M$, suppose to the contrary $K_4^* > 0$. Moving all common component capacity in market 2 to market 1 keeps the investment cost unchanged but increases the profit because $v_1 \geq v_4$ and $v_3 \geq v_2$, contradicting the optimality. Similarly for the case of $c_{T,2} \leq \Delta c_M$. ■

Lemma 2 Proof. Suppose to the contrary that $K_3^* > K_1^*$. Let $\varepsilon = K_3^* - K_1^*$, then ε can only be utilized by activity x_3. Moving this ε capacity to K_4^* gives a higher expected profit because $v_2 \geq v_3$, contradicting the assumed optimality. Similarly, we can prove that $K_4^* \leq K_2^*$. ■

Proposition 3 Proof. The proof for part (i) and (ii) proceeds in three steps. First, we establish the strict concavity of the optimization problem, i.e., the uniqueness of the optimal solution. Second, we provide the optimality conditions. Third, we derive the transportation cost threshold. We separate the proof for the medium and high Δp cases.

(1) Medium Δp

Step 1. Claim: There exists a unique K^* that solves the capacity investment problem.

Let F be the joint distribution function of $D_1$and D_2 and f be the density function. The greedy solution for the stage-2 contingent capacity allocation problem is:

$$x_1(K,D) = \min\{D_1, K_1, K_3\} = \min\{D_1, K_3\}, \quad x_2(K,D) = \min\{D_2, K_2, K_4\} = \min\{D_1, K_4\},$$
$$x_3(K,D) = \min\{D_2 - x_2, K_2 - x_2, K_3 - x_1\}, \quad x_4(K,D) = \min\{D_1 - x_1, K_1 - x_1, K_4 - x_2\}.$$

Partition the demand space as in Figure 8, such that the marginal value of each capacity is constant within each domain. The optimal activity vector and marginal values of capacities in each demand domain are displayed in Table 5.

Let H denote the Hessian matrix of $V(K; c_K)$. For the interior solution $K^* = (K_1^*, K_2^*, K_3^*, K_4^*)$,

$$\begin{aligned} H = D_K^2 V(K; c_K) &= D_K^2 \mathbb{E}\pi(K,D) = D_K \mathbb{E}(\nabla_K \pi(K,D)) = D_K \mathbb{E}\lambda, \\ \mathbb{E}\lambda &= \Lambda P, \end{aligned}$$

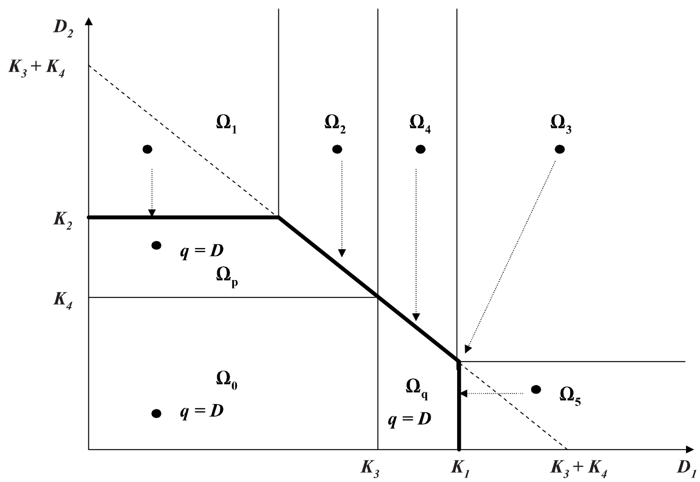

Figure 8: Partitioning of the demand space (assuming medium Δp)

Domain	Activity Vector x	Marginal Value λ
Ω_0	$(D_1, D_2, 0, 0)$	$(0, 0, 0, 0)$
Ω_p	$(D_1, K_4, D_2 - K_4, 0)$	$(0, 0, 0, v_2 - v_3)$
Ω_q	$(K_3, D_2, 0, D_1 - K_3)$	$(0, 0, v_1 - v_4, 0)$
Ω_1	$(D_1, K_4, K_2 - K_4, 0)$	$(0, v_3, 0, v_2 - v_3)$
Ω_2	$(D_1, K_4, K_3 - D_1, 0)$	$(0, 0, v_3, v_2)$
Ω_3	$(K_3, K_4, 0, 0)$	$(0, 0, v_1, v_2)$
Ω_4	$(K_3, D_2, 0, K_4 - D_2)$	$(0, 0, v_1, v_4)$
Ω_5	$(K_3, D_2, 0, K_1 - K_3)$	$(v_4, 0, v_1 - v_4, 0)$

Table 5: Optimal activity vector and marginal values of capacities in each demand domain

where

$$\Lambda = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & v_4 \\ 0 & 0 & 0 & v_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & v_1 - v_4 & 0 & v_3 & v_1 & v_1 & v_1 - v_4 \\ 0 & v_2 - v_3 & 0 & v_2 - v_3 & v_2 & v_2 & v_4 & 0 \end{pmatrix},$$

$$P = (P(\Omega_0(K^*)), P(\Omega_p(K^*)), P(\Omega_q(K^*)), P(\Omega_1(K^*)), P(\Omega_2(K^*)), P(\Omega_3(K^*)), P(\Omega_4(K^*)), P(\Omega_5(K^*)))'.$$

Hence,

$$H = \Lambda D_K P = \Lambda \begin{pmatrix} 0 & 0 & I_5 & I_3 \\ 0 & I_1 & I_2 & I_2 - I_3 \\ I_{10} & 0 & I_8 - I_5 & I_8 \\ 0 & -I_1 - I_6 & I_6 & I_6 \\ 0 & I_6 & I_7 - I_2 - I_6 & -I_2 - I_6 \\ 0 & 0 & -I_7 & -I_4 \\ I_9 & 0 & -I_8 - I_9 & I_4 - I_8 - I_9 \\ -I_9 - I_{10} & 0 & I_9 & I_9 \end{pmatrix},$$

where $I_i's$ are the marginal change in the probability of the demand domains as a result of the marginal change in the capacities. Since certain capacity changes impact multiple demand domains, multiple $I_i's$ are defined according to the boundaries of the domains. For example, I_1 represents the marginal increase in the probability of Ω_p as a result of an increase in K_2.

$$\begin{aligned} I_1 &= \int_0^{K_1+K_4-K_2} f(x, K_2)dx, \quad I_2 = \int_{K_1+K_4-K_2}^{K_1} f(x, K_1 + K_4 - x)dx, \\ I_3 &= \int_0^{K_3} f(x, K_4)dx, \quad I_4 = \int_{K_3}^{\infty} f(x, K_4)dx, \\ I_5 &= \int_0^{K_4} f(K_3, x)dx, \quad I_6 = \int_{K_2}^{\infty} f(K_1 + K_4 - K_2, x)dx, \\ I_7 &= \int_{K_4}^{\infty} f(K_3, x)dx, \quad I_8 = \int_{K_3}^{K_1} f(x, K_3 + K_4 - x)dx, \\ I_9 &= \int_{K_1}^{\infty} f(x, K_3 + K_4 - K_1)dx, \quad I_{10} = \int_0^{K_3+K_4-K_1} f(K_1, x)dx. \end{aligned}$$

Thus

$$H = \begin{pmatrix} -a_1 - a_2 & 0 & a_1 & a_1 \\ 0 & -a_3 - a_4 & a_4 & a_4 \\ a_1 & a_4 & -a_1 - a_4 - a_5 - a_6 & -a_1 - a_4 - a_5 \\ a_1 & a_4 & -a_1 - a_4 - a_5 & -a_1 - a_4 - a_5 - a_7 \end{pmatrix},$$

where

$$\begin{aligned} a_1 &= v_4 I_9, \quad a_2 = v_4 I_{10}, \\ a_3 &= v_3 I_1, \quad a_4 = v_3 I_6, \\ a_5 &= v_3 I_2 + v_4 I_8, \quad a_6 = (v_1 - v_3) I_7 + (v_1 - v_4) I_5, \\ a_7 &= (v_2 - v_3) I_3 + (v_2 - v_4) I_4. \end{aligned}$$

H is negative definite because it is symmetric and has a negative and dominant diagonal (To see the diagonal is dominant, let $\eta = (1, 1, 1/2, 1/2)$. We then have $|\eta_i H_{ii}| > \sum_{j \neq i} |\eta_j H_{ij}|$.) The uniqueness of the optimal solution follows from the negative definiteness of H.

Step 2. The optimality conditions follow from

$$\begin{aligned} \nabla_K V(K^*; c_K) &= \nabla_K \{\mathbb{E}\pi(K^*, D) - c_K' K^* + \mu' K^* + \theta_1 (K_1^* - K_3^*) + \theta_2 (K_2^* - K_4^*)\} \\ &= \mathbb{E}\lambda - c_K + \mu + \theta = 0. \end{aligned}$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ v_2 - v_3 \end{pmatrix} P(\Omega_p(K^*)) + \begin{pmatrix} 0 \\ 0 \\ v_1 - v_4 \\ 0 \end{pmatrix} P(\Omega_q(K^*)) + \begin{pmatrix} 0 \\ v_3 \\ 0 \\ v_2 - v_3 \end{pmatrix} P(\Omega_1(K^*)) + \begin{pmatrix} 0 \\ 0 \\ v_3 \\ v_2 \end{pmatrix} P(\Omega_2(K^*)$$

$$\begin{aligned} + \begin{pmatrix} 0 \\ 0 \\ v_1 \\ v_2 \end{pmatrix} P(\Omega_3(K^*) + \begin{pmatrix} 0 \\ 0 \\ v_1 \\ v_4 \end{pmatrix} P(\Omega_4(K^*) + \begin{pmatrix} v_4 \\ 0 \\ v_1 - v_4 \\ 0 \end{pmatrix} P(\Omega_5(K^*) &= c_K - \mu - \theta, \\ \mu' K^* &= 0, \\ \theta_i (K_i^* - K_{i+2}^*) &= 0, \ i \in \{1, 2\}, \end{aligned}$$

where $\mu \in R_+^4$ and $\theta_1, \theta_2 \in R_+$.

Step 3. Derivation of $\bar{c}_T$. Consider the boundary solution $\bar{K} = (\bar{K}_1, \bar{K}_2, \bar{K}_1, 0)$ and simplify the first-order conditions to

$$v_3 P(\Omega_2(\bar{K})) + v_1 P(\Omega_3(\bar{K})) = c_{K,1} + c_{K,3}, \quad (3)$$

$$v_3 P(\Omega_1(\bar{K})) = c_{K,2}, \quad (4)$$

$$v_2 - v_3 P(\Omega_{p+1}(\bar{K})) = c_{K,3} - \mu_{4.} \quad (5)$$

If $\mu_4 > 0$, $K_4^* = 0$ and thus $\bar{K}$ is the unique solution to the optimization problem. Combining equations (3)-(5), $\mu_4 > 0$ is equivalent to $c_{T,1} < \bar{c}_T$, where $\bar{c}_T$ is given by equation (1) with $\bar{P}_3 = \Pr(D_1 > \bar{K}_1)$.

Proof for part (iii). The market-focused configuration is optimal iff $P(\Omega_p(K^*)) = P(\Omega_q(K^*)) = P(\Omega_2(K^*)) = P(\Omega_4(K^*)) = 0$. The first-order conditions simplify to

$$\begin{pmatrix} 0 \\ v_3 \\ 0 \\ v_2 - v_3 \end{pmatrix} P(\Omega_1(\tilde{K})) + \begin{pmatrix} 0 \\ 0 \\ v_1 \\ v_2 \end{pmatrix} P(\Omega_3(\tilde{K})) + \begin{pmatrix} v_4 \\ 0 \\ v_1 - v_4 \\ 0 \end{pmatrix} P(\Omega_5(\tilde{K})) = \begin{pmatrix} c_{K,1} - \theta_1 \\ c_{K,2} - \theta_2 \\ c_{K,3} + \theta_1 \\ c_{K,3} + \theta_2 \end{pmatrix}.$$

Solving it yields

$$P(D_1 > \tilde{K}_1) = \frac{c_{K,1} + c_{K,3}}{v_1}, \qquad P(D_2 > \tilde{K}_2) = \frac{c_{K,2} + c_{K,3}}{v_2}.$$

By complementary slackness, the optimality of $\tilde{K}$ requires $\theta_i > 0$, for $i = 1, 2$. It follows from the above first-order conditions that

$$P(\Omega_1(\tilde{K})) < \frac{c_{K,2}}{v_3}, \qquad P(\Omega_5(\tilde{K})) < \frac{c_{K,1}}{v_4},$$

which are equivalent to

$$P(\Omega_3(\tilde{K})) > \min\left(\frac{c_{K,1} + c_{K,3}}{v_1} - \frac{c_{K,1}}{v_4}, \frac{c_{K,2} + c_{K,3}}{v_2} - \frac{c_{K,2}}{v_3}\right).$$

(2) High Δp

The proof follows a similar logic as in the case of medium Δp. Here we only provide the details for the steps that are different. The greedy solution for the stage-2 contingent capacity allocation problem is:

$$x_1(K, D) = \min\{D_1, K_1, K_3\} = \min\{D_1, K_1\}, \qquad x_4(K, D) = \min\{D_1 - x_1, K_1 - x_1, K_4\},$$

$$x_2(K, D) = \min\{D_2, K_2, K_4 - x_4\}, \qquad x_3(K, D) = \min\{D_2 - x_2, K_2 - x_2, K_3 - x_1\}.$$

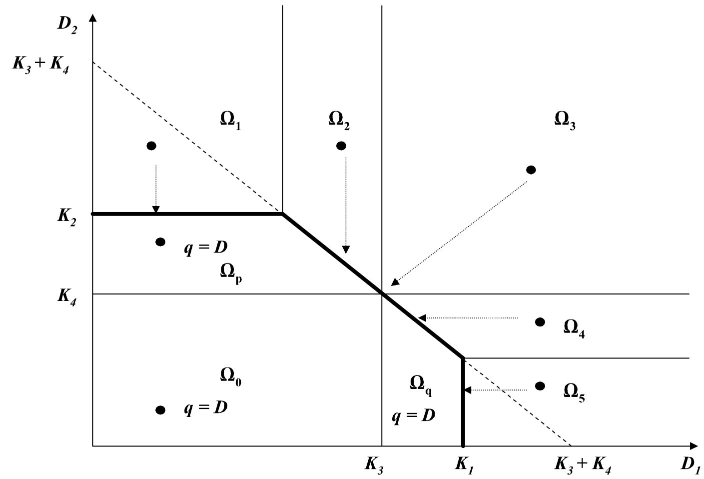

Figure 9: Partitioning of the demand space (assuming high Δp)

Partition the demand space as in Figure 9. A capacity investment vector $K^* \in R^4_+$ is optimal and the first-order conditions are

$$\begin{aligned}
(v_4 - v_2)P(\Omega_3(K^*)) + v_4 P(\Omega_5(K^*)) &= c_{K,1} - \mu_1 - \theta_1, \\
v_3 P(\Omega_1(K^*)) &= c_{K,1} - \mu_2 - \theta_2, \\
(v_1 - v_4)P(\Omega_{q+3+4+5}(K^*)) + v_3 P(\Omega_2(K^*)) + v_2 P(\Omega_{3+4}(K^*)) &= c_{K,3} - \mu_3 + \theta_1, \\
(v_2 - v_3)P(\Omega_{p+1}(K^*)) + v_2 P(\Omega_{2+3+4}(K^*)) &= c_{K,3} - \mu_4 + \theta_2, \\
\mu' K^* &= 0, \\
\theta_i(K_i^* - K_{i+2}^*) &= 0, \ i \in \{1, 2\},
\end{aligned}$$

The rest of the proof is similar to the medium Δp case. ■

Proposition 4 Proof. The proof is similar to the medium Δp case and thus omitted because both the demand partition and the greedy solution are identical. The modification is that we consider boundary solution $\underline{K} = (\underline{K}_1, \underline{K}_2, 0, \underline{K}_2)$ instead because boundary solution $\bar{K} = (\bar{K}_1, \bar{K}_2, \bar{K}_1, 0)$ does not exist for this case. This is evident from the expression of $\bar{c}_T$ derived previously. Since $\Delta p \leq \Delta c_M$, the numerator of $\bar{c}_T$ is strictly negative. Hence, $c_T < \bar{c}_T$ can never be satisfied. ■

Table 3 Proof. We prove the comparative statics for $\bar{c}_T$, the proof for $\underline{c}_T$ is omitted due to similarity. As mentioned in the main text, the proof boils down to determining the sign of $d\bar{K}_1/dy$. For boundary solution $\bar{K} = (\bar{K}_1, \bar{K}_2, \bar{K}_1, 0)$, the optimization problem is reduced to two dimensional. Thus, the Hessian matrix becomes

$$\begin{aligned}\bar{H} &= \bar{\Lambda} D_K \bar{P} = \begin{pmatrix} 0 & 0 & v_3 & v_1 \\ 0 & v_3 & 0 & 0 \end{pmatrix} \begin{pmatrix} I_2 & I_2 \\ I_6 & -I_1 - I_6 \\ -I_2 - I_6 + I_7 & I_6 \\ -I_7 & 0 \end{pmatrix} \\ &= \begin{pmatrix} -v_3(I_2 + I_6) - (v_1 - v_3)I_7 & v_3 I_6 \\ v_3 I_6 & -v_3(I_1 + I_6) \end{pmatrix}.\end{aligned}$$

Taking derivative w.r.t. $\tilde{c}_K = (c_{K,1}, c_{K,2}, c_{K,3})$ on both sides of the first-order condition and applying chain rule gives

$$D_K^2 \mathbb{E}\pi(K, D) D_{\tilde{c}_K} \bar{K} = D_{\tilde{c}_K} c_K = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \bar{H} D_{\tilde{c}_K} \bar{K}.$$

Hence,

$$\begin{aligned}D_{\tilde{c}_K} \bar{K} &= \bar{H}^{-1} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \\ &= \frac{1}{|\bar{H}|} \begin{pmatrix} -v_3(I_1 + I_6) & -v_3 I_6 & -v_3(I_1 + I_6) \\ -v_3 I_6 & -v_3(I_2 + I_6) - (v_1 - v_3)I_7 & -v_3 I_6 \end{pmatrix}.\end{aligned}$$

All elements of $D_{\tilde{c}_K} \bar{K}$ are negative, which follows from the negative definiteness of $\bar{H}$ (thus $|\bar{H}| > 0$). Therefore

$$\text{sign}(\frac{d\bar{K}_1}{dc_{K,i}}) = -1, \; i = 1, 2, 3.$$

Now we prove the part for $p_i's$ and $c'_{M,i}s$. From the first-order condition, we have

$$\bar{\Lambda} \bar{P} = c_K - \mu - \theta.$$

Taking derivative w.r.t. to $y (y = p_i \text{ or } c_{M,i})$,

$$\frac{\partial \bar{\Lambda}}{\partial y} \bar{P} + \bar{\Lambda} \frac{\partial \bar{P}}{\partial y} = \frac{\partial \bar{\Lambda}}{\partial y} \bar{P} + \bar{\Lambda} \nabla_K \bar{P} \frac{\partial \bar{K}}{\partial y} = 0.$$

Since

$$\begin{aligned}\bar{H} &= \bar{\Lambda} D_K \bar{P}, \\ \bar{\Lambda} &= \begin{pmatrix} 0 & 0 & v_3 & v_1 \\ 0 & v_3 & 0 & 0 \end{pmatrix}, \\ \frac{\partial \bar{K}}{\partial y} &= -\bar{H}^{-1} \frac{\partial \bar{\Lambda}}{\partial y} \bar{P},\end{aligned}$$

it follows that,

$$\begin{aligned}
\frac{\partial \bar{K}}{\partial p_1} &= -\bar{H}^{-1}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}\bar{P} = \frac{1}{|\bar{H}|}\begin{pmatrix} \bar{P}_3 v_3(I_1+I_6) \\ \bar{P}_3 v_3 I_6 \end{pmatrix} >> 0, \\
\frac{\partial \bar{K}}{\partial p_2} &= -\bar{H}^{-1}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\bar{P} \\
&= \frac{1}{|\bar{H}|}\begin{pmatrix} \bar{P}_1 v_3 I_6 + \bar{P}_2 v_3(I_1+I_6) \\ \bar{P}_1 v_3(I_2+I_6) + \bar{P}_1(v_1-v_3)I_7 + \bar{P}_2 v_3 I_6 \end{pmatrix} >> 0, \\
\frac{\partial \bar{K}}{\partial c_{M,1}} &= -\bar{H}^{-1}\begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & -1 & 0 & 0 \end{pmatrix}\bar{P} \\
&= \frac{1}{|\bar{H}|}\begin{pmatrix} -\bar{P}_1 v_3 I_6 - \bar{P}_2 v_3(I_1+I_6) - \bar{P}_3 v_3(I_1+I_6) \\ -\bar{P}_1 v_3(I_2+I_6) - \bar{P}_1(v_1-v_3)I_7 - \bar{P}_2 v_3 I_6 - \bar{P}_3 v_3 I_6 \end{pmatrix} << 0, \\
\frac{\partial \bar{K}}{\partial c_{M,2}} &= -\bar{H}^{-1}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\bar{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.
\end{aligned}$$

■

Property 1 Proof. It follows from a similar proof as in Proposition 3 and 4 of Van Mieghem (1998). ■

Property 2 Proof. The following proof applies to medium and low Δp cases (the high Δp case can be proved similarly). We proceed to prove the first half of the property. In the proof of Proposition 3, we have shown the Hessian matrix of $V(K)$ is

$$H = \begin{pmatrix} -a_1-a_2 & 0 & a_1 & a_1 \\ 0 & -a_3-a_4 & a_4 & a_4 \\ a_1 & a_4 & -a_1-a_4-a_5-a_6 & -a_1-a_4-a_5 \\ a_1 & a_4 & -a_1-a_4-a_5 & -a_1-a_4-a_5-a_7 \end{pmatrix},$$

where $a_i \geq 0$, $i = 1, ..., 7$. With a change of variable, let

$$\tilde{V}(\tilde{K}) = \tilde{V}(K_1, K_2, K_3, K_{\text{com}}) = V(K_1, K_2, K_3, K_{\text{com}} - K_3).$$

Calculating the cross partial derivatives, we get

$$\begin{aligned}
&\tfrac{\partial^2 \tilde{V}}{\partial K_1 \partial K_2} = \tfrac{\partial^2 V}{\partial K_1 \partial K_2} = 0, && \tfrac{\partial^2 \tilde{V}}{\partial K_1 \partial K_3} = \tfrac{\partial^2 V}{\partial K_1 \partial K_3} - \tfrac{\partial^2 V}{\partial K_1 \partial (K_{\text{com}}-K_3)} = 0 \\
&\tfrac{\partial^2 \tilde{V}}{\partial K_1 \partial K_{\text{com}}} = \tfrac{\partial^2 V}{\partial K_1 \partial (K_{\text{com}}-K_3)} = a_1, && \tfrac{\partial^2 \tilde{V}}{\partial K_2 \partial K_3} = \tfrac{\partial^2 V}{\partial K_2 \partial K_3} - \tfrac{\partial^2 V}{\partial K_2 \partial (K_{\text{com}}-K_3)} = 0 \\
&\tfrac{\partial^2 \tilde{V}}{\partial K_2 \partial K_{\text{com}}} = \tfrac{\partial^2 V}{\partial K_2 \partial (K_{\text{com}}-K_3)} = a_4, && \tfrac{\partial^2 \tilde{V}}{\partial K_3 \partial K_{\text{com}}} = \tfrac{\partial^2 V}{\partial K_3 \partial (K_{\text{com}}-K_3)} - \tfrac{\partial^2 V}{\partial (K_{\text{com}}-K_3)^2} = a_7.
\end{aligned}$$

Notice that all cross partials are nonnegative, satisfying the condition for supermodularity. Further,

$$\frac{\partial \tilde{V}}{\partial K_1} = \frac{\partial V}{\partial K_1} = E\lambda_1(K) - c_{K,1} + \theta_1, \qquad \frac{\partial \tilde{V}}{\partial K_2} = \frac{\partial V}{\partial K_2} = E\lambda_2(K) - c_{K,2} + \theta_2,$$
$$\frac{\partial \tilde{V}}{\partial K_3} = \frac{\partial V}{\partial K_3} - \frac{\partial V}{\partial K_4} = E\lambda_3(K) - E\lambda_4(K) - \theta_1 + \theta_2, \quad \frac{\partial \tilde{V}}{\partial K_{\text{com}}} = \frac{\partial V}{\partial K_4} = E\lambda_4(K) - c_{K,3} - \theta_2.$$

Taking partial derivative w.r.t. $c_{K,i}$ gives that all the cross partial terms are nonpositive, proving the increasing differences result. Given that

$$\begin{aligned} E\lambda_1(K) &= v_4 P(\Omega_5(K)), \\ E\lambda_2(K) &= v_3 P(\Omega_1(K)), \\ E\lambda_3(K) &= (v_1 - v_4)P(\Omega_{q+5}(K)) + v_3 P(\Omega_2(K)) + v_1 P(\Omega_{3+4}(K)), \\ E\lambda_4(K) &= (v_2 - v_3)P(\Omega_{p+1}(K)) + v_2 P(\Omega_{2+3}(K)) + v_4 P(\Omega_4(K)). \end{aligned}$$

Calculating the cross partial derivatives w.r.t. p_1 yields

$$\frac{\partial^2 \tilde{V}}{\partial K_1 \partial p_1} = \frac{\partial \mathbb{E}\lambda_1(K)}{\partial p_1} = P(\Omega_5(K)) \qquad \frac{\partial^2 \tilde{V}}{\partial K_2 \partial p_1} = \frac{\partial \mathbb{E}\lambda_2(K)}{\partial p_1} = 0$$
$$\frac{\partial^2 \tilde{V}}{\partial K_3 \partial p_1} = \frac{\partial \mathbb{E}\lambda_3(K)}{\partial p_1} - \frac{\partial \mathbb{E}\lambda_4(K)}{\partial p_1} = P(\Omega_3(K)) \quad \frac{\partial^2 \tilde{V}}{\partial K_{\text{com}} \partial p_1} = \frac{\partial \mathbb{E}\lambda_4(K)}{\partial p_1} = P(\Omega_4(K))$$

The nonnegativitity of all cross partial terms yields the increasing differences property. The second half of the property can be proved similarly. ■

Property 3 Proof. We have already established supermodularity and increasing differences property and the result is an application of Topkis' monotonicity theorem. ■

Proposition 5 Proof. We will show the proof for one of the two cases (the case stated in the bracket follows a similar logic). We proceed to prove part (ii) and (iii) (part (i) can be proved similarly). Let $\bar{K} = (\bar{K}_1, \bar{K}_2, \bar{K}_1, 0)$ and $\bar{K}' = (\bar{K}_1', \bar{K}_2', \bar{K}_1', 0)$ be the boundary solution for (D_1, D_2) and (D_1, D_2'), respectively.

Claim: $\bar{K}_2' > \bar{K}_2$. Suppose to the contrary that $\bar{K}_2' \leq \bar{K}_2$. Recall the optimality conditions for the boundary solution $\bar{K}$ are

$$\begin{aligned} v_3 P(\Omega_1(\bar{K})) &= c_{K,1} \\ v_3 P(\Omega_2(\bar{K})) + v_1 P(\Omega_3(\bar{K}) &= c_{K,1} + c_{K,3}, \\ (v_2 - v_3)(P(\Omega_{p+1}(\bar{K}))) + v_2(P(\Omega_{2+3}(\bar{K})) &= c_{K,3} - \mu_4. \end{aligned}$$

To keep $P(\Omega_1(K)) = P(D_1 < K_1 - K_2)P(D_2 > K_2)$ unchanged as we change the demands from (D_1, D_2) to (D_1, D_2'), we must have

$$\bar{K}_1' - \bar{K}_2' < \bar{K}_1 - \bar{K}_2,$$

which follows from that D_2' first-order stochastic dominates D_2 and the supposition that $\bar{K}_2' \leq \bar{K}_2$. Thus,

$$\begin{aligned} \bar{K}_1' &< \bar{K}_1 \\ P(\Omega_{2+3}(\bar{K}')) &= P(D_1 > \bar{K}_1' - \bar{K}_2', D_1 + D_2 > \bar{K}_1') > P(\Omega_{2+3}(\bar{K})), \\ P(\Omega_3(\bar{K}')) &= P(D_1 > \bar{K}_1') > P(\Omega_3(\bar{K})). \end{aligned}$$

It follows that

$$\begin{aligned} v_3 P(\Omega_2(\bar{K}')) + v_1 P(\Omega_3(\bar{K}')) &= v_3 P(\Omega_{2+3}(\bar{K}')) + (v_1 - v_3) P(\Omega_3(\bar{K}')) \\ &> v_3 P(\Omega_{2+3}(\bar{K})) + (v_1 - v_3) P(\Omega_3(\bar{K})) \\ &= v_3 P(\Omega_2(\bar{K})) + v_1 P(\Omega_3(\bar{K})) = c_1 + c_3, \end{aligned}$$

contradicting the optimality condition. Hence, $\bar{K}_2' > \bar{K}_2$. It follows that $\bar{K}_1' > \bar{K}_1$ in order to satisfy the second optimality condition. It follows that

$$\text{sign}(\frac{\partial \bar{c}_T}{\partial D_2}) = \text{sign}(\frac{\partial \bar{c}_T}{\partial \bar{P}_3}) \times \text{sign}(\frac{\partial \bar{P}_3}{\partial \bar{K}_1}) \times \text{sign}(\frac{\partial \bar{K}_1}{\partial D_2}) = 1 \times (-1) \times 1 = -1,$$

where ∂D_2 is in the sense of first-order stochastic dominance. Hence $\bar{c}_T > \bar{c}_T'$, which proves part (ii). Since it is monotonically decreasing in D_2 in the sense of first-order stochastic dominance, $\bar{c}_T$ becomes negative when D_2 is large enough, which will happen when $E(D_2)$ is large enough. This proves part (iii). ■

Property 4 Proof. The proof boils down to show $\partial\pi/\partial D_1 = v'\partial x/\partial D_1$ is decreasing in D_2. Consider three scenarios (see Figure 9): (i) if $0 < D_1 < K_3 + K_4 - K_2$ or $D_1 > K_1$, then, $v'\partial x/\partial D_1$ remains unchanged for any value of D_2; (ii) if $K_3 + K_4 - K_2 < D_1 < K_3$, then as D_2 increases, $v'\partial x/\partial D_1$ remains constant throughout Ω_0, Ω_p, and Ω_2,but decreases from v_1 to $v_1 - v_3$ when crossing the border of Ω_p and Ω_2; (iii) if $K_3 < D_1 < K_1$, then as D_2 increases, $v'\partial x/\partial D_1$ remains constant throughout Ω_q and Ω_4,but decreases from $v_1 + v_4$ to $v_1 + v_4 - v_3$ when crossing the border of Ω_q and Ω_2. ■

Proposition 6 Proof. (i) High and medium Δp. First notice that the total demand $k \geq \bar{K}_1$. Otherwise, both $\bar{P}_2$ and $\bar{P}_3$ are zero, violating the first order condition of the boundary solution. $k < \bar{K}_1$ then implies the following set of equations that determine the boundary solution $\bar{K}$.

$$\begin{aligned} v_3 \bar{P}_2 + v_1 \bar{P}_3 &= c_{K,1} + c_{K,3}, \\ v_3 \bar{P}_1 &= c_{K,2}, \\ \bar{P}_1 + \bar{P}_2 + \bar{P}_3 &= 1. \end{aligned}$$

Solving the equation gives $\bar{P}_1 = \frac{c_{K,2}}{v_3}$, $\bar{P}_2 = \frac{v_1(v_3 - c_{K,2}) - v_3(c_{K,1}+c_{K,3})}{v_3(v_1-v_3)}$, and $\bar{P}_3 = \frac{c_{K,1}+c_{K,2}+c_{K,3}-v_3}{v_1-v_3}$. It follows that

$$\begin{aligned}\bar{c}_T - c_T &= \frac{\Delta p \bar{P}_3 - \Delta c_M - c_{K,1}}{1-\bar{P}_3} - c_T \\ &= \frac{(\Delta p + c_T)(c_{M,2} + c_{K,2} + c_{K,3} - p_2)}{p_1 - c_{M,1} - (c_{K,1} + c_{K,2} + c_{K,3})} \\ &= \frac{(v_1 - v_3)(c_{K,2} + c_{K,3} - v_2)}{v_1 - (c_{K,1} + c_{K,2} + c_{K,3})}.\end{aligned}$$

In order to have a feasible boundary solution $\bar{K}$, we must have $\bar{P}_1 > 0$, $\bar{P}_2 > 0$, and $\bar{P}_3 \geq 0$. It follows from the expression of $\bar{P}_i's$ that $v_3 \leq c_{K,1}+c_{K,2}+c_{K,3}$ and $\frac{c_{K,1}+c_{K,3}}{v_1} + \frac{c_{K,2}}{v_3} < 1$. These two conditions further imply that $v_1 > c_{K,1} + c_{K,2} + c_{K,3}$. Suppose to the contrary that $v_1 \leq c_{K,1} + c_{K,2} + c_{K,3}$, then

$$\begin{aligned}\frac{c_{K,1} + c_{K,3}}{v_1} + \frac{c_{K,2}}{v_3} &\geq \frac{c_{K,1} + c_{K,3}}{c_{K,1} + c_{K,2} + c_{K,3}} + \frac{c_{K,2}}{v_3} \\ &\geq \frac{c_{K,1} + c_{K,3}}{c_{K,1} + c_{K,2} + c_{K,3}} + \frac{c_{K,2}}{c_{K,1} + c_{K,2} + c_{K,3}} = 1,\end{aligned}$$

contradicting the second condition above. Finally, centralization in the high-price market is optimal if and only if $\bar{c}_T > c_T$, which requires $v_2 < c_{K,2}+c_{K,3}$ given that $v_1 > c_{K,1}+c_{K,2}+c_{K,3}$ and $v_1 > v_3$. The proof for part (ii) is similar to (i). ■

Proposition 7 Proof. (i) Medium Δp. As the demand curve is reduced to a 45-degree line starting from the origin, there are four cases to consider depending on where in the demand partition space the demand curve passes through. Case(1): $K_2^* < K_3^* + K_4^* - K_2^*$, i.e. $P(\Omega_q) = P(\Omega_4) = P(\Omega_5) = 0$. The first order conditions reduce to

$$0 = c_{K,1} - \theta_1, \tag{6}$$

$$v_3 P(\Omega_1) = c_{K,2} - \theta_2, \tag{7}$$

$$v_3 P(\Omega_2) + v_1 P(\Omega_3) = c_{K,3} + \theta_1 - \mu_3, \tag{8}$$

$$(v_2 - v_3)P(\Omega_p) + (v_2 - v_3)P(\Omega_1) + v_2(P(\Omega_2) + P(\Omega_3)) = c_{K,3} + \theta_2 - \mu_4. \tag{9}$$

$\theta_1 = c_{K,1} > 0$ implies that $K_1^* = K_3^*$ and thus $P(\Omega_p) = P(\Omega_4) = 0$. (6) + (8) and (7) + (9) and rearranging give

$$\begin{aligned}\frac{v_3}{v_1}P(\Omega_2) + P(\Omega_3) + \frac{\mu_3}{v_1} &= \frac{c_{K,1} + c_{K,3}}{v_1}, \\ 1 - P(\Omega_0) - \frac{v_3}{v_2}P(\Omega_p) + \frac{\mu_4}{v_2} &= \frac{c_{K,2} + c_{K,3}}{v_2}.\end{aligned}$$

Since $\frac{v_3}{v1}P(\Omega_2) + P(\Omega_3) < 1 - P(\Omega_0) - \frac{v_3}{v_2}P(\Omega_p)$, this case is feasible only if $\frac{c_{K,1}+c_{K,3}}{v_1} < \frac{c_{K,2}+c_{K,3}}{v_2}$. Case (2): $K_2^* \geq K_3^* + K_4^* - K_2^*$ and $K_3^* \geq K_4^*$, i.e. $P(\Omega_q) = P(\Omega_1) = P(\Omega_4) = P(\Omega_5) = 0$. It follows from the FOCs above that $\theta_1 = c_{K,1}$ and $\theta_2 = c_{K,2}$. Hence, $P(\Omega_p) = P(\Omega_2) = 0$ and $K_1^* = K_3^* = K_2^* = K_4^*$. The FOCs simplify to

$$\begin{aligned} v_1 P(\Omega_3) &= c_{K,1} + c_{K,3} - \mu_3, \\ v_2 P(\Omega_3) &= c_{K,2} + c_{K,3} - \mu_4. \end{aligned}$$

Since $\mu_3 = \mu_4 = 0$, this case is feasible only if $\frac{c_{K,1}+c_{K,3}}{v_1} = \frac{c_{K,2}+c_{K,3}}{v_2}$. Case (3): $K_1^* \geq K_3^* + K_4^* - K_1^*$ and $K_4^* \geq K_3^*$, i.e., $P(\Omega_p) = P(\Omega_1) = P(\Omega_2) = P(\Omega_5) = 0$. Similar to Case (2), this case gives $K_1^* = K_3^* = K_2^* = K_4^*$ and it is feasible only if $\frac{c_{K,1}+c_{K,3}}{v_1} = \frac{c_{K,2}+c_{K,3}}{v_2}$. Case (4): $K_1^* < K_3^* + K_4^* - K_1^*$, i.e., $P(\Omega_p) = P(\Omega_1) = P(\Omega_2) = 0$. This case is a mirror image of Case (1). The simplified FOCs become

$$\begin{aligned} 1 - P(\Omega_0) - \frac{v_4}{v_1}P(\Omega_q) + \frac{\mu_3}{v_1} &= \frac{c_{K,1} + c_{K,3}}{v_1}, \\ P(\Omega_3) + \frac{v_4}{v_2}P(\Omega_4) + + \frac{\mu_4}{v_2} &= \frac{c_{K,2} + c_{K,3}}{v_2}. \end{aligned}$$

This case gives $K_2^* = K_4^*$ and is feasible only if $\frac{c_{K,1}+c_{K,3}}{v_1} > \frac{c_{K,2}+c_{K,3}}{v_2}$. ■

Proposition 8 Proof. We provide the proof for the stochastic demand case because the deterministic case is similar yet simpler. $c_{T,1} \leq -\Delta c_M - \Delta c_K$ implies that $v_2 - v_3 \leq -\Delta c_K$. Because $v_2 - v_3 \geq v_4 - v_1$, $v_2 - v_3$ is the upper bound of profit gain of moving one unit of common component from market 1 to market 2. When $\Delta c_K \leq 0$, $-\Delta c_K$ is the associated capacity cost increase. The inequality condition implies that investing in any amount of common component in market 2 would be suboptimal. Similarly for proving the optimality condition of centralization in market 2. ■