Mix, time and volume flexibility: Valuation and corporate diversification

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Abstract

This article examines the joint impact of three types of operational flexibility in a theoretical model of a two-product firm that makes capacity, production and pricing decisions at three points in time with an underlying continuous-time information evolution. Mix flexibility is measured by the cost of switching production between the two products. Volume flexibility is measured by the fraction of production cost that is variable at the time when the production decision is made. Finally, time flexibility is measured by the relative timing of the production decision. We show that mix and volume flexibilities are substitutes in creating firm value but both are complementary to time flexibility. We discuss the implications of these results for the optimal investment in different aspects of flexibility. We also relate these results to corporate strategy and show how different types of flexibility impact the benefits of corporate diversification.

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1   Introduction

Flexibility measures the ability to adapt to change and often has multiple dimensions that impact value jointly yet differently. This paper considers three specific types of flexibility and examines their value, relationships and implications for corporate diversification. We consider a two-product firm that makes capacity, production and pricing decisions at three decision epochs with an underlying continuous-time information evolution. Initially, the firm chooses product-specific capacity levels based on an imperfect forecast of future product demand curves. At a later point of time, called the "update time," the firm updates its forecast and chooses production levels. The output of each product is constrained by the existing capacity but the firm has the option to convert, at a cost, one product-specific capacity to another. Finally, when the selling season comes and actual demand curves are observed, the firm sets prices and realizes sales.

 We measure the firm's ability to adapt to dynamically evolving demand forecast along three dimensions:

1. *Mix flexibility*, also known as product or resource flexibility, is the ability to change production mix. We measure it by the cost of converting one unit of product-specific capacity to the other product. The lower this cost, the higher mix flexibility. In practice, mix flexibility depends on machine changeover costs, workforce cross-training, etc..

2. *Volume flexibility* is the ability to change production volume. We measure it by the fraction of total unit cost incurred after the update time. Two arguments explain why volume flexibility is high when this fraction is large. When unit capacity cost is low relative to the unit production cost, the firm installs relatively large capacity that is less likely to constrain production volume. In addition, the bulk of the unit cost is still variable at the update time, and therefore, the firm has the incentive to adapt its volume to the updated forecast. Factors

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that determine volume flexibility include capital intensity of the production process or the flexibility of supply contracts.

3. *Time flexibility* is the ability to delay the production decision until more accurate forecast becomes available. In our model, it is measured by the timing of the production decision between capacity selection and the selling season. In practice, time flexibility depends primarily on production lead times.

We show that mix and volume flexibilities are substitutes in creating firm value, but are both complements with time flexibility. Thus, e.g., a firm with inherently greater mix flexibility should invest more in time flexibility and less in volume flexibility. Furthermore, we show that the marginal values of mix and time flexibility are decreasing in demand correlation whereas the marginal value of volume flexibility increases in demand correlation. Therefore, as demand correlation increases, a firm should invest more in volume flexibility and less in mix and time flexibilities.

We also relate our results to corporate diversification. The strategy literature has recognized that diversification coupled with technological or organizational flexibility may create value by reducing uncertainty in capacity planning through demand pooling. In addition to showing that the value of diversification increases in mix and time flexibility and decreases in volume flexibility and demand correlation, we provide insights into the trade-offs between related and unrelated diversification.

Whereas the literature on mix (product, resource) flexibility is extensive (e.g., Fine and Freund 1990, Van Mieghem 1998, and Chod and Rudi 2005), time and volume aspects of flexibility have received less attention. Our model of volume flexibility is similar to postponement flexibility in Chod, Rudi and Van Mieghem (2010) who examine its relationship to financial hedging, whereas our concept of time flexibility builds on Chod and Rudi (2006) who examine the benefits of risk pooling under forecast updating.

In its research question, our paper is closely related to Goyal and Netessine (2011) who also ex-

amine the relationship between product (mix) and volume flexibility of a two-product price-setting firm. They model volume flexibility as the ability to adjust, at a quadratic cost, capacity levels up or down once demand uncertainty is resolved. Their model of product flexibility corresponds to the special case of our mix flexibility with zero capacity switching cost and full time flexibility. Netessine and Goyal show that adding product flexibility to volume flexibility does not necessarily benefit the firm, even if it is costless, because of possible diseconomies of scope. In our model, increasing product (mix) flexibility is always beneficial but, as volume flexibility increases, the *marginal* value of mix flexibility decreases.

Netessine and Goyal also show that whereas the value of product flexibility always decreases in demand correlation, the value of volume flexibility can increase or decrease in demand correlation depending on whether the products are complements or substitutes. In our model, we focus on the impact of demand correlation on the *marginal* value of each type of flexibility.

Our work is also related to the literature on corporate diversification (see e.g., Martin and Sayrak 2003 for a comprehensive survey). Among the first articles explaining corporate diversification with resource flexibility is Teece (1982) who studies a firm that chooses a product mix according to constantly changing market conditions which create opportunities in different markets at different times. Since then, the link between diversification and different types of flexibility has been discussed in several other papers (e.g., Levy and Haber 1986, Von Ungern-Sternberg 1989, and Matsusaka 2001). We contribute to this literature by formalizing in a rigorous mathematical model the idea that in the presence of demand uncertainty and transaction costs in the factor market, mix flexibility may justify corporate diversification. We also show how the diversification premium depends on the three dimensions of flexibility.

The remainder of this paper is structured as follows. Section 2 presents the model which is solved in Section 3. The value of flexibility and the interplay among different types of flexibility are

examined in Section 4. Section 5 discusses the link between flexibility and market diversification. Section 6 concludes. All proofs are relegated to Appendix 2.

2 Model

Consider a two-product firm that must make three decisions at three different points in time. Uncertainty and information availability are formalized by a standard probabilistic framework with a probability space $(\Omega, \mathcal{F}, P)$ and filtration $\mathbb{F} = \{\mathcal{F}_t, t \geq 0\}$ as primitives. Expectation conditional on $\mathcal{F}_t$ (information available at time t) is denoted as $\mathbb{E}_t$.

At time 0, the firm chooses a vector $\mathbf{K} \in \mathbb{R}_+^2$ of two product-specific capacity levels incurring a constant marginal capacity cost c_K. (We assume, for simplicity, that all costs, capacity consumption rates and demand parameters are identical for both products.) At the update time $\tau \in [0, T]$, the firm chooses the output vector $\mathbf{Q} \in \mathbb{R}_+^2$ that will be produced at a constant marginal production cost c_Q. Although the aggregate output cannot exceed total capacity, i.e., $Q_1 + Q_2 \leq K_1 + K_2$, the firm has the option to convert, or "switch", capacity i to capacity $j \neq i$. The cost of converting one unit of capacity is denoted as $c_S \geq 0$, and we refer to it as switching cost.

Finally, at time T, production is complete and uncertainty is resolved. The firm sets output prices $\mathbf{p} \in \mathbb{R}_+^2$ and sells the output vector $\mathbf{q}(\mathbf{p}, \mathbf{Q}) \leq \mathbf{Q}$ earning revenue $\pi(\mathbf{p}, \mathbf{Q}) = \mathbf{p}'\mathbf{q}(\mathbf{p}, \mathbf{Q})$, where the prime denotes transpose. Given that no model dynamics occur after the start of the sales season, we compress the latter into an instantaneous sales event at time T, after which the firm is liquidated. We suppress the time value of money so that the firm terminal value, denoted as $v(T)$, is equal to the sales revenue minus the capacity investment, capacity switching and production costs:

$$v\left(T; \mathbf{K}, \mathbf{Q}, \mathbf{p}\right) = \pi(\mathbf{p}, \mathbf{Q}) - \sum_{i=1}^{2} \left(c_K K_i + c_S \max\left(Q_i - K_i, 0\right) + c_Q Q_i\right). \tag{1}$$

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The firm makes the three decisions, $(\mathbf{K}, \mathbf{Q}, \mathbf{p})$, with the objective to maximize its value. The firm value at time $t \in [0, T]$, denoted as $v(t)$, is the expectation of the firm terminal value conditional on information available then, i.e., $v(t) = \mathbb{E}_t v(T)$. The optimality equations simplify as follows:

$$\mathbf{p}^*(\mathbf{Q}) = \underset{\mathbf{p} \in \mathbb{R}_+^2}{\arg\max} v(T; \mathbf{K}, \mathbf{Q}, \mathbf{p}), \tag{2}$$

$$\mathbf{Q}^*(\mathbf{K}) = \underset{\mathbf{Q} \in \mathbb{R}_+^2}{\arg\max} v(\tau; \mathbf{K}, \mathbf{Q}, \mathbf{p}^*(\mathbf{Q})) \text{ s.t. } Q_1 + Q_2 \leq K_1 + K_2, \tag{3}$$

$$\mathbf{K}^* = \underset{\mathbf{K} \in \mathbb{R}_+^2}{\arg\max} v(0; \mathbf{K}, \mathbf{Q}^*(\mathbf{K}), \mathbf{p}^*(\mathbf{Q}^*(\mathbf{K}))). \tag{4}$$

An optimal strategy $(\mathbf{K}^*, \mathbf{Q}^*, \mathbf{p}^*)$ is a solution $(\mathbf{K}^*, \mathbf{Q}^*(\mathbf{K}^*), \mathbf{p}^*(\mathbf{Q}^*(\mathbf{K}^*)))$ to (2)–(4). Finally, we let $v^*(t) = v(t; \mathbf{K}^*, \mathbf{Q}^*, \mathbf{p}^*)$ be the firm value at time t, when the optimal strategy is followed.

We assume that in each of the two product markets, the firm is a monopolist facing an iso-elastic demand curve that is subject to a multiplicative random shock $\epsilon_i(T)$, $i = 1, 2$. Thus, the inverse demand curve in market i at time T is

$$p_i = \epsilon_i(T) q_i^{1/b},$$

where $b \in (-\infty, -1)$ is the constant price elasticity of demand.

The vector of random shocks, $\boldsymbol{\epsilon} = \{\boldsymbol{\epsilon}(t), t \geq 0\}$, is assumed to follow a two-dimensional geometric Brownian motion. Thus, $\ln \boldsymbol{\epsilon}(t)$ is a bivariate normal random vector with mean $\ln \boldsymbol{\epsilon}(0)$ and covariance matrix $t\boldsymbol{\Sigma}$, where $\boldsymbol{\Sigma}_{ii} = \sigma^2$ and $\boldsymbol{\Sigma}_{ij} = \rho\sigma^2$ if $i \neq j$. The information available at time t includes the history of $\boldsymbol{\epsilon}(t)$ or, formally, the filtration $\mathbb{F}$ is generated by $\boldsymbol{\epsilon}$.

The firm's flexibility has the following three dimensions:

1. *Mix flexibility* φ is the firm's ability to convert one type of capacity to another after updating the demand forecast. Since this ability depends on the switching cost c_S, we define mix

flexibility as $\varphi = 1/c_S$. With zero mix flexibility ($\varphi = 0$), the firm does not have the option to switch capacity. With perfect mix flexibility ($\varphi \to \infty$), both products rely on the same capacity.

2. *Volume flexibility* γ is the ability to vary product volumes after the demand forecast is updated. We proxy volume flexibility by the fraction of the total unit cost (bar the switching cost) that is incurred after the update time, i.e., $\gamma = c_Q/(c_K + c_Q)$. With zero volume flexibility ($\gamma = 0$), the firm has incentive to always utilize full capacity, i.e., the aggregate production volume is fully determined at time zero. With perfect volume flexibility ($\gamma = 1$), the aggregate production volume is not constrained by capacity, and the average total cost (assuming no capacity switching) is independent of production volume. In general, the higher the volume flexibility, the larger the optimal capacity and the lower the impact of production volume on the average total cost.

3. *Time flexibility* τ measures how long the firm can wait before making the production decision. With no time flexibility ($\tau = 0$), the firm chooses the output quantities at time 0. With perfect time flexibility ($\tau = T$), the firm can postpone the production decision until all uncertainty is resolved.

3   Optimal strategy

We solve for the optimal strategy by backward induction starting with the pricing decision (2). It is a well-known property of the iso-elastic demand function that a monopoly always maximizes its revenue from a given output by selling all units at the market clearing price. In other words, $\mathbf{q}\left(\mathbf{p}^*\left(\mathbf{Q}\right), \mathbf{Q}\right) = \mathbf{Q}$ and

$$p_i^*\left(\mathbf{Q}\right) = \epsilon_i\left(T\right) Q_i^{1/b}, \, i = 1, 2. \tag{5}$$
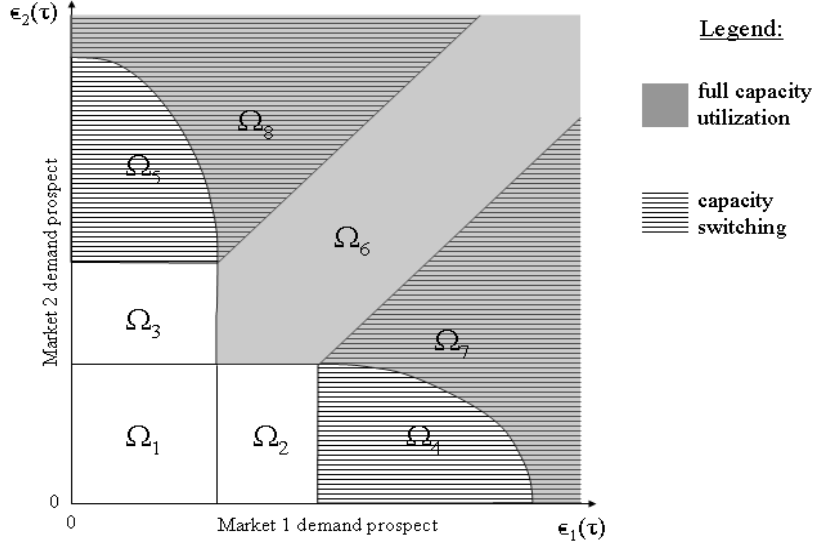
Figure 1: The state space of the demand prospects $\boldsymbol{\epsilon}\left(\tau\right)$ is partitioned into eight events that specify different optimal production vectors. If $\boldsymbol{\epsilon}\left(\tau\right) \in \Omega_{678}$, the total capacity is fully utilized. If $\boldsymbol{\epsilon}\left(\tau\right) \in \Omega_{4578}$, some capacity conversion or switching is optimal.

Therefore, the firm's revenue under the optimal pricing policy is $\pi\left(\mathbf{p}^*\left(\mathbf{Q}\right), \mathbf{Q}\right) = \sum_{i=1}^2 \epsilon_i\left(T\right) Q_i^{1+1/b}$.

The optimal output vector (3) maximizes the firm value at time τ, which can be written as

$$v\left(\tau; \mathbf{K}, \mathbf{Q}, \mathbf{p}^*\left(\mathbf{Q}\right)\right) = \sum_{i=1}^2 \left(\mathbb{E}_\tau \epsilon_i\left(T\right) Q_i^{1+1/b} - c_Q Q_i - c_S \max\left(Q_i - K_i, 0\right) - c_K K_i\right). \qquad (6)$$

Since $b < -1$, this objective function is strictly concave and the optimal output vector $\mathbf{Q}^*(\mathbf{K})$ is

the unique solution to the Kuhn-Tucker optimality conditions. To characterize the optimal output

vector, we partition the state space of the updated "demand prospects" $\boldsymbol{\epsilon}\left(\tau\right)$ into eight events

$\Omega_1, ..., \Omega_8$ as illustrated in Figure 1. We also define $\Omega_{i_1 i_2 \cdots i_n} \equiv \Omega_{i_1} \cup \Omega_{i_2} \cup \cdots \cup \Omega_{i_n}$. The formal

definitions of $\Omega_1, ..., \Omega_8$ as well as the corresponding optimal output vectors are characterized in

Appendix 1. The intuitive interpretation of these eight events is as follows:

Event $\Omega_1\left(\mathbf{K}\right)$: The prospects of both markets are poor and neither capacity is fully utilized: $Q_i^* < K_i$, $i = 1, 2$.

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Event $\Omega_2(\mathbf{K})$: The prospects of market 1 are relatively good while those of market 2 are poor. Capacity 1 is fully utilized but capacity 2 is not and no capacity switching occurs: $Q_1^* = K_1$ and $Q_2^* < K_2$. (The event Ω_3 is symmetric: $Q_1^* < K_1$ and $Q_2^* = K_2$.)

Event $\Omega_4(\mathbf{K})$: The prospects of market 1 are very good so that not only full capacity 1 but also some of capacity 2 is used to make product 1: $Q_1^* > K_1$. The prospects of market 2, however, are poor so that only a fraction of the remaining capacity 2 is used to make product 2: $Q_2^* < K_2 - (Q_1^* - K_1)$. (The event Ω_5 is symmetric: $Q_2^* > K_2$ and $Q_1^* < K_1 - (Q_2^* - K_2)$.)

Event $\Omega_6(\mathbf{K})$: The prospects of both markets are good enough to justify full utilization of both capacities and not sufficiently different to justify capacity conversion: $Q_i^* = K_i$, $i = 1, 2$.

Event $\Omega_7(\mathbf{K})$: The overall market prospects are very good so that both resources are fully utilized. Furthermore, the prospects of market 1 are significantly better than those of market 2 warranting conversion of some capacity 2 to make product 1: $Q_1^* > K_1$ and $Q_2^* = K_2 - (Q_1^* - K_1)$. (The event Ω_8 is symmetric: $Q_2^* > K_2$ and $Q_1^* = K_1 - (Q_2^* - K_2)$.)

The capacity investment problem (4) is also concave and the optimal capacity vector is characterized by the first order condition. If furthermore $c_S > 0$, the optimal capacity vector is unique. (If $c_S = 0$, only the *total* capacity matters, i.e., there is a continuum of optimal capacity vectors characterized by the first order conditions, all of which represent the same total capacity.)

Proposition 1 *If $c_S > 0$, there exists a unique optimal capacity vector $\mathbf{K}^*$ which satisfies $K_1 = K_2$ and*

$$
\Pr(\Omega_{78}(\mathbf{K})) \, \mathbb{E}\left(\sum_{i=1}^{2} \frac{\partial Q_i^*}{\partial K_1} \frac{\partial (p_i^* Q_i^*)}{\partial Q_i^*} - c_Q - c_S \frac{\partial |Q_1^* - K_1|}{\partial K_1} \Bigg| \Omega_{78}(\mathbf{K}) \right)
$$
$$
+ \quad \Pr(\Omega_{26}(\mathbf{K})) \, \mathbb{E}\left(\frac{\partial (p_1^* K_1)}{\partial K_1} - c_Q \Bigg| \Omega_{26}(\mathbf{K}) \right) + \Pr(\Omega_4(\mathbf{K})) \, c_S = c_K, \tag{7}
$$

where $\mathbf{p}^* = \mathbf{p}^* \left(\mathbf{Q}^* \left(\mathbf{K} \right) \right)$ *is given by (5) and* $\mathbf{Q}^* = \mathbf{Q}^* \left(\mathbf{K} \right)$ *is given in Appendix 1.*

Proof: All proofs can be found in Appendix 2.□

Optimality condition (7) sets the marginal value of capacity 1 equal to the unit capacity investment cost c_K. The marginal value of capacity (left-hand side of (7)) stems from the marginal sales revenue and the potential savings in the switching costs net of the marginal production cost and the potential increase in the switching cost, and it depends on the specific capacity allocation at the update time.

To evaluate the benefits of flexibility, it is useful to consider a firm without any flexibility, i.e., a firm that must choose its output vector at time zero. Formally, a non-flexible firm is a limiting case of the flexible firm with no time flexibility ($\tau = 0$) or, alternatively, with no mix and volume flexibilities ($\varphi = 0$ and $\gamma = 0$). Let $\tilde{\mathbf{p}}$, $\tilde{\mathbf{Q}}$ and $\tilde{\mathbf{K}}$ be the price, output and capacity vectors of a non-flexible firm. Furthermore, let $\tilde{c}_K \leq c_K$ and $\tilde{c}_Q \leq c_Q$ be the unit capacity and production costs of a non-flexible firm, reflecting the fact that flexibility typically comes at a cost. Finally, let $\tilde{v} \left(t; \tilde{\mathbf{K}}, \tilde{\mathbf{Q}}, \tilde{\mathbf{p}} \right)$, or $\tilde{v}(t)$ for short, denote the value of a non-flexible firm at time t, and let $\tilde{v}^* \left(t \right) \equiv \tilde{v} \left(t; \tilde{\mathbf{K}}^*, \tilde{\mathbf{Q}}^*, \tilde{\mathbf{p}}^* \right)$ be its value if the optimal strategy is followed. The optimal capacity and value of a non-flexible firm can be both obtained in closed form.

Corollary 1 *The optimal capacity vector and value of the non-flexible firm are, respectively,*

$$\tilde{K}_1^* = \tilde{K}_2^* = \left(\frac{1 + 1/b}{\tilde{c}_Q + \tilde{c}_K} \mathbb{E}_0 \epsilon_i \left(T \right) \right)^{-b} \qquad and \qquad \tilde{v}^* \left(0 \right) = \frac{\tilde{c}_Q + \tilde{c}_K}{|1 + b|} \left(\tilde{K}_1^* + \tilde{K}_2^* \right).$$

In the next section, we define the value of flexibility and discuss how it depends on the three dimensions of flexibility.

4 Value of flexibility and its drivers

We define the value premium for flexibility as the relative difference between the value of a flexible and a non-flexible firm:

$$\Delta_F \equiv \frac{v^*(0) - \tilde{v}^*(0)}{\tilde{v}^*(0)}.$$

The value premium for flexibility is in general difficult to study analytically. However, it can be examined analytically in the special case of perfect mix flexibility and zero volume flexibility $(c_Q = c_S = 0)$. In this case, the value of flexibility stems from risk pooling and the revenue maximizing option, and can be expressed in closed form.

Lemma 1 *If $c_Q = c_S = 0$, the value premium for flexibility simplifies into*

$$\Delta_F = \left(\frac{\tilde{c}_K}{c_K}\right)^{-b-1} \left(\frac{\|\epsilon(T)\|_\tau}{\mathbb{E}_0 \epsilon_i(T)}\right)^{-b} - 1, \ \text{where} \ \|\epsilon(T)\|_\tau = \mathbb{E}_0 \left\{ \left(\frac{\mathbb{E}_\tau^{-b} \epsilon_1(T) + \mathbb{E}_\tau^{-b} \epsilon_2(T)}{2}\right)^{-1/b} \right\}.$$

Whether flexibility creates value depends on the benefit of flexibility relative to its cost. We define the maximum sustainable cost of flexibility δ as the maximum ratio of the cost of flexible and nonflexible capacity for which flexibility creates value, i.e.,

$$\Delta_F > 0 \qquad \Leftrightarrow \qquad \frac{c_K}{\tilde{c}_K} < \left(\frac{\|\epsilon(T)\|_\tau}{\mathbb{E}_0 \epsilon_i(T)}\right)^{\frac{b}{1+b}} \equiv \delta.$$

It follows from the Minkowski inequality that $\|\epsilon(T)\|_\tau \geq \mathbb{E}_0 \epsilon_i(T)$, i.e., $\delta > 1$. This means that in general a firm benefits from flexibility as long as its cost is not too high. The next three lemmas characterize the effects of time flexibility, demand variability and demand correlation on the optimal capacity, firm value, flexibility premium and the maximum sustainable cost of flexibility δ. The longer the firm can wait before exercising the option to switch capacity, the higher the value of this option and the higher the optimal capacity investment.

Lemma 2 *If $c_Q = c_S = 0$, the optimal capacity, firm value, flexibility premium and the maximal sustainable cost of flexibility increase in time flexibility τ: $\frac{\partial}{\partial \tau}(K_1^* + K_2^*) \geq 0$, $\frac{\partial}{\partial \tau}v^*(0) \geq 0$, $\frac{\partial}{\partial \tau}\Delta_F \geq 0$, and $\frac{\partial}{\partial \tau}\delta \geq 0$.*

Similar to financial options, volatility increases the value of the option to switch and, hence, the value of a flexible firm. It also increases the optimal capacity level.

Lemma 3 *If $c_Q = c_S = 0$, the optimal capacity, firm value, flexibility premium and the maximal sustainable cost of flexibility increase in demand volatility σ: $\frac{\partial}{\partial \sigma}(K_1^* + K_2^*) \geq 0$, $\frac{\partial}{\partial \sigma}v^*(0) \geq 0$, $\frac{\partial}{\partial \sigma}\Delta_F \geq 0$, and $\frac{\partial}{\partial \sigma}\delta \geq 0$.*

As expected, higher demand correlation reduces the value of the switching option. It also leads to a lower capacity investment.

Lemma 4 *If $c_Q = c_S = 0$, the optimal capacity, firm value, flexibility premium and the maximal sustainable cost of flexibility decrease in demand correlation ρ: $\frac{\partial}{\partial \rho}(K_1^* + K_2^*) \leq 0$, $\frac{\partial}{\partial \rho}v^*(0) \leq 0$, $\frac{\partial}{\partial \rho}\Delta_F \leq 0$ and $\frac{\partial}{\partial \rho}\delta \leq 0$.*

While it is intuitive that higher demand volatility and lower demand correlation both increase the value of flexibility, it is less obvious that they also result in a higher optimal capacity level, $K_1^* + K_2^*$. In the newsvendor model of Eppen (1979), the optimal flexible capacity increases in demand volatility and correlation if, and only if, the capacity exceeds the expected demand. In that model, the effect of demand volatility and correlation on the optimal capacity depends on whether it increases or decreases the probability that all capacity will be used. With zero production cost ($c_Q = 0$), that probability is always one. However, with endogenous pricing, higher demand volatility and lower demand correlation increase the expected output prices and, hence, the marginal value of capacity. Next, we use a numerical analysis to examine the general case of nonnegative production and switching costs ($c_Q \geq 0$ and $c_S \geq 0$).

4.1 Three dimensions of flexibility: Complements or substitutes?

Both mix and volume flexibilities mitigate the expected cost of mismatch between capacity and demand resulting from the capacity investment being made under demand uncertainty. Mix flexibility reduces this mismatch cost by providing the opportunity to reallocate capacity based on the additional information revealed by the update time. Volume flexibility means cheaper and more abundant capacity which makes it easier to vary production volume at the update time when more information is available. Because mix and volume flexibility are two distinct ways of reducing the mismatch cost, one would expect them to be strategic substitutes. At the same time, both mix and volume flexibilities become more valuable as the update time moves closer to the selling season when more information is available. One would therefore expect both mix and volume flexibilities to be complementary with time flexibility. We can formalize these notions as follows.

Conjecture 1 *Mix and time flexibilities are strategic complements: $\frac{\partial^2 \Delta_F}{\partial \varphi \partial \tau} \geq 0$ for any volume flexibility.*

Conjecture 2 *Volume and time flexibilities are strategic complements: $\frac{\partial^2 \Delta_F}{\partial \tau \partial \gamma} \geq 0$ for any mix flexibility.*

Conjecture 3 *Volume and mix flexibilities are strategic substitutes: $\frac{\partial^2 \Delta_F}{\partial \varphi \partial \gamma} \leq 0$ for any time flexibility.*

Our numerical investigation supports all three conjectures, as illustrated in Figure 2 for a representative set of parameter values. (All numerical results are based on simulation using 50,000 demand scenarios. In the boundary cases that were also solved analytically, the simulation errors were below 1%.)

Figure 2a shows the value premium for flexibility Δ_F for volume flexibility $\gamma = 0.5$, time flexibility $\tau \in [0, 1]$ and mix flexibility $\varphi \in \{0, ..., \infty\}$ (i.e., switching cost $c_S \in \{0, ..., \infty\}$). We
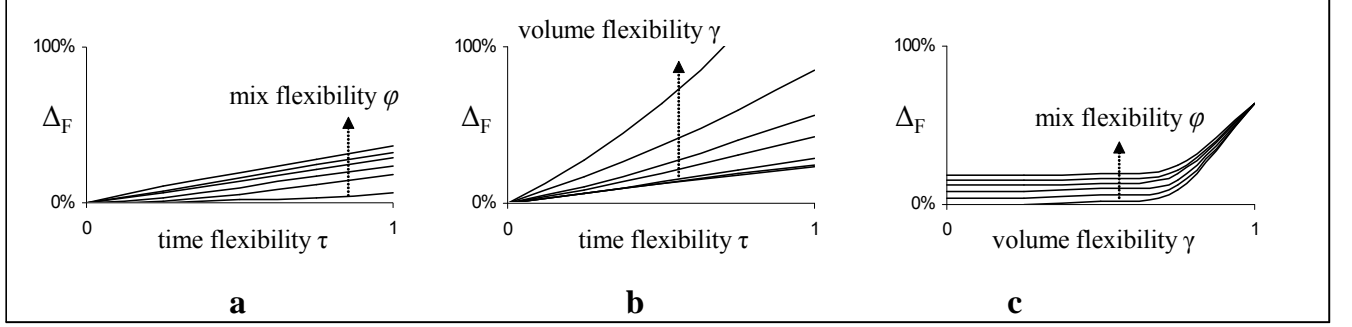
Figure 2: The effects of mix, volume and time flexibilities on the flexibility premium Δ_F for independent demands. ($T = 1$, $b = -2$, $c_K + c_Q = 0.2$, $\epsilon(0) = 1$, $\sigma = 1$ and $\rho = 0$.)

observe that the marginal value of time flexibility $\partial \Delta_F / \partial \tau$ increases in mix flexibility φ, which confirms Conjecture 1 that mix and time flexibilities are strategic complements. This has two managerial implications:

1. The closer to the selling season a firm chooses its output mix, or the more demand information it has at that time, the more it should invest in product-flexible technology, workforce cross-training and other enablers of mix flexibility.

2. The easier (cheaper) it is for a firm to convert one type of capacity to another, the more the firm should invest in obtaining accurate and timely market information and/or in postponing the point of product differentiation.

Figure 2b illustrates the complementarity of time and volume flexibilities for time flexibility $\tau \in [0, 1]$, volume flexibility $\gamma \in [0, 1]$ and mix flexibility $\varphi = 20$ (i.e., switching cost $c_S = 0.05$). We notice that the marginal value of time flexibility $\partial \Delta_F / \partial \tau$ increases in volume flexibility γ, confirming Conjecture 2. The managerial implication is again twofold:

1. The more information a firm can gain by waiting, the more it should strive to postpone purchasing, hiring or production decisions.

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2. And vice versa, the more of its quantity commitments a firm can postpone, the more it should invest in improving demand forecast prior to making these commitments.

Finally, Figure 2c shows how the flexibility premium Δ_F depends on volume flexibility $\gamma \in [0, 1]$ and mix flexibility $\varphi \in \{0, ..., \infty\}$ for time flexibility $\tau = 0.5$. The marginal value of volume flexibility $\partial \Delta_F / \partial \gamma$ decreases in mix flexibility φ indicating that mix and volume flexibilities are strategic substitutes as conjectured. The figure also indicates that mix flexibility has no value under perfect volume flexibility, whereas volume flexibility is valuable even under perfect mix flexibility. In other words, volume flexibility can deliver all benefits of mix flexibility but not vice versa. The following two managerial implications ensue:

1. The higher the cost of fixed assets (capacity), the more the firm should invest in its ability to switch between the production of different products. Thus, mix flexibility is particularly important in expensive, highly utilized capital equipment.

2. The higher the mix flexibility of a resource, the less can be gained from postponing its acquisition. In other words, it is more valuable to postpone the acquisition of product-specific resources than that of a product-flexible resource.

4.2 Three dimensions of flexibility and demand correlation

Positive demand correlation reduces the efficacy of risk pooling and thus diminishes the value of mix flexibility. This insight, which we stated in Lemma 4 for the boundary case of $c_Q = c_S = 0$ and which can be verified numerically for any value of c_Q and c_S, is intuitive and consistent with the literature (e.g., Fine and Freund 1990). However, it is less obvious how demand correlation affects the *marginal* value of different types of flexibility, which is important because it determines the optimal investment in flexibility. Given that risk pooling is the main value driver of mix flexibility
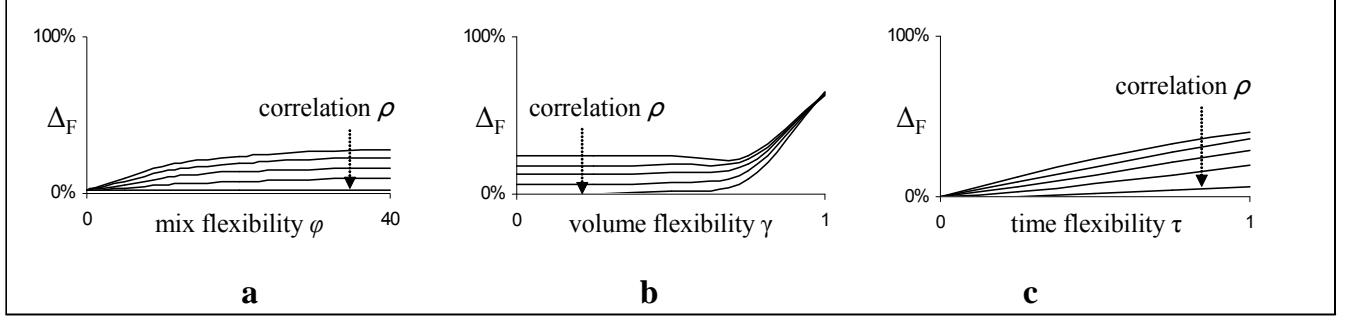
Figure 3: The effects of mix, volume and time flexibilities on the flexibility premium Δ_F under different demand correlations.

we expect the marginal value of mix flexibility to decrease in demand correlation. Similarly for the marginal value of time flexibility (which stems partially from increasing the benefits of risk pooling). Since volume and mix flexibilities are strategic substitutes, less risk pooling increases the marginal value of volume flexibility so we expect an increase in demand correlation to increase the marginal value of volume flexibility as well. We formalize our intuition in the following three conjectures:

Conjecture 4 *The value premium for flexibility is submodular in mix flexibility and demand correlation:* $\frac{\partial^2 \Delta_F}{\partial \varphi \partial \rho} \leq 0$ *for any cost and time flexibility.*

Conjecture 5 *The value premium for flexibility is supermodular in volume flexibility and demand correlation:* $\frac{\partial^2 \Delta_F}{\partial \gamma \partial \rho} \geq 0$ *for any mix and time flexibility.*

Conjecture 6 *The value premium for flexibility is submodular in time flexibility and demand correlation:* $\frac{\partial^2 \Delta_F}{\partial \tau \partial \rho} \leq 0$ *for any mix and volume flexibility.*

Our numerical investigation confirms the three conjectures, as illustrated in Figure 3. (This figure is based on the same parameter values as Figure 2 except that the demand correlation coefficient ρ is varied between -1 and 1.) We note that the value premium for flexibility Δ_F is strictly decreasing in demand correlation except for the following three cases in which demand

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correlation has no effect on this premium: (i) the firm has no mix flexibility; (ii) the firm has perfect volume flexibility (no capacity commitment has to be made under demand uncertainty); and (iii) firm's time flexibility is zero (output decision has to be made at time zero).

Figure 3a plots the flexibility premium Δ_F as a function of mix flexibility φ for given volume flexibility $\gamma = 0.5$ and time flexibility $\tau = 0.5$. It confirms our conjecture that the marginal value of mix flexibility $\partial \Delta_F / \partial \varphi$ decreases in demand correlation. The higher the demand correlation, the less capacity is likely to be converted and, therefore, the less value results from reducing the switching cost. As a result, the higher the demand correlation, the lower the optimal investment in mix flexibility (switching cost reduction).

Figure 3b graphs the flexibility premium Δ_F as a function of volume flexibility γ for given mix flexibility $\varphi = 20$ (switching cost $c_S = 0.05$) and time flexibility $\tau = 0.5$. As conjectured, the marginal value of volume flexibility $\partial \Delta_F / \partial \gamma$ increases in demand correlation. As demand correlation increases, mix flexibility becomes less effective in reducing the mismatch between capacity and demand, which makes volume flexibility relatively more important. Therefore, the higher the demand correlation, the more a firm should invest in volume flexibility.

Finally, Figure 3c shows how the flexibility premium Δ_F depends on time flexibility τ for given mix flexibility $\varphi = 20$ and volume flexibility $\gamma = 0.5$. As expected, the marginal value of time flexibility $\partial \Delta_F / \partial \tau$ decreases in demand correlation. More time flexibility means that more information is available before capacity conversion has to be made. As demand correlation increases, additional demand information is less likely to result in capacity conversion and, hence, is less valuable. Therefore, when demand correlation is high, the firm should invest less in time flexibility (forecasting, shorter lead times, etc.) than when demand correlation is low.

In the next section, we discuss the link between the three types of flexibility and corporate diversification.

5 Flexibility and market diversification

The strategy and economic literatures have recognized that diversification can create shareholder value in the presence of market imperfections such as transaction costs in the factor market (see e.g. Teece 1982, Levy and Haber 1986, and Matsusaka 2001). In particular, if capacity investment is irreversible, market diversification coupled with technological or organizational flexibility may create value by reducing the aggregate uncertainty in capacity planning through statistical aggregation or "pooling" of demands. We provide further insights into the relationship between diversification and flexibility by showing how the diversification premium depends on the specific type of flexibility.

We quantify the benefits of diversification by comparing the value of a two-product firm to the sum of the values of two single-product firms under the assumption of sufficiently high transaction costs in the factor market that prevent capacity trading or subcontracting. The optimal value of a two-product or "diversified" firm is $v^*(0)$. The sum of the values of two single-product firms that cannot trade capacity is equal to $v^*(0)$ with prohibitive capacity switching cost, or zero mix flexibility, i.e., $\varphi = 0$. We define the relative diversification premium as

$$\Delta_D \equiv \frac{v^*(0)|_{\varphi>0} - v^*(0)|_{\varphi=0}}{v^*(0)|_{\varphi=0}}. \tag{8}$$

In the special case of negligible production and switching costs, the diversification premium equals the flexibility premium characterized in Lemma 1. It then follows from Lemmas 2-4 that in this case the diversification premium Δ_D increases in time flexibility τ and demand volatility σ and decreases in demand correlation ρ.

In general, the key drivers of the diversification premium are demand volatility, demand correlation, time and volume flexibility (which are both assumed to be the same for the diversified firm and the single-product firms) and mix flexibility of the diversified firm. Note that even though

we are assuming the diversified firm and the two single-product firms to have the same time and volume flexibility, these flexibilities affect the diversification premium due to their complementarity/substitutability with mix flexibility.

The effects of the three types of flexibility and demand correlation are illustrated in Figure 4, which is based on the same parameter values as Figures 2 and 3. The benefits of diversification stem from the diversified firm's ability to switch capacity between the products based on the additional information revealed up to the update time. As a consequence, the diversification premium increases in time flexibility and the diversified firm's mix flexibility and it decreases in demand correlation. At the same time, the diversification premium decreases in the firms' volume flexibility. This is because volume and mix flexibilities are substitutes, i.e., as volume flexibility increases, additional mix flexibility of the diversified firm is less valuable. In conclusion, diversification creates more value if demand correlation is low, the diversified firm can switch capacity relatively close to the selling season and at a relatively low cost, and the irreversible capacity investment represents a considerable part of the firm's total cost.

Figure 4 further indicates the following complementarity and substitutability results. More time flexibility magnifies both the positive effect of mix flexibility ($\partial^2 \Delta_D / \partial \varphi \partial \tau \geq 0$) and the negative effect of volume flexibility ($\partial^2 \Delta_D / \partial \gamma \partial \tau \leq 0$). In addition, greater volume flexibility reduces the positive effect of mix flexibility ($\partial^2 \Delta_D / \partial \varphi \partial \gamma \leq 0$). Higher demand correlation reduces the positive effects of mix flexibility ($\partial^2 \Delta_D / \partial \varphi \partial \rho \leq 0$) and time flexibility ($\partial^2 \Delta_D / \partial \tau \partial \rho \leq 0$) and the negative effect of volume flexibility ($\partial^2 \Delta_D / \partial \gamma \partial \rho \geq 0$). Thus, the lower the correlation among the different business segments of a diversified firm, the more this firm should invest in technological and organizational flexibility that allows it employing its resources across different business segments.

One aspect of diversification that has attracted significant attention in the strategy literature is
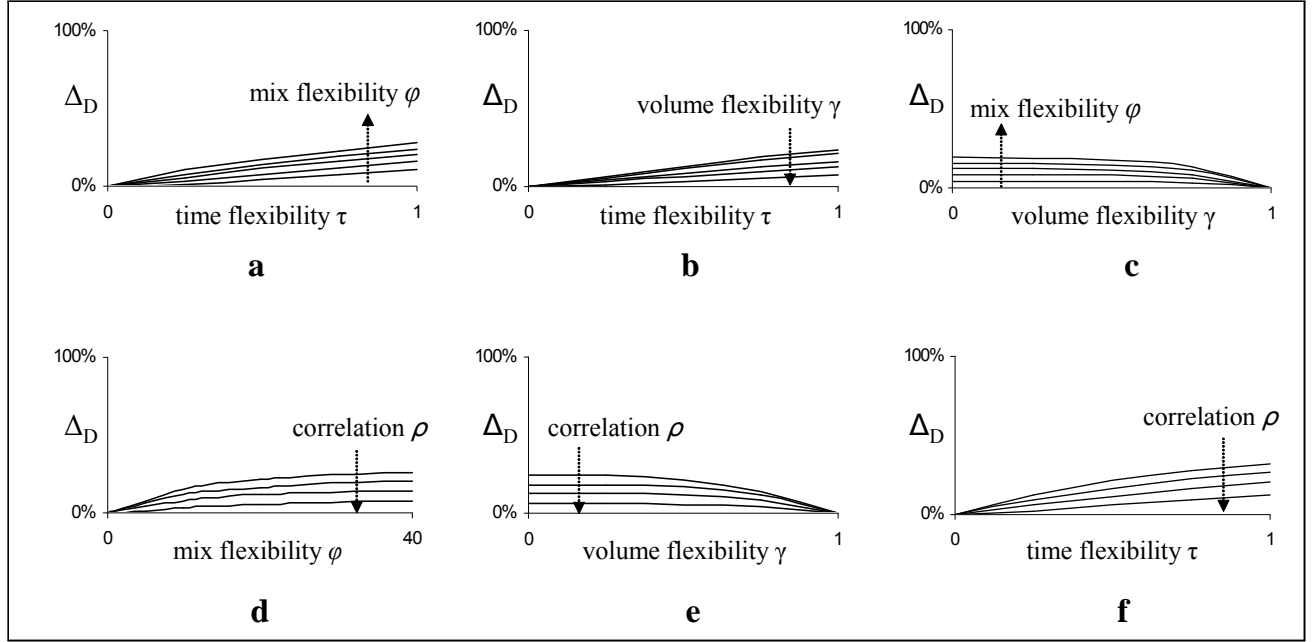
Figure 4: The effects of mix flexibility φ, volume flexibility γ, time flexibility τ and demand correlation ρ on the diversification premium Δ_D.

the relative benefit of diversification into related versus unrelated industries. Related diversification is defined as one "involving businesses that share related production or marketing technologies" (Lubatkin and O'Neill 1987). While resource sharing arguments favor related diversification, risk and internal capital market considerations support unrelated diversification. The extensive empirical evidence on the effect of diversification relatedness is also very fragmented (see e.g., Palich et al. 2000). This ambiguity stems partially from the fact that the notion of relatedness in the existing literature involves two aspects with very different implications for the value of diversification: relatedness of markets and relatedness of technology. Our model allows to clearly distinguish the two aspects of relatedness. Because more similar markets for the two products are likely to be more (positively) correlated, one can think of demand correlation as a proxy for *market relatedness* of diversification.

Because the switching cost is likely to be lower for technologically similar products, a natural proxy for *technological relatedness* of diversification is mix flexibility. An alternative proxy for tech-

nological relatedness would be time flexibility because it is presumably easier to postpone product differentiation until a later stage of the production process for technologically close products. No matter which of the two possible surrogates for technological relatedness we consider, Figure 4 shows that the diversification premium increases in technological relatedness and market unrelatedness and, furthermore, technological relatedness and market unrelatedness are complementary in increasing this premium. These findings are supported by the empirical evidence of Amit and Livnat (1989) who demonstrate that efficient diversifiers operate in related business segments that have differential responses to business cycle changes and thereby enjoy the benefits from (technologically) related diversification as well as from the portfolio effect (market unrelatedness).

6 Summary

This paper considers a two-product firm that make capacity, production, and pricing decisions at three different points in time while continuously updating its demand forecast. We identify three aspects of the firm's ability to respond to the dynamically evolving demand conditions, which we refer to as mix, volume and time flexibility. We show that while mix and volume flexibilities are substitutes, they are both complementary with time flexibility. This has important implications for the optimal level of each of these flexibilities. For example, a firm that has inherently more mix flexibility should invest more in time flexibility and less in volume flexibility. Furthermore, we show that as demand correlation increases, managers should invest more in volume flexibility and less in mix and time flexibilities. Finally, we link the value of flexibility to the value of market diversification and discuss the main drivers of the diversification premium such as the three types of flexibility and demand correlation.

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Appendix 1

The partitioning of the state space of $\boldsymbol{\epsilon}(\tau)$:

$$\Omega_1(\mathbf{K}) \equiv \left\{\mathbf{x} \in \mathbb{R}_+^2 : x_1 < \frac{c_Q K_1^{-1/b}}{(1 + 1/b)\,\mathbb{E}_0\epsilon_1(\tau, T)},\ x_2 < \frac{c_Q K_2^{-1/b}}{(1 + 1/b)\,\mathbb{E}_0\epsilon_2(\tau, T)}\right\},$$

$$\Omega_2(\mathbf{K}) \equiv \left\{\mathbf{x} \in \mathbb{R}_+^2 : \frac{c_Q K_1^{-1/b}}{(1 + 1/b)\,\mathbb{E}_0\epsilon_1(\tau, T)} < x_1 < \frac{(c_Q + c_S) K_1^{-1/b}}{(1 + 1/b)\,\mathbb{E}_0\epsilon_1(\tau, T)},\ x_2 < \frac{c_Q K_2^{-1/b}}{(1 + 1/b)\,\mathbb{E}_0\epsilon_2(\tau, T)}\right\},$$

$$\Omega_3(\mathbf{K}) \equiv \left\{\mathbf{x} \in \mathbb{R}_+^2 : x_1 < \frac{c_Q K_1^{-1/b}}{(1 + 1/b)\,\mathbb{E}_0\epsilon_1(\tau, T)},\ \frac{c_Q K_2^{-1/b}}{(1 + 1/b)\,\mathbb{E}_0\epsilon_2(\tau, T)} < x_2 < \frac{(c_Q + c_S) K_2^{-1/b}}{(1 + 1/b)\,\mathbb{E}_0\epsilon_2(\tau, T)}\right\},$$

$$\Omega_4(\mathbf{K}) \equiv \left\{\mathbf{x} \in \mathbb{R}_+^2 : x_1 > \frac{(c_Q + c_S) K_1^{-1/b}}{(1 + 1/b)\,\mathbb{E}_0\epsilon_1(\tau, T)},\ \left(\frac{x_1}{c_Q + c_S}\right)^{-b} + \left(\frac{x_2}{c_Q}\right)^{-b} < \frac{K_1 + K_2}{((1 + 1/b)\,\mathbb{E}_0\epsilon_1(\tau, T))^{-b}}\right\},$$

$$\Omega_5(\mathbf{K}) \equiv \left\{\mathbf{x} \in \mathbb{R}_+^2 : x_2 > \frac{(c_Q + c_S) K_2^{-1/b}}{(1 + 1/b)\,\mathbb{E}_0\epsilon_2(\tau, T)},\ \left(\frac{x_1}{c_Q}\right)^{-b} + \left(\frac{x_2}{c_Q + c_S}\right)^{-b} < \frac{K_1 + K_2}{((1 + 1/b)\,\mathbb{E}_0\epsilon_1(\tau, T))^{-b}}\right\},$$

$$\Omega_6(\mathbf{K}) \equiv \left\{\mathbf{x} \in \mathbb{R}_+^2 : x_1 > \frac{c_Q K_1^{-1/b}}{(1 + 1/b)\,\mathbb{E}_0\epsilon_1(\tau, T)},\ x_2 > \frac{c_Q K_2^{-1/b}}{(1 + 1/b)\,\mathbb{E}_0\epsilon_2(\tau, T)},\right.$$
$$\left.\left|x_1 K_1^{1/b} - x_2 K_2^{1/b}\right| < \frac{c_S}{(1 + 1/b)\,\mathbb{E}_0\epsilon_1(\tau, T)}\right\},$$

$$\Omega_7(\mathbf{K}) \equiv \left\{\mathbf{x} \in \mathbb{R}_+^2 : \left(\frac{x_1}{c_Q + c_S}\right)^{-b} + \left(\frac{x_2}{c_Q}\right)^{-b} > \frac{K_1 + K_2}{((1 + 1/b)\,\mathbb{E}_0\epsilon_1(\tau, T))^{-b}},\right.$$
$$\left.x_1 K_1^{1/b} - x_2 K_2^{1/b} > \frac{c_S}{(1 + 1/b)\,\mathbb{E}_0\epsilon_1(\tau, T)}\right\},$$

$$\Omega_8(\mathbf{K}) \equiv \left\{\mathbf{x} \in \mathbb{R}_+^2 : \left(\frac{x_1}{c_Q}\right)^{-b} + \left(\frac{x_2}{c_Q + c_S}\right)^{-b} > \frac{K_1 + K_2}{((1 + 1/b)\,\mathbb{E}_0\epsilon_1(\tau, T))^{-b}},\right.$$
$$\left.x_2 K_2^{1/b} - x_1 K_1^{1/b} > \frac{c_S}{(1 + 1/b)\,\mathbb{E}_0\epsilon_1(\tau, T)}\right\}.$$

The corresponding optimal output vector $\mathbf{Q}^*(\mathbf{K}, \boldsymbol{\epsilon}(\tau))$:

$$\text{If } \boldsymbol{\epsilon}(\tau) \in \Omega_1(\mathbf{K}),\ Q_i^* = \left(\frac{(1 + 1/b)\,\mathbb{E}_\tau\epsilon_i(T)}{c_Q}\right)^{-b}, i = 1, 2.$$

$$\text{If } \boldsymbol{\epsilon}(\tau) \in \Omega_2(\mathbf{K}),\ Q_1^* = K_1, Q_2^* = \left(\frac{(1 + 1/b)\,\mathbb{E}_\tau\epsilon_2(T)}{c_Q}\right)^{-b}.$$

$$\text{If } \boldsymbol{\epsilon}(\tau) \in \Omega_3(\mathbf{K}),\ Q_1^* = \left(\frac{(1 + 1/b)\,\mathbb{E}_\tau\epsilon_1(T)}{c_Q}\right)^{-b}, Q_2^* = K_2.$$

If $\epsilon(\tau) \in \Omega_4(\mathbf{K})$, $Q_1^* = \left(\dfrac{(1 + 1/b)\, \mathbb{E}_\tau \epsilon_1(T)}{c_Q + c_S} \right)^{-b}, Q_2^* = \left(\dfrac{(1 + 1/b)\, \mathbb{E}_\tau \epsilon_2(T)}{c_Q} \right)^{-b}.$

If $\epsilon(\tau) \in \Omega_5(\mathbf{K})$, $Q_1^* = \left(\dfrac{(1 + 1/b)\, \mathbb{E}_\tau \epsilon_1(T)}{c_Q} \right)^{-b}, Q_2^* = \left(\dfrac{(1 + 1/b)\, \mathbb{E}_\tau \epsilon_2(T)}{c_Q + c_S} \right)^{-b}.$

If $\epsilon(\tau) \in \Omega_6(\mathbf{K})$, $\mathbf{Q}^* = \mathbf{K}$.

If $\epsilon(\tau) \in \Omega_7(\mathbf{K})$, $\mathbf{Q}^*$ is the unique solution to $Q_1 + Q_2 = K_1 + K_2$ and

$$\mathbb{E}_\tau \epsilon_1(T)\, Q_1^{1/b} - \mathbb{E}_\tau \epsilon_2(T)\, Q_2^{1/b} = \frac{c_S}{(1 + 1/b)}.$$

If $\epsilon(\tau) \in \Omega_8(\mathbf{K})$, $\mathbf{Q}^*$ is the unique solution to $Q_1 + Q_2 = K_1 + K_2$ and

$$\mathbb{E}_\tau \epsilon_2(T)\, Q_2^{1/b} - \mathbb{E}_\tau \epsilon_1(T)\, Q_1^{1/b} = \frac{c_S}{(1 + 1/b)}.$$

Appendix 2

Proof of Proposition 1: For simplicity, we use $\mathbf{p}^*$ and $\mathbf{Q}^*$ as shorthand for $\mathbf{p}^*(\mathbf{Q}^*(\mathbf{K}))$ and $\mathbf{Q}^*(\mathbf{K})$, respectively, where $\mathbf{p}^*$ is given by (5) and $\mathbf{Q}^*$ is characterized in Appendix 1. Given the optimal pricing and output decisions, the firm value at time τ is

$$v(\tau; \mathbf{K}) = \sum_{i=1}^{2} \left(\mathbb{E}_\tau \epsilon_i(T)\, (Q_i^*)^{1+1/b} - c_Q Q_i^* - c_S \max(Q_i^* - K_i, 0) - c_K K_i \right). \tag{9}$$

The Hessian matrix of (9) with respect to $\mathbf{K}$ is

$$
H_{\mathbf{K}}v\left(\tau;\mathbf{K}\right) = \begin{cases} \begin{pmatrix} 0 & 0 \\ & \\ 0 & 0 \end{pmatrix} & \text{if } \boldsymbol{\epsilon}\left(\tau\right) \in \Omega_{145}, \\[2em] \frac{1+b}{b^2}\mathbb{E}_{\tau}\epsilon_1\left(T\right)K_1^{1/b-1} \begin{pmatrix} 1 & 0 \\ & \\ 0 & 0 \end{pmatrix} & \text{if } \boldsymbol{\epsilon}\left(\tau\right) \in \Omega_2, \\[2em] \frac{1+b}{b^2}\mathbb{E}_{\tau}\epsilon_2\left(T\right)K_2^{1/b-1} \begin{pmatrix} 0 & 0 \\ & \\ 0 & 1 \end{pmatrix} & \text{if } \boldsymbol{\epsilon}\left(\tau\right) \in \Omega_3, \\[2em] \begin{pmatrix} \frac{1+b}{b^2}\mathbb{E}_{\tau}\epsilon_1\left(T\right)K_1^{1/b-1} & 0 \\ & \\ 0 & \frac{1+b}{b^2}\mathbb{E}_{\tau}\epsilon_2\left(T\right)K_2^{1/b-1} \end{pmatrix} & \text{if } \boldsymbol{\epsilon}\left(\tau\right) \in \Omega_6, \\[2em] \frac{1+b}{b^2}\frac{\mathbb{E}_{\tau}\epsilon_1(T)\mathbb{E}_{\tau}\epsilon_2(T)\left(Q_1^*\right)^{1/b-1}\left(Q_2^*\right)^{1/b-1}}{\mathbb{E}_{\tau}\epsilon_1(T)\left(Q_1^*\right)^{1/b-1}+\mathbb{E}_{\tau}\epsilon_2(T)\left(Q_2^*\right)^{1/b-1}} \begin{pmatrix} 1 & 1 \\ & \\ 1 & 1 \end{pmatrix} & \text{if } \boldsymbol{\epsilon}\left(\tau\right) \in \Omega_{78}. \end{cases}
$$

Thus, $H_{\mathbf{K}}v\left(\tau;\mathbf{K}\right)$ is negative definite if $\boldsymbol{\epsilon}\left(\tau\right) \in \Omega_6$ and negative semidefinite otherwise. Therefore, $v\left(\tau;\mathbf{K}\right)$ is concave in $\mathbf{K}$ for any $\boldsymbol{\epsilon}\left(\tau\right)$ and the concavity is strict if $\boldsymbol{\epsilon}\left(\tau\right) \in \Omega_6$. This means that $v\left(0;\mathbf{K}\right) = \mathbb{E}_0 v\left(\tau;\mathbf{K}\right)$ is concave in $\mathbf{K}$ and the first-order optimality condition $\nabla_{\mathbf{K}}v\left(0;\mathbf{K}\right) = \mathbf{0}$ is sufficient. Furthermore, if $c_S > 0$, then $\Pr\left(\Omega_6\left(\mathbf{K}\right)\right) > 0$ and the concavity is strict, implying that $\mathbf{K}^*$ is unique. The uniqueness of $\mathbf{K}^*$ together with the symmetry of all parameters implies that $K_1^* = K_2^*$. Taking the derivative of $v\left(0;\mathbf{K}\right)$ with respect to K_1 yields

$$
\frac{\partial v\left(0;\mathbf{K}\right)}{\partial K_1} = \frac{\partial}{\partial K_1}\mathbb{E}_0 v\left(T;\mathbf{K}\right) = \frac{\partial}{\partial K_1}\sum_{i=1}^{8}\Pr\left(\Omega_i\left(\mathbf{K}\right)\right)\mathbb{E}_0\left(v\left(T;\mathbf{K}\right)|\,\Omega_i\left(\mathbf{K}\right)\right), \tag{10}
$$

where the firm terminal value, given the optimal pricing and output decisions, is

$$
v\left(T;\mathbf{K}\right) = \sum_{i=1}^{2}\left(Q_i^* p_i^* - c_P Q_i^* - c_S \max\left(Q_i^* - K_i, 0\right) - c_K K_i\right).
$$

Note that $v(T; \mathbf{K})$ is continuous in $\boldsymbol{\epsilon}(\tau)$ and, therefore, the terms from differentiating the boundaries of $\Omega_1, ..., \Omega_8$ with respect to K_1 in (10) cancel out. This leaves us with

$$\frac{\partial v(0; \mathbf{K})}{\partial K_1} = \sum_{i=1}^{8} \Pr(\Omega_i(\mathbf{K})) \mathbb{E}_0 \left(\frac{\partial v(T; \mathbf{K})}{\partial K_1} \bigg| \Omega_i(\mathbf{K}) \right).$$

Differentiating $v(T; \mathbf{K})$ with respect to K_1 and setting $\partial v(0; \mathbf{K})/\partial K_1 = 0$ results in (7). $\square$

Proof of Corollary 1: The result follows from Proposition 1 with $\tau = 0$, $c_K = \tilde{c}_K$ and $c_Q = \tilde{c}_Q$. $\square$

Proof of Lemma 1: It follows from Proposition 1 that if $c_S = c_Q = 0$, the optimal total capacity and firm value are, respectively,

$$K_1^* + K_2^* = \left[\frac{1 + 1/b}{c_K} \mathbb{E}_0 \left(\left(\mathbb{E}_\tau^{-b} \epsilon_1(T) + \mathbb{E}_\tau^{-b} \epsilon_2(T) \right)^{-1/b} \right) \right]^{-b} \text{ and } v^*(0) = \frac{c_K}{|1 + b|} (K_1^* + K_2^*).$$

This together with Corollary 1 gives the desired result. $\square$

Proof of Lemma 2: To simplify the notation, we normalize $T = 1$ and $\boldsymbol{\epsilon}(0) = \mathbf{1}$. To prove the desired results, it is sufficient to show that $\frac{\partial}{\partial \tau} \|\boldsymbol{\epsilon}(1)\|_\tau \geq 0$. Recall that $\|\boldsymbol{\epsilon}(1)\|_\tau = \mathbb{E}_0 \left\{ \left(\frac{\mathbb{E}_\tau^{-b} \epsilon_1(1) + \mathbb{E}_\tau^{-b} \epsilon_2(1)}{2} \right)^{-1/b} \right\}$ and $\ln \boldsymbol{\epsilon}(t) \sim N(\ln \boldsymbol{\epsilon}(0), t\boldsymbol{\Sigma})$. Using the fact that $\mathbb{E}_\tau \epsilon_i(1) = \epsilon_i(\tau) \exp\left(\frac{1}{2}\sigma^2(1 - \tau)\right)$, we can write

$$\begin{aligned}
\|\boldsymbol{\epsilon}(1)\|_\tau &= \mathbb{E}_0 \left[\left(\frac{\left(\epsilon_1(\tau) \exp\left(\frac{1}{2}\sigma^2(1 - \tau)\right)\right)^{-b} + \left(\epsilon_2(\tau) \exp\left(\frac{1}{2}\sigma^2(1 - \tau)\right)\right)^{-b}}{2} \right)^{-1/b} \right] \\
&= 2^{1/b} \exp\left(\frac{1}{2}\sigma^2(1 - \tau)\right) \mathbb{E}_0 \left[\left(\epsilon_1^{-b}(\tau) + \epsilon_2^{-b}(\tau) \right)^{-1/b} \right].
\end{aligned}$$

The normal vector $\ln \boldsymbol{\epsilon}(\tau)$ can be rewritten in terms of two independent standard normal random variables as $\ln \boldsymbol{\epsilon}(\tau) = \sqrt{\tau \boldsymbol{\Sigma}} \mathbf{Z}$, where $\mathbf{Z} \sim N(\mathbf{0}, \mathbf{I})$ and $\mathbf{I}$ is a 2×2 identity matrix. Since $\tau \boldsymbol{\Sigma}$ is positive definite, $\sqrt{\tau \boldsymbol{\Sigma}}$ exists and can be obtained using eigenvector decomposition, $\sqrt{\tau \boldsymbol{\Sigma}} =$

$$\sqrt{\tau}\sigma \begin{pmatrix} \sqrt{(1-\rho)/2} & \sqrt{(1+\rho)/2} \\ -\sqrt{(1-\rho)/2} & \sqrt{(1+\rho)/2} \end{pmatrix}.$$ Using this transformation, we obtain

$$\|\epsilon(1)\|_\tau = 2^{1/b} \exp\left(\frac{1}{2}\sigma^2(1-\tau)\right) \times$$

$$\mathbb{E}_0\left[\exp\left(\sqrt{\tau}\sigma\sqrt{(1+\rho)/2}Z_2\right)\left(\exp\left(-b\sqrt{\tau}\sigma\sqrt{(1-\rho)/2}Z_1\right) + \exp\left(b\sqrt{\tau}\sigma\sqrt{(1-\rho)/2}Z_1\right)\right)^{-1/b}\right].$$

Since Z_1 and Z_2 are independent, we can further simplify

$$\|\epsilon(1)\|_\tau = 2^{1/b} \exp\left(\frac{1}{2}\sigma^2 - \frac{1}{4}\tau\sigma^2 + \frac{1}{4}\tau\sigma^2\rho\right) \times$$

$$\mathbb{E}_0\left[\left(\exp\left(-b\sqrt{\tau}\sigma\sqrt{(1-\rho)/2}Z_1\right) + \exp\left(b\sqrt{\tau}\sigma\sqrt{(1-\rho)/2}Z_1\right)\right)^{-1/b}\right]. \tag{11}$$

Next, we take the derivative of (11) with respect to τ. After some algebra, we obtain

$$\frac{\partial}{\partial\tau}\|\epsilon(1)\|_\tau = \frac{1}{4}\sigma^2(\rho-1)\|\epsilon(1)\|_\tau + 2^{1/b}\exp\left(\frac{1}{2}\sigma^2 - \frac{1}{4}\tau\sigma^2 + \frac{1}{4}\tau\sigma^2\rho\right)\frac{\sigma\sqrt{(1-\rho)/2}}{\sqrt{\tau}} \times$$

$$\mathbb{E}_0\left[\left(\exp\left(-b\sqrt{\tau}\sigma\sqrt{(1-\rho)/2}Z_1\right) + \exp\left(b\sqrt{\tau}\sigma\sqrt{(1-\rho)/2}Z_1\right)\right)^{-1/b-1}\exp\left(-b\sqrt{\tau}\sigma\sqrt{(1-\rho)/2}Z_1\right)Z_1\right].$$

$$\tag{12}$$

To evaluate (12), we make use of the fact that for a differentiable function g and a standard normal random variable Z_1, $\mathbb{E}(g(Z_1)Z_1) = \mathbb{E}g'(Z_1)$ (Rubinstein 1976). Applying this result and some

algebra to (12), we obtain

$$\frac{\partial}{\partial \tau} \|\epsilon(1)\|_\tau = -\sigma^2 (1-\rho)(1+b) 2^{1/b} \exp\left(\frac{1}{2}\sigma^2 - \frac{1}{4}\tau\sigma^2 + \frac{1}{4}\tau\sigma^2\rho\right) \times$$

$$\mathbb{E}_0\left[\left(\exp\left(-b\sqrt{\tau}\sigma\sqrt{(1-\rho)/2}Z_1\right) + \exp\left(b\sqrt{\tau}\sigma\sqrt{(1-\rho)/2}Z_1\right)\right)^{-1/b-2}\right]$$

$$= -\sigma^2 (1-\rho)(1+b) 2^{1/b}\mathbb{E}_0\left[\left(\prod_{i=1,2}\mathbb{E}_\tau^{-b}\epsilon_i(1)\right)\left(\sum_{i=1,2}\mathbb{E}_\tau^{-b}\epsilon_i(1)\right)^{-1/b-2}\right]$$

$$\geq 0. \square$$

Proof of Lemma 3: The proof is similar to the proof of Lemma 2 and is omitted.$\square$

Proof of Lemma 4: The proof is similar to the proof of Lemma 2 and is omitted.$\square$

References

Amit, R. and Livnat, J. (1989), "Efficient Corporate Diversification: Methods and Implications", *Management Science*, 35(7): 879-897.

Chod, J. and Rudi, N. (2005), "Resource Flexibility with Responsive Pricing", *Operations Research*, 53(3): 533-548.

Chod, J., Rudi, N. (2006), "Strategic Investments, Trading and Pricing under Forecast Updating", *Management Science*, 52(12): 1913-1929.

Chod, J., Rudi, N. and Van Mieghem, J.A. (2010), "Operational Flexibility and Financial Hedging: Complements or Substitutes?", *Management Science*, 56(6): 1030-1045.

Eppen, G.D. (1979), "Effects of Centralization on Expected Costs on Multi-Location Newsboy Problem", *Management Science*, 25(5): 498-501.

Fine, C.H. and Freund, R.M. (1990), "Optimal Investment in Product-Flexible Manufacturing Capacity", *Management Science*, 36(4): 449-467.

Goyal, M. and Netessine, S. (2011), "Volume Flexibility, Product Flexibility, or Both: The Role of Demand Correlation and Product Substitution", *Manufacturing & Service Operations Management*, 13(2): 180-193.

Levy, D.T. and Haber, L.J. (1986), "An Advantage of the Multiproduct Firm: The Transferability of Firm-Specific Capital", *Journal of Economic Behaviour and Organization*, 7(3): 291-302.

Lubatkin, M. and O'Neill, H. (1987), "Merger Strategies and Capital Market Risk", *Academy of Management Journal*, 30(4): 665-684.

Martin, J.D. and Sayrak, A. (2003), "Corporate Diversification and Shareholder Value: A Survey of Recent Literature", *Journal of Corporate Finance*, 9(1): 37-57.

Matsusaka, J.G. (2001), "Corporate Diversification, Value Maximization, and Organizational Capabilities", *Journal of Business*, 74(3): 409-431.

Palich, L.E., Cardinal, L.B. and Miller, C.C. (2000), "Curvilinearity in the Diversification-Performance Linkage: An Examination of Over Three Decades of Research", *Strategic Management Journal*, 21(2): 155-174.

Rubinstein, M. (1976), "The Valuation of Uncertain Income Streams and the Pricing of Options", *The Bell Journal of Economics*, 7(2): 407-425.

Teece, D.J. (1982), "Towards an Economic Theory of the Multiproduct Firm", *Journal of Economic Behaviour and Organization*, 3(1): 39-63.

Van Mieghem, J.A. (1998), "Investment Strategies for Flexible Resources", *Management Science*, 44(8): 1071-1078.

Von Ungern-Sternberg, T. (1989), "The Flexibility to Switch between Different Products", *Economica*, 57(227): 355-369.