Global Dual Sourcing and Order Smoothing: The Impact of Capacity and Lead Times

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After decades of offshoring production across the world, companies are rethinking their global networks. Local sourcing is receiving more attention, but it remains challenging to balance the offshore sourcing cost advantage against the increased inventories, because of its longer lead time, and against the cost and (volume) flexibility of each source's capacity. To guide strategic allocation in this global network decision, this paper establishes reasonably simple prescriptions that capture the key drivers. We adopt a conventional discrete-time inventory model with a linear control rule that smoothes orders and allows an exact and analytically tractable analysis of single- and dual-sourcing policies under normal demand. Distinguishing features of our model are that it captures each source's lead time, capacity cost, and flexibility to work overtime. We use Lagrange's inversion theorem to provide exact and simple square-root bound formulae for the strategic sourcing allocations and the value of dual sourcing. The formulae provide structural insight on the impact of financial, operational, and demand parameters, and a starting point for quantitative decision making. We investigate the robustness of our results by comparing the smoothing policy with existing single- and dual-sourcing models in a simulation study that relaxes model assumptions.

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1. Introduction and Summary

Over the past few decades it became conventional wisdom that factory jobs could be done cheaply in some far-flung corner of the world. The original impetus behind offshoring for western firms was to reap savings from performing work in overseas countries with lower wages and materials costs. For several decades that strategy worked, often brilliantly, but now companies are rethinking their global networks. A special report in *The Economist* (2013) on offshoring and outsourcing describes the story of Lenovo, a Chinese technology group, opening a U.S. computer manufacturing line in North Carolina. The global labor "arbitrage" is running out: wages in China have nearly doubled since 2008, partly as a result of domestic minimum-wage policies (the country's 2011 five-year plan called for a 13% average annual minimum-wage increase, a rate some provinces already exceeded (George et al. 2014)). The cost of shipping goods around the world has been rising sharply, and goods spend weeks in transit. Offshoring also often requires substantial safety inventory, the holding cost of which can outweigh the labor and materials cost advantage. Today, greater emphasis is placed on proximity to demand: responding to customers' new-product requests, shorter delivery times, and swift corrections to improve designs and quality has magnified the need for responsive and flexible supply chains. Local sourcing therefore is receiving increasing attention.

A complete reversal to local sourcing, however, may be unlikely and ill-advised. Indeed, the concepts of global and local sourcing are not mutually exclusive. Rather, the combined use of multiple supply sources, each of which is different and possesses unique advantages, might be better than any single-sourcing strategy. Admittedly, multisourcing involves higher coordination costs, but a strategically configured portfolio of suppliers with complementary skills can often perform better than any individual supplier.

In this paper we analyze global sourcing for companies that have access to two sources with complementary competencies: a local source that is responsive but more expensive, and a global source that is (globally) more cost-efficient but with a longer lead time.[1]

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[1] Supply competencies can correspond to transportation modes so that our analysis also applies to balancing mixed-mode transportation or spot and forward market purchasing.

Although our policy can be used for two local sources, we adopt the setting where the low-cost source is offshore and far away from the local responsive source. Whereas the sourcing literature typically only considers sourcing and inventory related costs, we explicitly add each source's capacity cost and flexibility (modeled by its cost to work overtime). In the current economic climate of rising labor costs in some countries, and decreasing flexibility to go beyond regular working hours (driven by strong labor unions or by capacity rigidity due to high levels of automation), capacity cost and flexibility are increasingly relevant factors for sourcing decisions. *The Economist* (2013) mentions that one of the reasons why reshoring may less likely happen in Europe, compared to the United States, is because Europe's labor markets are still fairly inflexible and costly. Labor flexibility still varies greatly from country to country. In a global economy where firms can go where they want, these differences have an effect.

Our approach assumes a wholly owned global network where capacity costs and flexibility are directly relevant. We believe this approach provides a stepping stone to a decentralized system, where independent suppliers will in one way or another include a sourcing charge for capacity costs and flexibility, but a formal game-theoretic analysis is left for future research.

We adopt a conventional discrete-time inventory model with stochastic demand and a linear control rule that is capable of smoothing orders to both sources. The reasons for analyzing this policy, which we will refer to as dual-sourcing smoothing (DSS), are that smoothing policies are effective to reduce capacity requirements and are used in practice (see empirical evidence in §2) when companies face high labor or capital capacity costs. In addition, the linearity of smoothing policies provides analytic tractability with normally distributed demand and allows us to specify analytically the strategic sourcing allocations to both sources.

The optimality equations involve a polynomial of degree higher than the lead time difference L between the two sources. Given that a quartic is the polynomial of highest degree for which general finite analytic expressions for the roots can exist, we use Lagrange's inversion formula to solve the optimality equations for general L. Another technical contribution, for which we relied on Lagrange's technique, is the inclusion of general lead times for each source in a smoothing policy. To the best of our knowledge, the application of the Lagrange series to inventory theory appears novel.

The main contribution of this paper is to provide managerial guidelines for strategic sourcing (global, local, or dual sourcing) based on exact formulae and simple square-root bounds that capture the impact of each source's cost, lead time, capacity, and flexibility for

normally distributed demand. Specifically, we present a simple guideline that captures the trade-off between these four parameters when deciding between local or global single sourcing with the standard base-stock policy. We show that single sourcing with order smoothing dominates using a base-stock policy in the presence of capacity costs and performs close to the optimal sourcing policy under capacity costs. We then extend order smoothing to dual sourcing and present formulae and bounds that specify the optimal volume fraction ordered from the global source (the strategic offshoring allocation), its corresponding total landed cost, and the value of dual sourcing (over single sourcing).

We show that order smoothing policies shine when dual sourcing faces capacity costs, inflexibility, or longer lead time difference L between both sources. The maximal value of dual-sourcing smoothing then increases significantly and grows at the order of $L^{1/6}$. Moreover, the parameter region for which dual-sourcing smoothing dominates capacitated single sourcing widens as L increases. The square-root formulae that we present are sufficiently simple to provide a starting point for quantitative decision making to optimally trade off cost and responsiveness. Although simple, the formulae still capture the key parameters and thus provide structural insight on the impact of financial, operational, and demand parameters on dual-sourcing decisions.

Finally, a simulation study demonstrates the robustness of our results by relaxing the normality assumption and by comparing the policy with other policies studied in the literature.

2. Related Literature

Our work directly relates to two streams of research: dual-sourcing inventory models and order smoothing policies. The dual-sourcing literature refers to inventory models where replenishment occurs through a regular channel and/or a more expensive, but faster expedited channel. The objective is to minimize the expected sum of procurement, holding, and shortage costs over multiple periods. The dual-sourcing literature is very rich; we focus primarily on discrete review models. Fukuda (1964) shows that when the lead time difference is one period, dual-base-stock policies are optimal. In a dual-base-stock policy, an expedited order is placed to bring the inventory position up to a first (*expedite*) base-stock level, after which a regular order is placed to bring it up to a second and higher (*regular*) base-stock level. Fukuda uses first-order conditions to derive expressions for the base-stock levels. Whittemore and Saunders (1977) extend Fukuda's (1964) model and show that when lead times differ by more than one period, the optimal policy is no longer a dual base stock, but it depends on the entire ordering history and requires multidimensional dynamic programming.

Optimal dual-sourcing policies are, in general, highly complex. Therefore, various heuristic policies are proposed in the literature. Veeraraghavan and Scheller-Wolf (2008) introduce a dual-index dual-base-stock policy that tracks inventory positions over both regular and expedited lead times. Order-up-to levels for both inventory positions are computed using a simulation-based optimization procedure. The authors show that such a dual-index policy is nearly optimal when compared to state-dependent policies. Scheller-Wolf et al. (2006) consider single-index dual-base-stock policies, whose structure is identical to dual-index policies except that only one inventory position is tracked instead of two. The authors computationally show that its performance is comparable to the more complex dual-index policy. The single-index policy also allows closed-form cost expressions under certain distributional assumptions. As such, this is the only other policy from which you can obtain insights by looking at the expressions. Sheopuri et al. (2010) generalize the class of dual-index policies. They show that the "lost sales inventory problem" is a special case of the dual-sourcing problem and leverage this property to suggest new classes of policies with an order-up-to structure that perform equal to, or even slightly better than, the dual-index policy with the same computational requirements. One of their best performing policies is a base-stock policy for placing expedite orders and a vector-base-stock policy for regular orders (this policy was inspired by Zipkin 2008 whose experiments showed that the vector-base-stock policy outperforms the best base-stock policy for the lost sales inventory problem in a single-source setting).

Rosenshine and Obee (1976) consider a standing order policy, which orders at a constant rate from the regular source and uses a base-stock policy for the emergency replenishment. Tagaras and Vlachos (2001) extend this policy to allow emergency replenishment within the regular review period. Allon and Van Mieghem (2010) refer to a standing order policy as a tailored base-surge (TBS) policy, where the regular source supplies the "base" demand and the fast source supplies the remaining "surge" demand using a base-stock policy. It is noteworthy that, by definition, a TBS policy is independent of the slow source's lead time. This allows some mathematical tractability: Allon and Van Mieghem (2010) develop an analytical Brownian model that is asymptotically optimal for high sourcing volumes. Janakiraman et al. (2015) show that the TBS policy is optimal when demand comes from a two-point distribution and when the probability of the smaller (base) demand is sufficiently large. They also show that TBS performance, relative to the optimal policy, improves as the lead time of the regular source increases.

Recently, the dual-sourcing literature is adopted in the context of global sourcing strategies, thus combining the advantages of global low-cost sourcing and local quick response manufacturing. Allon and Van Mieghem (2010) provide guidelines for determining the "strategic allocations," i.e., how the average total sourcing volume should be allocated to the global and local sources when the standing order or TBS policy is used. Wu and Zhang (2014) develop a game-theoretic model where multiple firms in a competitive setting may choose between efficient sourcing and responsive sourcing; a key feature of the game is that depending on the sourcing strategy, a firm may observe different signals about the uncertain market demand. Liu and Nagurney (2013) address the impact of demand and cost uncertainty in a supply chain network with offshoring and quick-response production. Using variational inequality theory, the authors formulate the governing equilibrium conditions of the competing manufacturers, and a simulation study investigates the quantitative impact of demand and cost uncertainty. Recent empirical work by Jain et al. (2014) studies the impact of global sourcing and supplier diversification on inventory investment.

Our dual-sourcing model also relates to the choice of mixed-mode transportation systems where a shipper can use two transport modes together for a single commodity flow. Recently, Combes (2011) studied this problem by minimizing the total landed cost using approximations and simulations. Dual sourcing also relates to the dual sourcing of commodities on the spot market and using forward contracts. Goel and Gutierrez (2011) provide an algorithm to study a dynamic dual-base-stock policy that depends on the spot price state. Our analysis and formulae may be applicable to similar settings with a constant cost differential between the two sources.

In this paper we study the order allocation to the global and local sources by introducing a class of order smoothing policies. Smoothing is a well-known method to reduce variability. The benefit of order smoothing stems from the fact that the order pattern is less variable than the demand. Therefore, the total installed safety capacity is reduced compared to demand-replacing chase policies such as traditional base-stock policies. The introduction of order smoothing in a global dual-sourcing context is not new. The TBS policy can actually be interpreted as an order smoothing policy: the presumption by Allon and Van Mieghem (2010) is that the low-cost source cannot rapidly change volumes because of frictions such as long lead times or an inflexible level production process that is essential to achieve this cost advantage. Indeed, under a TBS policy the global source needs no safety capacity. Moreover, an increase in the standing order reduces the variability of the responsive order stream ("peak-shaving behavior") and thus the required safety capacity of the responsive sources also reduces. Veeraraghavan and Scheller-Wolf (2008) specify a capacitated scenario in

their dual-index policy with capacity limits at each source.

Smoothing is justified when production and holding costs are convex or when there is a cost of changing the level of production (Sobel 1969, 1971). Simon (1952) and Vassian (1955) did pioneering work on the development of smoothing rules using servomechanism (or control) theory and Laplace transform methods. Forrester (1961) and Magee (1958) suggest that production smoothing can be achieved by distributing the transient part of the required production over a number of successive periods. Bertrand (1986) extends this approach to a multiproduct multiphase production system.

Graves has contributed to the smoothing literature over the course of the last 25 years. Graves (1988) reviewed the literature on safety stocks for manufacturing systems and criticized its (lacking) consideration of the role of safety stocks in the presence of inflexibility in manufacturing systems. He characterized the need for additional safety stocks as a result of the smoothing or decoupling function within a manufacturing operation. Of particular interest to our work is the linear production control rule described in his paper, which smoothes the aggregate production and permits an explicit examination of the trade-off between safety stocks and production flexibility. A similar rule is used by Balakrishnan et al. (2004) who set the order quantity equal to a convex combination of the previously observed consumer demands. They make use of these order smoothing rules downstream in the chain to coordinate the entire supply chain. They also characterize the optimal smoothing parameter values and assess the potential cost savings that these order smoothing strategies can yield compared to the uncoordinated case when individual firms separately minimize their costs. In our paper we use the same linear control rules to allocate orders to the global and local source, thereby smoothing production over both sources.

Recent empirical work on production smoothing by Cachon et al. (2007) found, based on industry-level U.S. data, that order smoothing exists in the retail industry and in some manufacturing industries, but not in the wholesale industry. Chen and Lee (2012) show how the prevalence of capacity constraints in these industries (e.g., limited shelf/warehouse space and manufacturing capacity) drives order smoothing. Cantor and Katok (2012) use a series of laboratory experiments to demonstrate the Cachon et al. (2007) findings: when the cost of varying orders is higher than the cost of holding inventory, production and order smoothing is indeed a rational and cost-minimizing behavior. Bray and Mendelson (2012) study firm-level U.S. data and show that firms generally amplify last-minute shocks, yet smooth seasonal variations. Cui et al. (2014) present strong empirical evidence of order smoothing. There is also a large economics literature

preceding the work in operations management, which empirically investigates production smoothing—we refer to Cachon et al. (2007) for an overview and discussion.

3. Single Sourcing, Smoothing, and Capacity

As a first stepping stone toward capacitated dual sourcing, this section sets up the full model and notation by reviewing single-sourcing policies and discussing the impact of order smoothing when the source incurs capacity costs and has a general lead time. Section 3.1. presents the full model and notation and the remainder of §3 focuses on single sourcing. Dual-sourcing policies are developed from §4 on.

3.1. Sourcing Model

Consider a periodic-review inventory system that can be replenished from two sources. (As a first stepping stone, we can source from one of these two sources; the next section will generalize to dual sourcing.) Time is discrete and the sequence of events at each time $t = 0, 1, \ldots, T$ is as follows: First the demand D_t is observed and satisfied; unfilled demand is backlogged. Then, the net inventory I_t, which is the inventory on hand minus backorders, is observed and replenishment orders are placed. The analysis is simplified by letting q_t^i denote the order quantity *received* in period t from source i with $i \in \{l, g\}$ for, respectively, the local and global source. The orders face a delay of, respectively, L_l and L_g periods, which means that the quantity q_t^i that is received at time t must be ordered in period $t - L_i$ (and thus depend only on quantities observed up to $t - L_i$). When $L_i = 0$, the order is received in time to fill next period's demand; this is equivalent to saying that the replenishment is received by the end of the period in which its order is placed. Following Zipkin (2000, p. 404), we say that the risk period or total lead time is $L_i + 1$ periods (this risk period includes the one period review). The essence is that the sources have a lead time difference of $L_g - L_l = L \geq 1$.

Demand is stationary and i.i.d. with $\mathbb{E}(D_t) = \mu$, $\text{Var}(D_t) = \sigma^2$, and distribution Φ. Let ϕ_N and Φ_N denote the standard normal density and distribution and $I_N(z) = \phi_N(z) - z(1 - \Phi_N(z))$ the unit normal loss function.

For the inventory evolution, all that is needed is the demand process and the total quantity received $q_t = q_t^l + q_t^g$. Given this sequence of events, where we first satisfy demand, then observe inventory and finally place and receive orders, we have the following dynamics of the net inventory for $t = 1, \ldots, T$:

$$I_t = I_{t-1} + q_{t-1} - D_t. \tag{1}$$

The dynamics for the first period are $I_0 = I_{-1} - D_0$. The initial inventory I_{-1}, which is a constant, can be

decomposed into $I_{-1} = \sum_{i \in \{l, g\}} (L_i + 1) \mathbb{E} q_t^i + I_s$, where I_s denotes the safety stock. In addition to the net inventory, there is an outstanding pipeline inventory $I_t^p = \sum_{i \in \{l, g\}} \sum_{k=0}^{L_i-1} q_{t+k}^i$.

The total landed cost per period incurred from origin (one of either source) to destination (finished goods warehouse) includes a cost per unit sourced, but also the capacity cost at source i, and the inventory (holding and shortage) costs. The total landed cost is most clear in a centralized system, where the global network with multiple supply points is wholly owned and part of one organization. But also in a decentralized system, the supply capacity cost typically remains an important component of the total landed cost as the supplier would include a charge for it.

The sourcing cost equals c^i per unit sourced from source i, with the faster source being more expensive ($c^l > c^g$), which reflects the standard variable cost component in the total cost for units coming from location i. This component includes direct material as well as any labor cost that can directly be attributed to the order size. We assume sourcing costs are incurred at receipt (although it does not make a difference in our undiscounted model).

The capacity cost at location i reflects the standard fixed cost component in the total cost for units coming from location i per period. This includes capital, labor, and other overhead cost rates that remain unchanged over the time horizon $[0, T]$. The installed capacity K^i at source i incurs a cost per period of $C(K^i) = k^i K^i$, with k^i the constant, marginal cost rate to add one unit of capacity at source i. In the natural regime, local capacity is more expensive than global capacity (although the model works without those conditions). In practice, companies often have some volume flexibility to exceed the installed capacity, which we model as follows: Orders q_t^i can be produced up to the capacity K^i that is installed at time 0; any excess order $(q^i - K^i)^+$ requires overtime capacity at extra cost o^i per unit. Overtime reflects excess cost not covered in regular capacity costs nor standard direct labor. (Obviously, $k^i < o^i$, otherwise it would never be optimal to invest in capacity.) The ratio $o^i/k^i > 1$ measures the *rigidity* of the capacity constraint; equivalently, $k^i/o^i < 1$ can be interpreted as the degree of flexibility in quantity deviations beyond a source's installed capacity. The limit $o^i/k^i \to \infty$ represents the standard theoretical model of capacity as a hard constraint.

Finally, each period, inventory incurs a holding cost h per unit on hand or a backlog cost b per unit short. Given that we consider a wholly owned global network, we also charge holding cost to the pipeline inventory as it represents capital tied up in the network (regardless of

whether the supplier or the buyer has the inventory on his accounts). The average cost over horizon T becomes

$$C_{I_{-1}}^T = \frac{1}{T} \sum_{i \in \{l, g\}} \sum_{t=0}^{T} \left[c^i q_t^i + k^i K^i + o^i (q_t^i - K^i)^+ + h(I_t)^+ \right.$$
$$\left. + b(I_t)^- + h \sum_{k=0}^{L_i-1} q_{t+k}^i \right]. \quad (2)$$

We will focus on minimizing the average cost $C = \lim_{T \to \infty} C_{I_{-1}}^T$. Ideally, one would like to characterize the initial inventory I_{-1} and an admissible sourcing policy (which defines the replenishments $q_{t+L_l}^l$ and $q_{t+L_g}^g$ as a function of I_{t-k} and D_{t-k} for $t, k = 0, 1, \ldots$) that minimizes C. In the remainder of this section we review some principal single-sourcing policies and their average cost C that will serve as a stepping stone toward dual-sourcing policies.

3.2. Standard Single-Sourcing Base-Stock Policy

The standard single-sourcing base-stock policy is optimal in minimizing inventory related costs only (Zipkin 2000). Although this policy is not optimal to minimize the total cost C (which additionally includes sourcing and capacity costs), it is a well known and useful benchmark against other sourcing policies. Single sourcing from either the local or global source under a standard base-stock policy is a demand-replacement policy, where $q_t^i = D_{t-L_i}$, and the associated inventory process is $I_t = I_s + (L_i + 1)\mu - \sum_{i=t-L_i}^t D_i$. With normal demand, both the order and net inventory process are also normally distributed and the optimal capacity and safety stock levels then follow from a standard newsvendor solution:

$$K^{i*} = \mu + z_K^i \sigma, \quad \text{where } \Phi_N(z_K^i) = \frac{o^i - k^i}{o^i};$$

$$I_s^* = \sqrt{(L_i + 1)} z_I \sigma, \quad \text{where } \Phi_N(z_I) = \frac{b}{b+h}.$$

There is also pipeline inventory whose average cost follows from Little's law ($hL_i\mu$). The total average cost of this single-sourcing standard base-stock policy (which we denote by s) is

$$C^s = c^i \mu + k^i \mu + \kappa^i \sigma + \sqrt{(L_i + 1)} \kappa_I \sigma + hL_i\mu,$$

with κ^i and κ_I, respectively, the financial capacity and inventory cost parameters:

$$\kappa^i = k^i z_K^i + o^i I_N(z_K^i) = k^i \left[z_K^i + \frac{o^i}{k^i} I_N(z_K^i) \right], \quad (3)$$

$$\kappa_I = hz_I + (h+b)I_N(z_I) = h \left[z_I + \frac{(h+b)}{h} I_N(z_I) \right]. \quad (4)$$

The capacity cost parameter κ^i increases linearly in the source's unit capacity cost k^i and concavely in

the capacity rigidity ratio o^i/k^i. The inventory cost parameter κ_I increases in the holding cost h and the critical fractile z_I (or, equivalently, the ratio b/h).

Compared to single local sourcing, global single-sourcing benefits from lower sourcing (and often capacity unit) costs, while the total order variability remains the same (the installed capacity will depend on the unit capacity cost and the rigidity of the source). In contrast, inventory costs increase because of the longer lead time. To compare the two standard single-sourcing policies, we consider the scaled cost

$$\hat{C} = \frac{C - (c^l + k^l + hL_l)\mu}{\kappa_I \sigma}, \qquad (5)$$

and introduce the following dimensionless notation that will also be useful for dual sourcing:

$$\theta_l = \frac{\kappa^l}{\kappa_I}, \quad \theta_g = \frac{\kappa^g}{\kappa_I} \quad \text{and}$$
$$\theta_c = \frac{c^l - c^g + k^l - k^g - hL}{\kappa_I} \frac{\mu}{\sigma}. \qquad (6)$$

These three dimensionless parameters capture the "degrees of freedom" in the model and also the main trade-offs in devising a sourcing policy: θ_l and θ_g contain the ratio of, respectively, the local and global source's capacity cost and the rigidity of the source versus the holding cost and the service level (see Equations (3) and (4)); while θ_c captures the unit cost advantage of the global source (in sourcing and unit capacity vis-à-vis the increased pipeline inventory cost), compared to the unit holding cost and the inventory service level (both through κ_I) and the volatility in demand (as measured by its coefficient of variation, $CV = \sigma/\mu$). To disentangle the impact of a change in L or in cost differences between the sources, from here on we will have θ_c be the parameter capturing cost differences (meaning, if the lead time difference increases, the absolute cost advantage must also increase to keep the cost difference θ_c the same). It directly follows that the scaled cost for local single sourcing (*ls*) and global single sourcing (*gs*) using the base-stock policy,

$$\hat{C}^{ls} = \theta_l + \sqrt{L_l + 1}, \quad \hat{C}^{gs} = -\theta_c + \theta_g + \sqrt{L_g + 1}. \qquad (7)$$

This directly leads to the following simple guideline to balance global and local sourcing when the standard base-stock policy is in use:

PROPOSITION 1. *With normal demand, and when using a standard base-stock replenishment policy, global single sourcing dominates local single sourcing if and only if* $\theta_c + \theta_l - \theta_g > \sqrt{L_g + 1} - \sqrt{L_l + 1}.$

3.3. Order Smoothing Policies with Single Sourcing

The high cost of installed capacity has led to the development of ordering policies that dampen the variability in orders. One effective order policy uses exponential smoothing with smoothing level $\alpha \in [0, 1]$. For an easy introduction to the order smoothing policy, first consider the case when the lead time $L_i = 0$. The order policy $q_t = \alpha q_{t-1} + (1 - \alpha)D_t$, with $0 \le \alpha \le 1$ and $q_{-1} = \mu$ as an initial condition covers a set of policies that range between a chase and level strategy: If $\alpha = 0$, then $q_t = D_t$ is a standard base-stock or demand-replacement (*chase*) policy, and if $\alpha = 1$, then $q_t = q_{t-1} = q_{-1} = \mu$ is a level strategy that orders the average demand each period. Any in-between smoothing level is a compromise between both and smoothes the orders.

Iterating the recursion shows that the total order quantity received in period t is a linear combination of the observed demand process:

$$q_t = \sum_{k=0}^{t}(1 - \alpha)\alpha^k D_{t-k} + \alpha^{t+1}\mu. \qquad (8)$$

This order policy finds its origin in linear control theory (Forrester 1961, Magee 1958). It is in essence a generalized base-stock policy, where the inventory deficit is not recovered in one period, but instead spread out over time, with $1/(1 - \alpha)$ the *adjustment time* to recover the deficit. The order smoothing policy also relates to the linear inflation rule that Zipkin (2000, p. 393) describes to deal with defects or yield losses. Graves (1988) used this control rule to smooth the aggregate production and it is also proposed by Balakrishnan et al. (2004) to reduce the variability in orders in a single-source setting to reduce total supply chain costs (for $L_i = 0$).

The smoothing policy (8) can be extended to general lead times $L_i \ge 0$. For $t \ge L_i$,

$$q_t = \sum_{k=L_i}^{t}(1 - \alpha)\alpha^{k-L_i} D_{t-k} + \alpha^{t-L_i+1}\mu$$
$$= \sum_{k=0}^{t-L_i}(1 - \alpha)\alpha^k D_{t-L_i-k} + \alpha^{t-L_i+1}\mu. \qquad (9)$$

The linear control makes the policy analytically tractable. Taking expectations and variances of the orders yields

$$\mathbb{E}q_t = \mu, \quad \text{Var}(q_t) = [1 - \alpha^{2(t+1)}]\frac{1 - \alpha}{1 + \alpha}\sigma^2 \le \sigma^2.$$

The total order variance is convex decreasing in the smoothing level α and vanishes under the level strategy $\alpha = 1$. As the order streams have smaller variability than the demand (which is referred to as smoothing), the optimal installed capacity reduces to

$$K^{i*} = \mu + z_K^i \sigma \sqrt{\frac{1 - \alpha}{1 + \alpha}},$$

and its corresponding capacity cost reduces to $C_K(\alpha) = k^i\mu + \kappa^i\sigma\sqrt{(1-\alpha)/(1+\alpha)}$. The reduction in capacity costs compared to the traditional base-stock policies represents the marginal benefit of order smoothing:

$$MB(\alpha) = -C_K'(\alpha) = \frac{1}{(1+\alpha)\sqrt{1-\alpha^2}}\kappa^i\sigma.$$

The linear order structure also yields analytic tractability of the inventory process: the net inventory process is a linear combination of the demand process, so with normal demand, the net inventory process I_t is also normally distributed. (All proofs are relegated to the technical companion, available as supplemental material at http://dx.doi.org/10.1287/mnsc.2014.1992.)

PROPOSITION 2. *With normal demand, the net inventory process when single-sourcing smoothing with lead time L_i is a linear combination of the demand process:*

$$I_t = \begin{cases} I_{-1} - \displaystyle\sum_{k=0}^{t-L_i}\alpha^k D_{t-L_i-k} - \sum_{k=0}^{L_i-1}D_{t-k} & \text{if } \alpha < 1, \\ I_{-1} + t\mu - \displaystyle\sum_{i=0}^{t}D_i & \text{if } \alpha = 1, \end{cases} \tag{10}$$

$$\mathbb{E}\,I_t = I_s$$

$$\lim_{t\to\infty}\text{Var}(I_t) = \begin{cases} \dfrac{1}{1-\alpha^2}\sigma^2 + L_i\sigma^2 \geq \sigma^2 & \text{if } \alpha < 1 \\ \infty & \text{if } \alpha = 1. \end{cases}$$

The variance of the inventory process increases in the smoothing level α and grows without bound under level ordering with smoothing ($\alpha = 1$). (The source then supplies a constant quantity μ and demand remains random with same mean μ. The resulting net inventory process behaves as a random walk with null drift and is unstable.) This increased inventory variance comes at a cost of requiring more safety inventory. The optimal safety stock level follows from a newsvendor solution:[2]

PROPOSITION 3. *With normal demand and $\alpha < 1$, the long-run optimal safety stock under order smoothing with general lead time L_i is*

$$I_s^* = z_I\sigma\sqrt{L_i + \frac{1}{1-\alpha^2}}.$$

The associated inventory holding and backlogging cost rate $C_I = \kappa_I\sigma\sqrt{L_i + 1/(1-\alpha^2)}$ is convex increasing in α, representing the marginal cost of smoothing:

$$MC(\alpha) = C_I'(\alpha) = \kappa_I\sigma\frac{\alpha}{(1-\alpha^2)^{3/2}(1+L_i-L_i\alpha^2)^{1/2}}.$$

[2] This is not a standard newsvendor problem because the decision variable I_s is the mean of the distribution, but it can be reduced to a newsvendor model.

The total scaled cost of single-sourcing smoothing (denoted by ss),

$$\hat{C}^{ss}(\alpha) = \theta_i\sqrt{\frac{1-\alpha}{1+\alpha}} + \sqrt{L_i + \frac{1}{1-\alpha^2}},$$

is continuous in the interval $[0, 1)$ with $\hat{C}^{ss}(0) = \theta_i + \sqrt{1+L_i}$ and $\hat{C}^{ss}(1) = +\infty$, and can be convex-concave-convex. There is, however, a unique minimum that satisfies the optimality condition $MB(\alpha^*) = MC(\alpha^*)$.

PROPOSITION 4. *With normal demand, and for any $0 \leq L_i$ and $0 < \theta_i$, there is a unique optimal smoothing level for order smoothing, satisfying the fixed point equation*

$$\alpha^* = \frac{\theta_i}{\theta_i + 1/\sqrt{1+L_i(1-\alpha^{*2})}}, \tag{11}$$

which is bounded by

$$\alpha_0 = \frac{\theta_i}{\theta_i + 1} \leq \alpha^* \leq \alpha_1 = \frac{\theta_i}{\theta_i + (1+L_i)^{-1/2}}. \tag{12}$$

Order smoothing sourcing outperforms the standard single-sourcing base-stock policy for any L_i if $\theta_i > 0$. The value of smoothing increases in θ_i if lead time $L_i = 0$.

If $L_i = 0$, Equation (11) reduces to $\alpha^* = \theta_i/(1+\theta_i)$ and its scaled cost $\hat{C}^{ss}(\alpha^*) = \sqrt{1+2\theta_i}$. The relative cost benefit of smoothing compared to the standard base stock can be quantified as

$$0 \leq \frac{\hat{C}^s - \hat{C}^{ss}}{\hat{C}^s} = 1 - \frac{\sqrt{1+2\theta_i}}{1+\theta_i} \leq 1 - \frac{1}{\sqrt{1+\theta_i}},$$

which increases in θ_i toward a maximum of 100%. The left panel of Figure 1 shows that the relative cost improvement of order smoothing compared to using the standard base-stock policy holds for lead times $L_i \geq 0$. When capacity is costly, the value of order smoothing can be substantial.[3]

Based on (12), we can derive the following bounds on the optimal cost (the right panel of Figure 1 shows the accuracy of $\hat{C}^{ss}(\alpha_1)$ compared to the true optimal costs $\hat{C}^{ss}(\alpha^*)$):

$$\hat{C}^{ss}(\alpha_0) = \theta_i\sqrt{\frac{1}{1+2\theta_i}} + \sqrt{L_i + \frac{(1+\theta_i)^2}{1+2\theta_i}},$$

$$\hat{C}^{ss}(\alpha_1) = \theta_i\sqrt{\frac{1}{1+2\theta_i\sqrt{1+L_i}}} \tag{13}$$

$$+ \sqrt{L_i + \frac{(1+\theta_i\sqrt{1+L_i})^2}{1+2\theta_i\sqrt{1+L_i}}}.$$

[3] Whereas Figure 1 compares order smoothing with the standard base stock using the same source, one can also similarly compare order smoothing from one source with the standard base-stock policy from another source using Equations (7) and (13). As such, it may for instance turn out to be beneficial for a firm that faces capacity supply costs to smooth orders from the local source, rather than source globally at low cost.

Figure 1 (Color online) (Left Panel) The Value of Order Smoothing Compared to the Standard Base-Stock Policy Increases as the Capacity Cost θ_i Increases for Any Lead Time L_i; (Right Panel) The Relative Cost Increase When Bound α_1 Is Used Instead of the Optimal α^* Is Small
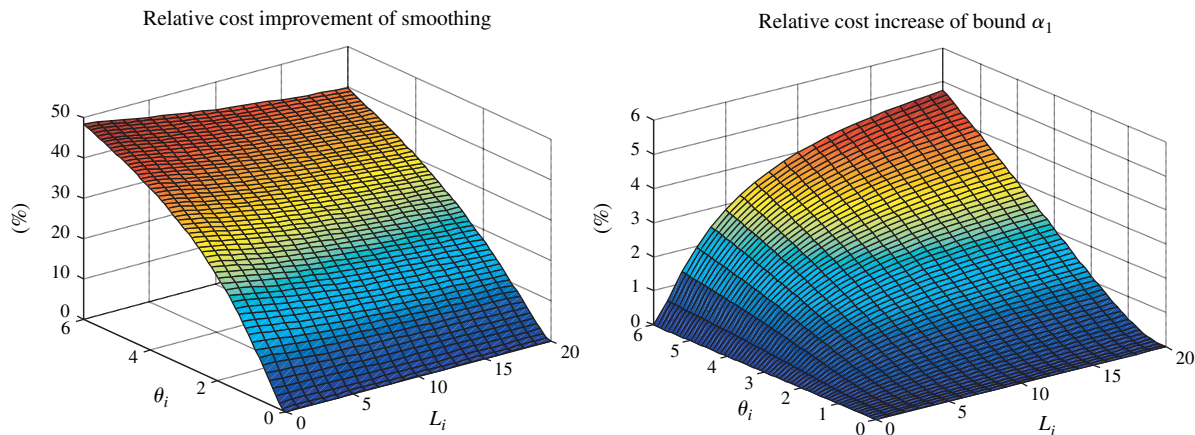

The right panel of Figure 1 shows that the cost penalty of using bound α_1 is modest.

3.4. Optimal Capacitated Single Sourcing

Although the order smoothing policy does not guarantee optimality in minimizing the total landed cost C, its linear structure makes it attractive because of its analytic tractability. This allows us to derive closed-form solutions and comes with the benefit of gaining insight into the cost drivers of the sourcing policy. For $L_i = 0$, we can numerically show that order smoothing closely tracks the optimal policy. The presence of capacity costs as modeled above is actually equivalent to piecewise linear convex order costs. The associated optimal policy for $L_i = 0$ is then characterized by a dual-base-stock policy. (For $L_i > 0$ the optimal policy uses the full history of orders and is therefore more complex.) When the inventory position exceeds the higher base-stock level, no order is placed. When the inventory position is below the higher base stock, we first use up the regular capacity K. If this raises the inventory position to above the lower base stock, we do not use overtime; otherwise, we use overtime to raise the inventory position to the lower base-stock level. In other words, there is a region of "inaction" where we order maximal K but less than the demand. (The marginal overtime cost exceeds the marginal benefit of raising inventory in terms of reducing backlogging relative to holding. In other words, it is better to wait and replenish in the future at regular cost versus now at overtime cost.) This is not a demand replacing policy and there are no simple solutions for the optimal base-stock levels and capacity level K.

Given that the optimal capacitated single-sourcing (SCC) policy cannot be optimized analytically, we numerically optimized its simulated cost. Figure 2 shows the total scaled cost of local single sourcing when $L_i = 0$ using the standard base-stock policy, order smoothing, and the optimal dual base stock

Figure 2 (Color online) Single Sourcing Using the Standard Base-Stock Policy, Order Smoothing, and the Optimal Capacitated Dual-Base-Stock Policy (with 95% Confidence Intervals) When $L_i = 0$
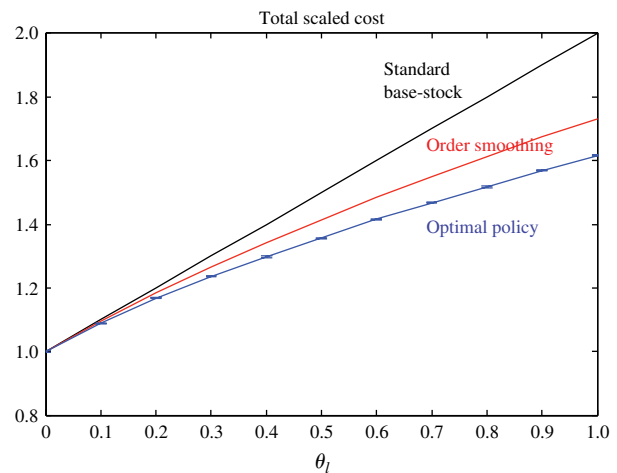
described previously. Although order smoothing does not guarantee optimality, it proves its value in the presence of capacity costs and closely tracks the optimal policy.

4. Dual Sourcing and Order Smoothing

4.1. Dual-Sourcing Model

Consider the dual-sourcing setting where units can be ordered from a local source and/or from a global source. As before, let $i \in \{l, g\}$ refer to the local or global source; in particular, L_i denotes the lead time of source i, where $0 \le L_l < L_g$. With two sources, we decompose the total order quantity received in period t as $q_t = q_t^l + q_t^g$, where q_t^i denotes the order quantity *received* in period t from source i. With lead times $L_l < L_g$, the received quantity q_t^i is based on information older than L_i periods. Correspondingly, we define a

DSS policy by splitting up the total smoothed order stream q_t as follows:

$$q_t^g = \sum_{k=L_g}^{t} (1-\alpha)\alpha^{k-L_l}D_{t-k} = \sum_{k=L}^{t-L_l} (1-\alpha)\alpha^k D_{t-L_l-k}, \quad (14)$$

$$q_t^l = \sum_{k=L_l}^{L_g-1} (1-\alpha)\alpha^{k-L_l}D_{t-k} = \sum_{k=0}^{L-1} (1-\alpha)\alpha^k D_{t-L_l-k}, \quad (15)$$

where $L = L_g - L_l > 0$ denotes the lead time difference between both sources. Observe that for any lead time difference $L > 1$ between the global and local source, local orders placed within $L-1$ units of time after a global order will be received prior to the global order's receipt. Our model thus includes order crossing. Given that our orders are a linear combination of the demands only, they do not depend on any state variables or interaction between local and global inventory position. Therefore, order crossing does not pose any problems or complications to our analysis.

For $t \to \infty$, the variability in orders is independent of L_l (and the same as when $L_l = 0$):

$$\mathbb{E}q_t^g = \alpha^L\mu, \quad \text{Var}(q_t^g) = \frac{1-\alpha}{1+\alpha}\alpha^{2L}\sigma^2,$$

$$\mathbb{E}q_t^l = (1-\alpha^L)\mu, \quad \text{Var}(q_t^l) = \frac{1-\alpha}{1+\alpha}(1-\alpha^{2L})\sigma^2.$$

Notice that (i) the strategic allocation a (the fraction of average total orders allocated to the global source) is α^L, which is *different* and smaller than the smoothing level α; (ii) the variance of each order stream is less than the demand variance (consistent with order smoothing). The total order variance is convex and decreasing in the smoothing level α and vanishes under the level strategy $\alpha = 1$ (i.e., single sourcing with constant order $q_t = q_t^g = \mu$ from the global source). The pipeline inventory holding cost is $hL_g\alpha^L\mu + hL_l(1-\alpha^L)\mu = hL_l\mu + hL\alpha^L\mu$. The inventory dynamics under this dual-sourcing smoothing policy equal those under single-sourcing smoothing with lead time L_l (because total receipts $q_t = q_t^l + q_t^g$ are equal). Thus, the inventory is independent of the lead time difference L but it does depend on the local lead time L_l:

$$\text{Var}(I_t) = \frac{1}{1-\alpha^2}\sigma^2 + L_l\sigma^2.$$

The total absolute and scaled (as defined by Equation (5)) costs thus become

$$C = (c^g + k^g)\alpha^L\mu + (c^l + k^l)(1-\alpha^L)\mu + \kappa^g\sigma\alpha^L\sqrt{\frac{1-\alpha}{1+\alpha}}$$

$$+ \kappa^l\sigma\sqrt{\frac{1-\alpha}{1+\alpha}(1-\alpha^{2L})} + \kappa_I\sigma\sqrt{L_l + \frac{1}{1-\alpha^2}}$$

$$+ hL_l\mu + hL\alpha^L\mu,$$

$$\hat{C} = -\theta_c\alpha^L + \theta_g\alpha^L\sqrt{\frac{1-\alpha}{1+\alpha}} + \theta_l\sqrt{\frac{1-\alpha}{1+\alpha}(1-\alpha^{2L})}$$

$$+ \sqrt{L_l + \frac{1}{1-\alpha^2}}.$$

The trade-offs are clearly shown in Figure 3: A higher smoothing level α implies a larger reliance on the global source and reduces sourcing and local capacity costs but increases inventory costs. (Global capacity costs initially increase reflecting a higher needed safety capacity as the global allocation increases; yet they decrease to zero as $\alpha \to 1$, which corresponds to a standing constant order $q_t^g = \mu$ and no safety capacity is needed.) This directly raises the question whether there is an optimal trade-off and, if so, how to characterize it. To that end, let $\hat{C}^* = \hat{C}(\alpha^*) \le \hat{C}(0) = \theta_l + \sqrt{L_l+1}$ denote the minimal cost (which exists because $\hat{C}(\alpha)$ is continuous in the interval $[0,1)$ with $\hat{C}(1) = \infty$) and $\alpha^* \in [0,1)$ an optimal smoothing level. Similarly, let $a^* = (\alpha^*)^L$ denote the corresponding strategic allocation.

### 4.2. Impact of Sourcing Cost Difference θ_c and Lead Times

To highlight the impact of the cost and lead time differences between both sources, we first consider the uncapacitated system, where $\theta_l = \theta_g = 0$. Recall that, to disentangle the impact of both, θ_c only captures the cost difference. As Figure 4 illustrates, the total cost and thus optimal trade-off depend jointly on the smoothing level α and lead time difference L in nonobvious ways: First, the cost increases as the lead time difference L increases and the boundary solution

Figure 3 (Color online) More Smoothing Implies a Larger Reliance on the Global Source, Which Decreases Sourcing and Local Capacity Costs But Increases Inventory Costs; There Is a Unique Optimal Smoothing Level α^* or Offshoring Allocation α^{*L}
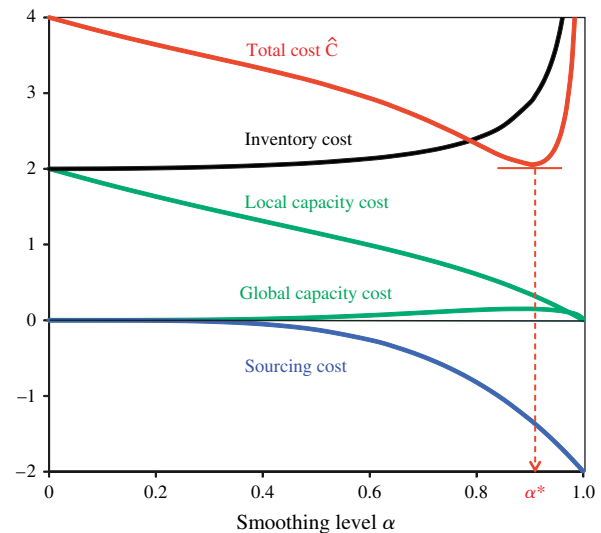
Note. Parameters: $\theta_c = 2$, $\theta_l = 2$, $\theta_g = 1$, $L_l = 3$, $L_g = 7$.

Figure 4 (Color online) (Left Panel) Total Cost and Corresponding Optimal Smoothing Level α^* as a Function of L; (Right Panel) The Optimal Strategic Allocation α^L Decreases as L Increases
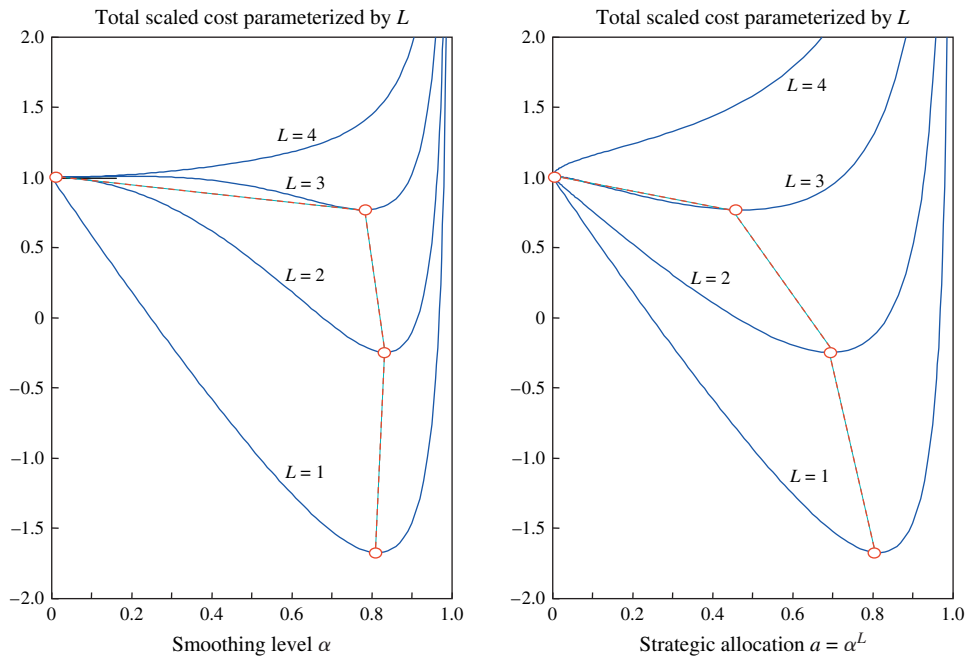
Notes. Both shown for $L_l = 0$.

$\alpha^* = 0$ is optimal above a certain threshold value of L (which is $L = 4$ in Figure 4). Second, while the cost is convex for $L = 1$, it is concave-convex for $L = 2$, and convex-concave-convex for $L > 2$, and can have a local maximum and minimum (as illustrated for $L = 3$). Third, the optimal smoothing level is not monotone in L, but the optimal strategic allocation is. We will generalize these observations in this section.

Optimal dual-sourcing smoothing requires that $\alpha^* \in (0, 1)$ where α^* satisfies the first-order condition (FOC)

$$MB(\alpha^*) = MC(\alpha^*)$$
$$\Leftrightarrow L\theta_c(\alpha^*)^{L-1} = \frac{\alpha^*}{(1 - \alpha^{*2})^{3/2}(1 + L_l(1 - \alpha^{*2}))^{1/2}}. \quad (16)$$

Manipulating the FOC shows the impact of three essential parameters:

PROPOSITION 5. *Under our model assumptions, as the local lead time L_l or the lead time difference L increases, the optimal cost $\hat{C}^*$ increases. The optimal offshoring allocation a^* increases as L_l increases, but decreases as L increases. As the standardized sourcing cost advantage θ_c increases, the optimal cost $\hat{C}^*$ decreases at rate $-1 < (d/d\theta_c)\hat{C}(\alpha^*) = -\alpha^{*L} < 0$ and the optimal smoothing level α^*, and thus offshoring allocation a^*, increases.*

By investigating how model parameters impact θ_c and L, the proposition confirms the intuitive impact of these parameters on the optimal offshoring allocation α^{*L} and cost.

1. As expected, if the global sourcing unit cost advantage $c^l - c^g$ increases, then θ_c and thus α^{*L} increase while cost decreases.

2. If the demand volatility, measured by its coefficient of variation $CV = \sigma/\mu$, increases, then θ_c and thus α^{*L} decrease while the cost increases proportionally.

3. If h increases, then κ_I increases[4] so that θ_c and thus α^{*L} decrease and cost increases.

These comparative statics give guidance on how to tailor and adapt the sourcing strategy to the financial, customer service, and demand characteristics. The tractability of our model allows additional insight and quantification: substituting $x = 1 - \alpha^2$ in the first-order condition (16) yields

$$f(x) = x^3(1 + L_l x)(1 - x)^{L-2} = (L\theta_c)^{-2}. \quad (17)$$

Before we specify the optimal smoothing level and offshoring allocation in exact analytic terms, notice that $x^* = 1 - \alpha^{*2}$ is easily found graphically as the intersection of the horizontal line at $(\theta_c L)^{-2}$ with the upward part of f in Figure 5. As f has a maximum for $L \geq 3$, the existence of this intersection (and thus an interior solution to the FOC) requires that $(\theta_c L)^{-2}$ does not exceed the maximum of f, which gives a lower bound $\underline{\theta}_{L, L_l}$ on θ_c. Also, the maximizer of f provides an upper bound on x^*, and thus lower

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[4] $d\kappa_I/dh = (d/dh)[hz_I + (h + b)I_N(z_I)] = \phi_N(z_I) + \Phi_N(z_I)z_I > 0$ for all z_I as optimal functions of h.

Figure 5 (Color online) The Optimal $x^* = 1 - \alpha^{*2}$ Is Easily Found Graphically as a $f^{-1}((\theta_c L)^{-2})$, Shown Here for $L_l = 1$
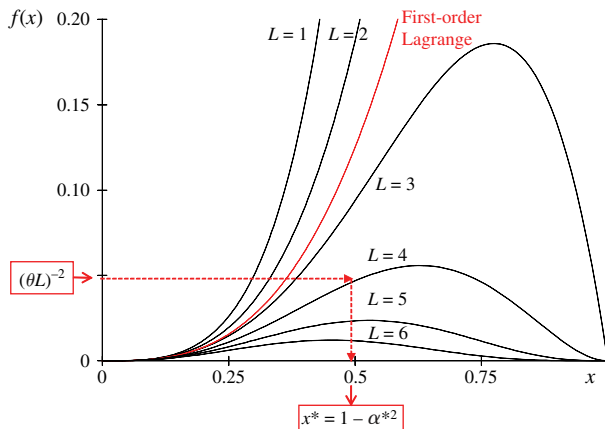
bound $\underline{\alpha}_{L, L_l}$ on α^*. (Proposition 17 in the technical companion provides expressions for these bounds.) Whereas $\theta_c \geq \underline{\theta}_{L, L_l}$ guarantees that the FOC have an interior solution $\alpha^* > 0$, for dual-sourcing smoothing to be optimal, this local cost minimum must have a cost below the single-sourcing cost $\hat{C}(0) = \sqrt{L_l + 1}$. This requires a more stringent condition $\theta_c \geq \underline{\theta}^*_{L, L_l} = \inf\{\theta_c : \hat{C}(\alpha^*; \theta_c, L) < \sqrt{L_l + 1}\} \geq \underline{\theta}_L$.

For $L_l = 0$ and $L = 2$, Equation (17) has a simple, unique solution if $(2\theta_c)^{-2} \leq 1$: $x^* = (\theta_c L)^{-2/3}$ so that

$$\alpha^* = \sqrt{1 - (2\theta_c)^{-2/3}} \quad \text{and} \quad \hat{C}(\alpha^*) = \tfrac{3}{2}(2\theta_c)^{1/3} - \theta_c. \quad (18)$$

Other special cases include the following: If $L_l = 0$ and $L = 1$, this is a cubic equation with a unique root in $[0, 1)$ for any $\theta_c \geq 0$ that can be solved exactly using the Cardano-Tartaglia formula. For $L \geq 3$, the cost function is convex-concave-convex and the FOC has two solutions in $[0, 1]$, representing two local extrema x^*. The local minimum corresponds to the smallest root x^* of a polynomial of degree $L + 2$ if $L_l > 0$ and of degree $L + 1$ otherwise. If the degree is 4, the root of the quartic can be solved using Ferrari's formula. (The Cardano and Ferrari formulas are relegated to the technical companion.) If the FOC is a polynomial of degree greater than 4, Galois (1846) showed that there exists no general "simple" formula (i.e., using only a finite number of the usual algebraic operations and radicals) to specify its root x^*. Next we apply the inversion theorem of Lagrange (1770), which provides a Taylor series, expanded around x_0, for the inverse of an analytic function. We set $x_0 = 0$ and use this powerful technique to specify the root $f^{-1}((\theta_c L)^{-2})$ as an infinite series. (The appendix shows the derivation.) We further show that its first-order term is a bound that is asymptotically correct as $L\theta_c \to \infty$.

Proposition 6. *Under normal demand and without capacity costs* ($\theta_g = \theta_l = 0$), *the optimal smoothing level* $\alpha^* = 0$ *if* $\theta_c \leq \underline{\theta}^*_{L, L_l}$, *otherwise*[5]

$$\alpha^* = \begin{cases} \sqrt{1 - \sum_{n=1}^\infty \frac{1}{n} \binom{\frac{-n}{3}}{n-1} L_l^{n-1} (2\theta_c)^{-\frac{2n}{3}}} & \text{if } L = 2, \\[2ex] \sqrt{1 - \sum_{n=1}^\infty \frac{1}{n} \sum_{i=0}^{n-1} (-1)^{n-1-i} \binom{\frac{-n}{3}}{i} \binom{\frac{-n(L-2)}{3}}{n-1-i} L_l^i (L\theta_c)^{-\frac{2n}{3}}} & \text{if } L \neq 2. \end{cases} \quad (19)$$

The Lagrange series (19) shows that the lead time difference L has a first-order impact, whereas L_l only has a second-order impact: $\alpha^* \simeq [1 - (L\theta_c)^{-2/3} + ((L - L_l - 2)/3)(L\theta_c)^{-4/3}]^{1/2}$. In addition, its first-order term provides a simple approximation for α^*, which is also a bound for $L_l = 0$ or 1 as illustrated in 5. (For $L_l \geq 2$, the first-order approximation of f^{-1} no longer cleanly partitions the exact f^{-1} for different values of L.)

Proposition 7. *With normal demand and if* $L\theta_c > 1$, *then the optimal smoothing level* α^* *has a square-root approximation* α_0,

$$\alpha_0 = \sqrt{1 - (L\theta_c)^{-2/3}},$$

that is asymptotically correct: $\alpha_0 \to \alpha^*$ *as* $L\theta_c \to \infty$. *If* $L_l = 0$ *or 1, the approximation is a lower bound* ($\alpha_0 \leq \alpha^*$) *if* $L \leq 2$ *and is an upper bound* ($\alpha_0 \geq \alpha^*$) *if* $L \geq 3$. *It is exact if* $L = 2$ *and* $L_l = 0$. *Similarly, the optimal cost* $\hat{C}(\alpha^*)$ *has an upper bound* $\hat{C}(\alpha_0)$ *that is exact for* $L = 2$ *and* $L_l = 0$ *and asymptotically correct as* $L\theta_c \to \infty$, *where*[6]

$$\hat{C}(\alpha_0) = -\theta_c(1 - (L\theta_c)^{-2/3})^{L/2} + (L_l + (L\theta_c)^{2/3})^{1/2}$$

$$= \frac{3}{2}(L\theta_c)^{1/3} - \theta_c - O((L - 2 - 4L_l)(L\theta_c)^{-1/3}).$$

Figure 6 compares the square-root allocation $a_0 = \alpha_0^L$ with the optimal allocation $a^* = \alpha^{*L}$ for $L = 1$ (left panel) and $L = 3$ (right panel), both for $L_l = 0$. In addition to providing a fine approximation of the allocation, the square-root formula's cost penalty $\hat{C}(\alpha_0) - \hat{C}(\alpha^*)$ is very low: for $L = 3$, its maximal cost penalty is 0.027 (at $\theta_c = \underline{\theta}^*_3 = 1.15$ when $\hat{C}(\alpha^*) = 1$) and diminishes quickly as θ_c increases (it is 0.011 at $\theta_c = 2$, and 0.003 at $\theta_c = 5$). For $L = 1$, the cost penalty starts higher at 0.35 (at $\theta_c = 1$ when $\hat{C}(\alpha_0) = 1$) but again diminishes quickly to 0.04 at $\theta_c = 2$ and 0.01 at $\theta_c = 5$. For $L = 1$, an expansion of the FOC for θ_c near 0 gives a better lower bound $\alpha_1 = \theta_c/\sqrt{1 + 3\theta_c^2}$.

The square-root approximation and bound also highlight the first-order impact of the standardized sourcing

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[5] The generalized binomial coefficient is defined for $x \in \mathbb{R}$: $\binom{x}{0} = 1$ and $\binom{x}{k} = (x(x-1)\cdots(x-k+1))/k!$ for $k = 1, 2, \ldots$.

[6] The Landau notations specify functions $o(f)$ that are of smaller order than f, and $O(g)$, which is of similar order as g. Formally, $\lim_{x\to\infty} o(f)(x)/f(x) = 0$, and $\lim_{x\to\infty} O(g)(x)/g(x)$ is a finite constant.

Figure 6 **(Color online) The Optimal Allocation $a^* = \alpha^{*L}$ Compared to the Square-Root Allocation $a_0 = \alpha_0^L$ for $L = 1$ (Left Panel) and $L = 3$ (Right Panel); Both for $L_I = 0$**
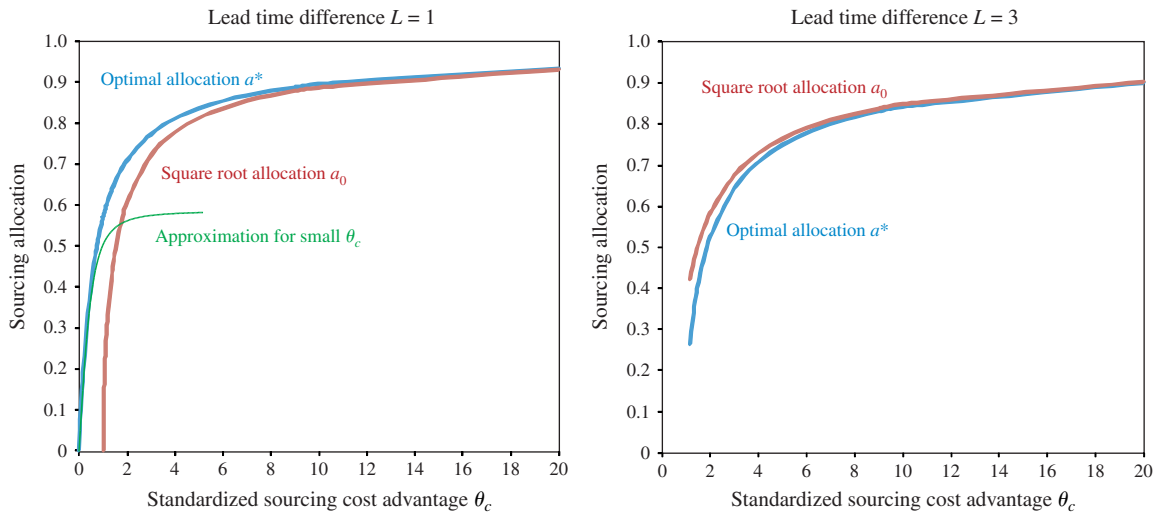
Figure 7 **(Color online) The Impact of Lead Time Difference L and Coefficient of Variation CV on the Global Allocation $a_0 = \alpha_0^L$**
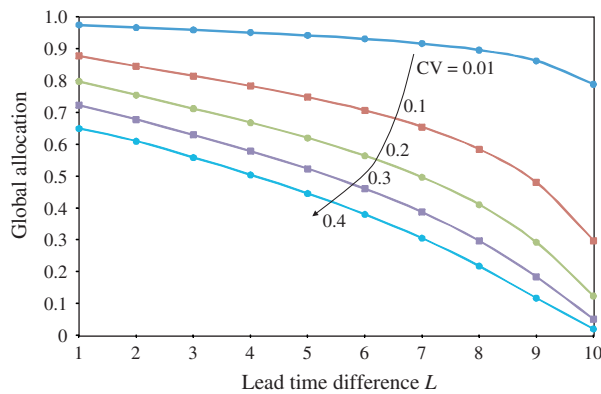
Notes. Parameters: $c^l - c^g = 1$, $h = 0.09$, $\kappa_I = 1$.

cost advantage θ_c and the lead time difference L on the global allocation:

global (offshoring) allocation a_0

$$= \alpha_0^L = \sqrt{(1 - (L\theta_c)^{-2/3})^L} = \left(1 - \left(\frac{c^l - c^g - hL}{\kappa_I CV} L\right)^{-2/3}\right)^{L/2}.$$

Figure 7 depicts this first-order impact of the lead time difference and variability (as measured by the CV) on offshoring.

4.3. Impact of Capacity Costs θ_l and θ_g

With capacitated sources, any optimal dual-sourcing smoothing level $\alpha^* \in (0, 1)$ satisfies $\widehat{MB}(\alpha^*) = \widehat{MC}(\alpha^*)$, where the marginal inventory cost remains as before, but the marginal benefit is augmented with the marginal capacity benefits:

$$\widehat{MB}(\alpha) = \theta_c L\alpha^{L-1} - \theta_g \alpha^{L-1}\frac{L(1-\alpha)(1+\alpha) - \alpha}{(1-\alpha)^{1/2}(1+\alpha)^{3/2}}$$

$$+ \theta_l \frac{L\alpha^{2L-1}(1-\alpha)(1+\alpha) + (1-\alpha^{2L})}{(1-\alpha^{2L})^{1/2}(1-\alpha)^{1/2}(1+\alpha)^{3/2}}.$$

Notice that the marginal global capacity benefit (term in θ_g) is the only term that can be negative (for small smoothing levels $\alpha < (\sqrt{1+4L} - 1)/(2L)$). We have the following comparative statics:

PROPOSITION 8. *Under our model assumptions, as the lead time difference L increases, the optimal cost $\hat{C}^*$ increases if $\theta_c \geq \theta_l + \theta_g$. The optimal smoothing level and offshoring allocation increase when θ_c or θ_l increase, and decrease when θ_g increases if $\alpha^* \leq (\sqrt{1+4L} - 1)/(2L)$. However, if $\alpha^* > (\sqrt{1+4L} - 1)/(2L)$, the optimal smoothing level and offshoring allocation increase when θ_g increases. The corresponding rate of change in the optimal cost $\hat{C}^*$ is bounded by the optimal allocation $\alpha^{*L} < 1$ or by 1:*

$$\frac{d\alpha^*}{d\theta_c} \geq 0, \quad -1 \leq \frac{\partial \hat{C}}{\partial \theta_c}(\alpha^*) = -\alpha^{*L} \leq 0,$$

$$\frac{d\alpha^*}{d\theta_l} \geq 0, \quad 0 \leq \frac{\partial \hat{C}}{\partial \theta_l}(\alpha^*) = \sqrt{\frac{1-\alpha^*}{1+\alpha^*}}(1-\alpha^{*2L}) < 1,$$

$$\frac{d\alpha^*}{d\theta_g} = \begin{cases} \leq 0 & \text{if } \alpha^* \leq (\sqrt{1+4L} - 1)/(2L), \\ > 0 & \text{otherwise,} \end{cases}$$

$$0 \leq \frac{\partial \hat{C}}{\partial \theta_g}(\alpha^*) = \alpha^{*L}\sqrt{\frac{1-\alpha^*}{1+\alpha^*}} < \alpha^{*L} \leq 1.$$

The comparative statics are as expected, except for $d\alpha^*/d\theta_g$, which reflects the counterbalancing forces that are at play when the global capacity cost increases. Increasing global capacity costs at first sight favors local sourcing. Yet, increased capacity costs also induce more smoothing. (Since the smoothing level is linked to the global sourcing allocation, this explains the counterbalancing forces. In contrast, increasing local

capacity costs favors more global sourcing and more smoothing, both leading to an increase in α^*.) When $\alpha = (\sqrt{1+4L}-1)/(2L)$ the global capacity cost reaches its maximum. Hence, if $\alpha < (\sqrt{1+4L}-1)/(2L)$ its marginal cost is positive. Given that an increase in θ_g increases marginal costs proportionally, the optimal allocation must reduce. If $\alpha^* > (\sqrt{1+4L}-1)/(2L)$, however, we get the opposite effect: here, an increase in smoothing decreases global capacity costs because less safety capacity is needed (recall that as $\alpha \to 1$, the order policy becomes constant).

By investigating how the model parameters impact the standardized capacity costs θ_l and θ_g, we find how those parameters impact the optimal smoothing level α^* and offshoring allocation α^{*L}:

1. If h increases, then κ_l increases[7] and both θ_c and θ_l decrease. If $\alpha^* < (\sqrt{1+4L}-1)/(2L)$, then θ_g increases and thus α^* decreases (less offshoring).

2. If k^l increases while $z_K^l > 0$ (which is typical), or when z_K^l increases while $k^l > 0$ (less flexibility in local source), then θ_c and θ_l increase[8] and thus α^* increases (more offshoring).

3. If k^g increases while $z_K^g > 0$ (which is typical), or when z_K^g increases while $k^g > 0$ (less flexibility in global source) then κ^g and θ_g increase while θ_c decreases. The overall effect depends on their relative magnitudes but typically the latter effect dominates and α^* decreases (less offshoring).

The limiting case $L \to \infty$ gives an upper bound on C^* and insight into the benefits of smoothing:

PROPOSITION 9. *With normal demand, as $L \to \infty$, the optimal smoothing level and cost converge to that of single local sourcing smoothing:*

$$\alpha^* \to \alpha_\infty^* = \frac{\theta_l}{\theta_l + (1 + L_l(1 - \alpha_\infty^{*2}))^{1/2}},$$

so that the optimal strategic allocation $\alpha^{*L} \simeq \alpha_\infty^{*L} \to 0$ as $L \to \infty$.

This means that for large lead times it remains optimal to smooth at about α_∞^*, reflecting the lower safety capacity needs (compared to the modest safety stock increase). This limiting smoothing level α_∞^* decreases as inventory costs increase relative to capacity costs. Yet, the reliance on the global source decreases exponentially. Indeed, recall that the strategic allocation is α^L and the optimal allocation decreases as L increases to a small but positive level $(\alpha_\infty^*)^L$. This convergence is quicker when inventory costs increase relative to capacity costs (then α_∞^* decreases). Theoretically, this means

that a dual sourcing strategy with very small reliance on the global source remains optimal for high lead times. In practice, this suggests single local sourcing with order smoothing (see Equation (11)).

Given that the optimal cost increases as L increases, the second insight from the proposition is that dual-sourcing smoothing dominates single local sourcing smoothing (and thus also single local sourcing using a base-stock policy given Proposition 4). This insight is strengthened when considering only local capacity costs, as we show next.

4.4. Local Capacity Costs ($\theta_l > 0$ But $\theta_g = 0$)

To quantify the impact of capacity, we first consider local capacitated supply and uncapacitated global supply (i.e., $\theta_g = 0$). That setting is not only more tractable but also more relevant when local capacity is less flexible than global. (Capacity in high-cost countries typically is subject to significant labor regulations and strong unions, or a high degree of automation to substitute for high labor cost. Either way, such "local" capacity tends to be more expensive and inflexible than in low-cost countries that feature cheap and abundant labor that yield significant capacity flexibility.) The optimal smoothing level can be specified using Lagrange's inversion series:

PROPOSITION 10. *With normal demand, and if $\theta_g = 0$, the optimal smoothing level α^* and allocation α^{*L} depend on L, θ_c and θ_l: $\alpha^* = \theta_l/(1+\theta_l) + O((\theta_l/(1+\theta_l))^2)$ if $L\theta_c + \sqrt{L}\theta_l \leq 1$ and elsewhere*

$$\alpha^* = \left[1 - (L\theta_c + \sqrt{L}\theta_l)^{-2/3} + \frac{L-L_l-2}{3}(L\theta_c + \sqrt{L}\theta_l)^{-4/3} \right. $$
$$\left. + O((L\theta_c + \sqrt{L}\theta_l)^{-5/3})\right]^{1/2}. \quad (20)$$

This reveals four insights: First, the key metric that drives the optimal smoothing level and offshoring allocation decision is $L\theta_c + \sqrt{L}\theta_l$, so that θ_c and θ_l are substitutes, up to a factor $\sqrt{L}$. Thus, local capacity costs have a similar impact as the standardized cost advantage. Second, the local lead time L_l continues to play a second-order role. Third, the Lagrange formula provides again a simple approximation and bound for α^*:

PROPOSITION 11. *With normal demand, if $L\theta_c + \sqrt{L}\theta_l > 1$, then the optimal smoothing level α^* has a square-root approximation α_0, where*

$$\alpha_0 = \sqrt{1 - (L\theta_c + \sqrt{L}\theta_l)^{-2/3}}, \quad (21)$$

that is asymptotically correct as $L\theta_c + \sqrt{L}\theta_l \to \infty$. If $L_l = 0$, the approximation is a lower bound ($\alpha_0 \leq \alpha^*$) if $L \leq 2$, and an asymptotic upper bound otherwise.

---

[7] $d\kappa_l/dh = (d/dh)[hz_l + (h+b)I_N(z_l)] = \phi_N(z_l) + \Phi_N(z_l)z_l > 0$ for all z_l as optimal functions of h.

[8] $d\kappa^i/dk^i = (d/dk^i)[k^i z_K^i + o^i I_N(z_K^i)] = z_K^i$ for all z_K^i as optimal functions of k^i.

The fourth insight from the Lagrange formulae is that dual sourcing with the DSS policy is significantly more attractive when local supply is capacitated and lead times increase. In contrast to uncapacitated sourcing, DSS then always dominates local single-sourcing LS (and increasingly so as local capacity costs increase) and also global single sourcing (GS) using the standard base-stock policy over a parameter domain that enlarges for large lead time differences. This finding can be corroborated and generalized analytically:

PROPOSITION 12. *With normal demand, in the presence of local capacity costs, DSS outperforms LS and GS if*

$$1 < L\theta_c + \sqrt{L}\theta_l$$
$$< \left(\tfrac{2}{3}\sqrt{L+1}\right)^3 + O((L\theta_c + \sqrt{L}\theta_l)^{-1/3}), \quad (22)$$

and the maximal relative value $\bar{V}$ of dual sourcing over LS ($V = 1 - \hat{C}^*/\hat{C}^l$) is an increasing value of the lead time difference:

$$\bar{V} = 1 - \frac{3}{2}\frac{(\sqrt{L}(\sqrt{L+1}-1))^{1/3}}{\sqrt{L+1}}$$
$$+ \frac{((L-2)/8)(\sqrt{L}(\sqrt{L+1}-1))^{-1/3}}{\sqrt{L+1}} + O(L^{-2.5}). \quad (23)$$

The parameter domain (22), where DSS outperforms single sourcing, essentially is a simplex that increases as the lead time difference increases. The maximal value (23) increases as the lead time difference increases as shown in Figure 8 and the numerical accuracy quickly improves (e.g., when $L = 10$, Equation (23) yields 27.7% while $\bar{V} = 28.4\%$.) The reason behind the impact of lead time difference is that, the higher the lead time difference between the sources, the higher the benefit of combining two sources using smoothing. The latter reduces the overall variance, and thus the capacity costs, and this reduction is larger as the lead time difference increases.

Figure 8 **(Color online) With Local Capacitated Supply, the Maximal Relative Value of DSS Over LS and GS Increases as the Lead Time Difference *L* Increases**
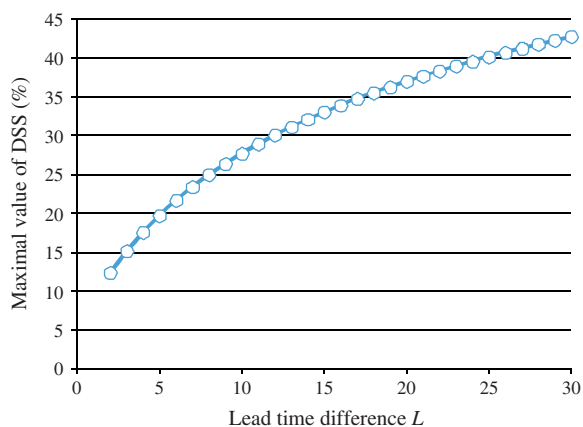
The proposition compares DSS with single-sourcing policies using a base-stock policy, but its message extends to single-sourcing smoothing and the SCC policy (discussed in §3.4). Given that SCC cannot be optimized analytically, we numerically optimized the simulated cost under this policy and compared it to the analytically optimized cost under the DSS policy. Figure 9 shows the magnitude of the cost reduction under DSS relative to the cost under the SCC policy. With no sourcing cost advantage ($\theta_c = 0$) and $L = 1$, the optimal SCC policy (by definition) outperforms the DSS policy, but for moderate θ_c, DSS performs better than SCC. As θ_l rises, local capacity becomes more constrained and both the gain, and the domain where DSS outperforms SCC, shrink.

4.5. Local and Global Capacity Costs ($\theta_l > 0$ and $\theta_g > 0$)

When both sources are capacitated, local and global capacity costs have counteracting effects on smoothing and offshoring as shown earlier in Proposition 8. For an interior, dual-sourcing solution $\alpha^* \in (0, 1)$ to exist, θ_c or θ_l must be sufficiently positive to offset a positive θ_g. Indeed, notice that if $\theta_g > 0$ while $\theta_c = \theta_l = 0$, the cost is

$$\hat{C}(\alpha) = \theta_g\alpha^L\sqrt{\frac{1-\alpha}{1+\alpha}} + \sqrt{L_l + \frac{1}{1-\alpha^2}} \geq \hat{C}(0),$$

so that $\alpha^* = 0$ and LS is optimal. When $\theta_g > 0$ and $\theta_c + \theta_l > 0$, there exist no general simple formulae to express the optimal smoothing level. (Even when $L = 1$, the FOC is a sixth order polynomial.) Even the Lagrange solutions become exceedingly complex (and we relegate them to the technical companion) but they do suggest that the optimal solution is a function of $L\theta_c + \sqrt{L}\theta_l$ and θ_g. However, given that α^* decreases as θ_g increases (Proposition 8), the exact solutions in the previous section (only local capacity costs) provide upper bounds for the general case.

Additional results are found by approximating or bounding the marginal costs. For $L = 1$, a linear approximation of the FOC around $\alpha = 0$ (light offshoring) yields

$$\widehat{MB}(\alpha) = \theta_c + \theta_g(-1 + 2\alpha) + \theta_l + o(\alpha) = \widehat{MC}(\alpha)$$
$$= (1 + L_l)^{-1/2}\alpha + o(\alpha).$$

For $L = 1$, the marginal costs are also bounded:

$$\widehat{MB}(0) = \theta_c + \theta_l - \theta_g = (1 - \underline{\alpha}_1^2)^{-3/2} \leq \widehat{MB}(\alpha^*)$$
$$= \widehat{MC}(\alpha^*) \leq (1 - \alpha^{*2})^{-3/2}.$$

A similar bounding can be done for $L > 1$, which yields the following.

Figure 9 (Color online) The Relative Value of Dual-Sourcing Smoothing Over Local Single-Sourcing-Capacitated SCC (Which Is the Optimal Strategy When $\theta_c = 0$ and $\theta_l < 0.6$, as Shown in the Left Panel); When the Global Source Has a Sourcing Cost Advantage (Right Panel), the DSS Policy Outperforms SCC
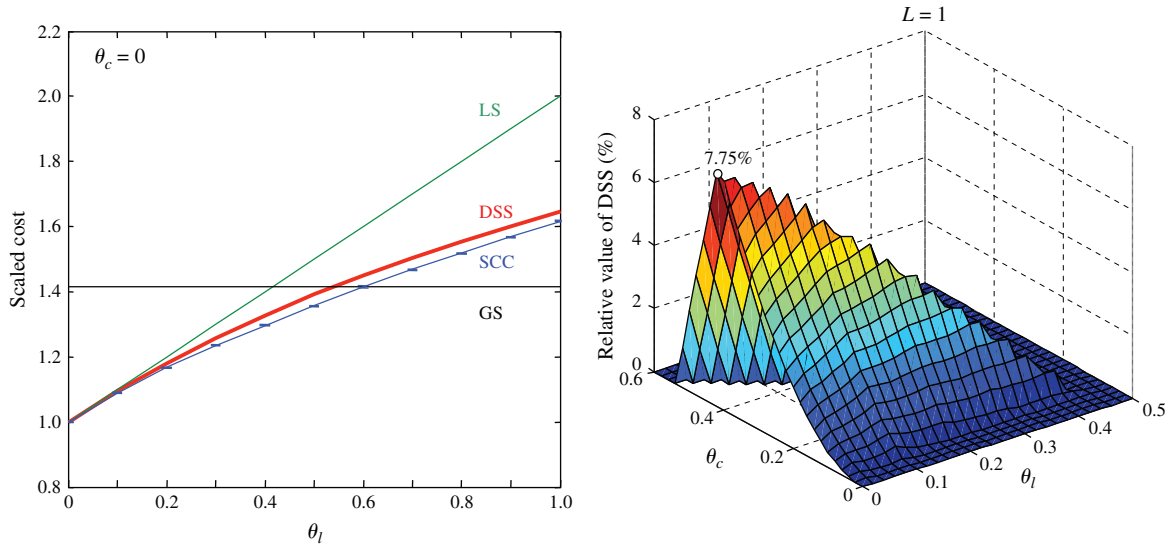
Notes. $L = 1$, $L_l = 0$, $\theta_g = 0$.

PROPOSITION 13. *Under normal demand, the following bounds apply to the general case $\theta_i > 0$:*

1. For $L = 1$, if $\theta_c + \theta_l - \theta_g > 1$, then the optimal smoothing level and offshoring allocation has a lower bound

$$\alpha^* \geq \underline{\alpha}_1 = \sqrt{1 - (\theta_c + \theta_l - \theta_g)^{-2/3}} \qquad (24)$$

that is asymptotically tight as $\theta_c + \theta_l - \theta_g \to \infty$. Otherwise, if $0 \leq \theta_c + \theta_l - \theta_g < 1$ and $\theta_g < \frac{1}{2}$:

$$\alpha^* = \frac{\theta_c + \theta_l - \theta_g}{(1 + L_l)^{-1/2} - 2\theta_g} + o\left(\frac{\theta_c + \theta_l - \theta_g}{(1 + L_l)^{-1/2} - 2\theta_g}\right). \quad (25)$$

2. For any $L > 1$, if $(4/(3\sqrt{3}))\theta_l - \theta_g(e^{-1/2}/2) \cdot (L/\sqrt{L-1}) > 1$, then the optimal smoothing level has a lower bound

$$\alpha^* \geq \underline{\alpha}_2 = \sqrt{1 - \left(\frac{4}{3\sqrt{3}}\theta_l - \theta_g \frac{e^{-1/2}}{2} \frac{L}{\sqrt{L-1}}\right)^{-2/3}}. \quad (26)$$

Proposition 13 presents a useful lower bound for $L = 1$ that captures the three key parameters where local and global capacity costs counteract each other (θ_l and θ_g have opposite signs). The bound (26) shows that this counteraction extends to general L. Unfortunately, we have not been able to generate additional insightful analytic results.

Despite the limited tractability of the general case, Proposition 13 does shine a light on recent evolutions in global labor markets. A growing number of American companies are moving their manufacturing back to the United States because of higher Chinese labor costs. Although workers in developing Asian countries are slowly acquiring more rights (increasing θ_g), there are signs that labor in rich countries is

becoming more flexible (decreasing θ_l) (*The Economist* 2013). At the same time, *The Economist* reports that Europe's inflexible and costly labor markets, vis-à-vis the United States, is one of the reasons why reshoring is largely an American phenomenon (compared to Europe). Only when national government is making the business environment attractive enough, companies will want to come back. Spurred by the Euro crisis, some European countries have now introduced substantial labor-market reforms to remain competitive (e.g., western car workers are willing to work in night shifts again). Our results predict that this will indeed work in favor of backshoring work to the developed countries.

5. Robustness and Comparison with Other Dual-Sourcing Policies

5.1. Robustness for Non-Normal Demand
Given that our analysis assumes normally distributed demand, we want to understand how sensitive the results are to this distributional assumption. Figure 10 shows the optimal smoothing level α^* for the uncapacitated and capacitated setting when demand follows a gamma, lognormal, (discrete) geometric, and (continuous) uniform distribution, in comparison with the optimal smoothing level assuming normal demand, α_N^*, with identical average and *CV*. The functional dependence of α^* on the standardized cost advantage or local capacity cost follows that of the normally distributed demand with identical *CV*. More importantly, the cost difference of a misestimate of α^* in case of non-normal demand is minimal (see Figure 11).

Figure 10 (Color online) The Optimal Smoothing Level α^* for Non-Normal Demand Follows the Optimal Smoothing Level α_N^* Assuming Normally Distributed Demand with Identical Relative Uncertainty ($\theta_g = 0$, $L_l = 0$)
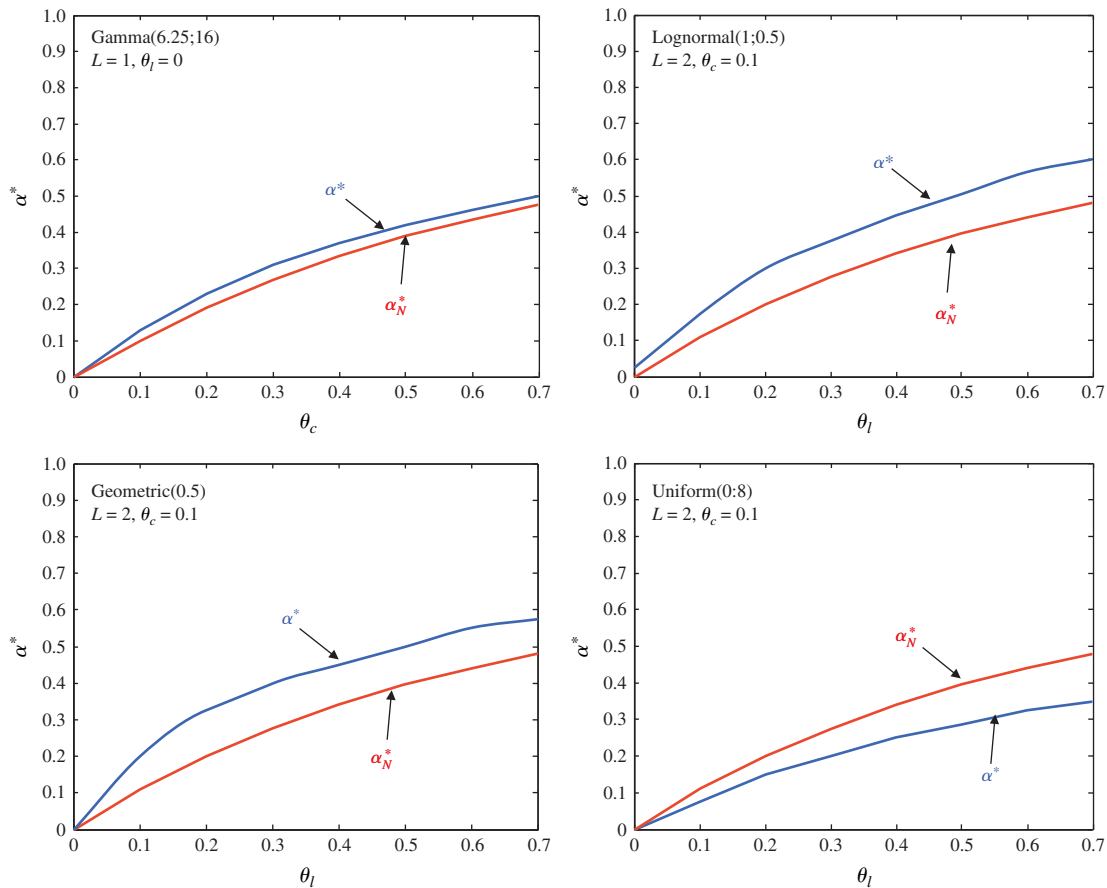
Of course, the distributional assumptions are less important as the variability decreases. Therefore, the reported cases not only give almost worst-case comparisons, but given that the probability of negative demand exceeds 0.6% for CV exceeding 0.4, they also are pushing the limit of the normal distribution assumption. Nevertheless, even with the assumed geometric distribution ($CV = 1.4$), the analytic formulae (assuming normal distribution) are still remarkably good approximations. If average demand and standard deviation are scaled up in the conventional sense (as, e.g., in Allon and Van Mieghem 2010), we expect the appropriately scaled version of our results to be asymptotically optimal. The scaled system with lead times is however nontrivial and would be a research project in its own.

These results provide some numerical evidence that our analysis remains valid to give guidance in the strategic sourcing allocations in function of the financial parameters, in practical settings regardless of the distributional assumptions of the demand.

5.2. Performance of DSS Policy Compared to Other Dual-Sourcing Policies

In this section we compare the performance of the dual-sourcing smoothing policy with other existing policies that have been shown to perform well in a dual-sourcing setting.

The dual-base-stock policies are shown to be optimal in minimizing sourcing and inventory costs for a lead time difference $L = 1$ (Fukuda 1964) and near optimal for longer lead time differences (Veeraraghavan and Scheller-Wolf 2008, Scheller-Wolf et al. 2006). An important distinction is the state dimension: the single-index (SI) policy uses one state variable, being the total inventory position, whereas the dual-index (DI) policy tracks two state variables, i.e., the inventory position of the local source and the total inventory position to place, respectively, local and global orders. The vector-base-stock (VBS) policy keeps track of the local inventory position and the recently placed global orders.

These policies have been shown to perform well if only sourcing and inventory costs are considered, and Veeraraghavan and Scheller-Wolf (2008), Scheller-Wolf et al. (2006) and Sheopuri et al. (2010) respectively present efficient solution procedures to find the optimal base-stock levels for the DI, SI, and VBS policies. However, they do not explicitly take into account capacity costs nor capacity flexibility. To minimize the

Figure 11   **(Color online) The Total Scaled Cost with Non-Normal Demand $\hat{C}(\alpha^*)$ Compared to the Scaled Cost Assuming the α_N^* Under Normal Demand with Identical Relative Uncertainty ($\theta_g = 0, L_l = 0$)**
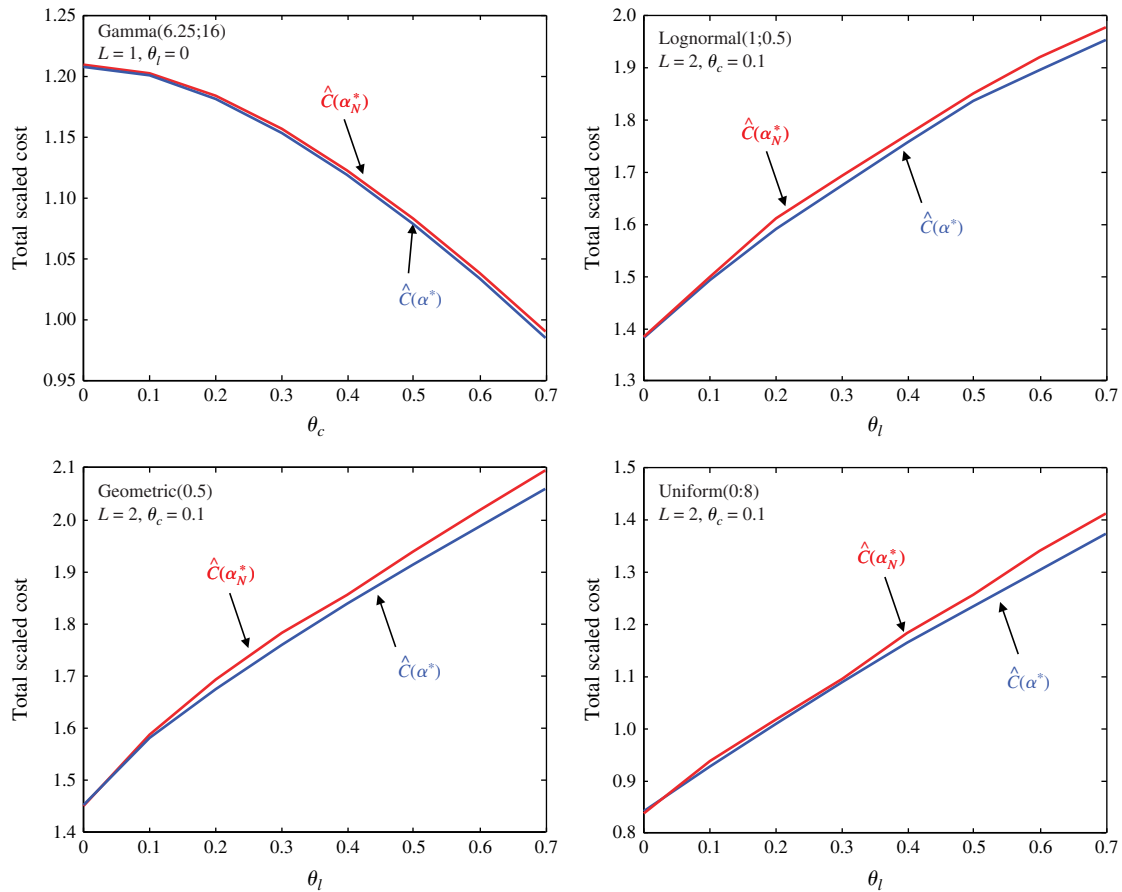
sum of average sourcing, inventory, and capacity costs per unit, as defined in (2), we therefore numerically optimized the target base-stock levels and the installed capacities at each source. There is no general closed-form distribution for the orders and net inventory in these policies (the global order may push the inventory position above its target level, causing an overshoot, so that no local order is placed), hence we resort to simulation for performance analysis and optimization.

We also benchmark with the standing order, or tailored base-surge (TBS) policy, which is as well a base-stock-type policy: the global source supplies every period a constant rate $q^g < \mu$, and an additional order to the local source is placed only when the inventory falls below the target level. In the TBS policy, the standing order may exceed the observed demand, resulting in excess inventory excursions above its target level. In essence, the "excess inventory process" behaves like a regulated random walk and the safety stock increases as the standing order to the global source goes up (since the exponential tail of the regulated random walk will be longer as q^g increases). The TBS policy requires the evaluation of a $G/D/1$ queue, where the interarrival time has the per-period demand distribution and the

processing time is the fixed order size from the global source. For certain choices of the demand distribution, this is a tractable problem (as shown by Janakiraman et al. 2015), but for general demand distributions we need to rely on a simulation analysis to optimize the target base-stock and capacity levels.

We conducted an extensive simulation study to analyze the impact of capacity and lead time differences in the uncapacitated and local capacitated setting ($\theta_g = 0$ and $L_l = 0$). Figure 12 shows representative findings, which can be summarized as follows:

(a) Without capacity costs ($\theta_l = \theta_g = 0$), DSS is outperformed by the other dual-sourcing policies. This is to be expected given that dual-base-stock policies are optimal in minimizing sourcing and inventory costs when $L = 1$, and near optimal for longer lead time differences. Their performance superiority derives from their ability to react promptly to demand fluctuations. However, DSS is smoothing out the demand fluctuations in its orders (see Equation (9)); TBS is constrained by its standing global order but has dynamic local orders. Because of its smoothing behavior, DSS (and to a lesser extent TBS) needs more safety stock to act as a buffer between volatile demand and smooth replenishments, leading

Figure 12   (Color online) Scaled Cost Under Five Dual-Sourcing Policies Under Normal Demand ($\theta_g = 0$, $L_l = 0$)
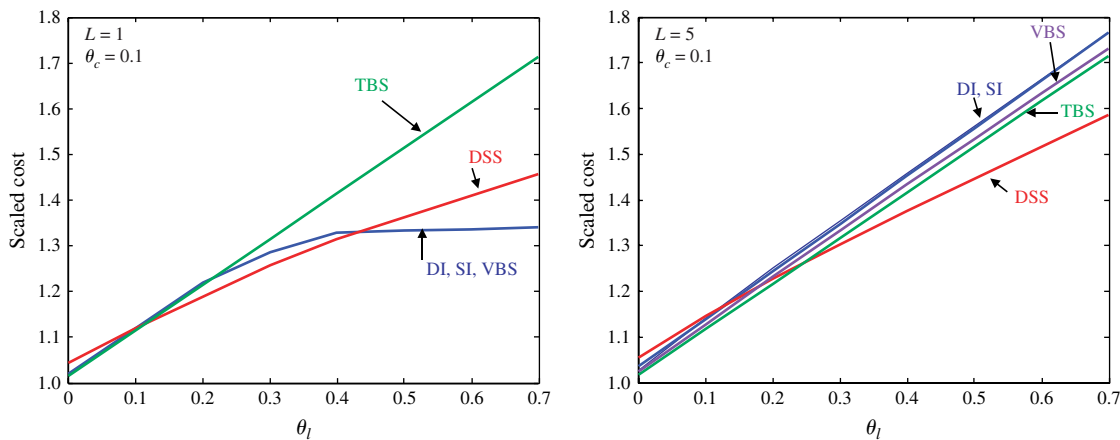
to increased inventory holding costs compared to the dual-base-stock policies. The TBS policy improves as the lead time difference increases (as also shown by Janakiraman et al. 2015).

(b) With capacity costs, DSS can outperform the dual-base-stock policies because smoothing reduces both local and global order variability and thus safety capacity requirements and costs. (Note that TBS completely smoothens (levels) global orders and requires no global but more local safety capacity; the dual-base-stock policies do not smooth at all and hence require higher local and global safety capacity.) This reduction in capacity costs may compensate the increased inventory holding costs of smoothing. DSS outperforms the other policies when capacity costs are dominating: the higher the sourcing cost advantage θ_c, the more the capacity cost θ_l should dominate for DSS to outperform the other policies. At the same time, the dual-base-stock policies converge naturally to global single sourcing when θ_l increases, which may in turn outperform DSS (DSS does not naturally converge to single global sourcing).

(c) DSS performs best for longer lead time differences L: DSS rapidly outperforms the other policies, even for low values of the capacity cost θ_l: in that case, the reduced capacity requirements quickly compensate for the (relatively modest) increase in safety stocks due to order smoothing.

These findings are valid under both normal and non-normal demand. Figure 13 illustrates the performance of the DSS policy compared to these reference policies under a lognormal, (continuous) uniform, (discrete) geometric, and normal demand distribution. These distributions are inspired by the abovementioned reference papers in the dual-sourcing literature that use the same distributions in their numerical experiments (Veeraraghavan and Scheller-Wolf 2008, Scheller-Wolf et al. 2006, Sheopuri et al. 2010).

In summary, DSS is a reasonable policy that is typically competitive to the reference dual-sourcing policies

and can perform better in the presence of high capacity costs and high lead time differences.

6.   Summary and Discussion

The main contribution of this paper is the inclusion of capacity cost and flexibility in a strategic global sourcing decision, in addition to considering sourcing cost and lead times (as in the existing literature). We propose a linear control policy that performs well in both a single- and dual-sourcing setting. An attractive feature of our analysis is the identification and analysis of three dimensionless parameters that, in addition to the lead times, are the four key forces behind the sourcing decision: θ_c (which captures the financial trade-off, the inventory service level, the volatility in demand) and θ_l and θ_g (which contain the ratio of capacity cost versus holding cost of, respectively, the local and global source, and their flexibility to work in overtime). These data are typically available inside the firm (although one may have to search for them in different departments), and we are now able to quantify how they contribute to strategic sourcing allocation decisions based on these data. These parameters also allow the comparison of different firms with different scales.

A numerical study shows that our analytic results for normally distributed demand are robust for other distributions and that the smoothing policy is competitive to existing single- and dual-sourcing policies in terms of total cost minimization. The study also confirms expectations that the smoothing policy performs better in the presence of high capacity costs and high lead time differences given that it explicitly takes the capacity costs and lead time difference into account.

As with any analytic study, ours begs for extensions. The assumption of normally distributed demand is essential in deriving all the analytical results: Direct extensions could study demand distributions with higher coefficients of variation (e.g., between one and two, or even up to three (Muckstadt 1997)), or with fat

Figure 13 (Color online) Scaled Cost Under Five Dual-Sourcing Policies Under Normal and Non-Normal Demand ($\theta_g = 0, L_l = 0$)
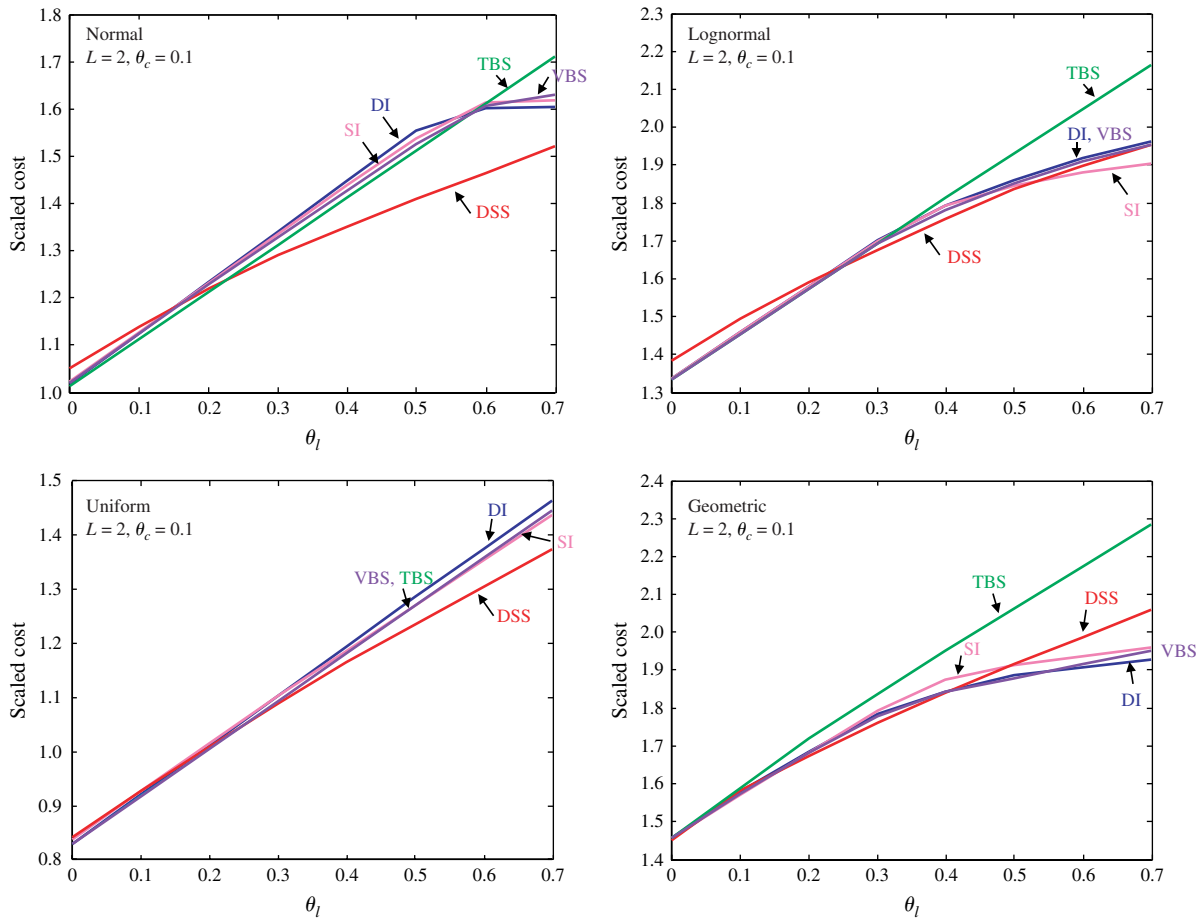
tails, which we expect to favor more local sourcing, or with serial demand correlation, where negative correlation will favor smoothing and global sourcing. Another extension could analyze nonstationary demand patterns to model, for example, how the sourcing allocation should adapt to the product life cycle.[9] A third extension could insert unit cost or quantity uncertainty to model, for example, currency exchange rate risk or supply quality risk. Last, but not least, would be a game-theoretic multidecision-maker model where one firm (the buyer) sources from one or two independent firms. These nonobvious model extensions are full research projects themselves.

Supplemental Material
Supplemental material to this paper is available at http://dx.doi.org/10.1287/mnsc.2014.1992.

Acknowledgments

Appendix. Lagrange's Inversion Theorem and Proof of Equation (19)
Markushevich (1985, Vol. II, p. 88) gives Lagrange's series for the inverse of a complex function $f(z)$ that is analytic around z_0 with $f'(z_0) \neq 0$ as

$$f^{-1}(z) = z_0 + \sum_{n=1}^{\infty} \frac{1}{n!} \left[\frac{d^{n-1}}{dz^{n-1}} \left(\frac{z - z_0}{f(z) - f(z_0)} \right)^n \right] \Bigg|_{z=z_0} \cdot (z - f(z_0))^n. \tag{27}$$

If $f^{(m)}(z_0) = 0$ but $f^{(k)}(z_0) \neq 0$ for all $m < k$, then (27) generalizes to Markushevich (1985, Vol. II, p. 92)

$$f^{-1}(z) = z_0 + \sum_{n=1}^{\infty} \frac{1}{n!} \left[\frac{d^{n-1}}{dz^{n-1}} \left(\frac{z - z_0}{[f(z) - f(z_0)]_s^{1/k}} \right)^n \right] \Bigg|_{z=z_0} \cdot (z - f(z_0))^{n/k}, \tag{28}$$

where the subscript s denotes any fixed single-valued branch of the k-valued function $[f(z) - f(z_0)]^{1/k}$.

[9] One could apply our formula for each phase separately to predict that the sourcing allocation will move from emphasizing local during product introduction (low mean but high variability in demand) to offshore during maturity and back to local during decline. However, this remains an unverified extrapolation of a stationary result.

If $L_l = 0$, consider the function $f(z) = z^3(1-z)^m$, which is analytic in z because $m = L - 2 \geq 0$ is integer. Given that $f'(0) = f''(0) = 0$ but $f^{(3)}(0) = 3! > 0$, (28) yields around $z_0 = 0$:

$$\left. \frac{d^j}{dz^j}(1-z)^{-n(m/3)} \right|_{z=0} = \frac{\Gamma((nm)/3 + j)}{\Gamma((nm)/3)}(1-z)^{-(nm)/3-j}\Bigg|_{z=0}$$

$$= \frac{\Gamma((nm)/3 + j)}{\Gamma((nm)/3)},$$

and we obtain the Lagrange series for the inverse function,

$$f^{-1}(z) = \sum_{n=1}^{\infty} \frac{\Gamma(n(m/3+1)-1)}{\Gamma((nm)/3)} \frac{z^{n/3}}{n!}, \qquad (29)$$

where $m = L - 2$. (The technical companion shows that the radius of convergence is exactly $\theta \geq \underline{\theta}_L$.) Here we were able to calculate each term in the series explicitly due to the simple form of f. If $L_l > 0$, the derivation becomes more complex but is still doable in closed form; with capacity costs, however, the derivation is term by term and cumbersome, as shown in the technical companion. □

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