Tail Risk Premia and Return Predictability*

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Abstract

The variance risk premium, defined as the difference between the actual and risk-neutral expectations of the forward aggregate market variation, helps predict future market returns. Relying on new essentially model-free estimation procedure, we show that much of this predictability may be attributed to time variation in the part of the variance risk premium associated with the special compensation demanded by investors for bearing jump tail risk, consistent with idea that market *fears* play an important role in understanding the return predictability.

Keywords: Variance risk premium; time-varying jump tails; market sentiment and fears; return predictability.

JEL classification: C13, C14, G10, G12.

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"When the VIX is high, it's time to buy, when the VIX is low, it's time to go."

Wall Street adage

1 Introduction

The VIX is popularly referred to by market participants as the "investor fear gauge." Yet, on average only a small fraction of the VIX is arguably attributable to market *fears*. We show that rather than simply buying (selling) when the VIX is high (low), the genuine *fear* component of the index provides a much better guide for making "good" investment decisions.

Volatility clustering in asset returns is ubiquitous. This widely documented temporal variation in volatility (Schwert, 2011; Andersen, Bollerslev, Christoffersen, and Diebold, 2013) represents an additional source of risk over and above the variation in the actual asset prices themselves.[1] For the market as whole, this risk is also rewarded by investors, as directly manifest in the form of a wedge between the actual and risk-neutralized expectations of the forward variation of the return on the aggregate market portfolio (Bakshi and Kapadia, 2003). Not only is the variance risk premium on average significantly different from zero, like the variance itself it also fluctuates non-trivially over time (Carr and Wu, 2009; Todorov, 2010). Mounting empirical evidence further suggests that unlike the variance, the variance risk premium is useful for predicting future aggregate market returns over and above the predictability afforded by more traditional predictor variables such as the dividend-price and other valuation ratios, with the predictability especially strong over relatively short quarterly to annual horizons (Bollerslev, Tauchen, and Zhou, 2009).[2]

The main goals of the present paper are twofold. First, explicitly recognizing the prevalence of different types of market risks, we seek to nonparametrically decompose their sum

[1]Following the classical ICAPM of Merton (1973), variance risk has traditionally been associated with changes in the investment opportunity set, which in turn induce a hedging component in the asset demands.

[2]Recent studies corroborating and extending the predictability results in Bollerslev, Tauchen, and Zhou (2009) include Drechsler and Yaron (2011), Han and Zhou (2011) Du and Kapadia (2012), Eraker and Wang (2014), Almeida, Vicente, and Guillen (2013), Bekaert and Hoerova (2014), Bali and Zhou (2014), Camponovo, Scaillet, and Trojani (2013), Kelly and Jiang (2014), Li and Zinna (2014), Vilkov and Xiao (2013) and Bollerslev, Marrone, Xu, and Zhou (2014), among others. The empirical results in Andreou and Ghysels (2013) and Bondarenko (2014) also suggest that the variance risk premium cannot be explained by other traditional risk factors.

total as embodied in the variance risk premium into separate diffusive and jump risk components with their own distinct economic interpretations. Second, relying on this new decomposition of the variance risk premium, we seek to clarify where the inherent market return predictability is coming from and how it plays out over different return horizons and for different portfolios with different risk exposures.

Extending the long-run risk model of Bansal and Yaron (2004) to allow for time-varying volatility-of-volatility, Bollerslev, Tauchen, and Zhou (2009) and Drechsler and Yaron (2011) have previously associated the temporal variation in the variance risk premium with notions of time-varying economic uncertainty. On the other hand, extending the habit formation type preferences of Campbell and Cochrane (1999), Bekaert and Engstrom (2010) and Bekaert, Hoerova, and Lo Duca (2013) have argued that the variance risk premium may be interpreted as a proxy for aggregate risk-aversion. Meanwhile, as emphasized by Bollerslev and Todorov (2011b), the variance risk premium formally reflects the compensation for two very different types of risks: continuous and discontinuous price moves. The possibility of jumps, in particular, adds an additional unique source of market variance risk stemming from the locally non-predictable nature of jumps. This risk is still present even if the investment opportunity set does not change over time (i.e., even in a static economy with independent and identically distributed returns), and it remains a force over diminishing investment horizon (i.e., even for short time-intervals where the investment opportunity set is approximately constant). As discussed more formally below, these distinctly different roles played by the two types of risks allows us to uniquely identify the part of the variance risk premium attributable to market *fears* and the special compensation for jump tail risk.

Our estimation of the separate components of the variance risk premium builds on and extends the new econometric procedures recently developed by Bollerslev and Todorov (2014). The basic idea involves identifying the shape of the risk-neutral jump tails from the rate at which the prices of short maturity options decay for successively deeper out-of-the-money contracts. Having identified the shape of the tails, their levels are easily determined by the actual prices of the options. In contrast to virtually all parametric jump-diffusion models hitherto estimated in the literature, which restrict the shape of the tail decay to be constant over time, we show that the shapes of the nonparametrically estimated jump tails vary significantly over time, and that this variation contributes non-trivially to the temporal variation of the variance risk premium. The statistical theory underlying our new estimation procedure is formally based on an increasing cross-section of options. Importantly, this allows for a genuine predictive analysis avoiding the look-ahead bias which invariably plagues other more traditional parametric-based estimation procedures relying on long-span asymptotics for the tail estimation.

The two separately estimated components of the variance risk premium each exhibit their own unique dynamic features. Although both increase during times of financial crisis and distress (e.g., the 1997 Asian crisis, the 1998 Russian default, the 2007-08 global financial crisis, and the 2010 European sovereign debt crisis), the component due to jump risk typically remains elevated for longer periods of time.[3] By contrast, the part of the variance risk premium attributable to "normal" risks rises significantly during other time periods that hardly register in the jump risk component (e.g., the end of the dotcom era in 2002-03). Counter to the implications from popular equilibrium-based asset pricing models, nonparametric regression analysis also suggests that neither of the two components of the variance risk premium can be fully explained as nonlinear functions of the aggregate market volatility.[4] Hence, nonlinearity of the pricing kernel cannot be the sole explanation for the previously documented predictability inherent in the variance risk premium.[5]

The distinctly different dynamic dependencies in the two components of the variance risk premium also naturally suggests that the return predictability for the aggregate market portfolio afforded by the total variance risk premium may be enhanced by separately considering the two components in the return predictability regressions. Our empirical results confirm this conjecture. In particular, we find that most of the predictability for the aggregate market portfolio previously ascribed to the variance risk premium stems from the jump tail risk component, and that this component drives out most of the predictability stemming from the part of the variance risk premium associated with "normal" sized price

[3]The overall level of the market volatility also tends to mean revert more quickly than the jump risk premia following all of these events.

[4]The habit persistence model of Campbell and Cochrane (1999), for example, and its extension in Du (2010), imply such a nonlinear relationship.

[5]Similarly, nonlinearity cannot explain the empirically weak mean-variance tradeoff widely documented in the literature; see, e.g., Bollerslev, Sizova, and Tauchen (2012) and the many references therein.

fluctuations. Replicating the predictability regressions for the aggregate market portfolio for size, value, and momentum portfolios comprised of stocks sorted on the basis of their market capitalizations, book-to-market values, and past annual returns, we document even greater increases in the degree of return predictability by separately considering the two variance risk premium components. The predictability patterns for the corresponding zero-cost high-minus-low arbitrage portfolios are generally also supportive of our interpretation of the jump tail risk component of the variance risk premium as providing a proxy for market *fears*.

Our empirical findings pertaining to the predictability of the aggregate market portfolio are related to other recent empirical studies, which have argued that various options-based measures of jump risk are useful for forecasting future market returns. Santa-Clara and Yan (2010), in particular, find that an estimate of the equity risk premium due to jumps, as implied from options and a one-factor stochastic volatility jump diffusion model, significantly predict subsequent market returns. Similarly, Andersen, Fusari, and Todorov (2014) relying on a richer multi-factor specification find that a factor directly related to the risk-neutral jump intensity helps forecast future market returns. Allowing for both volatility jumps and self-exciting jump intensities, Li and Zinna (2014) report that the predictive performance of the variance risk premium estimated within their model may be improved by separately considering the estimated jump component. All of these studies, however, rely on specific model structures and long time-span asymptotics for parameter estimation and extraction of the state variables that drive the jump and stochastic volatility processes. By contrast, our empirical investigations are distinctly non-parametric in nature, thus imbuing our findings with a built-in robustness against model misspecification.[6] Moreover, our approach for estimating the temporal variation in the jump tail risk measures is based on the cross-section of options at a given point-in-time, thus circumventing the usual concerns about

[6]A plethora of competing parametric models have been used in the empirical option pricing literature. For instance, while one factor models, as in, e.g., Pan (2002), Broadie, Chernov, and Johannes (2007), and Santa-Clara and Yan (2010), are quite common, the empirical evidence in Bates (2000), Christoffersen, Heston, and Jacobs (2009) among others, clearly suggests that multiple volatility factors are needed. Correspondingly, in models that do allow for jumps, the jump arrival rates are typically taken to be constant, although the estimates in Christoffersen, Jacobs, and Ornthanalai (2012), Andersen, Fusari, and Todorov (2014) among others, clearly point to time-varying jump intensities. Related to this, Duffie, Pan, and Singleton (2000), Eraker (2004) among others, further advocate allowing for volatility jumps. Moreover, despite ample empirical evidence favoring log-volatility formulations when directly modeling returns, virtually all parametric option pricing models have been based on either affine or linear-quadratic specifications.

structural-stability and "look-ahead" biases that invariably plague conventional parametric-based procedures.

Other related nonparametric-based approaches includes Vilkov and Xiao (2013), who argue that a conditional Value-at-Risk (VaR) type measure extracted from options through the use of Extreme Value Theory (EVT) predicts future market returns, although the predictability documented in that study is confined to relatively short weekly horizons. Also, Du and Kapadia (2012) find that a tail index measure for jumps defined as the difference between the sum of squared log-returns and the square of summed log-returns affords some additional predictability for the market portfolio over and above that of the variance risk premium. In contrast to these studies, the new nonparametric jump risk measures proposed and analyzed here are all economically motivated, with direct analogs in popular equilibrium consumption-based asset pricing models. Moreover, the predictability results for the market portfolio and the interpretation thereof are further corroborated by our new empirical findings pertaining to other portfolio sorts and priced risk factors.

The rest of the paper is organized as follows. Section 2 presents our formal setup and definitions of the variance risk premium and its separate components. We also discuss how the jump tail risk component manifests within two popular stylized equilibrium setups. Section 3 outlines our new estimation strategy for nonparametrically extracting the jump tails. Section 4 describes the data that we use in our empirical analysis. The actual estimation results for the new jump tail risk measures is discussed in Section 5. Section 6 presents the results from the return predictability regressions, beginning with the aggregate market portfolio followed by the results for the different portfolio sorts and systematic risk factors. Section 7 concludes.

2 General Setup and Assumptions

The continuous-time dynamic framework, and corresponding variation measures, underlying our empirical investigations is very general. It encompasses almost all parametric asset pricing models hitherto used in the literature as special cases.

2.1 Returns and Variance Risk Premium

Let X_t denote the price of some risky asset defined on the filtered probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $(\mathcal{F}_t)_{t\geq 0}$ refers to the filtration. We will assume the following dynamic continuous-time representation for the instantaneous arithmetic return on X,

$$\frac{dX_t}{X_{t-}} = a_t dt + \sigma_t dW_t + \int_{\mathbb{R}} (e^x - 1)\widetilde{\mu}^{\mathbb{P}}(dt, dx), \tag{2.1}$$

where the drift and diffusive processes, a_t and σ_t, respectively, are both assumed to have càdlàg paths, but otherwise left unspecified, W_t is a standard Brownian motion, and $\mu(dt, dx)$ is a counting measure for the jumps in X with compensator $dt \otimes \nu_t^{\mathbb{P}}(dx)$, so that $\widetilde{\mu}^{\mathbb{P}}(dt, dx) \equiv \mu(dt, dx) - dt \otimes \nu_t^{\mathbb{P}}(dx)$ is a martingale measure under $\mathbb{P}$.[7]

The continuously compounded return from time t to $t + \tau$, say $r_{[t,t+\tau]} \equiv \log(X_{t+\tau}) - \log(X_t)$, implied by the formulation in (2.1) may be expressed as,

$$r_{[t,t+\tau]} = \int_t^{t+\tau} (a_s + q_s) ds + \int_t^{t+\tau} \sigma_s dW_s + \int_t^{t+\tau} \int_{\mathbb{R}} x \widetilde{\mu}^{\mathbb{P}}(ds, dx), \tag{2.2}$$

where q_t represents the standard convexity adjustment term associated with the transformation from arithmetic to logarithmic returns. Correspondingly, the variability of the price over the $[t, t + \tau]$ time-interval is naturally measured by the quadratic variation,

$$QV_{[t,t+\tau]} = \int_t^{t+\tau} \sigma_s^2 ds + \int_t^{t+\tau} \int_{\mathbb{R}} x^2 \mu(ds, dx). \tag{2.3}$$

Even though the diffusive price increments associate with σ and the jumps controlled by the counting measure μ both contribute to the total variation of returns and the pricing thereof, they do so in distinctly different ways.

In order to more formally investigate the separate pricing of the diffusive and jump components, we will assume the existence of the alternative risk-neutral probability measure $\mathbb{Q}$, under which the dynamics of X takes the form,

$$\frac{dX_t}{X_{t-}} = (r_{f,t} - \delta_t) dt + \sigma_t dW_t^{\mathbb{Q}} + \int_{\mathbb{R}} (e^x - 1)\widetilde{\mu}^{\mathbb{Q}}(dt, dx), \tag{2.4}$$

where $r_{f,t}$ and δ_t refer to the instantaneous risk-free rate and the dividend yield, respectively, $W_t^{\mathbb{Q}}$ is a Brownian motion under $\mathbb{Q}$, and $\widetilde{\mu}^{\mathbb{Q}}(dt, dx) \equiv \mu(dt, dx) - dt \otimes \nu_t^{\mathbb{Q}}(dx)$ where $dt \otimes \nu_t^{\mathbb{Q}}(dx)$

[7] This implicitly assumes that X_t does not have fixed times of discontinuities. This assumption is satisfied by virtually all asset pricing models hitherto used in the literature.

denotes the compensator for the jumps under $\mathbb{Q}$. The existence of $\mathbb{Q}$ follows directly from the lack of arbitrage under mild technical conditions (see, e.g., the discussion in Duffie, 2001). Importantly, while the no-arbitrage condition restricts the diffusive volatility process σ_t to be the same under the $\mathbb{P}$ and $\mathbb{Q}$ measures, the lack of arbitrage puts no restrictions on the $dt \otimes \nu_t^{\mathbb{Q}}(dx)$ jump compensator for the "larger" (in absolute value) sized jumps. In that sense, the two different sources of risk manifest themselves in fundamentally different ways in the pricing of the asset.

Consider the (normalized by horizon) variance risk premium on X defined by,

$$VRP_{t,\tau} = \frac{1}{\tau}\left(\mathbb{E}_t^{\mathbb{P}}(QV_{[t,t+\tau]}) - \mathbb{E}_t^{\mathbb{Q}}(QV_{[t,t+\tau]})\right). \tag{2.5}$$

This mirrors the definition of the variance risk premium most commonly used in the options pricing literature (see, e.g., Carr and Wu, 2009), where the difference is also sometimes referred to as a volatility spread (see, e.g., Bakshi and Madan, 2006).[8] Let

$$CV_{[t,t+\tau]} = \int_t^{t+\tau} \sigma_s^2 ds,$$

denote the total continuous variation over the $[t, t+\tau]$ time-interval, and denote the corresponding total predictable jump variation under the $\mathbb{P}$ and $\mathbb{Q}$ probability measures by,[9]

$$JV_{[t,t+\tau]}^{\mathbb{P}} = \int_t^{t+\tau}\int_{\mathbb{R}} x^2 \nu_s^{\mathbb{P}}(dx)ds \qquad JV_{[t,t+\tau]}^{\mathbb{Q}} = \int_t^{t+\tau}\int_{\mathbb{R}} x^2 \nu_s^{\mathbb{Q}}(dx)ds.$$

The variance risk premium may then be decomposed as,

$$\begin{aligned} VRP_{t,\tau} &= \frac{1}{\tau}\left(\mathbb{E}_t^{\mathbb{P}}(CV_{[t,t+\tau]} + JV_{[t,t+\tau]}^{\mathbb{P}}) - \mathbb{E}_t^{\mathbb{Q}}(CV_{[t,t+\tau]} + JV_{[t,t+\tau]}^{\mathbb{Q}})\right) \\ &= \frac{1}{\tau}\left[\left(\mathbb{E}_t^{\mathbb{P}}(CV_{[t,t+\tau]}) - \mathbb{E}_t^{\mathbb{Q}}(CV_{[t,t+\tau]})\right) + \left(\mathbb{E}_t^{\mathbb{P}}(JV_{[t,t+\tau]}^{\mathbb{P}}) - \mathbb{E}_t^{\mathbb{Q}}(JV_{[t,t+\tau]}^{\mathbb{P}})\right)\right] \\ &\quad + \frac{1}{\tau}\left(\mathbb{E}_t^{\mathbb{Q}}(JV_{[t,t+\tau]}^{\mathbb{P}}) - \mathbb{E}_t^{\mathbb{Q}}(JV_{[t,t+\tau]}^{\mathbb{Q}})\right). \end{aligned}$$

[8] This difference also corresponds directly to the expected payoff on a (long) variance swap contact. Empirically, the variance risk premium for the aggregate market portfolio as defined in (2.5) is on average negative. In the discussion of the empirical results below we will refer to our estimate of $-VRP_{t,\tau}$ as the *variance risk premium* for short.

[9] The quadratic variation due to jumps equals $\int_t^{t+\tau}\int_{\mathbb{R}} x^2\mu(ds,dx)$, which does not depend on the probability measure. $JV_{[t,t+\tau]}^{\mathbb{P}}$ and $JV_{[t,t+\tau]}^{\mathbb{Q}}$ denote the *predictable* components of the jump variation, which do depend on the respective probability measure. By contrast, for the continuous component $CV_{[t,t+\tau]}$ the quadratic variation and its predictable component coincide.

The first parenthesis inside the square brackets on the right-hand-side involves the differences between the $\mathbb{P}$ and $\mathbb{Q}$ expectations of the continuous variation. Analogously, the second parenthesis inside the square brackets involves the differences between the $\mathbb{P}$ and $\mathbb{Q}$ expectations of the *same* $\mathbb{P}$ jump variation measure. These two terms account for the pricing of the temporal variation in the diffusive risk σ_t^2 and the jump intensity process $\nu_t^{\mathbb{P}}(dx)$, respectively. For the aggregate market portfolio, these differences in expectations under the $\mathbb{P}$ and $\mathbb{Q}$ measures are naturally associated with investors willingness to hedge against changes in the investment opportunity set. By contrast, the very last term on the right-hand-side in the above decomposition involves the difference between the expectations of the objective $\mathbb{P}$ and risk-neutral $\mathbb{Q}$ jump variation measures evaluated under the *same* probability measure $\mathbb{Q}$. As such, this term is effectively purged from the compensation for time-varying jump intensity risk. It has no direct analogue for the diffusive price component, but instead reflects the "special" treatment of jump risk.[10]

Without additional parametric assumptions about the underlying model structure it is generally impossible to empirically identify and estimate the separate diffusive and jump risk components.[11] However, by focussing on the jump "tails" of the distribution, it is possible (under very weak additional semi-nonparametric assumptions) to estimate a measure that parallels the second term in the above decomposition and the part of the variance risk premium due to the special compensation for jump tail risk. Moreover, as we argue in the

[10]Formally, the total quadratic variation in (2.3) may alternatively be expressed as,

$$QV_{[t,t+\tau]} = \langle \log(X), \log(X) \rangle_{[t,t+\tau]} + \int_t^{t+\tau} \int_{\mathbb{R}} x^2 \widetilde{\mu}^{\mathbb{P}}(ds, dx),$$

where the first term on the right-hand side corresponds to the so-called predictable quadratic variation, and the second term is a martingale; see e.g., Protter (2004). The first predictable quadratic variation term captures the risk associated with the temporal variation in the stochastic volatility and its analogue for the jumps; i.e., the jump intensity $\nu_t^{\mathbb{P}}(dx)$. The second martingale term associated with the compensated, or demeaned, jump process $\widetilde{\mu}^{\mathbb{P}}(dt, dx) \equiv \mu(dt, dx) - dt \otimes \nu_t^{\mathbb{P}}(dx)$ stems solely from the the fact that jumps, or price discontinuities, may occur. This term has no analogue for the diffusive price component. The "special" compensation for jumps refer to the price attached to this second term. In theory, all jumps, "small" and "large," will contribute to this term. Empirically, however, with discretely sample prices and options data, it is impossible to uniquely identify and distinguish the "small" jumps from continuous price moves. Hence, in our empirical investigations, we restrict our attention to the "special" compensation for *jump tail* risk.

[11]Andersen et al. (2014) have recently estimated the separate components based on a standard two-factor stochastic volatility model augmented with a third latent time-varying jump intensity factor.

next section, this new measure may be interpreted as a proxy for investor *fears*.[12]

2.2 Jump Tail Risk

The general dynamic representations in (2.1) and (2.4) do not formally distinguish between different sized jumps. However, there is ample anecdotal as well as more rigorous empirical evidence that "large" sized jumps, or tail events, are viewed very differently by investors than more "normal" sized price fluctuations (see, e.g. Bansal and Shaliastovich, 2011, and the references therein). Motivated by this observation, we will focus on the pricing of unusually "large" sized jumps, with the notion of "large" defined in a relative sense compared to the current level of risk in the economy.[13] Empirically, of course, without an explicit parametric model it would also be impossible to separately identify the "small" jump moves from the diffusive price increments.

Specifically, define the left and right risk-neutral jump tail variation over the $[t, t+\tau]$ time-interval by,

$$LJV^{\mathbb{Q}}_{[t,t+\tau]} = \int_t^{t+\tau} \int_{x<-k_t} x^2 \nu^{\mathbb{Q}}_s(dx) ds, \qquad RJV^{\mathbb{Q}}_{[t,t+\tau]} = \int_t^{t+\tau} \int_{x>k_t} x^2 \nu^{\mathbb{Q}}_s(dx) ds, \tag{2.6}$$

where $k_t > 0$ is a time-varying cutoff pertaining to the log-jump size.[14] Let the corresponding left and right jump tail variation measures under the actual probability measure $\mathbb{P}$, say $LJV^{\mathbb{P}}_{[t,t+\tau]}$ and $RJV^{\mathbb{P}}_{[t,t+\tau]}$, be defined analogously from the $dt \otimes \nu^{\mathbb{P}}_t(dx)$ jump tail compensator. In parallel to the definition of the variance risk premium in (2.5), the (normalized by horizon) left and right jump tail risk premia are then naturally defined by,

$$\begin{aligned} LJP_{t,\tau} &= \tfrac{1}{\tau}\left(\mathbb{E}^{\mathbb{P}}_t(LJV^{\mathbb{P}}_{[t,t+\tau]}) - \mathbb{E}^{\mathbb{Q}}_t(LJV^{\mathbb{Q}}_{[t,t+\tau]})\right), \\ RJP_{t,\tau} &= \tfrac{1}{\tau}\left(\mathbb{E}^{\mathbb{P}}_t(RJV^{\mathbb{P}}_{[t,t+\tau]}) - \mathbb{E}^{\mathbb{Q}}_t(RJV^{\mathbb{Q}}_{[t,t+\tau]})\right), \end{aligned} \tag{2.7}$$

[12]Intuitively, for $\tau \downarrow 0$,

$$\lim_{\tau\downarrow 0} VRP_{t,\tau} = \int_{\mathbb{R}} x^2(\nu^{\mathbb{P}}_t(dx) - \nu^{\mathbb{Q}}_t(dx)),$$

corresponding to the second term on the right-hand-side in the decomposition of $VRP_{t,\tau}$, and the lack of compensation for changes in the investment opportunity set over diminishing horizons.

[13]That is, our definition of what constitute "large" sized jumps and our jump tail risk measures are relative as opposed to absolute concepts.

[14]The use of a time-varying cutoff k_t for identifying the "large" jumps directly mirrors the use of a time-varying threshold linked to the diffusive volatility σ_t in the tests for jumps based on high-frequency intraday data pioneered by Mancini (2001).

both of which contribute to $VRP_{t,\tau}$. Correspondingly, the difference $VRP_{t,\tau} - (LJP_{t,\tau} + RJP_{t,\tau})$ may be interpreted as the part of the variance risk premium attributable to "normal" sized price fluctuations.

Mimicking the decomposition of the variance risk premium discussed in the previous section, the left and right tail jump premia defined above may be decomposed as,

$$LJP_{t,\tau} = \frac{1}{\tau}\left[\mathbb{E}_t^{\mathbb{P}}(LJV_{[t,t+\tau]}^{\mathbb{P}}) - \mathbb{E}_t^{\mathbb{Q}}(LJV_{[t,t+\tau]}^{\mathbb{P}})\right] + \frac{1}{\tau}\left[\mathbb{E}_t^{\mathbb{Q}}(LJV_{[t,t+\tau]}^{\mathbb{P}}) - \mathbb{E}_t^{\mathbb{Q}}(LJV_{[t,t+\tau]}^{\mathbb{Q}})\right],$$

and

$$RJP_{t,\tau} = \frac{1}{\tau}\left[\mathbb{E}_t^{\mathbb{P}}(RJV_{[t,t+\tau]}^{\mathbb{P}}) - \mathbb{E}_t^{\mathbb{Q}}(RJV_{[t,t+\tau]}^{\mathbb{P}})\right] + \frac{1}{\tau}\left[\mathbb{E}_t^{\mathbb{Q}}(RJV_{[t,t+\tau]}^{\mathbb{P}}) - \mathbb{E}_t^{\mathbb{Q}}(RJV_{[t,t+\tau]}^{\mathbb{Q}})\right],$$

respectively. The first term on the right-hand-side in each of the two expressions involves the difference between the $\mathbb{P}$ and $\mathbb{Q}$ expectations of the *same* jump variation measures. Again, this directly mirrors the part of the variance risk premium associated with the difference between the $\mathbb{P}$ and $\mathbb{Q}$ expectations of the future diffusive risk $CV_{[t,t+\tau]}$. By contrast, the second term on the right-hand-side in each of the two expressions involves the difference between the expectations of the respective $\mathbb{P}$ and $\mathbb{Q}$ jump tail variation measures under the *same* probability measure $\mathbb{Q}$, reflecting the "special" treatment of jump tail risk.[15]

Under the additional assumption that the $\mathbb{P}$ jump intensity process is approximately symmetric for "large" sized jumps, we have $LJV_{[t,t+\tau]}^{\mathbb{P}} \approx RJV_{[t,t+\tau]}^{\mathbb{P}}$. Hence, the first terms on the right-hand-sides in the above decompositions of $LJP_{t,\tau}$ and $RJP_{t,\tau}$ will be approximately the same.[16] Therefore, for sufficiently large values of the cutoff k_t, the *difference* between the two jump tail premia,

$$LJP_{t,\tau} - RJP_{t,\tau} \approx \frac{1}{\tau}\left[\mathbb{E}_t^{\mathbb{Q}}(LJV_{[t,t+\tau]}^{\mathbb{P}}) - \mathbb{E}_t^{\mathbb{Q}}(LJV_{[t,t+\tau]}^{\mathbb{Q}})\right] - \frac{1}{\tau}\left[\mathbb{E}_t^{\mathbb{Q}}(RJV_{[t,t+\tau]}^{\mathbb{P}}) - \mathbb{E}_t^{\mathbb{Q}}(RJV_{[t,t+\tau]}^{\mathbb{Q}})\right],$$

[15]In parallel to the expression for the variance risk premium above, it follows that for $\tau \downarrow 0$,

$$\lim_{\tau \downarrow 0} LJP_{t,\tau} = \int_{x<-k_t} x^2(\nu_t^{\mathbb{P}}(dx) - \nu_t^{\mathbb{Q}}(dx)), \qquad \lim_{\tau \downarrow 0} RJP_{t,\tau} = \int_{x>k_t} x^2(\nu_t^{\mathbb{P}}(dx) - \nu_t^{\mathbb{Q}}(dx)),$$

corresponding to the second term on the right-hand-side in the respective decompositions.

[16]The assumption that the $\mathbb{P}$ jump intensity process is approximately symmetric deep in the tails is supported empirically by the EVT-based estimates for the S&P 500 market portfolio reported in Bollerslev and Todorov (2011a). This evidence, however, is based on jumps of much smaller magnitude than the cutoffs k_t that we use below. As such, the statistical uncertainty associated with the symmetry of the $\mathbb{P}$ jump tail intensities remains nontrivial. Nevertheless, given the small size of the $\mathbb{P}$ jumps relative to their $\mathbb{Q}$ counterparts, some asymmetry in the $\mathbb{P}$ jump tail intensities will not materially affect the results.

will be largely void of the compensation for temporal variation in jump intensity risk. As such, $LJP_{t,\tau} - RJP_{t,\tau}$ may be interpreted as a proxy for investor *fears.* This mirrors the arguments behind the investor *fear index* proposed by Bollerslev and Todorov (2011b).[17] However, in contrast to the estimates reported in Bollerslev and Todorov (2011b), which restrict the shape of the jump tails to be time-invariant, we explicitly allow for empirically more realistic time-varying tail shape parameters, relying on the information in the cross-section of options for identifying the temporal variation in the $\mathbb{Q}$ jump tails.

Going one step further, it follows readily that for approximately symmetric $\mathbb{P}$ jump tails,

$$LJP_{t,\tau} - RJP_{t,\tau} \approx \frac{1}{\tau}\mathbb{E}_t^{\mathbb{Q}}(RJV_{[t,t+\tau]}^{\mathbb{Q}}) - \frac{1}{\tau}\mathbb{E}_t^{\mathbb{Q}}(LJV_{[t,t+\tau]}^{\mathbb{Q}}),$$

thus expressing the *fear* component of the tail risk premia as a function of the $\mathbb{Q}$ jump tails alone.[18] As such, this conveniently avoids any tail estimation under $\mathbb{P}$, which inevitably is plagued by a dearth of "large" sized jumps and a "law-of-small-numbers," or Peso-type problem. Moreover, for the aggregate market portfolio the magnitude of the risk-neutral left jump tail dwarfs that of the right jump tail, so that empirically $LJP_{t,\tau} - RJP_{t,\tau}$ is approximately equal to the $\mathbb{Q}$ expectation of the negative left jump variation only,

$$LJP_{t,\tau} - RJP_{t,\tau} \;\approx\; -\frac{1}{\tau}\mathbb{E}_t^{\mathbb{Q}}(LJV_{[t,t+\tau]}^{\mathbb{Q}}), \tag{2.8}$$

affording a particularly simple expression for the *fear* component.

2.3 Equilibrium Interpretations of the Jump Tail Measures

The definition of the jump tail risk premia and their interpretation discussed above hinge solely on the general continuous-time specification for the price process in (2.1) and the

[17]A similar decomposition has recently been explored by Li and Zinna (2014) within a more restrictive fully parametric framework. The interpretation of the difference between the left and right jump tail variation as a proxy for investor *fears* is also broadly consistent with the stylized partial equilibrium model in Gabaix (2012), discussed further below, although the underlying one-factor representation does not formally distinguish between the different variation measures explicitly defined here. Also, Schneider (2012) has argued that empirically the *fear index* is highly correlated with the fixed leg of a simple skew swap trading strategy.

[18]Of course, this same approximate expression for $LJP_{t,\tau} - RJP_{t,\tau}$ also holds true under assumption that the $\mathbb{Q}$ jump tails are orders of magnitude larger than the $\mathbb{P}$ jump tails, even if the $\mathbb{P}$ jump tails are not necessarily symmetric. For the values of the cutoff k_t used in the empirical analysis below this is clearly the case. Note also that in order to reach this approximation from (2.7), we do not need the preceding additional decompositions of $LJP_{t,\tau}$ and $RJP_{t,\tau}$. We merely include these additional steps to help illustrate the different types of risk premia embodied in $LJP_{t,\tau}$ and $RJP_{t,\tau}$, and the fact that the compensation for changes in the investment opportunity set, in particular, approximately cancels out in their difference.

corresponding no-arbitrage condition. Importantly, our empirical estimation of the different measures also do not require us to to specify any other aspects of the underlying economy. Nonetheless, in order to gain some intuition for the different measures, and $LJP_{t,\tau}$ in particular, we briefly consider their manifestation within the context of two popular stylized equilibrium consumption-based asset pricing frameworks.

To begin, we consider a setup build on a representative agent with time non-separable Epstein-Zin preferences and affine dynamics for consumption and dividends. This setup has been analyzed extensively by Eraker and Shaliastovich (2008). It includes the long-run risks models of Bansal and Yaron (2004) and Drechsler and Yaron (2011), as well as the rare disaster model with time-varying probabilities for disasters of Gabaix (2012) and Wachter (2013) as special cases. In this general setup, the jump intensity under the statistical probability measure $\mathbb{P}$ may be conveniently expressed as,

$$\nu_t^{\mathbb{P}}(dx) = \left(\nu_{t,1}^{\mathbb{P}} * \cdots * \nu_{t,i}^{\mathbb{P}} * \cdots * \nu_{t,n}^{\mathbb{P}}\right)(dx), \tag{2.9}$$

where $*$ denotes the convolution operator, $\nu_{t,i}^{\mathbb{P}}$ controls the intensity of different sources of jumps in the economy (e.g., jumps in consumption growth), which by assumption takes the form,

$$\nu_{t,i}^{\mathbb{P}}(x) = (\alpha_i' \mathbf{V}_t)\nu_i^{\mathbb{P}}(x), \tag{2.10}$$

for some time-invariant jump intensity measures $\nu_i^{\mathbb{P}}(x)$ and the $\mathbf{V}_t$ vector of state variables that drive the dynamics of the fundamentals in the economy. The pricing kernel in this economy in turn implies that the jump intensity process under the risk-neutral probability measure $\mathbb{Q}$ takes the form,

$$\nu_t^{\mathbb{Q}}(dx) = \left(\nu_{t,1}^{\mathbb{Q}} * \cdots * \nu_{t,i}^{\mathbb{Q}} * \cdots * \nu_{t,n}^{\mathbb{Q}}\right)(dx), \tag{2.11}$$

where

$$\nu_{t,i}^{\mathbb{Q}}(x) = e^{\lambda_i x}\nu_{t,i}^{\mathbb{P}}(x). \tag{2.12}$$

Comparing (2.9) and (2.10) with (2.11) and (2.12), the pricing of all jump risk in this economy is formally based on exponential tilting of the $\mathbb{P}$ jump distribution, with the extent of the tilting and the pricing of the different sources of risks determined by the λ_i-s. The actual values of the λ_i-s will depend on the structural parameters and the risk aversion of the representative agent in particular. Importantly, the temporal variation in the priced jump risk is driven by the *same* factors that drive the actual market jump risks.

Further specializing this setup along the lines of the recent rare disaster models of Gabaix (2012) and Wachter (2013) involving a single source of (negative) jumps, the expression for the $\mathbb{Q}$ jump intensity simplifies to $\nu_t^{\mathbb{Q}}(x) = e^{-\gamma x}\nu_t^{\mathbb{P}}(x)$, where γ refers to the risk-aversion of the representative agent. It follows readily from the definition of $LJP_{t,\tau}$ that in this situation,

$$LJP_{t,\tau} = \frac{1}{\tau}\int_t^{t+\tau}\int_{\mathbb{R}} x^2\left(\mathbb{E}_t^{\mathbb{P}}(\nu_s^{\mathbb{P}}(dx)) - \mathbb{E}_t^{\mathbb{Q}}(\nu_s^{\mathbb{P}}(dx))\right)ds + \frac{1}{\tau}\int_t^{t+\tau}\int_{\mathbb{R}}(1-e^{-\gamma x})x^2\mathbb{E}_t^{\mathbb{Q}}(\nu_s^{\mathbb{P}}(dx)). \tag{2.13}$$

The second term on the right-hand-side arises solely from the representative agent's special attitude towards jump risk. Moreover, as this expression shows, any variation in this term is intimately related to the state variables that drive the fundamentals in the economy.

As an alternative equilibrium framework, consider now the generalization of the habit formation model of Campbell and Cochrane (1999) recently proposed by Du (2010), in which the representative agent faces disaster risks in consumption. In this setup consumption growth is assumed to be *i.i.d.* and subject to the possibility of rare disasters in the form of extreme negative jumps, while the agent's risk-aversion γ_t varies with the level of (external) habits determined by aggregate consumption. Correspondingly, the risk-neutral jump intensity may be expressed as,

$$\nu_t^{\mathbb{Q}}(x) = f(\gamma_t)\nu_t^{\mathbb{P}}(dx), \tag{2.14}$$

for some nonlinear function $f(\cdot)$. Within this model the pricing of jump risk is therefore directly related to γ_t and the pricing of risk in the economy more generally. In contrast to the framework based on an agent with Epstein-Zin preferences, the jump distribution also does not change between the $\mathbb{P}$ and $\mathbb{Q}$ measures. Again, from the definition of $LJP_{t,\tau}$ it follows that in this situation,

$$LJP_{t,\tau} = \frac{1}{\tau}\int_t^{t+\tau}\int_{\mathbb{R}} x^2(\mathbb{E}_t^{\mathbb{P}}(\nu_s^{\mathbb{P}}(dx)) - \mathbb{E}_t^{\mathbb{Q}}(\nu_s^{\mathbb{P}}(dx)))ds + \frac{1}{\tau}\int_t^{t+\tau}\int_{\mathbb{R}} x^2\mathbb{E}_t^{\mathbb{Q}}[(1-f(\gamma_s))\nu_s^{\mathbb{P}}(dx)]ds. \tag{2.15}$$

Thus, unlike the Epstein-Zin setup discussed above where the temporal variation in the second term that reflects the special attitude towards jump risk is driven solely by $\nu_t^{\mathbb{P}}(x)$, this term now also varies explicitly with the time-varying risk-aversion of the representative agent.

However, since $\nu_t^{\mathbb{P}}(x)$ and $f(\gamma_t)$ both depend nonlinearly on the risk-aversion coefficient, $LJP_{t,\tau}$ may simply be expressed as a nonlinear function of γ_t. The market volatility in this economy also depends nonlinearly on γ_t. Consequently, $LJP_{t,\tau}$ and the market volatility are effectively "tied" together in a nonlinear relationship.

Even though the exact form and interpretation of the $LJP_{t,\tau}$ measure differ across the different equilibrium settings, it clearly conveys important information about the pricing of tail risk in the economy. We turn next to a discussion of the new tail approximations and related estimation procedures that we use for empirically quantifying $LJP_{t,\tau}$ and the other tail risk measures introduced above.

3 Jump Tail Estimation

Our estimation of the $\mathbb{Q}$ jump tail measures builds on the specification for the $\nu_t^{\mathbb{Q}}(dx)$ jump intensity process proposed by Bollerslev and Todorov (2014),

$$\nu_t^{\mathbb{Q}}(dx) = \left(\phi_t^+ \times e^{-\alpha_t^+ x} 1_{\{x>0\}} + \phi_t^- \times e^{-\alpha_t^- |x|} 1_{\{x<0\}}\right) dx. \tag{3.1}$$

This specification explicitly allows the left ($^-$) and right ($^+$) jump tails to differ. Although it formally imposes the same structure on *all* sized jumps, the results that follow only requires that $\nu_t^{\mathbb{Q}}(x)$ satisfies (3.1) for "large" jumps beyond some threshold, say $|x| > k_t$.

The specification in (3.1) is very general, allowing for two separate sources of independent variation in the jump tails, in the form of "level shifts" governed by $\phi_t^{\pm}$, and shifts in the rate of decay, or the "shape," of the tails governed by $\alpha_t^{\pm}$. By contrast, the assumption of constant tail shape parameters, or $\alpha_t^+ = \alpha_t^- = \alpha$, employed in essentially all parametric models estimated in the literature to date imply that the relative importance of differently sized jumps is time invariant, so that the only way for the intensity of "large" sized jumps to change over time is for the intensity of *all* sized jumps to change proportionally.[19] In most models hitherto employed in the literature that do allow for temporal variation in the jump intensity process $\nu_t^{\mathbb{Q}}(dx)$, it is also assumed that the dynamic dependencies in the left and right tails may be described by the identical level-shift process, with the temporal variation

[19]This includes the affine jump diffusion models of Duffie, Pan, and Singleton (2000), the time-changed tempered stable models of Carr, Geman, Madan, and Yor (2003), along with the nonparametric estimation procedure employed in Bollerslev and Todorov (2011b).

in $\phi_t^+ = \phi_t^-$ driven by a simple affine function of the diffusive variance σ_t^2.[20] By contrast, the temporal variation in $\phi_t^\pm$ is left completely unspecified in the present setup.

The jump intensity process in (3.1) readily allows for closed-form solutions for the integrals that define $LJV_{[t,t+\tau]}^{\mathbb{Q}}$ and $RJV_{[t,t+\tau]}^{\mathbb{Q}}$ in equation (2.6) in terms of the $\alpha_t^\pm$ and $\phi_t^\pm$ tail parameters and the cutoff k_t defining "large" jumps. In particular, assuming that the tail parameters remain constant over the horizon τ, the left and right jump tail variation measures may be succinctly expressed as,

$$\begin{aligned} LJV_{[t,t+\tau]}^{\mathbb{Q}} &= \tau\phi_t^- e^{-\alpha_t^-|k_t|}(\alpha_t^- k_t(\alpha_t^- k_t - 2) + 2)/(\alpha_t^-)^3, \\ RJV_{[t,t+\tau]}^{\mathbb{Q}} &= \tau\phi_t^+ e^{-\alpha_t^+|k_t|}(\alpha_t^+ k_t(\alpha_t^+ k_t + 2) + 2)/(\alpha_t^+)^3. \end{aligned} \tag{3.2}$$

Our estimation of $\alpha_t^\pm$ and $\phi_t^\pm$, and in turn the $LJV_{[t,t+\tau]}^{\mathbb{Q}}$ and $RJV_{[t,t+\tau]}^{\mathbb{Q}}$ measures, will be based on out-of-the-money (OTM) puts and calls for the left and right tails, respectively. Intuitively, the $\alpha_t^\pm$ parameters may be uniquely identified from the rate at which the prices of the options decay in the tail, while for given tail shapes the $\phi_t^\pm$ parameters may be inferred from the actual option price levels.

Formally, let $O_{t,\tau}(k)$ denote the time t price of an OTM option on X with time to expiration τ and log-moneyness k. It follows then from Bollerslev and Todorov (2011b) that for two put options with the same maturity $\tau \downarrow 0$, but different strikes $k_1 \downarrow -\infty < k_2 \downarrow -\infty$, $\log(O_{t,\tau}(k_2)/O_{t,\tau}(k_1)) \approx (1 + \alpha_t^-)(k_2 - k_1)$. Similarly, for two call options with strikes $k_1 \uparrow \infty < k_2 \uparrow \infty$, $\log(O_{t,\tau}(k_2)/O_{t,\tau}(k_1)) \approx (1 - \alpha_t^+)(k_2 - k_1)$. Utilizing these approximations, Bollerslev and Todorov (2014) show how the time-varying tail shape parameters $\alpha_t^\pm$ may be consistently estimated from an ever increasing number of deep OTM short-maturity options by,[21]

$$\widehat{\alpha}_t^\pm = \operatorname{argmin}_{\alpha^\pm} \frac{1}{N_t^\pm} \sum_{i=1}^{N_t^\pm} \left| \log\left(\frac{O_{t,\tau}(k_{t,i})}{O_{t,\tau}(k_{t,i-1})}\right)(k_{t,i} - k_{t,i-1})^{-1} - \big(1 \pm (-\alpha^\pm)\big) \right|, \tag{3.3}$$

where $N_t^\pm$ denotes the total number of calls (puts) used in the estimation with moneyness $0 < k_{t,1} < ... < k_{t,N_t^+}$ $(0 < -k_{t,1} < ... < -k_{t,N_t^-})$. In the results reported on below, we implement this estimator on a weekly basis, thus implicitly assuming that the $\alpha_t^\pm$ parameters only change from week to week.

[20] This approach is exemplified by the jump-diffusion models estimated in Pan (2002) and Eraker (2004).

[21] The use of a robust M-estimator effectively downweighs the influence of any "outliers."

The estimates for $\alpha_t^\pm$ in (3.3) put no restrictions on the $\phi_t^\pm$ parameters that shift the level of the jump intensity process through time. Meanwhile, let $r_{t,\tau}$ denote the risk-free interest rate over the $[t, t+\tau]$ time-interval, and $F_{t,\tau}$ the time t futures price of $X_{t+\tau}$. It then follows from Bollerslev and Todorov (2014) that for $\tau \downarrow 0$ and $k < 0$, $e^{r_{t,\tau}} O_{t,\tau}(k)/F_{t-,\tau} \approx \tau\phi_t^- e^{k(1+\alpha_t^-)}/(\alpha_t^-(\alpha_t^- + 1))$, while for $k > 0$, $e^{r_{t,\tau}} O_{t,\tau}(k)/F_{t-,\tau} \approx \tau\phi_t^+ e^{k(1-\alpha_t^+)}/(\alpha_t^+(\alpha_t^+ - 1))$. Utilizing these approximations, the "level shift" parameters may be estimated in a second step by,

$$
\begin{aligned}
\widehat{\phi}_t^\pm = \operatorname{argmin}_{\phi^\pm} \frac{1}{N_t^\pm} \sum_{i=1}^{N_t^\pm} \Bigg| \log\left(\frac{e^{r_{t,\tau}} O_{t,\tau}(k_{t,i})}{\tau F_{t-,\tau}}\right) - \left(1 \mp \widehat{\alpha}_t^\pm\right) k_{t,i} \\
+ \ \log\left(\widehat{\alpha}_t^\pm \mp 1\right) + \log\left(\widehat{\alpha}_t^\pm\right) - \log(\phi^\pm) \Bigg|.
\end{aligned} \tag{3.4}
$$

Taken together these estimates completely characterize the $\mathbb{Q}$ jump intensity process in (3.1), and in turn all of the jump tail risk measures defined in Section 2.

4 Data

The data used in our empirical analysis comes from three different sources. The raw options data is obtained from OptionMetrics, and consists of closing bid and ask quotes for all S&P 500 options traded on the Chicago Board of Options Exchange (CBOE), along with the corresponding zero coupon rates. The options span the period from January 1996 to August 2013, for a total of 4,445 trading days.[22]

The estimates for the jump tail parameters in (3.3) and (3.4) formally rely on an increasing number of arbitrarily short-lived OTM options to eliminate the impact of the diffusive price component. In an effort to best mimic this condition, we restrict our analysis to options with no more than 45 days until expiration. To help alleviate the impact of market microstructure complications for the shortest lived options, we also rule out any options with less than eight days to maturity. In practice, of course, for a given fixed maturity, these OTM option prices will still reflect some diffusive risk. To help mitigate this risk, for the estimation of the left jump tail parameters, we only use puts with log-moneyness

[22]Following standard "cleaning" procedures to rule out arbitrage, starting from the closest at-the-money options we omit any out-of-the-money options for which the midquotes do not decrease with the strike price. We also omit any zero bid option prices.

less than minus two-and-a-half times the maturity-normalized Black-Scholes at-the-money implied volatility. Similarly, for the right jump tail parameters, we only use call options with log-moneyness in excess of the maturity-normalized Black-Scholes implied volatility.[23] In the end, this leaves us with an average of 100.2 and 51.0 puts and calls per week, respectively, over the full sample.

Our construction of the actual realized variation measures and the variance risk premium rely on high-frequency S&P 500 futures prices obtained from Tick Data Inc. The intraday prices are recorded at five-minute intervals, starting at 8:35 CST until the last price of the day at 15:15 CST, for a total of 81 observations per trading day. We also use these same high-frequency data in testing whether the option-based $\mathbb{Q}$ jump tail expectations are consistent with the subsequently observed $\mathbb{P}$ jump tail realizations.

Our aggregate market return predictability regressions are based on a broad value-weighted portfolio of all CRSP firms incorporated in the U.S. and listed on the NYSE, AMEX, or NASDAQ stock exchanges. The relevant time series of daily returns are obtained from Kenneth R. French's data library.[24] We also rely on that same data source for daily returns on various size, book-to-market and momentum sorted portfolios. Lastly, we obtain data on the monthly dividend-price ratio for the aggregate market from CRSP.

5 Empirical Tail Measures

The left and right $\mathbb{Q}$ jump variation measures introduced above, including the approximate *fear* component in (2.8), may all be expressed as explicit functions of the jump tail parameters in (3.1). We begin our empirical analysis with a discussion of these parameters and the time-varying left and right "large" jump intensities implied by the estimates.

[23]By explicitly relating the threshold of the moneyness for the options used in the estimation to the overall level of the volatility, we screen out more relatively close to at-the-money options in periods of high volatility, thereby effectively minimizing the impact of the on average larger diffusive price component in the OTM option price when the volatility is high. Since the market for call options is less liquid than the market for puts, we rely on a more lenient cutoff for the right tail estimation.

[24]Website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french.

5.1 Tail Parameters

Our estimates for the weekly left and right jump tail "shape" parameters are based on equation (3.3) and all of the qualifying options within each calendar week. The resulting sample mean of $\hat{\alpha}_t^-$ equals 16.23 compared to 61.81 for $\hat{\alpha}_t^+$, indicative of the on average much slower tail decay inherent in the put versus call OTM option prices. Further to this effect, the top two panels in Figure 1 show $1/\hat{\alpha}_t^{\pm}$ corresponding to the left and right jump *tail indexes*. The estimates for the left tail index varies almost ten-fold over the sample, ranging from a low of around 0.03 in 1997 and 2007, to a high of more than 0.25 in 2008-09 at the height of the recent financial crisis. Although less dramatic, the estimates for the right tail index also exhibit substantial variation over time. These temporal dependencies are directly manifest in the form of first order autocorrelations for the left and right tail "shape" parameters equal to 0.59 and 0.67, respectively.[25]

The jump intensity process, of course, also depends on the "level" parameters. Our weekly estimates for these are based on the expression in (3.4). Rather than plotting the estimates for $\phi_t^{\pm}$, the bottom two panels in Figure 1 show the annualized left and right "large" jump intensities implied by $\widehat{\alpha}_t^{\pm}$ and $\widehat{\phi}_t^{\pm}$,

$$LJI_t = \int_{x<-|k_t|} \nu_t^{\mathbb{Q}}(dx) = \widehat{\phi}_t^- e^{-\widehat{\alpha}_t^-|k_t|}/\widehat{\alpha}_t^-, \qquad RJI_t = \int_{x>|k_t|} \nu_t^{\mathbb{Q}}(dx) = \widehat{\phi}_t^+ e^{-\widehat{\alpha}_t^+|k_t|}/\widehat{\alpha}_t^+. \quad (5.1)$$

The calculation of these measures also necessitates a choice for the cutoff k_t pertaining to the log-jump size and the start of the jump "tails." For both of the plots in the figure, as well as the RJV_t and LJV_t jump variation measures reported on below, we fix k_t at 6.868 times the normalized Black-Scholes ATM volatility at time t. This specific cutoff corresponds to the median strike price for the deepest OTM puts in the sample.[26]

Allowing the $\alpha_t^{\pm}$ tail "shape" parameters to vary over time, results in fairly stable and

[25] Our finding of time-varying $\alpha_t^{\pm}$ parameters is consistent with the evidence for serially correlated "extreme" returns based on the so-called extremogram estimator in Davis and Mikosch (2009) and Davis et al. (2012). The recent cross-sectional based tail index estimates reported in Chollete and Lu (2011), Kelly and Jiang (2014) and Ruenzi and Weigert (2011) also point to strong dynamic dependencies. All of these studies, however, pertain to the actual return distributions and the shape of the tails under $\mathbb{P}$. Recent studies that have estimated somewhat simpler dynamic dependencies in the tails under $\mathbb{Q}$ include Almeida, Vicente, and Guillen (2013), Du and Kapadia (2012), Hamidieh (2011), Siriwardane (2013), and Vilkov and Xiao (2013).

[26] We also experimented with other choices for this "tail" cutoff, resulting in qualitatively very similar dynamic features and predictability regressions to the ones reported below. Further details concerning these additional results are available in a Supplementary Appendix.

mildly serially correlated intensities for the "large" negative jumps. Meanwhile, there is a sense of "euphoria" and relatively high jump intensities for the "large" positive jumps embedded in the OTM call option prices leading up to the financial crisis. Of course, the right jump tail intensities are orders of magnitude less than those for the left jump tail. We turn next to a discussion of the jump tail variation measures and risk premia implied by these estimates for the $\nu_t^{\mathbb{Q}}(dx)$ "large" jump intensity process.

5.2 Jump Tail Variation Measures

Our estimates for the weekly left and right $\mathbb{Q}$ jump variation measures, as implied by equation (3.2), are depicted in Figure 2. Looking first at LJV_t in the top panel, the measure inherits many of the same key dynamic dependencies evident in the left tail index shown in the top left panel in Figure 1. However, referring to Panel B in Table 1, the sample correlation between LJV_t and LJI_t is only equal to 0.26. By contrast, the correlation between RJV_t and the right tail intensity RJI_t equals 0.89. Of course, as Figure 2 and Table 1 both make clear, RJV_t is orders of magnitude less than LJV_t, so the *fear* component defined as the difference between the two is effectively equal to $-LJV_t$, as previously stated in (2.8).

To underscore the importance of explicitly allowing both the "shape" and the "level" of the jump tails to change over time in the estimation of this new *fear* component, the left panel in Figure 3 shows the estimates for the left jump tail variation LJV_t^* obtained by restricting $\alpha_t^- = \alpha^-$ to be constant, but allowing ϕ_t^- to change over time. Correspondingly, the right panel shows the estimates for LJV_t^{**} obtained by restricting $\phi_t^- = \phi^-$ to be constant, but allowing α_t^- to be time-varying. Restricting the "shape" parameter to be constant, as is commonly done in the literature, clearly mutes the temporal variation and cuts the sample standard deviation of LJV_t^* in half compared to LJV_t. By contrast, restricting the temporal variation to be solely driven by the "shape" of the jump tails, results in an even more dramatic increase in the magnitude of the *fear* component during the recent financial crisis. Along these lines, it is also worth noting that the first order sample autocorrelation for LJV_t is larger than the autocorrelations of both LJV_t^* and LJV_t^{**}. Consistent with the return predictability results discussed below, LJV_t also correlates more strongly with LJV_t^{**} than

LJV_t^*.[27]

The stylized equilibrium models discussed in Section 2.3 imply that the variation in LJV_t, is a direct, possibly nonlinear, function of the spot volatility. To investigate this conjecture empirically, Figure 4 presents the results from a nonparametric kernel regression of our nonparametric estimate of LJV_t on the at-the-money implied variance from the shortest-maturity options available on the day (with at least eight days to maturity), where the latter serves as a proxy for the unobservable spot volatility.[28,29] As the figure shows, there is a substantial amount of variation in LJV_t that cannot be explained by the current market volatility, even when allowing for a highly nonlinear relation between the two series. Further, as directly seen from the right panel in Figure 4, forcing LJV_t to have the same value for a given market volatility produces a fitted variation measure with a much more pronounced spike than the actual LJV_t series in the aftermath of the dot-com bubble and the mild economic recession in the early 2000s. On the other hand, since the spot volatility is generally faster mean reverting than the actual LJV_t series, the nonlinear projection of LJV_t on the volatility series results in a shorter-lived impact of the recent financial crises. In sum, LJV_t contains its own unique dynamic dependencies and which cannot be spanned by the volatility.

5.3 Jump Tail Variation and Return Correlations

The sample correlations between the weekly returns on the aggregate market portfolio MRK and the different jump tail variation measures, reported in the first row in Panel B of Table 1, are all negative.[30] This mirrors the contemporaneous asymmetric return-volatility relationship, or so-called "leverage effect," widely documented in the literature for other volatility measures and models; see, e.g., the discussion in Bollerslev, Sizova, and Tauchen (2012) and

[27]The correlation between LJV_t and the *fear index* estimated in Bollerslev and Todorov (2011b) relying on long-span asymptotics and the more restrictive assumption of constant tail "shape" parameters equals 0.75.

[28]The reported kernel density estimates are based on a Gaussian kernel with the bandwidth parameter set according to the prescription in Bowman and Azzalini (1997). We also experimented with the use of alternative nonparametric estimates for the spot volatility obtained from high-frequency data on the S&P 500 index futures, resulting in very similar nonparametric regression estimates for LJV_t.

[29]As the time to maturity converges to zero, the at-the-money implied volatility formally converges to the diffusive spot volatility; see, e.g., Durrleman (2008).

[30]All of the weekly variation measures are based on data available at the 15:15 CST close of the CBOE on Fridays, while the weekly aggregate market returns span the period from 16:00 EST the previous Monday to 16:00 EST the following Monday.

the references therein. At the same time, the contemporaneous correlations between the different tail variation measures and the weekly returns on the SMB, HML and WML zero-cost portfolios, further analyzed below, are all smaller (in absolute value) and some even positive.

Meanwhile, with the exceptions of RJV_t, the sample correlations between the jump tail variation measures and the market return over the *subsequent* week, reported in the first row in Panel C of Table 1, are all positive. This suggests that a risk-return tradeoff, or "volatility feedback effect," may also be operative, whereby an increase (decrease) in one of the variation measures causes an immediate drop (rise) in the price in order to allow for higher (lower) future returns as a compensation for the increased (decreased) risk. Of course, these unconditional sample correlations do not distinguish whether the higher (lower) returns are indeed associated with an increase (decrease) in systematic risk or a change in the attitude towards risk, or both.

5.4 Tail "Shape" Variation: Risk or Attitude to Risk?

Our interpretation of LJV_t as a measure of market *fears* hinges on the standard no-arbitrage condition and the fact that it does not restrict the form of the $dt \otimes \nu_t^{\mathbb{Q}}(dx)$ jump compensator for the "large" sized jumps in (2.4) vis-a-vis the $dt \otimes \nu_t^{\mathbb{P}}(dx)$ jump compensator in (2.1). If, on the other hand, jumps and diffusive price moves were treated as identical risks by investors, the jump intensity process should be the same under the $\mathbb{P}$ and $\mathbb{Q}$ measures. Consequently, the mapping from $\nu_t^{\mathbb{P}}(dx)$ to $\nu_t^{\mathbb{Q}}(dx)$ directly reflects the "special" compensation for jump tail risk in the economy, as exemplified by the exponential tilting in equation (2.12) implied by the stylized long-run risk and rare disaster models, or the proportional shift from $\mathbb{P}$ to $\mathbb{Q}$ in equation (2.14) implied by the habit formation model.

The estimation of a general process for $\nu_t^{\mathbb{P}}(dx)$ that parallels that of $\nu_t^{\mathbb{Q}}(dx)$ in (3.1) is inevitable plagued by a dearth of "large" jump tail realizations over short weekly time intervals. Instead, as a way to meaningfully test whether the "shape" of the risk-neutral and actual jump tails are indeed the same, as implied for example by the habit persistence model with measure change given in (2.14), we consider the time-series of actual high-frequency-based tail realizations over the full sample. In particular, let η_s denote the threshold for defining the "large" negative jump realizations. Provided the jump tail "shape" parameter for $\nu_t^{\mathbb{P}}(dx)$ equals α_t^-, the integral pertaining to the realized jumps,

$$\int_t^{t+\tau} \int_{x<-\eta_s} \left[|x| - \frac{(1+\alpha_t^- \eta_s)}{\alpha_t^-} \right] \mu(ds, dx),$$

should then be a martingale under the statistical probability measure $\mathbb{P}$.[31] Importantly, this condition does not depend on the overall "level" of the actual jump tails, but only on their "shape."

Substituting the weekly estimates of $\hat{\alpha}_t^-$ for the $\mathbb{Q}$ jump tails in place of α_t^-, the above martingale condition is readily operationalized as a test for identical jump tail "shapes" under the $\mathbb{P}$ and $\mathbb{Q}$ measures. Specifically, define the vector of sample moments,

$$\widehat{\mathbf{m}} = \frac{1}{T} \left(\sum_{s \in [0,T): \Delta \log(X_s) < -\eta_s} \left[|\Delta \log(X_s)| - \frac{(1+\hat{\alpha}_s^- \eta_s)}{\hat{\alpha}_s^-} \right] \otimes \psi_s \right),$$

where ψ_s refers to any vector of valid instruments. Also, let $\widehat{W}$ denote an estimate for the corresponding asymptotic variance-covariance matrix for $\widehat{\mathbf{m}}$.[32] If the martingale condition is satisfied, $T\widehat{\mathbf{m}}'\widehat{W}^{-1}\widehat{\mathbf{m}}$ should be (asymptotically for increasing sample size T) distributed as a Chi-square distribution with degrees of freedom equal to the dimension of the instrument vector ψ_s.

In implementing the test we rely on high-frequency intraday five-minute returns along with a time-varying threshold for determining the "large" sized jumps.[33] In particular, fixing η_s at six times the estimated continuous five-minute return variation, together with a two-dimensional instrument vector comprised of a constant and an estimate of the integrated volatility over the previous day relative to the occurrence of a jump at time s, results in a test statistic of 175.6, thus strongly rejecting the null hypothesis that diffusive and jump

[31] An analogous martingale condition obviously holds for the right jump tails.

[32] By standard arguments, the covariance matrix may be consistently estimated by,

$$\widehat{W} = \frac{1}{T} \left(\sum_{s \in [0,T): \Delta \log(X_s) < -\eta_s} \left(|\Delta \log(X_s)| - \frac{(1+\hat{\alpha}_s^- \eta_s)}{\hat{\alpha}_s^-} \right)^2 \otimes \psi_s \psi_s' \right).$$

[33] The threshold that we use explicitly adjust for the temporal variation in the daily continuous volatility based on the realized bipower variation over the previous day as well as the strong intraday volatility pattern based on an estimate of the time-of-day effect; for additional details see Bollerslev and Todorov (2011b).

risks are treated the same by investors.[34]

At a general level, the test therefore also supports the idea that LJV_t affords a "cleaner" proxy for market *fears* than the variance risk premium, let alone the popularly used VIX "investor fear gauge." To further explore this, we turn next to the results from a series of standard monthly-based return predictability regressions for returns horizons ranging up to a year, using different variation measures as explanatory variables.

6 Return Predictability Regressions

Several recent studies have argued for the existence of a statistically significant link between the variance risk premium and future returns on the aggregate market portfolio, with this predictive relationship especially strong over 3-6 months horizons (see, e.g., Bollerslev, Tauchen, and Zhou, 2009; Drechsler and Yaron, 2011; Du and Kapadia, 2012; Bekaert and Hoerova, 2014; Bollerslev, Marrone, Xu, and Zhou, 2014; Camponovo, Scaillet, and Trojani, 2013; Vilkov and Xiao, 2013, among several other studies). The results from our predictability regressions complement and expand on these findings by explicitly considering the new jump tail variation measures, and the part of the variance risk premium due to jump tail risk, as separate predictor variables.

Following standard practice in the literature, we rely on a monthly observation frequency for all of our return predictability regressions. Specifically, let $r_{[t,t+\tau]}$ denote the continuously compounded return from time t to $t+\tau$ formally defined in equation (2.2) above, with the unit time interval corresponding to a month. The return regressions discussed below may then be expressed as,

$$r_{[t,t+h]} \;=\; a_h \;+\; b_h V_t \;+\; u_{t,t+h}, \qquad t = 1, 2, ..., T-h, \tag{6.1}$$

where V_t refers to one or more of the variation measures and other explanatory variables, and the horizon range from $h = 1$ (one-month) to $h = 12$ (one year). To account for the overlap that occur for $h > 1$, we rely on the standard robust Newey-West t-statistics with

[34] This particular choice of threshold results in a total of 285 left tail jumps over the full sample. We also experimented with other choices of η_s, resulting in equally strong rejections. The corresponding test for the right jump tails equals 87.0, which also strongly rejects the null of identical jump tail "shapes" under the $\mathbb{P}$ and $\mathbb{Q}$ measures.

a lag length equal to two times the return horizon. In addition, to help alleviate concerns about size distortions and over-rejections known to plague inference in overlapping return regressions with persistent predictor variables (see, e.g., Ang and Bekaert, 2007), following Hodrick (1992) we also report robust t-statistics for the null of no predictability from the reverse regressions.[35] Further, to aid with the interpretation of the results from the multiple regressions involving more than one explanatory variable, we report Newey-West and Hodrick Wald-tests for the null of no predictability by *any* of the predictor variables included in the regression.[36] We turn next to a discussion of the specific explanatory variables considered in the return predictability regressions.

6.1 Variation Measures and Other Explanatory Variables

Our estimation of the jump tail parameters $\hat{\alpha}_t^{\pm}$ and $\hat{\phi}_t^{\pm}$, and the resulting new jump tail variation measures discussed above, closely follows the approach developed in Bollerslev and Todorov (2014) and the choice of a weekly estimation frequency advocated therein. Even though this results in unbiased parameter estimates, the estimates are invariably somewhat noisy. To help smooth out this estimation error, and directly match the observation frequency of the jump tail variation measures to the observation frequency of the return regressions, in the empirical results reported on below we rely on the monthly variation measures obtained by averaging the within month weekly values.[37] Comparing the plot for the resulting monthly LJV_t series given in the top panel in Figure 5 to the weekly series shown in the top panel in Figure 2 clearly shows the effect of smoothing out the estimation error, and the implicit use of a coarser monthly estimation frequency. Still, the weekly and monthly series obviously share the same general features and dynamic dependencies.

[35] As forcefully emphasized by Hodrick (1992), the Hodrick-based t-statistics are only formally valid under the null of *no* predictability, not just by the regressor in question but by *any* predictor variable. As such, they are difficult to interpret even in simple regressions when more than one explanatory variable is useful for predicting the returns, let alone in multiple regressions. By contrast, the Newey-West t-statistics are always (asymptotically) justified and interpretable.

[36] With two predictor variables, the five- and one-percent critical values in the corresponding asymptotic Chi-square distribution with two degrees of freedom equal 5.991 and 9.210, respectively.

[37] For reasons of symmetry, we apply the same monthly averaging to all of the variation measures used as explanatory variables in the return regressions. We also performed predictive regressions where VRP_t and VIX_t^2 were not averaged over the month with the results being very similar to the ones reported below for their monthly-averaged counterparts.

The second panel in Figure 5 plots the monthly VIX_t^2 series. The VIX, of course, represents an approximation to the risk-neutral expectation of the total quadratic variation in (2.3), and as such reflects the compensation for both time-varying diffusive volatility and jump intensity risks, as well as market expectations about future jump tail events and the "special" pricing thereof. Meanwhile, comparing our LJV_t *fear* proxy to the VIX_t^2 reveals a strong coherence between the two series. At the same time, however, there are also some important differences. In particular, even though both series attained their maximum in-sample values around October 2008, the VIX fairly quickly reverted to pre-crisis levels, while LJV_t remained elevated for a much longer period of time. LJV_t also experienced another more prolonged period of elevated jump tail risk in 2010 in connection with the European sovereign debt crisis. Similarly, LJV_t remained unusually high over a longer period of time than VIX_t^2 in the aftermath of the East Asian crises starting in July 1997 and the August 1998 Russian default. Conversely, the collapse of the tech bubble and the declining equity valuations in 2002 clearly resulted in higher values of the VIX, but hardly affected the left jump variation.[38]

The third panel in Figure 5 shows the variance risk premium minus the left jump tail variation $VRP_t - LJV_t$, or the part of the variance risk premium attributable to "normal" sized price fluctuations.[39] Our estimate of the variance risk premium is based on the difference between the VIX_t^2 and the expectation of the forward variation aggregated over the month, as derived from the same multivariate forecasting model for the high-frequency based realized variation measures developed in Bollerslev and Todorov (2011b).[40] Interestingly, the figure reveals a much less dramatic impact of the recent financial crisis on $VRP_t - LJV_t$ than on LJV_t and VIX_t^2. On the other hand, $VRP_t - LJV_t$ was relatively high for part of 2002-03, while this period hardly registered for the left jump tail variation measure. These differences in the variation measures also directly manifest in the return predictability regressions

[38]These differences between the VIX and LJV mirror the differences between the option-based estimates for the time-varying diffusive and jump intensity risks in the fully parametric three-factor stochastic volatility model recently estimated by Andersen et al. (2014).

[39]Following a number of recent studies in the empirical asset pricing literature, we will refer to $-VRP_t$ as the variance risk premium, and correspondingly $VRP_t - LJV_t$ as the part of the premium due to "normal" sized price moves. This, of course, is immaterial for the fit of the return predictability regressions.

[40]Consistent with the idea that on average it is profitable to sell volatility, the sample mean of VRP_t equals 1.20 in annualized percentage form.

discussed below.

As previously noted, the log dividend-price ratio arguable represents the most widely used predictor variable in the literature. Although, it is commonly thought that the dividend-price ratio offers the most predictability over longer multi-year horizons, the results in Ang and Bekaert (2007) suggest that the predictability is actually the strongest over shorter within-year horizons as analyzed here. In parallel to the variation-based measures, the log dividend-price ratio $log(D_t/P_t)$ shown in the bottom panel in Figure 5 also increased quite dramatically during the recent financial crisis and the accompanying precipitous drop in equity valuations. Meanwhile, comparing the plot of $log(D_t/P_t)$ to the plots of the variation-based measures in the first three panels reveals a much more smoothly evolving series void of most of the other clearly discernible peaks associated with other readily identifiable economic and financial events.[41]

The next section presents the results from the return predictability regressions based on our new LJV_t measure and these other explanatory variables. We begin by discussing the results for the aggregate market portfolio MRK.

6.2 Aggregate Market Return

As noted above, the predictability afforded by the variance risk premium appears to be the strongest over intermediate 3-6 months return horizons. Focussing on the 6-months horizon, the left panel in Table 2 reports the results from simple predictability regressions with standard Newey-West t-statistics in parentheses and Hodrick t-statistics in square brackets. The sixth column, in particular, confirms the existing empirical evidence that a higher (lower) variance risk premium tends to be associated with higher (lower) returns over the next six months. Meanwhile, comparing the results to the regression reported in the first column, the degree of predictability inherent in VRP_t is dominated by that of the LJV_t left jump tail variation measure. Further to this effect, subtracting LJV_t from VRP_t results in less significant t-statistic for $VRP_t - LJV_t$, and lowers the R^2 relative to the regression based on VRP_t by a factor of a-half.

[41]The first order autocorrelation for each of the four monthly series shown in Figure 5 equal 0.67, 0.79, 0.53, and 0.96, respectively.

The regressions for the left jump variation measures LJV_t^* and LJV_t^{**} that restrict either α_t^- or ϕ_t^- to be constant show that allowing for temporal variation in both the "shape" and the "level" of the jump tails enhances the predictability. Indeed, the regression coefficient associated with the LJV_t^* jump tail variation measure extracted under the conventional assumption of time-invariant tail "shapes" is insignificant.[42] Also, the right jump variation measure RJV_t does not help in predicting the future returns.

The results from the multiple regressions reported in the right panel of the table further corroborates these findings. Including either RJV_t or $VRP_t - LJV_t$ in a multiple regression together with LJV_t, leaves only LJV_t significant.[43] Also, even though the inclusion of VIX_t^2 in the multiple regression with LJV_t renders the t-statistics for LJV_t insignificant at the conventional five-percent level, the joint Wald-tests for the null of no predictability are overwhelmingly significant. Moreover the R^2 from the multiple regression based on LJV_t and VIX_t^2 hardly increases relative to the R^2 from the simple regression based on LJV_t only.

Altogether, this suggests that much of the return predictability previously ascribed to the variance risk premium is effectively coming from the part of the premium due to the left jump tail variation. Further along these lines, it is worth noting that replacing LJV_t with the nonparametric kernel density estimate thereof discussed in Section 5.2, reduces the R^2 from 6.54 to 4.54. Moreover, when including both the fitted value of LJV_t and the corresponding residual from the nonparametric regression as explanatory variables in the same return predictability regression, both remain significant with Newey-West t-statistics of 2.17 and 2.13, respectively, underscoring the fact that LJV_t and the predictability therein cannot be spanned by the market spot volatility.

As an additional robustness check, we also include the log dividend-price ratio as a possible predictor variable. In line with the existing literature (see, e.g., Ang and Bekaert, 2007, and the many additional references therein), the t-statistics for $log(D_t/P_t)$ in the simple

[42]The plots of $1/\hat{\alpha}_t^-$ and LJI_t previously discussed in Figure 1 also suggest that most of the discernable variation in LJV_t reside in the "shape" as opposed to the intensity of the "large" negative jumps.

[43]Including LJV_t and VRP_t in the same regression, obviously results in the same R^2 as the regressions based on LJV_t and $VRP_t - LJV_t$. The Newey-West t-statistics for VRP_t and $VRP_t - LJV_t$ are also the same, while the t-statistics for LJV_t drops from 4.39 for the regression reported in Table 2 to 3.66 for the regression based on LJV_t and VRP_t.

regression reported in the last column in the first part of Table 2 are highly significant. Consistent with the results for the return horizons in excess of three months reported in Bollerslev, Tauchen, and Zhou (2009), the R^2 of 18.9 also far exceeds that of the variance risk premium and the other return variation measures. Meanwhile, including $log(D_t/P_t)$ and LJV_t in the same multiple regression further increases the R^2 to 21.4, and even though the t-statistics for $log(D_t/P_t)$ and LJV_t are reduced somewhat compared to the two simple regressions, the Wald tests for their joint significance are highly significant. As such, this supports the idea that the dividend yield is not able to fully span the predictive information in the LJV tail variation measure. This, of course, is also the case for the stylized theoretical equilibrium models discussed in Section 2.3.

Turning to Table 3 and the regression results for other return horizons reveal the same general patterns vis-a-vis the predictability in the variance risk premium and its jump tail component. In particular, while $VRP_t - LJV_t$ is significant in all of the simple regressions, except for the one-month horizon, the simple regressions based on LJV_t generally results in larger t-statistics and the R^2s are also much higher.[44] The plots of the corresponding Newey-West t-statistics and adjusted regression R^2s for all of the one through 12-months return regressions shown in Figure 6 further illustrate this. The t-statistics from the simple regressions based on LJV_t are all significant, and the R^2s increase with the return horizons. On the other hand, the R^2s from the simple regressions based on $VRP_t - LJV_t$ plateaus at the four-month horizon, while the R^2s from the multiple regressions based on both predictor variables are fairly close to those from the simple regressions based on LJV_t only.

The general setup and discussion in Section 2, along with the test in Section 5.4, suggest that the $VRP_t - LJV_t$ predictor variable may be interpreted as a measure of economic uncertainty, while LJV_t is more readily associated with notions of market *fears* and the special compensation for jump tail risk. To further buttress this interpretation and help explain where the predictability is coming from, we turn next to a series of predictability regressions for various portfolio sorts and the three common Fama-French-Carhart risk factors.

[44] To put the monthly R^2s reported in the table into perspective, Huang, Jiang, Tu, and Zhou (2013) have recently shown that a properly aligned version of the investor sentiment index originally proposed by Baker and Wurgler (2006) results in a statistically and economically significant predictive relationship for the monthly aggregate market returns with an R^2 of 1.54 percent.

6.3 Portfolio Sorts and Risk Factors

Different portfolios may respond differently to changes in risk and risk aversion depending on their risk exposures. To this end, Table 4 reports the results from the same multiple regressions based on LJV_t and $VRP_t - LJV_t$ reported in Tables 2 and 3, replacing the aggregate market portfolio with portfolios comprised of stocks sorted according to their market capitalization, book-to-market value, and most recent annual return. For considerations of space, we focus our discussion on the six equally weighted portfolios made up of the top and bottom quintiles for each of the three different sorts.

Beginning with the results pertaining to the size-sorted portfolios reported in the first two columns of Table 4, a clear distinction emerges in the influences of LJV_t and $VRP_t - LJV_t$. In particular, while $VRP_t - LJV_t$ is not as significant as LJV_t for the aggregate market portfolio, it is the most significant predictor for the small-stock portfolio. The regression R^2s for the small-stock portfolio reach an impressive 19.3 percent at the annual level. Meanwhile, the results for the large-stock portfolio more closely mirror those for the aggregate market, with the predictability primarily driven by LJV_t.

To purge the returns from systematic market risks, the third column reports the regression results from the Small Minus Big (SMB) zero-cost portfolio defined by the returns on the previously discussed small minus large-stock portfolios. In contrast to the aggregate market portfolio, the longer-run predictability for this portfolio is exclusively coming from the $VRP_t - LJV_t$ variation measure. The results from the simple regressions for the SMB portfolio shown in Figure 7 further underscore this point. While the t-statistics associated with $VRP_t - LJV_t$ for predicting the three through 12-months returns are all significant at conventional levels, the t-statistics for LJV_t are all insignificant. Moreover, the R^2s from the multiple regressions that include both $VRP_t - LJV_t$ and LJV_t, shown in the bottom panel of the figure, are very close to the R^2s from the simple regressions based on $VRP_t - LJV_t$ only.

The SMB portfolio has previously been associated with variables that describe changes in the investment opportunity set (e.g., Petkova, 2006). As such, our finding that the predictability of the SMB portfolio is solely driven by $VRP_t - LJV_t$ is consistent with the idea that this measure is most directly associated with notions of economic uncertainty. Along these lines, a number of studies have also argued that smaller firms tend to be more strongly affected by credit market conditions than larger firms and therefore also more susceptible to general economic conditions (see, e.g., Perez-Quiros and Timmermann, 2000), thus further helping to explain why $VRP_t - LJV_t$ is a better predictor for the return on the small-stock portfolio.

The results for the value and growth portfolios along with the corresponding High Minus Low (HML) book-to-market portfolio reported in the next three columns in Table 4 convey a very different picture. The predictability pattern for the low book-to-market portfolio, in particular, fairly closely mirror that of the aggregate market portfolio. On the other hand, the R^2s for the HML portfolio in Figure 8 appear to be maximized at the intermediate four months horizon, with *all* of the predictability attributable to the LJV_t jump tail variation measure.

The returns on portfolios comprised of growth stocks have previously been related to measures of funding liquidity risk (e.g., Asness et al., 2013). Thus, to the extend that liquidity conditions reflect market sentiment, the results in Figure 8 for the the HML portfolio again indirectly corroborate the interpretation of LJV_t as a measure of market *fears*.

Turning to the results for the momentum portfolios, reported in the last three columns in Table 4, reveal some very high R^2s for the "loser" portfolio made up of the stocks that performed the worst over the previous 12 months. As shown in Figure 9, this also translates into very impressive R^2s for the Winners Minus Losers (WML) portfolio in excess of thirty percent at the 6-8 months horizons. In contrast to the predictability of the SMB portfolio, which appears to be driven solely by $VRP_t - LJV_t$, and the predictability of HML, which is solely attributable to LJV_t, both of the two variation measures contribute to the predictability of the WML portfolio.

The return on momentum portfolios, and "loser" portfolios in particular, have also previously been associated with funding liquidity risk (e.g., Pastor and Stambaugh, 2003; Korajczyk and Sadka, 2004). The returns on momentum portfolios have also been shown to exhibit option like characteristics and be predictable by the "state" of the economy and the volatility of the aggregate market (e.g., Daniel et al., 2012; Daniel and Moskowitz, 2013), as well as the magnitude of the volatility risk premium (Nagel, 2012; Fan et al., 2013, e.g.,). Consistent with these earlier findings, the much stronger predictability for the "loser" and WML portfolio based on the decomposition of VRP_t reported here again support the interpretation of $VRP_t - LJV_t$ and LJV_t as separate proxies for market uncertainty and *fears*, respectively, both of which help predict the momentum returns.

7 Conclusion

The variance risk premium, defined as the difference between the actual and risk-neutral expectations of the forward variation of the aggregate market portfolio, is naturally decomposed into two fundamentally different sources of market variance risk: "normal" sized price fluctuations and jump tail risk. We argue that the part of the variance risk premium associated with the compensation for left jump tail risk, in particular, may be seen as a proxy for market *fears*. We develop new procedures for empirically estimating this component of the variance risk premium from the options surface at a given point in time. Consistent with the idea that this jump risk component represents separate state variables that drive the market risk premium, we find that the explanatory power of return predictability regressions based on the total variance risk premium, as previously reported in the literature, may be significantly enhanced by including the jump tail risk component as a separate predictor variable. Our empirical findings corroborate the theoretical equilibrium-based interpretation of the jump tail risk component as being more closely associated with changes in risk aversion, as opposed to changes in market risks. The predictability patterns observed for other commonly studied portfolio sorts and systematic risk factors convey a similar message: the predictability of the approximately market-neutral small-minus-big portfolio appears to be driven solely by the non-jump component, the high-minus-low book-to-market portfolio is only predicted by the jump risk component, while both components are highly significant for predicting the winners-minus-losers portfolio returns.

Our new jump tail risk measure is obviously related to the pricing of tail risk in the economy and investors' attitude toward risk more generally. It would be interesting to more fully explore these relations, and how the new measure relates to other directly observable economic variables and market-based sentiment indicators. Further along these lines, while the theoretical arguments underlying our interpretation of the variance risk premium components and the econometric procedures underlying their separate estimation are both essentially model-free, a more structural-based estimation might afford a deeper understanding of the economic mechanisms that drive the temporal variation in the separate components and help shed new light on the validity of competing equilibrium-based asset pricing models. We leave further work along these lines for future research.

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Table 1: Summary Statistics

The table reports summary statistics for weekly returns, estimated jump tail parameters, and jump tail variation measures. The data ranges from January 1996 to August 2013. The variation measures are recorded at the end-of-the-week, with the returns spanning the corresponding Monday close-to-close. The returns are in weekly percentage form. All of the variation measures are in annualized percentage form, except for the right tail variation measure which is in annualized percentage square form.

Panel A: Univariate Statistics

	MRK	SML	HML	WML	α^-	α^+	LJI	RJI	LJV	RJV	LJV*	LJV**
Mean	0.14	0.03	0.06	0.21	16.23	61.81	3.61	0.55	0.45	0.02	0.37	0.52
Std. Dev	2.89	2.15	2.50	4.83	5.33	19.68	3.19	2.04	0.54	0.05	0.23	0.72
Skewness	-0.33	-0.83	0.11	-0.95	0.43	0.65	1.94	6.10	5.41	5.03	2.21	6.02
Kurtosis	6.35	10.32	5.24	11.48	3.00	3.52	8.90	44.99	48.62	32.80	12.04	63.49
Max	14.12	9.98	12.07	28.76	33.57	145.92	28.32	18.68	7.20	0.46	2.14	10.40
Min	-15.06	-14.83	-11.33	-34.40	3.87	18.93	0.02	0.00	0.01	0.00	0.00	0.00
AR(1)	-0.16	0.07	-0.10	-0.03	0.59	0.67	0.24	0.07	0.69	0.11	0.28	0.52

Panel B: Contemporaneous Correlations

	MRK	SML	HML	WML	α^-	α^+	LJI	RJI	LJV	RJV	LJV*	LJV**
MRK	1.00	-0.15	0.07	-0.25	-0.02	0.01	-0.16	-0.02	-0.23	-0.05	-0.26	-0.11
SML		1.00	0.36	0.00	0.05	0.04	-0.03	0.00	-0.09	-0.01	-0.03	-0.02
HML			1.00	-0.32	0.03	0.06	0.01	0.01	-0.13	-0.02	-0.03	-0.06
WML				1.00	0.05	0.05	0.01	-0.02	0.06	0.00	0.06	0.05
α^-					1.00	0.68	0.17	0.17	-0.53	0.12	-0.00	-0.54
α^+						1.00	0.42	0.10	-0.35	-0.01	0.12	-0.25
LJI							1.00	0.45	0.26	0.44	0.71	0.21
RJI								1.00	-0.02	0.89	0.10	0.04
LJV									1.00	0.04	0.42	0.72
RJV										1.00	0.19	0.06
LJV*											1.00	0.18
LJV**												1.00

Panel C: One-Week-Ahead Return Correlations

	α^-	α^+	LJI	RJI	LJV	RJV	LJV*	LJV**
MRK	-0.01	-0.03	0.02	-0.01	0.05	-0.00	0.06	0.01
SML		-0.00	-0.05	-0.02	-0.12	-0.03	-0.09	-0.06
HML			-0.02	0.03	-0.05	0.06	-0.05	-0.01
WML				-0.00	-0.02	-0.01	0.06	-0.04

Table 2: Market Return Predictability Regressions

The table reports the estimated regression coefficients and R^2s from return predictability regressions for the six-month excess return on the aggregate market portfolio MRK. The returns are observed monthly with the sample period ranging from January 1996 to August 2013. Newey-West and Hodrick t-statistics and $\chi^2(2)$ Wald tests for the significance of the estimated regression coefficients accounting for the overlap in the regressions are reported in parentheses and square brackets, respectively.

Constant	-0.749	2.756	-0.755	-0.237	-0.172	0.906	2.145	-14.372	-1.173	-1.523	-0.617	-13.236
	-0.290	1.095	-0.166	-0.081	-0.075	0.416	1.115	-2.719	-0.405	-0.550	-0.274	-2.156
	-1.069	0.460	-0.628	-0.680	-0.698	-0.373	0.234	-2.618	-1.283	-1.085	-1.098	-2.013
LJV	8.784								8.749	8.418	9.741	4.645
	(4.373)								(4.370)	(4.387)	(1.533)	(1.275)
	[3.943]								[3.925]	[2.400]	[1.362]	[1.536]
RJV		28.64							25.32			
		(0.833)							(0.920)			
		[0.812]							[1.264]			
LJV*			10.88									
			(1.159)									
			[1.155]									
LJV**				6.646								
				(2.960)								
				[2.738]								
VIX2					0.624						-0.103	
					(3.891)						(-0.185)	
					[3.128]						[-0.441]	
VRP						1.896						
						(2.943)						
						[3.237]						
VRP-LJV							1.416			1.202		
							(2.116)			(1.919)		
							[2.471]			[0.442]		
log(D/P)								25.74				20.99
								(3.572)				(1.911)
								[3.330]				[1.299]
R^2	11.38	0.434	1.910	9.588	7.116	7.194	3.538	18.85	11.72	13.92	11.45	21.39
$Wald$									(19.19)	(21.80)	(23.67)	(27.68)
									[16.19]	[19.38]	[19.61]	[22.78]

Table 3: Market Return Predictability Regressions at Different Horizons

The table reports one- to twelve-months return predictability regressions for the aggregate market portfolio, analogous to the six-month regressions reported in Table 2.

	One Month			Three Months			Nine Months			Twelve Months		
Constant	-0.049	0.308	-0.226	-0.163	0.923	-0.692	-0.615	3.245	-1.664	-0.422	4.346	-1.648
	-0.114	0.878	-0.465	-0.127	1.011	-0.530	-0.151	1.073	-0.379	-0.082	1.061	-0.297
	-0.114	0.878	-0.465	-0.532	0.485	-0.708	-1.081	-0.005	-1.064	-1.238	-0.120	-1.187
LJV	1.343		1.245	3.999		3.729	11.69		11.21	14.21		13.67
	(2.689)		(2.667)	(2.613)		(2.835)	(3.502)		(3.335)	(3.812)		(3.579)
	[2.689]		[2.667]	[2.539]		[1.849]	[3.229]		[1.692]	[3.249]		[1.466]
VRP-LJV		0.329	0.294		0.949	0.851		1.879	1.600		2.174	1.836
		(1.503)	(1.337)		(2.454)	(2.382)		(2.416)	(2.121)		(2.762)	(2.378)
		[1.503]	[1.337]		[2.488]	[1.647]		[2.416]	[0.128]		[2.661]	[0.109]
R^2	1.707	1.234	2.688	5.125	3.471	7.891	12.97	3.997	15.85	14.18	3.949	16.98
$Wald$			(8.498)			(17.61)			(12.42)			(15.75)
			[8.498]			[12.36]			[14.19]			[15.82]

Table 4: Portfolio Return Predictability Regressions

The table reports the results from one- to twelve-months return predictability regressions for size (twenty percent smallest and biggest firms), book-to-market (twenty percent highest and lowest B/M ratios), and momentum (twenty percent top and bottom performers over the past 2-12 months) portfolios, along with the corresponding zero-cost portfolios, analogous to the results for the aggregate market in Tables 2 and 3.

	Small	Big	SMB	High	Low	HML	Winners	Losers	WML
	One Month Returns								
Constant	0.569	0.012	0.557	0.601	-0.180	0.781	1.132	-2.278	3.410
	(0.980)	(0.024)	(1.462)	(1.126)	(-0.368)	(2.395)	(1.163)	(-1.273)	(2.369)
	[0.980]	[0.024]	[1.462]	[1.126]	[-0.368]	[2.395]	[1.163]	[-1.273]	[2.369]
LJV	0.027	1.179	-1.151	0.260	1.430	-1.169	1.281	4.327	-3.045
	(0.037)	(2.666)	(-2.040)	(0.463)	(3.244)	(-2.761)	(1.337)	(1.606)	(-1.189)
	[0.037]	[2.666]	[-2.040]	[0.463]	[3.244]	[-2.761]	[1.337]	[1.606]	[-1.189]
VRP-LJV	0.411	0.252	0.159	0.303	0.377	-0.074	0.270	1.817	-1.547
	(1.744)	(1.163)	(0.960)	(1.321)	(1.792)	(-0.425)	(0.601)	(2.060)	(-1.692)
	[1.744]	[1.163]	[0.960]	[1.321]	[1.792]	[-0.425]	[0.601]	[2.060]	[-1.692]
R^2	1.134	2.449	1.780	0.900	3.867	2.551	0.521	4.518	4.156
Wald	(3.042)	(7.795)	(7.592)	(1.865)	(13.73)	(8.388)	(1.842)	(5.066)	(2.871)
	[3.042]	[7.795]	[7.592]	[1.865]	[13.73]	[8.388]	[1.842]	[5.066]	[2.871]
	Three Months Returns								
Constant	0.546	0.174	0.373	1.459	-0.548	2.008	3.656	-10.59	14.25
	(0.362)	(0.133)	(0.381)	(1.001)	(-0.420)	(1.918)	(1.448)	(-2.019)	(3.389)
	[0.144]	[-0.141]	[0.451]	[0.765]	[-0.724]	[2.106]	[1.054]	[-2.003]	[3.137]
LJV	2.473	3.248	-0.776	1.366	4.880	-3.514	3.317	19.46	-16.14
	(1.686)	(2.358)	(-0.567)	(1.089)	(3.412)	(-2.423)	(0.953)	(3.534)	(-2.409)
	[0.723]	[1.623]	[-1.015]	[0.278]	[2.650]	[-2.709]	[0.719]	[2.559]	[-2.041]
VRP-LJV	1.277	0.703	0.574	0.987	0.723	0.264	0.690	6.232	-5.542
	(2.710)	(2.108)	(2.311)	(2.451)	(2.079)	(1.032)	(1.069)	(2.554)	(-2.275)
	[2.428]	[1.440]	[2.598]	[2.166]	[1.139]	[1.594]	[0.756]	[2.359]	[-2.323]
R^2	4.644	6.374	1.531	3.420	9.862	6.477	1.183	19.93	22.78
Wald	(13.59)	(11.11)	(7.248)	(6.371)	(19.39)	(11.38)	(1.784)	(13.63)	(6.129)
	[13.30]	[8.299]	[6.886]	[6.023]	[14.19]	[8.387]	[1.497]	[13.46]	[6.701]
	Six Months Returns								
Constant	-0.339	0.357	-0.696	2.078	-0.522	2.600	5.871	-23.27	29.14
	(-0.112)	(0.127)	(-0.386)	(0.614)	(-0.200)	(1.127)	(1.222)	(-2.085)	(3.660)
	[-0.516]	[-0.522]	[-0.164]	[0.188]	[-0.993]	[1.367]	[0.555]	[-2.189]	[2.902]
LJV	7.697	7.285	0.412	5.339	9.196	-3.857	9.253	50.34	-41.09
	(2.663)	(3.708)	(0.167)	(1.772)	(4.723)	(-1.303)	(1.833)	(3.669)	(-2.587)
	[0.940]	[2.323]	[-1.238]	[0.991]	[2.762]	[-2.046]	[1.104]	[2.420]	[-1.894]
VRP-LJV	2.437	0.861	1.576	1.462	0.917	0.545	1.536	7.693	-6.157
	(2.970)	(1.419)	(3.474)	(1.673)	(1.569)	(0.969)	(1.486)	(2.484)	(-2.401)
	[2.004]	[0.145]	[3.879]	[1.337]	[-0.093]	[1.668]	[0.429]	[0.771]	[-0.831]
R^2	12.93	10.33	5.835	6.651	14.34	3.836	3.850	32.93	34.22
Wald	(17.76)	(14.40)	(12.38)	(3.732)	(24.52)	(7.408)	(6.223)	(13.96)	(7.439)
	[17.76]	[12.89]	[16.65]	[3.732]	[21.90]	[5.839]	[4.639]	[15.17]	[9.702]
	Nine Months Returns								
Constant	0.015	1.107	-1.092	3.275	0.137	3.138	8.196	-27.825	36.021
	(0.003)	(0.240)	(-0.368)	(0.617)	(0.032)	(0.877)	(1.030)	(-1.903)	(4.291)
	[-0.512]	[-0.605]	[-0.076]	[0.016]	[-0.963]	[1.155]	[0.185]	[-1.951]	[2.798]
LJV	11.20	9.582	1.621	8.055	11.48	-3.421	13.96	63.60	-49.64
	(2.182)	(2.975)	(0.561)	(1.382)	(3.912)	(-0.806)	(2.703)	(3.364)	(-2.862)
	[0.649]	[1.759]	[-1.083]	[0.793]	[1.956]	[-1.875]	[1.200]	[1.767]	[-1.707]
VRP-LJV	2.976	1.111	1.865	1.755	1.335	0.421	2.581	8.964	-6.383
	(2.802)	(1.446)	(2.506)	(1.658)	(1.959)	(0.573)	(2.013)	(2.701)	(-2.402)
	[1.082]	[-0.129]	[2.586]	[0.770]	[-0.141]	[1.485]	[0.479]	[0.085]	[0.093]
R^2	16.07	11.01	5.620	8.480	14.726	1.777	6.116	33.26	29.55
Wald	(15.03)	(8.862)	(8.707)	(2.897)	(16.06)	(6.497)	(10.11)	(13.40)	(8.979)
	[10.26]	[10.06]	[8.404]	[2.516]	[19.85]	[7.204]	[9.543]	[13.25]	[9.221]
	Twelve Months Returns								
Constant	0.872	2.020	-1.148	5.120	0.818	4.303	10.76	-28.06	38.83
	(0.171)	(0.339)	(-0.261)	(0.812)	(0.145)	(0.924)	(1.029)	(-1.752)	(4.902)
	[-0.458]	[-0.799]	[0.352]	[-0.140]	[-1.110]	[1.002]	[-0.121]	[-1.742]	[2.489]
LJV	13.18	11.61	1.577	9.995	13.86	-3.860	18.99	68.61	-49.63
	(2.245)	(3.122)	(0.458)	(1.399)	(4.325)	(-0.675)	(2.989)	(3.449)	(-2.848)
	[0.428]	[1.547]	[-1.754]	[0.822]	[1.744]	[-1.054]	[1.318]	[1.421]	[-1.387]
VRP-LJV	3.714	1.147	2.566	1.450	1.571	-0.121	2.953	9.317	-6.364
	(3.222)	(1.382)	(2.817)	(1.341)	(2.012)	(-0.120)	(1.928)	(3.089)	(-2.399)
	[0.967]	[-0.354]	[3.675]	[-0.028]	[-0.300]	[0.871]	[0.061]	[-0.159]	[0.249]
R^2	19.31	11.16	7.616	8.101	15.30	1.537	7.629	30.30	22.72
Wald	(26.43)	(9.768)	(12.03)	(2.580)	(18.98)	(0.839)	(10.30)	(18.33)	(9.889)
	[9.841]	[11.89]	[16.83]	[2.520]	[18.13]	[1.979]	[9.894]	[10.99]	[6.422]

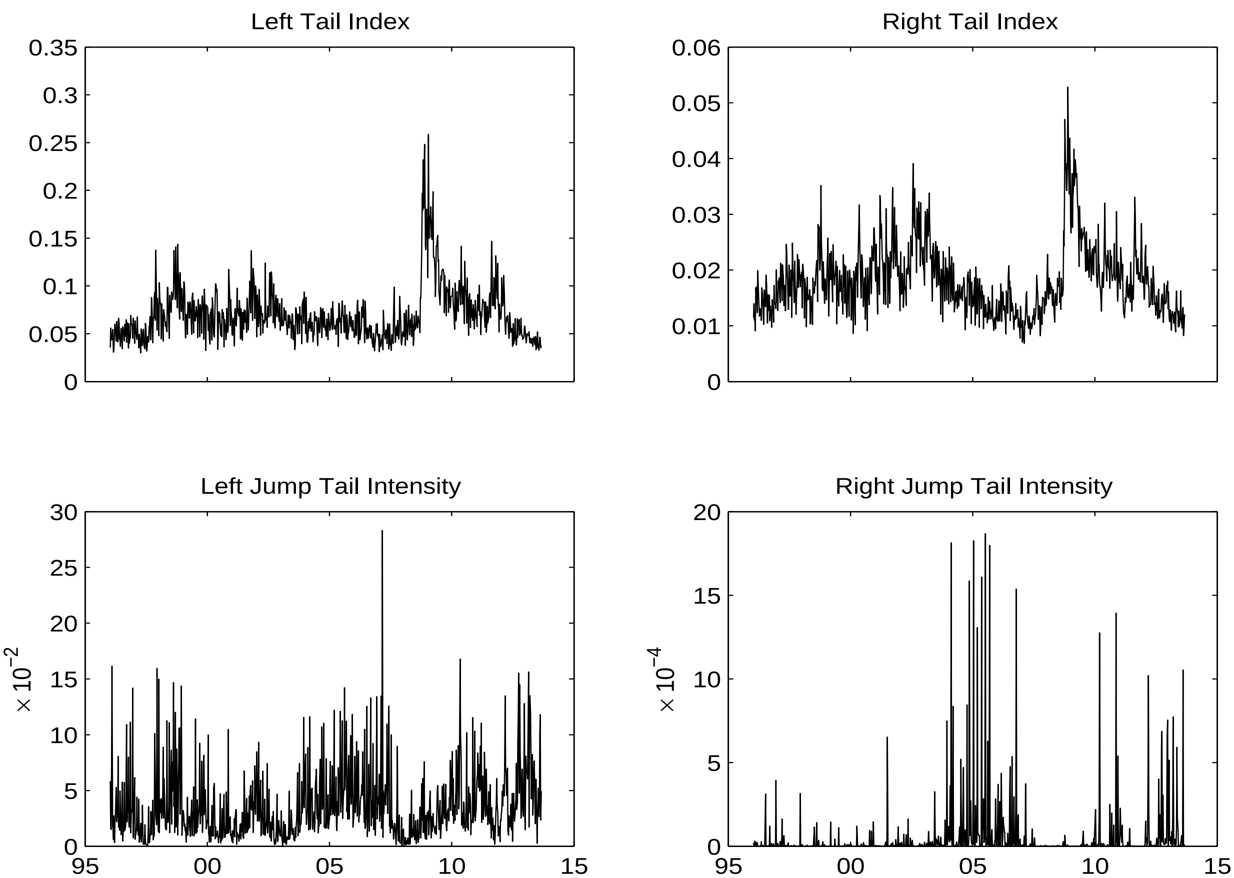

Figure 1: Tail Shape Parameters and Jump Intensities

The top two panels plot the estimated left and right tail indexes, $1/\alpha_t^{\pm}$, defined by the estimator in equation (3.3). The bottom two panels plot the estimated left and right jump intensities for "large" jumps beyond the threshold k_t, $LJI_t = \widehat{\phi}_t^- e^{-\widehat{\alpha}_t^- |k_t|}/\widehat{\alpha}_t^-$ and $RJI_t = \widehat{\phi}_t^+ e^{-\alpha_t^+ |k_t|}/\widehat{\alpha}_t^+$, defined in equation (5.1). LJI and RJI are both reported in annualized square form.

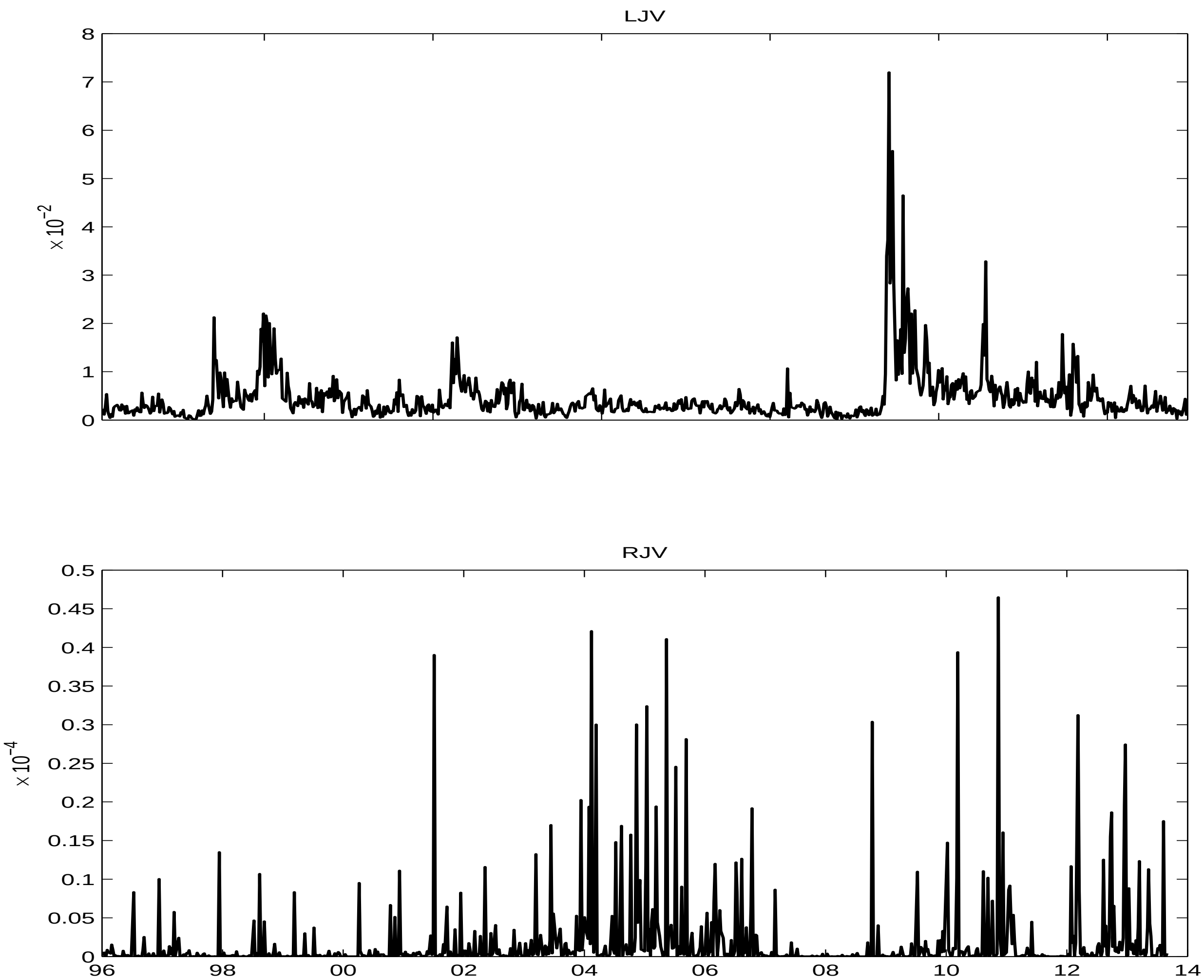

Figure 2: Jump Variation

This figure plots the left and right jump variation, LJV_t and RJV_t, defined in equation (3.2). LJV and RJV are both reported in annualized square form.

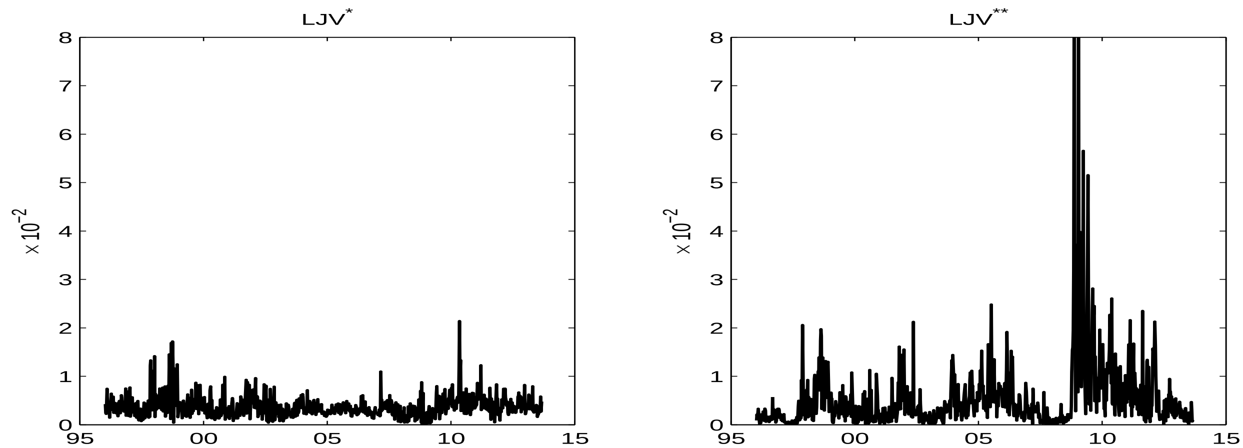

Figure 3: Left Jump Variation with Restricted Tail Parameters

The left panel plots the left jump variation LJV_t^* obtained by restricting the "shape" parameter α^- to be constant, but allowing ϕ_t^- to change over time. The right panel shows LJV_t^{**} obtained by restricting the "level" parameter ϕ^- to be constant, but allowing α_t^- to be time-varying. The estimates are reported in annualized square form.

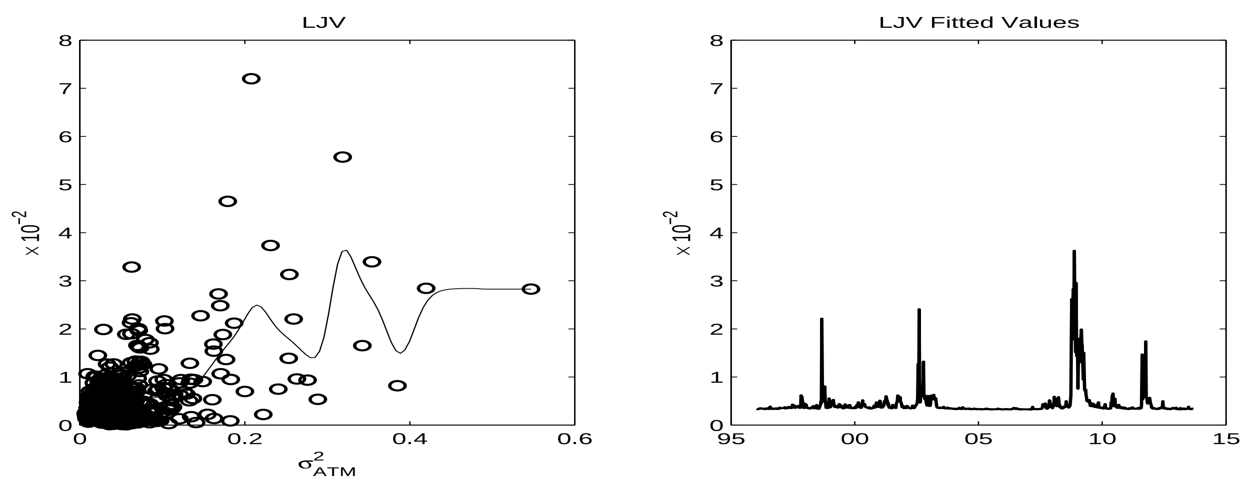

Figure 4: Nonparametric Regression of Left Jump Variation on Volatility

The left panel displays a scatterplot of LJV_t against $\sigma^2_{ATM,t}$ (circles), along with the fit from a nonparametric kernel regression of LJV_t on $\sigma^2_{ATM,t}$ (line), where $\sigma_{ATM,t}$ denotes the at-the-money implied volatility from the shortest-maturity options available on day t. The right panel shows a time series plot of the corresponding fitted values of LJV_t.

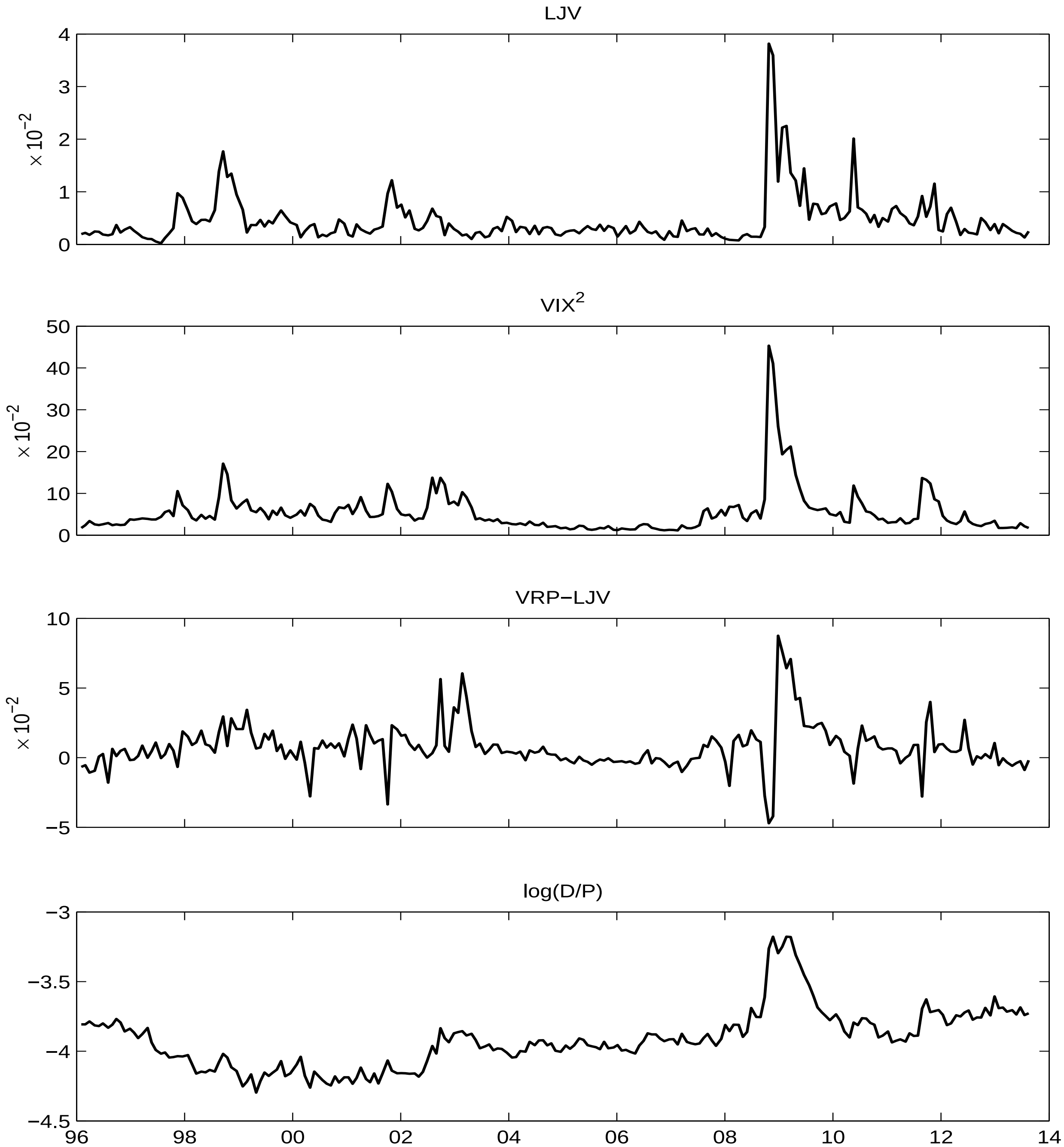

Figure 5: Predictor Variables

All of the series are plotted at a monthly frequency, and span the period from January 1996 to August 2013. The top panel shows the estimated left jump tail variation measure LJV_t. The second panel shows the CBOE VIX_t^2 volatility index. The third panel plots the estimated variance risk premium VRP_t minus LJV_t. All of the monthly variation measures are aggregated from the weekly option-based estimates and reported in annualized squared form. The bottom panel plots the log dividend-price ratio for the aggregate market $log(D_t/P_t)$.

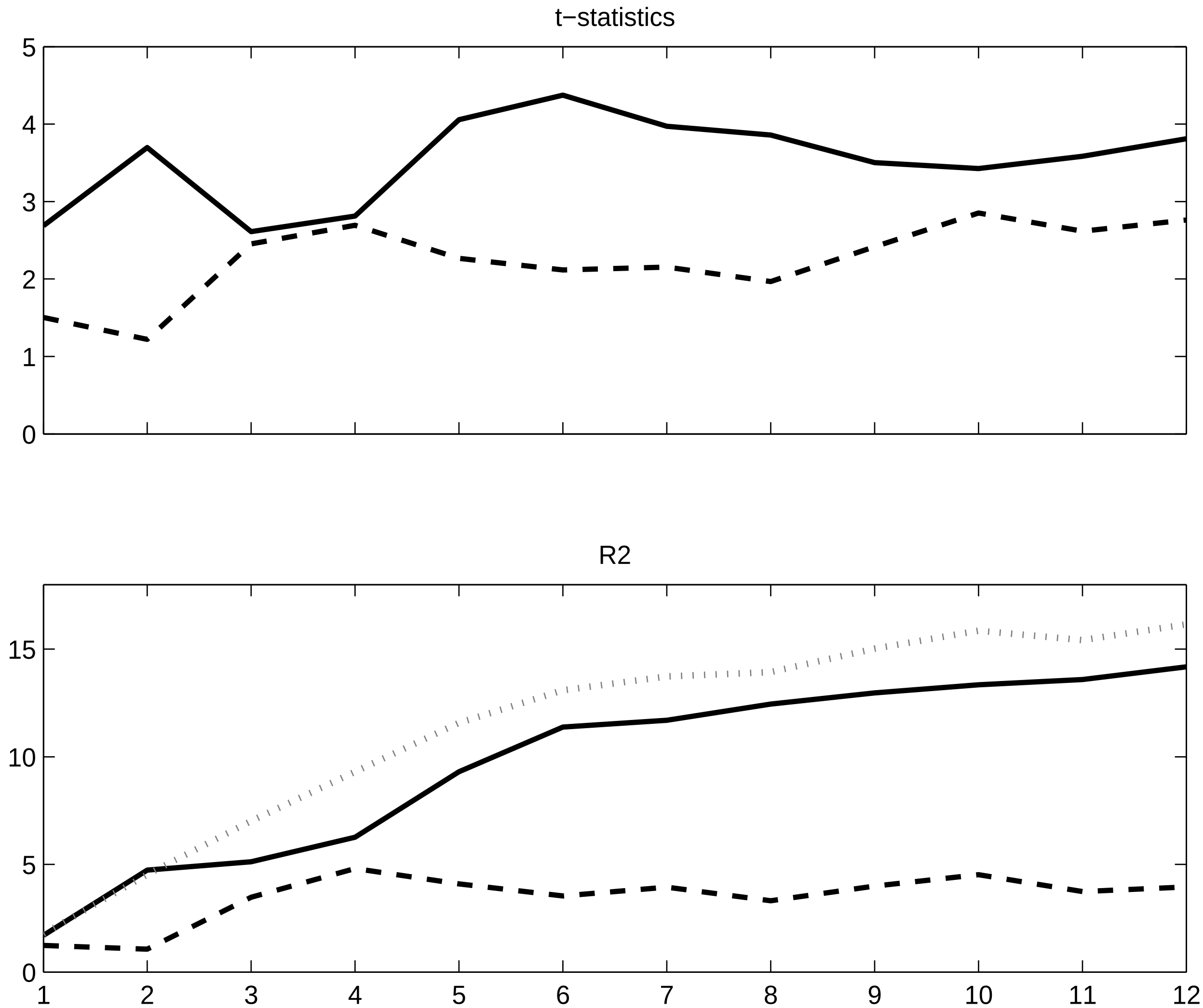

Figure 6: Market Return Predictability Regressions

The top panel shows the Newey-West t-statistics from from simple return predictability regressions for the aggregate market portfolio MRK based on the left jump tail variation LJV (solid line) and the difference between the variance risk premium and the left jump variation $VRP - LJV$ (dashed line). The bottom panel shows the corresponding R^2s, along with the R^2s from multiple regressions including both LJV and $VRP - LJV$ (dotted line).

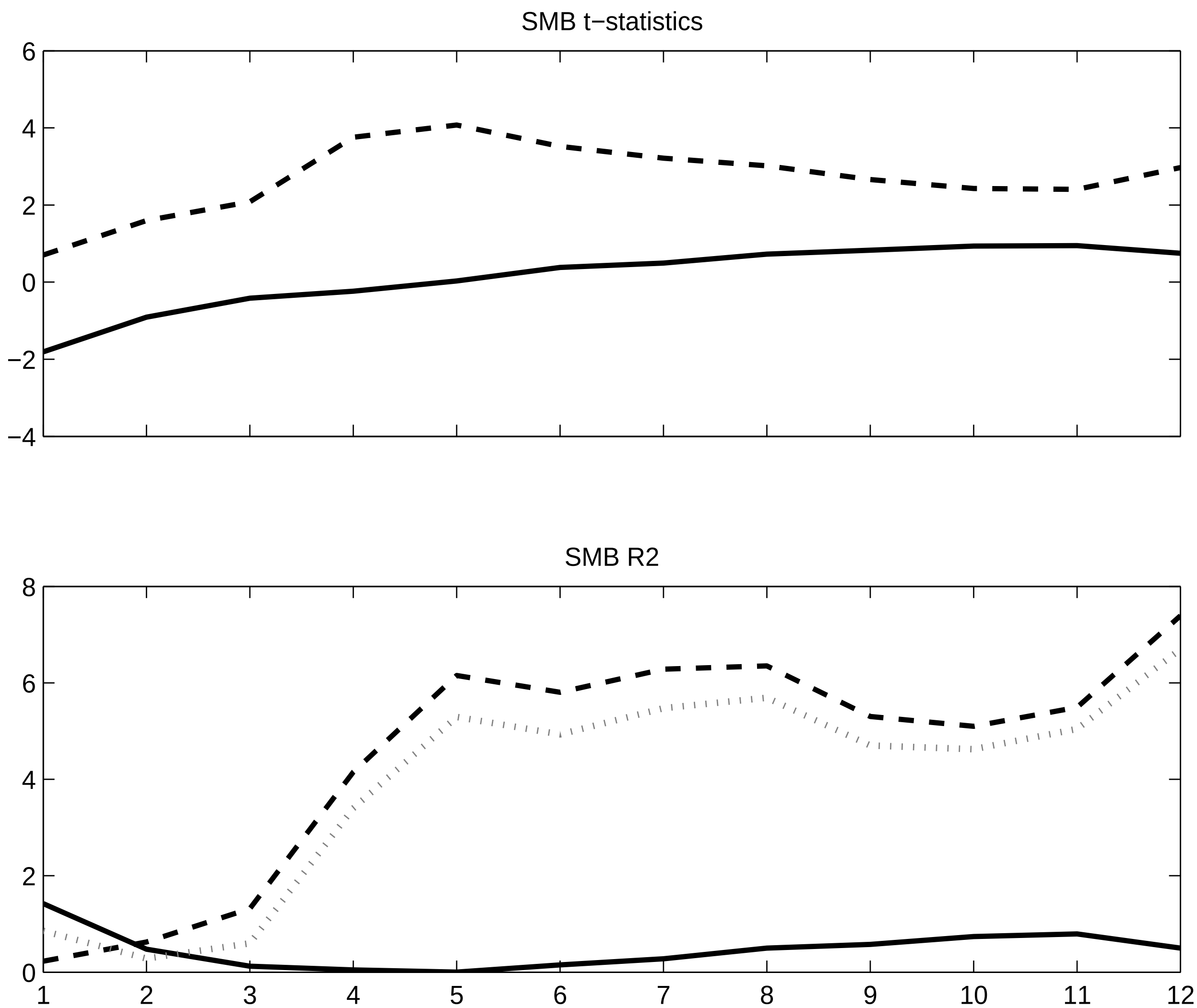

Figure 7: SMB Return Predictability Regressions

The top panel shows the Newey-West t-statistics from simple return predictability regressions for the Small Minus Big (SMB) market capitalization sorted zero-cost portfolio based on the left jump tail variation LJV (solid line) and the difference between the variance risk premium and the left jump variation $VRP - LJV$ (dashed line). The bottom panel shows the corresponding R^2s, along with the R^2s from multiple regressions including both LJV and $VRP - LJV$ (dotted line).

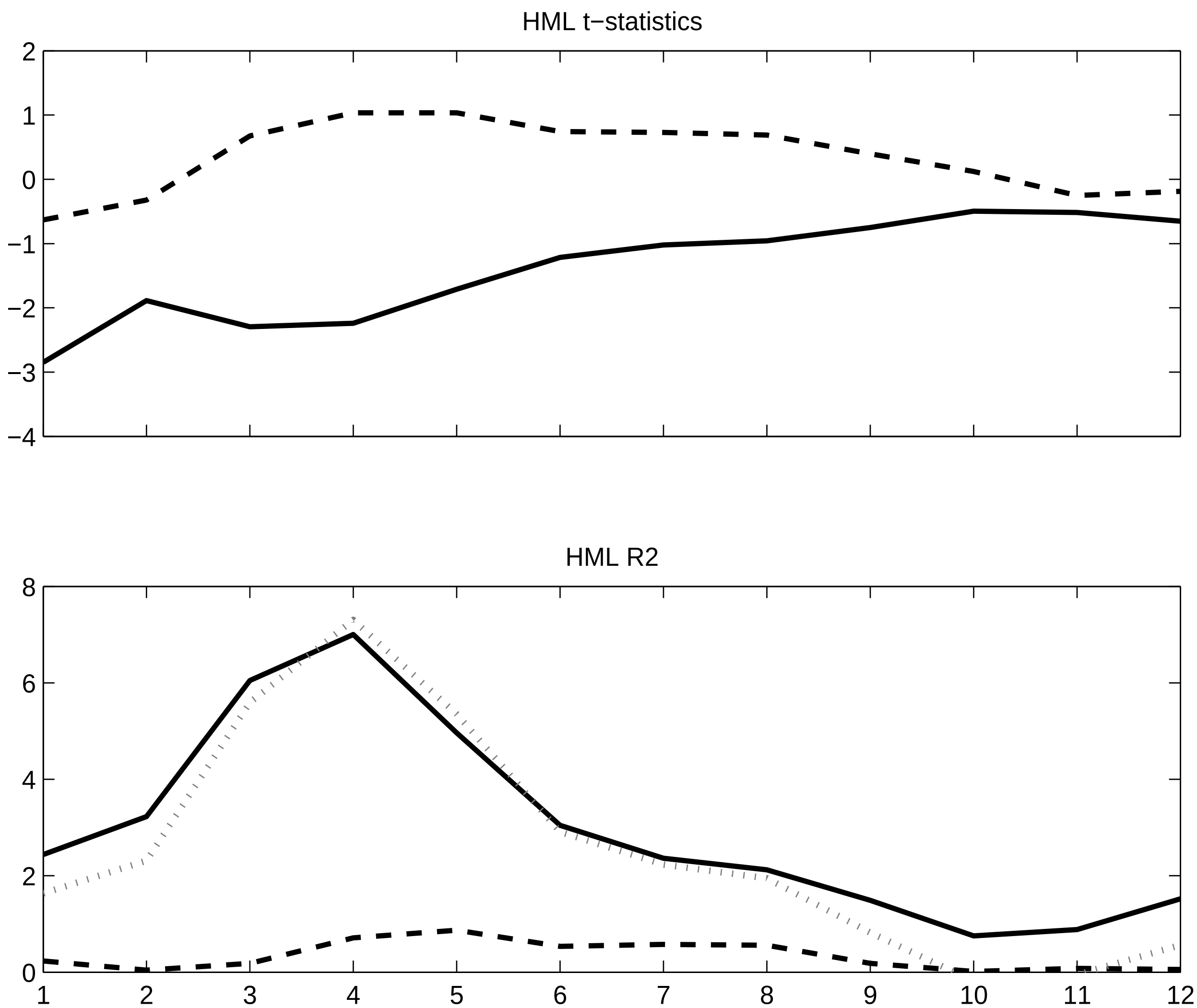

Figure 8: HML Return Predictability Regressions

The top panel shows the Newey-West t-statistics from simple return predictability regressions for the High Minus Low (HML) book-to-market sorted zero-cost portfolio based on the left jump tail variation LJV (solid line) and the difference between the variance risk premium and the left jump variation $VRP - LJV$ (dashed line). The bottom panel shows the corresponding R^2s, along with the R^2s from multiple regressions including both LJV and $VRP - LJV$ (dotted line).

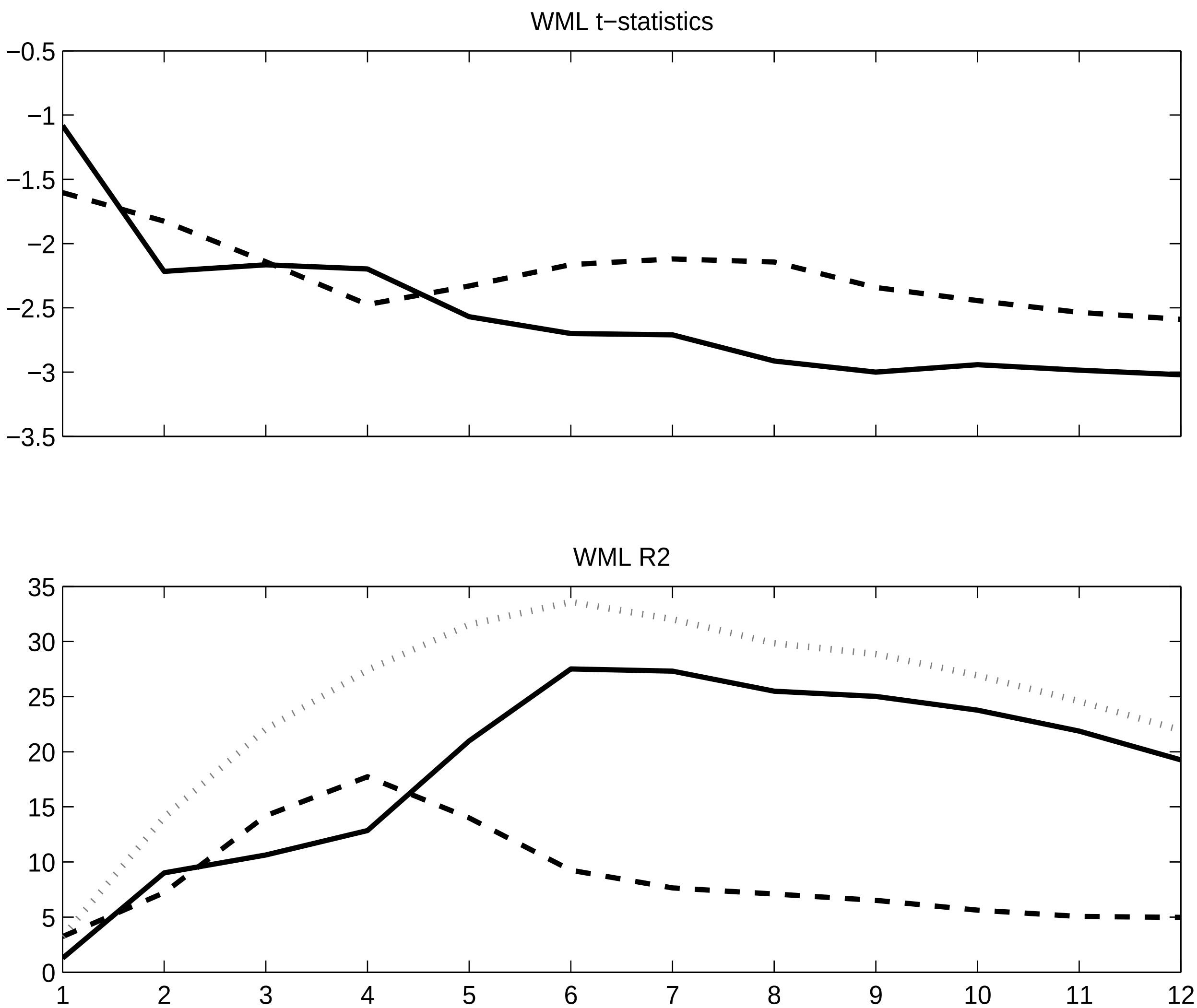

Figure 9: WML Return Predictability Regressions

The top panel shows the Newey-West t-statistics from simple return predictability regressions for the Winners Minus Losers (WML) sorted zero-cost portfolio based on the left jump tail variation LJV (solid line) and the difference between the variance risk premium and the left jump variation $VRP - LJV$ (dashed line). The bottom panel shows the corresponding R^2s, along with the R^2s from multiple regressions including both LJV and $VRP - LJV$ (dotted line).