Supplementary Appendix to
“The Risk Premia Embedded in Index Options”
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A The Non-Linear Factor Structure of Option Surfaces

In a standard option pricing model, the Black-Scholes implied volatility is given by \( \kappa(k, \tau, S_t, \theta) \) where \( S_t \) is the state vector driving the option surface dynamics (\( S_t \) equals \( (V_{1,t}, V_{2,t}, U_t) \) for our three-factor model in the paper). This makes the option surface obey a nonlinear factor structure. The goal of this section is to illustrate in a simple setting the amount of this nonlinearity and further illustrate the extra dynamics in the option surface induced by it.

We use simulated data from the Heston model for this analysis. We recall the Heston model is a one-factor constrained version of our general stochastic volatility model in (3.1) in the paper with no price and volatility jumps. The parameters in the simulation are set to: \( \theta_1 = 0.0225 \), \( \kappa_1 = 4 \), \( \sigma_1 = 0.3 \) and \( \rho_1 = -0.9 \) (for simplicity we assume the same model under \( \mathbb{P} \) and \( \mathbb{Q} \)). We simulate the process over a period of five years and, at the end of every week on the simulated trajectory, we compute the theoretical price of an OTM put option with fixed moneyness and maturity equal to 0.1 years. We report results for two alternative ways of defining moneyness: fixed moneyness of \( K/F_{t,t+\tau} = 0.9 \) and volatility adjusted moneyness of \( \log(K/F_{t,t+\tau})/(\sqrt{\tau} \sqrt{V_{1,t}}) = -2 \).

For each of the days in the simulated sample and for the two different ways of fixing the moneyness, we then compute the derivative of the theoretical Black-Scholes implied volatility with respect to the current level of the stochastic variance, i.e., we compute

\[
\frac{\partial \kappa(k, \tau, V_{1,t}, \theta)}{\partial V_{1,t}}.
\]

The results are reported on Figures A.1 and A.2. As seen from the figures, the sensitivity of the option implied volatility with respect to current spot variance is rather nontrivial, regardless of the method of fixing the moneyness, even in the very simple Heston stochastic volatility model. This variability plotted on Figures A.1 and A.2 is essentially ignored when conducting linear factor analysis on the Black-Scholes implied volatility surface.

B Additional Results and Robustness Checks for the Parametric Modeling of the Option Panel

B.1 Estimation Results for the General Model

The model is given by:

2
Figure A.1: **Sensitivity of Option Implied Volatility to the level of Volatility in the Heston Model: Case of Fixed Moneyness.** Top Panel: simulated variance path from the Heston model. Bottom Panel: sensitivity of OTM Put option’s implied volatility with respect to the the current level of variance, for a fixed level of moneyness $K/F_{t,t+\tau} = 0.9$. The straight line on the bottom panel corresponds to the average value of the derivative in the sample.
Figure A.2: Sensitivity of Option Implied Volatility to the level of Volatility in the Heston Model: Case of Volatility-Adjusted Moneyness. Top Panel: simulated variance path from the Heston model. Bottom Panel: sensitivity of OTM Put option’s implied volatility with respect to the current level of variance, for a volatility adjusted level of moneyness \(\log(K/F_{t,t+\tau})/(\sqrt{\tau}\sqrt{V_1,t}) = -2\). The straight line on the bottom panel corresponds to the average value of the derivative in the sample.
\[
\frac{dX_t}{X_t} = (r_t - \delta_t) dt + \sqrt{V_{1,t}} dW_{1,t}^Q + \sqrt{V_{2,t}} dW_{2,t}^Q + \eta \sqrt{U_t} dW_{3,t}^Q + \int_{\mathbb{R}^2} (e^x - 1) \tilde{\mu}^Q(dt, dx, dy),
\]
\[
dV_{1,t} = \kappa_1 (\nu_t - V_{1,t}) dt + \sigma_1 \sqrt{V_{1,t}} dB_{1,t}^Q + \mu_1 \int_{\mathbb{R}^2} x^2 1_{\{x<0\}} \mu(dt, dx, dy),
\]
\[
dV_{2,t} = \kappa_2 (\nu_t - V_{2,t}) dt + \sigma_2 \sqrt{V_{2,t}} dB_{2,t}^Q,
\]
\[
dU_t = -\kappa_u U_t dt + \mu_u \int_{\mathbb{R}^2} [(1 - \rho_u) x^2 1_{\{x<0\}} + \rho_u y^2] \mu(dt, dx, dy),
\]

where \((W_{1,t}^Q, W_{2,t}^Q, W_{3,t}^Q, B_{1,t}^Q, B_{2,t}^Q)\) is a five-dimensional Brownian motion with \(\text{corr} (W_{1,t}^Q, B_{1,t}^Q) = \rho_1\) and \(\text{corr} (W_{2,t}^Q, B_{2,t}^Q) = \rho_2\), while the remaining Brownian motions are mutually independent. The jump compensator is given by,

\[
\nu_t^Q(dx, dy) = \left\{
\begin{array}{ll}
(c^- (t) \cdot 1_{\{x<0\}} \lambda_- e^{-\lambda_- |x|} + c^+ (t) \cdot 1_{\{x>0\}} \lambda_+ e^{-\lambda_+ x}) , & \text{if } y = 0, \\
c^- (t) \lambda_- e^{-\lambda_- |y|} , & \text{if } x = 0 \text{ and } y < 0,
\end{array}
\right.
\]

with

\[
c^-(t) = c_0^- + c_1^- V_{1,t-} + c_2^- V_{2,t-} + U_{t-}, \quad c^+(t) = c_0^+ + c_1^+ V_{1,t-} + c_2^+ V_{2,t-} + c_u^+ U_{t-}.
\]

Figure B.1: **Recovered** \(U\). Solid dark line: \(U\) corresponding to model (B.1) with no restrictions on coefficients. Dotted light line: \(U\) corresponding to model (B.1) with \(c_0^- = c_2^- = c_u^+ = \eta = 0\).
Table B.1: Estimation results for model (B.1).

<table>
<thead>
<tr>
<th>Panel A: Parameter Estimates</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1$</td>
<td>-0.962</td>
<td>0.157</td>
<td>$\sigma_2$</td>
<td>0.168</td>
<td>0.007</td>
<td>$c_1^+$</td>
<td>25.178</td>
<td>5.246</td>
</tr>
<tr>
<td>$\bar{v}_1$</td>
<td>0.003</td>
<td>0.000</td>
<td>$\eta$</td>
<td>0.001</td>
<td>0.013</td>
<td>$c_2^-$</td>
<td>0.233</td>
<td>3.391</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>10.968</td>
<td>0.204</td>
<td>$\mu_u$</td>
<td>7.3425</td>
<td>29.736</td>
<td>$c_2^+$</td>
<td>82.151</td>
<td>7.051</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.257</td>
<td>0.049</td>
<td>$\kappa_u$</td>
<td>0.084</td>
<td>0.146</td>
<td>$c_u^+$</td>
<td>0.007</td>
<td>0.007</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>12.161</td>
<td>0.261</td>
<td>$\rho_u$</td>
<td>0.385</td>
<td>5.438</td>
<td>$\lambda_-$</td>
<td>25.941</td>
<td>0.212</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>-0.989</td>
<td>0.036</td>
<td>$c_0^-$</td>
<td>0.005</td>
<td>0.132</td>
<td>$\lambda_+$</td>
<td>36.543</td>
<td>0.826</td>
</tr>
<tr>
<td>$\bar{v}_2$</td>
<td>0.010</td>
<td>0.000</td>
<td>$c_0^+$</td>
<td>0.348</td>
<td>0.029</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>1.828</td>
<td>0.122</td>
<td>$c_1^-$</td>
<td>110.324</td>
<td>5.139</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Summary Statistics

- RMSE: 1.71%
- Mean jump intensity (yearly) (-/+) 5.55/1.79
- Mean jump size (-/+) 3.85%/2.74%

Note: Estimation period is January 1996-July 2010.
B.2 Estimation Results on Subsamples

B.2.1 From January 1996 to December 2006

Table B.2: Estimation results for model (B.1).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1$</td>
<td>-0.974</td>
<td>0.271</td>
<td>$\nu_2$</td>
<td>0.016</td>
<td>0.000</td>
<td>$c_0^+$</td>
<td>0.384</td>
<td>0.029</td>
</tr>
<tr>
<td>$\nu_1$</td>
<td>0.001</td>
<td>0.000</td>
<td>$\kappa_2$</td>
<td>1.677</td>
<td>0.109</td>
<td>$c_1^-$</td>
<td>176.055</td>
<td>10.385</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>13.028</td>
<td>0.287</td>
<td>$\sigma_2$</td>
<td>0.223</td>
<td>0.005</td>
<td>$c_1^+$</td>
<td>9.098</td>
<td>4.419</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.144</td>
<td>0.048</td>
<td>$\mu_u$</td>
<td>9.013</td>
<td>71.215</td>
<td>$c_2^+$</td>
<td>84.003</td>
<td>7.611</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>10.983</td>
<td>0.347</td>
<td>$\kappa_u$</td>
<td>0.162</td>
<td>0.373</td>
<td>$\lambda_-$</td>
<td>26.306</td>
<td>0.238</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>-0.858</td>
<td>0.018</td>
<td>$\rho_u$</td>
<td>0.887</td>
<td>7.651</td>
<td>$\lambda_+$</td>
<td>37.218</td>
<td>0.735</td>
</tr>
</tbody>
</table>

Panel B: Summary Statistics

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>0.99%</td>
</tr>
<tr>
<td>Mean jump intensity (yearly) (-/+)</td>
<td>6.65/1.54</td>
</tr>
<tr>
<td>Mean jump size (-/+</td>
<td>3.80%/2.69%</td>
</tr>
</tbody>
</table>

Note: Estimation period is January 1996-December 2006.
### Panel A: Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1$</td>
<td>-0.987</td>
<td>0.081</td>
<td>$\varpi_2$</td>
<td>0.007</td>
<td>0.000</td>
<td>$c_0^+$</td>
<td>0.571</td>
<td>0.066</td>
</tr>
<tr>
<td>$\varpi_1$</td>
<td>0.004</td>
<td>0.000</td>
<td>$\kappa_2$</td>
<td>2.061</td>
<td>0.155</td>
<td>$c_1^-$</td>
<td>112.519</td>
<td>6.845</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>11.962</td>
<td>0.318</td>
<td>$\sigma_2$</td>
<td>0.169</td>
<td>0.011</td>
<td>$c_1^+$</td>
<td>18.786</td>
<td>4.863</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.322</td>
<td>0.034</td>
<td>$\mu_u$</td>
<td>9.163</td>
<td>23.635</td>
<td>$c_2^+$</td>
<td>87.929</td>
<td>11.474</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>12.630</td>
<td>0.347</td>
<td>$\kappa_u$</td>
<td>0.056</td>
<td>0.119</td>
<td>$\lambda_-$</td>
<td>25.720</td>
<td>0.353</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>-0.990</td>
<td>0.054</td>
<td>$\rho_u$</td>
<td>0.323</td>
<td>3.982</td>
<td>$\lambda_+$</td>
<td>35.814</td>
<td>0.995</td>
</tr>
</tbody>
</table>

### Panel B: Summary Statistics

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>2.16%</td>
</tr>
<tr>
<td>Mean jump intensity (yearly) (-/+)</td>
<td>5.59/1.94</td>
</tr>
<tr>
<td>Mean jump size (-/+)</td>
<td>3.89%/2.79%</td>
</tr>
</tbody>
</table>

*Note: Estimation period is January 2007-July 2010.*
Figure B.2: Top Panel: $U_t$ from the full sample estimation from January 1996 to July 2010. Bottom Panel: $U_t$ from the two sub-sample estimations: January 1996 to December 2006 and January 2007 to July 2010.
B.3 Effect of $\rho_u$ Parameter

Figure B.3: The effect of $\rho_u$ on $U$. Solid dark line is $U$ given parameter estimates of model (B.1) reported in Table 4 in the paper with $\rho_u$ set to zero. Dotted light line is $U$ given parameter estimates of model (B.1) reported in Table 4 in the paper with $\rho_u$ set to one.
**B.4 Effect of Volatility Fit Penalization in Estimation**

**B.4.1 $\lambda = 0.1$**

Table B.4: Estimation results for model (B.1).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1$</td>
<td>$-0.929$</td>
<td>0.076</td>
<td>$\nu_2$</td>
<td>0.011</td>
<td>0.000</td>
<td>$c^+_0$</td>
<td>0.461</td>
<td>0.027</td>
</tr>
<tr>
<td>$\nu_1$</td>
<td>0.003</td>
<td>0.000</td>
<td>$\kappa_2$</td>
<td>2.091</td>
<td>0.101</td>
<td>$c^-_1$</td>
<td>110.244</td>
<td>5.327</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>11.066</td>
<td>0.245</td>
<td>$\sigma_2$</td>
<td>0.203</td>
<td>0.004</td>
<td>$c^+_1$</td>
<td>22.415</td>
<td>4.024</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.242</td>
<td>0.024</td>
<td>$\mu_u$</td>
<td>5.512</td>
<td>30.187</td>
<td>$c^+_2$</td>
<td>78.805</td>
<td>5.548</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>12.188</td>
<td>0.254</td>
<td>$\kappa_u$</td>
<td>0.267</td>
<td>0.148</td>
<td>$\lambda_-$</td>
<td>25.982</td>
<td>0.273</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>$-0.992$</td>
<td>0.019</td>
<td>$\rho_u$</td>
<td>0.480</td>
<td>7.157</td>
<td>$\lambda_+$</td>
<td>36.329</td>
<td>0.588</td>
</tr>
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</table>

Panel B: Summary Statistics

<p>| | |</p>
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>1.71%</td>
</tr>
<tr>
<td>Mean jump intensity (yearly) (-/+)</td>
<td>5.54/1.78</td>
</tr>
<tr>
<td>Mean jump size (-/+)</td>
<td>3.84%/2.75%</td>
</tr>
</tbody>
</table>

*Note: Estimation period is January 1996-July 2010. We set $\lambda$ in the objective function given in (4.1) in the paper to 0.1.*
Figure B.4: Solid dark line is $U$ based on estimation with $\lambda = 0.2$ and dotted light line is $U$ based on estimation with $\lambda = 0.1$. Estimation period is January 1996 to July 2010.
B.4.2 \( \lambda = 0.3 \)

Table B.5: Estimation results for model \((B.1)\).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_1)</td>
<td>-0.948</td>
<td>0.075</td>
<td>(\overline{\nu}_2)</td>
<td>0.008</td>
<td>0.000</td>
<td>(c_0^+)</td>
<td>0.368</td>
<td>0.026</td>
</tr>
<tr>
<td>(\overline{\nu}_1)</td>
<td>0.003</td>
<td>0.000</td>
<td>(\kappa_2)</td>
<td>1.865</td>
<td>0.086</td>
<td>(c_1^-)</td>
<td>106.353</td>
<td>4.498</td>
</tr>
<tr>
<td>(\kappa_1)</td>
<td>10.896</td>
<td>0.210</td>
<td>(\sigma_2)</td>
<td>0.163</td>
<td>0.004</td>
<td>(c_1^+)</td>
<td>13.742</td>
<td>3.751</td>
</tr>
<tr>
<td>(\sigma_1)</td>
<td>0.251</td>
<td>0.026</td>
<td>(\mu_u)</td>
<td>6.188</td>
<td>25.739</td>
<td>(c_2^+)</td>
<td>80.797</td>
<td>5.612</td>
</tr>
<tr>
<td>(\mu_1)</td>
<td>12.023</td>
<td>0.239</td>
<td>(\kappa_u)</td>
<td>0.076</td>
<td>0.123</td>
<td>(\lambda^-)</td>
<td>25.984</td>
<td>0.234</td>
</tr>
<tr>
<td>(\rho_2)</td>
<td>-0.987</td>
<td>0.023</td>
<td>(\rho_u)</td>
<td>0.467</td>
<td>5.220</td>
<td>(\lambda^+)</td>
<td>36.747</td>
<td>0.604</td>
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Panel B: Summary Statistics

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>RMSE</td>
<td>1.75%</td>
</tr>
<tr>
<td>Mean jump intensity (yearly) ((-/+))</td>
<td>5.72/1.63</td>
</tr>
<tr>
<td>Mean jump size ((-/+))</td>
<td>3.85%/2.72%</td>
</tr>
</tbody>
</table>

Note: Estimation period is January 1996-July 2010. We set \(\lambda\) in the objective function given in (4.1) in the paper to 0.3.
Figure B.5: Solid dark line is $U$ based on estimation with $\lambda = 0.2$ and dotted light line is $U$ based on estimation with $\lambda = 0.3$. Estimation period is January 1996 to July 2010.
B.5  Alternative Models and/or Constrained Versions of the Model

B.5.1  Estimation Results for 1FGSJ Model

1FGSJ Model is given by:

\[
\frac{dX_t}{X_{t-}} = (r_t - \delta_t) dt + \sqrt{V_{1,t}} dW_{1,t}^Q + \int_{\mathbb{R}^2} (e^x - 1) \bar{\mu}^Q(dt, dx, dy),
\]

\[
dV_{1,t} = \kappa_1 (\bar{v}_1 - V_{1,t}) dt + \sigma_1 \sqrt{V_{1,t}} dB_{1,t}^Q + \int_{\mathbb{R}^2} y \mu(dt, dx, dy),
\]

where \((W_{1,t}^Q, B_{1,t}^Q)\) is a two-dimensional Brownian motion with \(\text{corr}(W_{1,t}^Q, B_{1,t}^Q) = \rho_1\).

The risk-neutral compensator for the jump measure is

\[
\nu_t^Q(dx, dy) = \left\{ (c_0 + c_1 V_{1,t-}) e^{\frac{(x - \mu_x - \rho_j y)^2}{2\sigma_x^2}} \frac{e^{-y/\mu_v}}{\mu_v} 1_{\{y>0\}} \right\} dx \otimes dy.
\]

Table B.6: Estimation results for the 1FGSJ model \[B.2\].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_1)</td>
<td>-0.899</td>
<td>0.009</td>
<td>(c_0)</td>
<td>0.011</td>
<td>0.002</td>
<td>(\sigma_x)</td>
<td>0.138</td>
<td>0.001</td>
</tr>
<tr>
<td>(\bar{v}_1)</td>
<td>0.028</td>
<td>0.001</td>
<td>(c_1)</td>
<td>20.517</td>
<td>0.347</td>
<td>(\mu_v)</td>
<td>0.025</td>
<td>0.001</td>
</tr>
<tr>
<td>(\kappa_1)</td>
<td>0.686</td>
<td>0.017</td>
<td>(\mu_x)</td>
<td>-0.076</td>
<td>0.001</td>
<td>(\rho_j)</td>
<td>-0.621</td>
<td>0.048</td>
</tr>
<tr>
<td>(\sigma_1)</td>
<td>0.154</td>
<td>0.003</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Summary Statistics

- RMSE: 3.14%
- Mean jump intensity (yearly): 0.63
- Mean jump size: -9.25%

Note: Estimation period is January 1996-July 2010.
B.5.2 Estimation Results for 2FGSJ Model

2FGSJ Model is given by:

\[
\begin{align*}
\frac{dX_t}{X_{t-}} &= (r_t - \delta_t) dt + \sqrt{V_{1,t}} dW_{1,t}^Q + \sqrt{V_{2,t}} dW_{2,t}^Q + \int_{\mathbb{R}^2} (e^x - 1) \tilde{\mu}^Q(dt, dx, dy), \\
\frac{dV_{1,t}}{V_{1,t}} &= \kappa_1 (\bar{V}_1 - V_{1,t}) dt + \sigma_1 \sqrt{V_{1,t}} dB_{1,t}^Q + \int_{\mathbb{R}^2} y\mu(dt, dx, dy), \\
\frac{dV_{2,t}}{V_{2,t}} &= \kappa_2 (\bar{V}_2 - V_{2,t}) dt + \sigma_2 \sqrt{V_{2,t}} dB_{2,t}^Q,
\end{align*}
\]  

\hspace{1cm} \text{(B.3)}

where \((W_{1,t}^Q, W_{2,t}^Q, B_{1,t}^Q, B_{2,t}^Q)\) is a four-dimensional Brownian motion with \(W_{1,t}^Q \perp W_{2,t}^Q, W_{1,t}^Q \perp B_{2,t}^Q,\) and \(W_{2,t}^Q \perp B_{1,t}^Q,\) while \(\text{corr}(W_{1,t}^Q, B_{1,t}^Q) = \rho_{d,1}\) and \(\text{corr}(W_{2,t}^Q, B_{2,t}^Q) = \rho_{d,2}\).

The risk-neutral compensator for the jump measure is

\[
\nu_t^Q(dx, dy) = \begin{cases} 
(c_0 + c_1 V_{1,t-} + c_2 V_{2,t-}) & e^{-\frac{(x-\mu_x-\rho_j y)^2}{2\sigma_x^2}} e^{-y/\mu_v} 1\{y>0\} \\
\end{cases} dx \otimes dy.
\]

Table B.7: Estimation results for the 2FGSJ model \(\text{(B.3)}\).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_1)</td>
<td>-0.969</td>
<td>0.031</td>
<td>(\bar{\nu}_2)</td>
<td>0.008</td>
<td>0.000</td>
<td>(c_2)</td>
<td>6.363</td>
<td>0.297</td>
</tr>
<tr>
<td>(\bar{\nu}_1)</td>
<td>0.009</td>
<td>0.000</td>
<td>(\kappa_2)</td>
<td>17.976</td>
<td>0.782</td>
<td>(\mu_x)</td>
<td>-0.193</td>
<td>0.003</td>
</tr>
<tr>
<td>(\kappa_1)</td>
<td>1.341</td>
<td>0.021</td>
<td>(\sigma_2)</td>
<td>0.074</td>
<td>0.054</td>
<td>(\sigma_x)</td>
<td>0.063</td>
<td>0.002</td>
</tr>
<tr>
<td>(\sigma_1)</td>
<td>0.138</td>
<td>0.006</td>
<td>(c_0)</td>
<td>0.006</td>
<td>0.004</td>
<td>(\mu_v)</td>
<td>0.047</td>
<td>0.001</td>
</tr>
<tr>
<td>(\rho_2)</td>
<td>-0.742</td>
<td>0.525</td>
<td>(c_1)</td>
<td>25.557</td>
<td>0.575</td>
<td>(\rho_j)</td>
<td>0.638</td>
<td>0.023</td>
</tr>
</tbody>
</table>

Panel B: Summary Statistics

RMSE 2.56%
Mean jump intensity (yearly) 0.62
Mean jump size -16.30%

Note: Estimation period is January 1996-July 2010.
B.5.3 Estimation Results for 2FESJ Model

2FESJ Model is given by:

\[
\frac{dX_t}{X_{t^-}} = (r_t - \delta_t) dt + \sqrt{V_{1,t}} dW_{1,t}^Q + \sqrt{V_{2,t}} dW_{2,t}^Q + \int_{\mathbb{R}^2} (e^x - 1) \tilde{\mu}(dt, dx),
\]

\[
dV_{1,t} = \kappa_1 (\bar{\nu}_1 - V_{1,t}) dt + \sigma_1 \sqrt{V_{1,t}} dB_{1,t}^Q + \mu_1 \int_{\mathbb{R}^2} x^2 1_{\{x<0\}} \mu(dt, dx),
\]

\[
dV_{2,t} = \kappa_2 (\bar{\nu}_2 - V_{2,t}) dt + \sigma_2 \sqrt{V_{2,t}} dB_{2,t}^Q,
\]

where \((W_{1,t}^Q, W_{2,t}^Q, B_{1,t}^Q, B_{2,t}^Q)\) is a four-dimensional Brownian motion with \(W_{1,t}^Q \perp W_{2,t}^Q, W_{1,t}^Q \perp B_{2,t}^Q,\) and \(W_{2,t}^Q \perp B_{1,t}^Q,\) while \(\text{corr} \left(W_{1,t}^Q, B_{1,t}^Q\right) = \rho_{d,1}\) and \(\text{corr} \left(W_{2,t}^Q, B_{2,t}^Q\right) = \rho_{d,2}.\)

The risk-neutral compensator for the jump measure is

\[
\nu_t^Q(dx, dy) = \left\{ \left( c^- 1_{\{x<0\}} \lambda_- e^{-\lambda_- |x|} + c^+ 1_{\{x>0\}} \lambda_+ e^{-\lambda_+ x} \right) \right\} dx,
\]

\[c^- = c^-_0 + c^-_1 V_{1,t-} + c^-_2 V_{2,t-}, \quad c^+ = c^+_0 + c^+_1 V_{1,t-} + c^+_2 V_{2,t-}.\]

In our estimation we set \(c^-_0\) to zero.

Table B.8: Estimation results for 2FESJ model [B.4].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_1)</td>
<td>-0.762</td>
<td>0.091</td>
<td>(\kappa_2)</td>
<td>0.169</td>
<td>0.014</td>
<td>(c^+_2)</td>
<td>52.795</td>
<td>4.597</td>
</tr>
<tr>
<td>(\bar{\nu}_1)</td>
<td>0.004</td>
<td>0.000</td>
<td>(\sigma_2)</td>
<td>0.129</td>
<td>0.002</td>
<td>(\lambda_-)</td>
<td>16.943</td>
<td>0.070</td>
</tr>
<tr>
<td>(\kappa_1)</td>
<td>12.831</td>
<td>0.227</td>
<td>(c^-_0)</td>
<td>2.315</td>
<td>0.063</td>
<td>(\lambda_+)</td>
<td>51.818</td>
<td>0.220</td>
</tr>
<tr>
<td>(\sigma_1)</td>
<td>0.247</td>
<td>0.031</td>
<td>(c^-_1)</td>
<td>69.208</td>
<td>1.856</td>
<td>(\mu_1)</td>
<td>6.262</td>
<td>0.143</td>
</tr>
<tr>
<td>(\rho_2)</td>
<td>-0.945</td>
<td>0.009</td>
<td>(c^+_1)</td>
<td>13.161</td>
<td>11.356</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\bar{\nu}_2)</td>
<td>0.059</td>
<td>0.004</td>
<td>(c^-_2)</td>
<td>97.656</td>
<td>1.091</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Summary Statistics

- RMSE: 2.07%
- Mean jump intensity (yearly) (-/+): 2.6/3.36
- Mean jump size (-/+): -5.9%/1.93%

Note: Estimation period is January 1996-July 2010.
B.5.4 Estimation Results for 3FESJ-V Model

3FESJ-V Model is given by:

\[
\frac{dX_t}{X_t} = (r_t - \delta_t) dt + \sqrt{V_{1,t}} dW_{1,t}^Q + \sqrt{V_{2,t}} dW_{2,t}^Q + \sqrt{V_{3,t}} dW_{3,t}^Q + \int_{\mathbb{R}^2} (e^x - 1) \tilde{\mu}^Q(dt, dx, dy),
\]

\[
dV_{1,t} = \kappa_1 (\overline{\sigma}_1 - V_{1,t}) dt + \sigma_1 \sqrt{V_{1,t}} dB_{1,t}^Q + \mu_1 \int_{\mathbb{R}^2} x^2 1_{\{x<0\}} \mu(dt, dx, dy),
\]

\[
dV_{2,t} = \kappa_2 (\overline{\sigma}_2 - V_{2,t}) dt + \sigma_2 \sqrt{V_{2,t}} dB_{2,t}^Q,
\]

\[
dV_{3,t} = -\kappa_3 V_{3,t} dt + \mu_3 \int_{\mathbb{R}^2} [(1 - \rho_3) x^2 1_{\{x<0\}} + \rho_3 y^2] \mu(dt, dx, dy),
\]

\[(B.5)\]

where \((W_{1,t}^Q, W_{2,t}^Q, B_{1,t}^Q, B_{2,t}^Q)\) is a four-dimensional Brownian motion with \(W_{1,t}^Q \perp W_{2,t}^Q\), \(W_{1,t}^Q \perp B_{2,t}^Q\), and \(W_{2,t}^Q \perp B_{1,t}^Q\), while \(\text{corr}\left(W_{1,t}^Q, B_{1,t}^Q\right) = \rho_{d,1}\) and \(\text{corr}\left(W_{2,t}^Q, B_{2,t}^Q\right) = \rho_{d,2}\).

The risk-neutral compensator for the jump measure is

\[
\nu_t^Q(dx) = \left\{ \left(c^- 1_{\{x<0\}} \lambda_- e^{-\lambda_- |x|} + c^+ 1_{\{x>0\}} \lambda_+ e^{-\lambda_+ x} \right) 1_{\{y=0\}} + c^- 1_{\{x=0, y<0\}} \lambda_- e^{-\lambda_- |y|} \right\} dx \otimes dy,
\]

\[
c^- = c_0^- + c_1^- V_{1,t}^- + c_2^- V_{2,t}^- + c_3^- V_{3,t}^- , \quad c^+ = c_0^+ + c_1^+ V_{1,t}^- + c_2^+ V_{2,t}^- + c_3^+ V_{3,t}^- .
\]

In our estimation we set \(c_0^-\) and \(c_3^+\) to zero.
Table B.9: Estimation results for 3FESJ-V model (B.5)

**Panel A: Parameter Estimates**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1$</td>
<td>$-0.888$</td>
<td>0.025</td>
<td>$\sigma_2$</td>
<td>$0.093$</td>
<td>0.008</td>
<td>$c_2^-$</td>
<td>$146.355$</td>
<td>1.315</td>
</tr>
<tr>
<td>$\nu_1$</td>
<td>$0.015$</td>
<td>0.001</td>
<td>$\mu_3$</td>
<td>$5.583$</td>
<td>0.369</td>
<td>$c_2^+$</td>
<td>$26.180$</td>
<td>36.532</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>$4.053$</td>
<td>0.146</td>
<td>$\kappa_3$</td>
<td>$17.296$</td>
<td>0.623</td>
<td>$c_3^-$</td>
<td>$116.395$</td>
<td>4.642</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>$0.347$</td>
<td>0.011</td>
<td>$\rho_3$</td>
<td>$0.470$</td>
<td>0.028</td>
<td>$\lambda_-$</td>
<td>$17.162$</td>
<td>0.105</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>$-0.989$</td>
<td>0.058</td>
<td>$c_0^+$</td>
<td>$6.301$</td>
<td>0.205</td>
<td>$\lambda_+$</td>
<td>$78.369$</td>
<td>0.425</td>
</tr>
<tr>
<td>$\nu_2$</td>
<td>$0.003$</td>
<td>0.000</td>
<td>$c_1^-$</td>
<td>$0.362$</td>
<td>1.556</td>
<td>$\mu_1$</td>
<td>$3.244$</td>
<td>0.486</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>$1.287$</td>
<td>0.043</td>
<td>$c_1^+$</td>
<td>$292.907$</td>
<td>49.268</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Panel B: Summary Statistics**

- **RMSE**: 1.86%
- **Mean jump intensity (yearly) (-/+)**: $2.72/9.26$
- **Mean jump size (-/+)**: $-5.83%/1.28%$

*Note: Estimation period is January 1996-July 2010.*
C  Option Pricing

C.1 Pricing Method

In this section we describe how we obtain the option prices for the most general model outlined in equation (B.1). Since the model belongs to the class of affine models, the option prices can be obtained using standard Fourier methods. We employ here the Fourier-cosine series expansion introduced by Fang and Oosterlee (2008). To apply the method, all we need is the conditional characteristic function of the log-prices. The latter is not available in closed form, but it can be easily obtained through the solution of a system of Ordinary Differential Equations (ODEs), as described in Section 2.3 of Duffie et al. (2000) and Theorem 2.7 of Duffie et al. (2003).

More specifically, for \( y_t = \log(X_t) \) and \( u \in \mathbb{C} \), we define

\[
g(u, y_t, V_{1,t}, V_{2,t}, U_t, \tau) = \mathbb{E}_{t}^{Q}[e^{uy_t+\tau}].
\]

Since, the model in (B.1) is in the affine class, we have

\[
g(u, y_t, V_{1,t}, V_{2,t}, U_t) = e^{\alpha(u, \tau) + \beta_{1}(u, \tau)V_{1,t} + \beta_{2}(u, \tau)V_{2,t} + \beta_{3}(u, \tau)U_t + uy_t},
\]

for some coefficients \( \alpha(u, \tau), \beta_{1}(u, \tau), \beta_{2}(u, \tau), \) and \( \beta_{3}(u, \tau) \) to be determined. The Feynman-Kac theorem implies that \( g(u, y_t, V_{1,t}, V_{2,t}, U_t) \) solves (we omit the subscript \( t \) for the state variables and the prime indicates the derivative with respect to time-to-expiration \( \tau \)):

\[
- \alpha' - \beta_{1}'V_{1} - \beta_{2}'V_{2} - \beta_{3}'U_{t} + u[r_t - \delta_t - \frac{1}{2}(V_{1} + V_{2} + \eta^{2}U_{t}) - c_{t}^{-}(\Theta^{nc}(u, 0, 0) - 1) - c_{t}^{+}(\Theta^{p}(u, 0, 0) - 1)] + \beta_{1}(k_{1}(v_{1} - V_{1})) + \beta_{2}(k_{2}(v_{2} - V_{2})) - k_{u}U_{t}\beta_{3} + \frac{1}{2}u^{2}(V_{1} + V_{2} + \eta^{2}U_{t}) + \frac{1}{2}\sigma_{1}^{2}V_{1}\beta_{1}^{2} + \frac{1}{2}\sigma_{2}^{2}V_{2}\beta_{2}^{2} + \beta_{1}u\sigma_{1}\beta_{1}V_{1} + \beta_{2}u\sigma_{2}\beta_{2}V_{2} + c_{t}^{-}(\Theta^{nc}(u, \beta_{1}, \beta_{3}) - 1) + c_{t}^{+}(\Theta^{ai}(0, 0, \beta_{3}) - 1) + c_{t}^{+}(\Theta^{p}(u, 0, 0) - 1) = 0,
\]

where

\[
\Theta^{nc}(q_{0}, q_{1}, q_{3}) = \int_{-\infty}^{0} e^{q_{0}z + q_{1}\mu_{1}z^{2} + q_{3}(1-\rho_{u})\mu_{u}z^{2}}\lambda^{-} e^{z\lambda^{-}} dz \tag{C.2}
\]

\[
\Theta^{ai}(q_{3}) = \int_{-\infty}^{0} e^{q_{3}\rho_{u}\mu_{u}z^{2}}\lambda^{-} e^{z\lambda^{-}} dz \tag{C.3}
\]

\[
\Theta^{p}(q_{0}) = \int_{0}^{+\infty} e^{q_{0}z}\lambda^{+} e^{-z\lambda^{+}} dz. \tag{C.4}
\]

For \( \Theta^{nc}(q_{0}, q_{1}, q_{3}) \) we have

\[
\Theta^{nc}(q_{0}, q_{1}, q_{3}) = \lambda^{-} \int_{-\infty}^{0} e^{z^{2}(q_{1}\mu_{1} + q_{3}(1-\rho_{u})\mu_{u} + \frac{2(q_{0} + \lambda^{-})}{(q_{1} + q_{3}(1-\rho_{u})\mu_{u})})} \int_{q_{0} + \lambda^{-}}^{\infty} e^{2z(q_{1}\mu_{1} + q_{3}(1-\rho_{u})\mu_{u})} \left( z + \frac{q_{0} + \lambda^{-}}{q_{1}(1-q_{3}(1-\rho_{u})\mu_{u})} \right)^{2} dz \tag{C.5}
\]

\[
= \lambda^{-} e^{q_{1}\mu_{1} + q_{3}(1-\rho_{u})\mu_{u}} \int_{-\infty}^{0} e^{z(q_{1}\mu_{1} + q_{3}(1-\rho_{u})\mu_{u})} \left( z + \frac{q_{0} + \lambda^{-}}{q_{1}(1-q_{3}(1-\rho_{u})\mu_{u})} \right)^{2} dz.
\]
and defining:
\[ a = -(q_1 \mu_1 + q_3 (1 - \rho_u) \mu_u), \quad b = \frac{q_0 + \lambda^-}{2(q_1 \mu_1 + q_3 (1 - \rho_u) \mu_u)} \]
we obtain:
\[
\Theta^{nc}(q_0, q_1, q_3) = \lambda^- e^{ab^2} \int_{-\infty}^{0} e^{-a(z+b)^2} dz = \lambda^- e^{ab^2} \int_{-\infty}^{b} e^{-ay^2} dy \]
\[
= \lambda^- e^{ab^2} \frac{1}{\sqrt{a}} \int_{-\infty}^{\sqrt{ab}} e^{-(\sqrt{a}y)^2} d(\sqrt{ay}) = \lambda^- e^{ab^2} \frac{1}{\sqrt{a}} \text{erf}(\sqrt{ab}),
\]
where \( \text{erf}(.) \) is the complex error function. \( \Theta^{ni}(q_3) \) can be computed following the steps in equations (C.5) and (C.6), while \( \Theta^p(q_0) \) is given by the characteristic function of the exponential distribution.

Finally, (C.1) can be solved by collecting terms containing \( V_{1,t}, V_{2,t}, \) and \( U_t \) and setting the coefficients in front of these variables. This leads to the following system of ODEs that we solve numerically:
\[
\alpha' = u[r - \delta - c_0^-(\Theta^m(u, 0, 0) - 1) - c_0^+(\Theta^p(u, 0, 0) - 1)] + \beta_1 k_1 v_1 + \beta_2 k_2 v_2 +
\]
\[
c_0^- (\Theta^{nc}(u, \beta_1, \beta_3) - 1) + c_0^- (\Theta^m(0, 0, \beta_3) - 1) + c_0^- (\Theta^p(u) - 1)
\]
\[
\beta'_1 = u[-\frac{1}{2} - c_1^- (\Theta^m(u, 0, 0) - 1) - c_1^+ (\Theta^p(u, 0, 0) - 1)] - \beta_1 k_1 + \frac{1}{2} u^2 \frac{1}{2} \sigma_1^2 \beta_1 + \beta_1 u \sigma_1 \rho_1 +
\]
\[
c_1^- (\Theta^{nc}(u, \beta_1, \beta_3) - 1) + c_1^- (\Theta^m(0, 0, \beta_3) - 1) + c_1^- (\Theta^p(u) - 1)
\]
\[
\beta'_2 = u[-\frac{1}{2} - c_2^- (\Theta^m(u, 0, 0) - 1) - c_2^+ (\Theta^p(u, 0, 0) - 1)] - \beta_2 k_2 + \frac{1}{2} u^2 \frac{1}{2} \sigma_2^2 \beta_2 + \beta_2 u \sigma_2 \rho_2 +
\]
\[
c_2^- (\Theta^{nc}(u, \beta_1, \beta_3) - 1) + c_2^- (\Theta^m(0, 0, \beta_3) - 1) + c_2^- (\Theta^p(u) - 1)
\]
\[
\beta'_3 = u[-\frac{1}{2} \eta^2 - c_3^- (\Theta^m(u, 0, 0) - 1) - c_3^+ (\Theta^p(u, 0, 0) - 1)] - \beta_3 k_u
\]
\[
c_3^- (\Theta^{nc}(u, \beta_1, \beta_3) - 1) + c_3^- (\Theta^m(0, 0, \beta_3) - 1) + c_3^- (\Theta^p(u) - 1)
\]
\[
(C.7)
\]
All the other models presented in the paper are special cases of the model in (B.1).

### C.2 Monte Carlo Exercise

In this section we compare the pricing obtained using the numerical procedure described in Section C.1 with option prices computed via a Monte Carlo simulation using 10,000,000 simulated paths (simulation is done via Euler discretization with length of the discretization interval of 1/5 of a day). We use the same parameters reported in Table 5 in the paper and we set the state vector at the sample median values from our estimation. Figure C.1 shows the corresponding implied volatility curve for moneyness \( m \) between \([-8, 4]\) and maturity equal to seven days.

---

1For example in Matlab the function ode45.m works very well.
Figure C.1: Monte Carlo Exercise. Comparison between the implied volatility curve obtained using the Fourier-cosine series expansion and Monte Carlo simulation with 10,000,000 simulated paths for the 3FESJ model in Equation [B.1]. Parameters as in Table 5 and state vector equal to the median of the daily filtered states vectors.
D Additional Details and Results for the Predictive Regressions

D.1 Filtered Jumps from High-Frequency Data

Figure D.1: Weekly Number of “big” Negative and Positive Market Jumps. The figure plots \( \hat{LT}_{t,t+\tau}^{K,i} \) and \( \hat{RT}_{t,t+\tau}^{K,i} \), defined in (9.11) in the paper with \( K = 0.5\% \) and \( \tau \) equal to one week. The total number of “big” negative and positive jumps in the sample is 224 and 235 respectively.
D.2 Predictive Regressions for the 2FESJ Model

Figure D.2: Predictive Regressions for Volatility and Jump Risks. The volatility and jump risk measures are defined in (9.11)-(9.12) in the paper. For each regression, the top panels depict the t-statistics for the individual parameter estimates while the bottom panels indicate the regression $R^2$. The predictive variables are $V_1$ (dotted line), $V_2$ (dashed line).
Figure D.3: Predictive Regressions for Equity and Variance Risk Premia. For each regression, the top panels depict the t-statistics for the individual parameter estimates while the bottom panels indicate the regression $R^2$. The predictive variables are $V_1$ (dotted line), $V_2$ (dashed line).
D.3 Predictive Regression for Model (B.1) Using CRSP Monthly Returns

Figure D.4: Predictive Regression for Future Excess Returns from CRSP. Monthly returns and the corresponding risk-free rate have been downloaded from Ken French website: [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)
References

