The Jump Leverage Risk Premium[⋆]

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Abstract

Jumps in asset prices are ubiquitous, yet the apparent high price of jump risk observed empirically is commonly viewed as puzzling. We develop new model-free short-time risk-neutral variance expansions, allowing us to clearly delineate the importance of jumps in generating both price and variance risks. We find that simultaneous jumps in the price and the stochastic volatility and/or jump intensity of the market commands a sizeable risk premium. The existence of "jump leverage" risk premium may be rationalized in the context of equilibrium-based models by jumps in the conditional moments of the underlying fundamentals and/or changes in investors' risk aversion.

Keywords:
Jumps, Options, Tail Risk, Leverage Effect, Risk Premiums, VIX Index
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1. Introduction

There is ample empirical evidence for the existence of jumps, or discontinuities, in the price of the market portfolio. There is also an extensive literature suggesting that the risk associated with market price jumps is priced differently from the risk associated with "smooth," or continuous, price moves.[1] Accordingly, jumps also feature prominently in many equilibrium-based models that seek to rationalize the joint behavior of stock and

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[1]See, e.g., Bakshi et al. (1997), Bates (2000, 2019), Pan (2002), Eraker et al. (2003), Eraker (2004), Broadie et al. (2007), Santa-Clara and Yan (2010), Bollerslev and Todorov (2011), Bollerslev et al. (2015), Andersen et al. (2015b), Aït-Sahalia et al. (2020), Aït-Sahalia et al. (2021) and Aleti (2022), among others.

option markets.[2] These existing empirical analyses and theoretical models notwithstanding, the apparent "high" price of market jump risk is widely regarded as puzzling.[3]

Can this puzzling "high" price of market jump risk be explained by the risk being accompanied by other risks that are similarly disliked by investors? Indeed, as widely documented in the literature, market price jumps are often accompanied by jumps in market volatility, typically proxied by the VIX index, in the opposite direction.[4] We refer to this risk as the jump leverage effect. Meanwhile, positive jumps in the VIX are clearly disliked by investors, as evidenced by the pronounced right skew in implied volatilities extracted from short-dated VIX index options. Correspondingly, one might naturally expect a nontrivial risk premium for said co-jumps in price and volatility. The goal of this paper is to provide a model-free theoretical analysis and empirical assessment of this jump leverage risk premium, that is the risk premium demanded by investors for the jump leverage effect.

The term "leverage effect" is commonly used more generally to refer to the dependence between return and volatility. Consistent with this usage of the term, we rely on the notion of quadratic covariation to assess this dependence.[5] More specifically, let x denote the logarithmic market price, and $\mathcal{V}$ refer to some proxy for the market spot variance. The stochastic leverage effect process for the market is then naturally defined by $[x, \mathcal{V}]_t$, where $[\cdot, \cdot]_t$ denotes the time 0 to time t quadratic covariation. The jump leverage effect that we are after, as formally defined below, effectively obtains by restricting the calculation of the quadratic covariation to simultaneous jumps in the price and the volatility proxy. In our analyses we will rely on the instantaneous counterpart to the VIX volatility index as our $\mathcal{V}$. This definition of the leverage effect also directly mirrors the definition previously used by Andersen et al. (2015a) and Kalnina and Xiu (2017) among others. Doing so has the obvious advantage of allowing for reliable model-free inference procedures. Importantly, however, as discussed in more detail below, as long as positive jumps in the specific volatility measure used in the definition, whether defined under the true or the risk neutral probability measure, are disliked by investors, our key theoretical and empirical results will remain the same.[6]

We are not the first to explicitly highlight the existence of jump leverage effects, or the tendency for negative market price jumps to trigger positive jumps in market volatility and/or market jump intensity, see, e.g., Jacod and Todorov (2010), Todorov and Tauchen

[2]See, e.g., Eraker and Shaliastovich (2008), Du (2011), Drechsler and Yaron (2011), Wachter (2013), Dew-Becker et al. (2017), Martin (2017), Seo and Wachter (2019), Schreindorfer (2020), Dew-Becker et al. (2021) and Eraker and Yang (2022), among others.

[3]This also echoes longstanding empirical evidence for the "expensiveness" of deep out-of-the-money put options; see, e.g., Bondarenko (2014b), among others.

[4]See, e.g., Todorov and Tauchen (2011), Andersen et al. (2015a) and Kalnina and Xiu (2017).

[5]See, e.g., Wang and Mykland (2014), Andersen et al. (2015a), Aït-Sahalia et al. (2017), and Kalnina and Xiu (2017).

[6]Most parametric stochastic volatility models used in prior work also imply a tight connection between alternate volatility proxies, as illustrated by the specific equilibrium models discussed in Section 5 below.

(2011), Bandi and Renò (2012), Bandi and Renò (2016), Aït-Sahalia et al. (2017), Jacod et al. (2017), among others. There is also a large existing literature concerned with the nonparametric estimation of leverage effects more generally without separately considering the impact of jumps; see, e.g., Bollerslev et al. (2006), Ait-Sahalia et al. (2013), Wang and Mykland (2014), Andersen et al. (2015a) and Kalnina and Xiu (2017), among others. Our paper is perhaps most closely related to the recent work by Orlowski et al. (2023), who measure the skewness risk premium through the average profits of a trading strategy that creates exposure to said risk in a model-free manner. However, the pricing implications of the jump leverage effect and its role in explaining the seemingly "high" price of market jump risk has not previously been thoroughly studied in the literature. We seek to fill this void and precise the impact of the jump leverage effect for explaining market jump risk premiums. In so doing, we find that allowing for volatility jumps and price-volatility co-jumps are both necessary for properly understanding the propagation of market jump risk and the pricing thereof.[7]

Our empirical approach is decidedly non-parametric, relying on the rich information in short-dated options with different tenors for extracting information about the pricing of the jump leverage effect.[8] Intuitively, while asymmetry in the jump distribution and the jump leverage effect will both contribute to asymmetry in the distributions of returns over very short horizons, it is possible to separate the two effects by also looking at slightly longer horizons and the term structure of skewness, as the leverage effect should manifest more strongly over slightly longer horizons.[9] However, we purposely do not seek to explicitly model the dynamics of the spot volatility and jump intensities of the price jumps and the price-volatility co-jumps, as would be required to study the propagation of risks over "long" horizons. Instead, we rely solely on the non-parametric model-free procedures and focus on the short-term pricing. This also clearly distinguishes our paper from other existing work.

We begin our analysis by nonparametrically identifying the risk-neutral expectation of the jump leverage effect. We do so by developing new short-term expansions for two alternative risk-neutral variance measures: (i) the risk-neutral conditional expectation of the log-price, mirroring the measure used by the Chicago Board Options Exchange (CBOE) in

[7]By contrast, earlier work based on the estimation of parametric stochastic volatility models and options with longer tenors than the ones used here often report much smaller economic gains from incorporating volatility jumps, and jump leverage effects, in the price dynamics; see, e.g., Broadie et al. (2007). Our results are also in line with the recent study by Beason and Schreindorfer (2022), and the finding that most existing equilibrium-based models as traditionally calibrated are not consistent with where in the domain of the return space the equity premium is actually earned.

[8]Jackwerth and Vilkov (2019) have also recently proposed an estimator for the risk-neutral leverage effect based on a parametric copula model and options on the S&P 500 and VIX index. By contrast, our method is fully nonparametric and seeks to estimate the instantaneous jump leverage effect.

[9]Note, this is distinctly different from the realized skewness of returns over very long horizons analyzed in the work by Neuberger (2012).

their computation of the VIX index, and (ii) the risk-neutral conditional log-return variance. While both of our expansions are linear functions of the return time horizon and its square, with the leading term in both being equal to the risk-neutral spot variance, the slopes of the two expansions differ. Moreover, this difference is almost exclusively attributable to the jump leverage effect, allowing for its nonparametric estimation from actively traded short-dated S&P 500 index options. Comparing the resulting risk-neutral estimates with their realized counterparts estimated from high-frequency intraday S&P 500 index returns, in turn reveal strong compensation for short-term leverage risk, with more than half of the risk premium in third return moments at a horizon of two weeks attributable to the jump leverage effect.

From a theoretical perspective, this pricing of the jump leverage effect is naturally associated with instantaneous changes in the investment opportunity set induced by jumps in the volatility and/or jump intensity. Accordingly, shocks to the stochastic volatility and/or jump intensity should be priced. Consistent with this thesis, we find that at a horizon of two weeks, close to one-fifth of the variance risk premium may be directly attributed to compensation for time variation in the stochastic volatility and/or jump intensity, as opposed to compensation for instantaneous variance risk. Further corroborating the non-trivial pricing of the jump leverage effect, we find that the term structure of the variance risk premium substantially flattens for horizons beyond two weeks.[10]

At a broader level, our new model-free empirical results naturally calls into question the ability of existing equilibrium-based asset pricing models to explain the observed phenomena. Focussing on popular consumption-based models that have previously been proposed in the literature for explaining option prices, two different economic channels that could be at work stand out: (i) jumps in the consumption volatility combined with preferences for early resolution of uncertainty by the representative agent in the economy; or (ii) jumps in the risk-aversion of the representative agent possibly triggered by habit formation and jumps in the level of consumption. While both of these channels in theory could account for the jump leverage effect, and the non-trivial pricing thereof, the equilibrium-based models hitherto studied in the literature are invariably too stylized to warrant direct estimation or any exact quantification of the effects that we document.

The rest of the paper is organized as follows. We begin in Section 2 with a discussion of our general theoretical setup and formal definition of the jump leverage effect. Section 3 develops the short-time expansions for the risk-neutral variance measures that allow us to quantify the relevant features from options in a model-free manner. Section 4 formally defines the jump and leverage risk premiums. Section 5 considers possible equilibrium-based foundations for the existence of jump leverage risk. Section 6 discusses our practical implementation of the different measures based on short-dated S&P 500 options and high-frequency intraday S&P

[10]This flattening of the term structure over longer horizons is also in line with the empirical evidence of Dew-Becker et al. (2017).

500 returns. Section 7 presents our main empirical findings pertaining to the pricing of the jump leverage effect, including the results from a series of simple return predictability regressions directly highlighting the practical relevance of the new measures, together with additional supportive empirical evidence related to the pricing of VIX index options. All of the proofs, along with Monte Carlo simulation evidence underscoring the accuracy of the new variance expansions, are deferred to Appendices.

2. Formal Setup and Definitions

This section formally defines the key features that we are after, including the jump leverage effect, within a general theoretical setting. To this end, let X_t denote the underlying price process for the market, with the corresponding log-price process denoted by x_t. We will assume that X_t is defined on the probability space $\big(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P}\big)$, and that the price obeys the following general Itô semimartingale dynamics

$$\frac{dX_t}{X_{t-}} = \alpha_t dt + \sigma_t dW_t + \int_{R^3} (e^x - 1)(\mu - \nu^{\mathbb{P}})(dt, dx, dy, dz), \tag{1}$$

where W_t is a Brownian motion, μ is an integer-valued jump measure counting the jumps in X with compensator $\nu^{\mathbb{P}}(dt, dx, dy, dz)$, α_t denotes the drift capturing the instantaneous expected return, and σ_t refers to the instantaneous stochastic volatility. We further assume that the jump compensator is of the form

$$\nu^{\mathbb{P}}(dt, dx, dy, dz) = \phi_t^{\mathbb{P}} dt F^{\mathbb{P}}(x, y, z) dx dy dz + \widetilde{\phi}_t^{\mathbb{P}} dt \widetilde{F}^{\mathbb{P}}(y, z) \epsilon_0(dx) dy dz, \tag{2}$$

for some predictable processes $\phi_t^{\mathbb{P}}$ and $\widetilde{\phi}_t^{\mathbb{P}}$, some functions over the jump sizes $F^{\mathbb{P}}$ and $\widetilde{F}^{\mathbb{P}}$, and ϵ_0 denoting the Dirac measure at zero.[11] The two separate components of the jump compensator capture the jump arrival intensity of the jumps that occur in the price, and the ones that do not, but still trigger jumps in volatility.[12] We will further express the volatility dynamics under $\mathbb{P}$ as

$$d\sigma_t^2 = b_t^{\sigma} dt + \eta_t^{\sigma} dW_t + \widetilde{\eta}_t^{\sigma} d\widetilde{W}_t + \int_{R^3} y(\mu - \nu^{\mathbb{P}})(dt, dx, dy, dz), \tag{3}$$

[11] Defining the jump measure over a three-dimensional jump size space makes it easier to represent the jumps in the diffusive volatility and jump intensity captured by our measures.

[12] Our analysis allows for more general forms of $\nu^{\mathbb{P}}(dt, dx, dy, dz)$ in which each triple (x, y, z) might have its own source of variation. We do not consider such a general representation here in order to help simplify the exposition. Most asset pricing models used in prior work also satisfy the specification for $\nu^{\mathbb{P}}(dt, dx, dy, dz)$ considered here.

for some suitable coefficients b_t^σ, η_t^σ and $\widetilde{\eta}_t^\sigma$. We will assume that the pricing kernel, denoted by Λ, evolves according to

$$\begin{aligned}\frac{d\Lambda_t}{\Lambda_{t-}} &= -\lambda_t dW_t - \widetilde{\lambda}_t d\widetilde{W}_t \\ &+ \int_{\mathbb{R}^3} \left(\frac{\phi_t^{\mathbb{Q}} F^{\mathbb{Q}}(x,y,z)}{\phi_t^{\mathbb{P}} F^{\mathbb{P}}(x,y,z)} 1_{\{x\neq 0\}} + \frac{\widetilde{\phi}_t^{\mathbb{Q}} \widetilde{F}^{\mathbb{Q}}(y,z)}{\widetilde{\phi}_t^{\mathbb{P}} \widetilde{F}^{\mathbb{P}}(y,z)} 1_{\{x=0\}} - 1 \right) (\mu - \nu^{\mathbb{P}})(dt,dx,dy,dz) + \mathcal{R}_t,\end{aligned} \tag{4}$$

for some processes $\phi_t^{\mathbb{Q}}$ and $\widetilde{\phi}_t^{\mathbb{Q}}$, and functions $F^{\mathbb{Q}}$ and $\widetilde{F}^{\mathbb{Q}}$, and a process $\mathcal{R}$ that is orthogonal (in a martingale sense) to W_t, $\widetilde{W}_t$ and μ. The inclusion of the $\mathcal{R}$ process allows for the possibility that other shocks, in addition to the ones generated by W_t, $\widetilde{W}_t$ and μ, could be priced, although we do not explicitly consider such shocks here. For some of our analysis, we also need the dynamics of the $\phi_t^{\mathbb{Q}}$ process under $\mathbb{P}$.[13] We will assume that

$$d\phi_t^{\mathbb{Q}} = b_t^\phi dt + \eta_t^\phi dW_t + \widetilde{\eta}_t^\phi d\widetilde{W}_t + \int_{R^3} z(\mu - \nu^{\mathbb{P}})(dt,dx,dy,dz), \tag{5}$$

for some suitable coefficients b_t^ϕ, η_t^ϕ and $\widetilde{\eta}_t^\phi$.

As shown in Appendix Appendix A, given the pricing kernel defined above, the dynamics of X under the risk-neutral probability measure, denoted with $\mathbb{Q}$, may be expressed as[14]

$$\frac{dX_t}{X_{t-}} = \sigma_t dW_t^{\mathbb{Q}} + \int_{\mathbb{R}} (e^z - 1)(\mu - \nu^{\mathbb{Q}})(dt,dx,dy,dz), \tag{6}$$

where $W_t^{\mathbb{Q}}$ is a $\mathbb{Q}$-Brownian motion, and $\nu^{\mathbb{Q}}$ denotes the jump compensator of μ under $\mathbb{Q}$

$$\nu^{\mathbb{Q}}(dt,dx,dy,dz) = [\phi_t^{\mathbb{Q}} dt F^{\mathbb{Q}}(x,y,z) dxdydz + \widetilde{\phi}_t^{\mathbb{Q}} dt \widetilde{F}^{\mathbb{Q}}(y,z)\epsilon_0(dx)dydz].$$

The specification of the pricing kernel in (4), and the resulting risk-neutral distribution for X, is extremely general. It encompasses essentially all pricing kernels implied by existing equilibrium models, as well as the pricing kernels used in prior reduced-form no-arbitrage models. The λ_t and $\widetilde{\lambda}_t$ parameters represent the prices attached to the two Brownian motions W_t and $\widetilde{W}_t$, respectively, while the price of jump risk is determined by the ratios of $\phi_t^{\mathbb{Q}} F^{\mathbb{Q}}(x,y,z)$ to $\phi_t^{\mathbb{P}} F^{\mathbb{P}}(x,y,z)$ and $\widetilde{\phi}_t^{\mathbb{Q}} \widetilde{F}^{\mathbb{Q}}(y,z)$ to $\widetilde{\phi}_t^{\mathbb{P}} \widetilde{F}^{\mathbb{P}}(y,z)$ that essentially reweight the

[13] The process $\phi_t^{\mathbb{Q}}$, which drives the risk-neutral jump compensator, contributes to the risk-neutral measures of variance that we rely on; see, e.g., equations (9) and (14) and the accompanying discussion below.

[14] Since we will be focussing on short return horizons, for notational simplicity we have set the risk-free rate and the dividend yield identically equal to zero. The extension to allow for nonzero interest rate and/or dividend yield is discussed in Appendix Appendix C. This appendix also shows that the effect from ignoring the risk-free interest rate and the dividend yield in our analysis is small.

arrival intensity of the jumps under the $\mathbb{P}$ and $\mathbb{Q}$ measures. In many reduced form representations and equilibrium-generated pricing kernels, these ratios are either assumed or implied to be time-invariant.[15]

To formally define the leverage effect, we need a specific proxy for volatility. Following Andersen et al. (2015a) and Kalnina and Xiu (2017), we will rely on the instantaneous counterpart to the VIX volatility index for this purpose. More specifically, the conditional risk-neutral mean of the log-return over a horizon T,

$$\mathcal{V}^{\mathbb{Q}}_{t,t+T} \equiv -\frac{2}{T}\mathbb{E}^{\mathbb{Q}}_t(x_{t+T} - x_t) = \frac{1}{T}\mathbb{E}^{\mathbb{Q}}_t\left(\int_t^{t+T} \sigma_s^2 ds + 2\int_t^{t+T}\int_{\mathbb{R}^3}(e^x - 1 - x)\mu(ds, dx, dy, dz)\right), \quad (7)$$

where the second equality follows by an application of Itô's lemma. For T corresponding to a month, this directly mirrors the variance measure used by the CBOE in the construction of their popular VIX volatility index. Accordingly, we define the instantaneous counterpart of the VIX measure as

$$\mathcal{V}^{\mathbb{Q}}_t \equiv \lim_{\Delta\to 0} \mathcal{V}^{\mathbb{Q}}_{t,t+\Delta}, \quad (8)$$

which may also be formally expressed as:

$$\mathcal{V}^{\mathbb{Q}}_t = \sigma_t^2 + \mathtt{v}^{\mathbb{Q}}\phi^{\mathbb{Q}}_t, \qquad \mathtt{v}^{\mathbb{Q}} = 2\int_{\mathbb{R}^3}(e^x - 1 - x)F^{\mathbb{Q}}(x, y, z)dxdydz. \quad (9)$$

As discussed in the introduction, the leverage effect, as commonly defined, measures the covariation between price and volatility. Correspondingly, for arbitrary processes y and z, we denote their quadratic covariation by:

$$[y, z]_t = \operatorname*{plim}_{n\to\infty} \sum_{i=1}^{\lfloor nt\rfloor}[(y_{i/n} - y_{(i-1)/n})(z_{i/n} - z_{(i-1)/n})]. \quad (10)$$

Given the price, volatility and jump intensity dynamics specified above, it follows that

$$\begin{aligned}
[x, \mathcal{V}^{\mathbb{Q}}]_t &= \int_0^t \sigma_s(\eta_s^\sigma + \mathtt{v}^{\mathbb{Q}}\eta_s^\phi)ds + \int_0^t\int_{\mathbb{R}^3} x(y + \mathtt{v}^{\mathbb{Q}}z)\mu(ds, dx, dy, dz) \\
&= \int_0^t \sigma_s(\eta_s^\sigma + \mathtt{v}^{\mathbb{Q}}\eta_s^\phi)ds + \int_0^t\int_{\mathbb{R}^3} x(y + \mathtt{v}^{\mathbb{Q}}z)\nu^{\mathbb{P}}(ds, dx, dy, dz) + \mathbb{P}\text{-martingale} \quad (11) \\
&= \int_0^t \sigma_s(\eta_s^\sigma + \mathtt{v}^{\mathbb{Q}}\eta_s^\phi)ds + \int_0^t\int_{\mathbb{R}^3} x(y + \mathtt{v}^{\mathbb{Q}}z)\nu^{\mathbb{Q}}(ds, dx, dy, dz) + \mathbb{Q}\text{-martingale.}
\end{aligned}$$

[15] As a case in point, in equilibrium models based on a representative agent with Epstein-Zin preferences, the ratios are constant determined by exponential tilting; see, e.g., Section 4.4 in Eraker and Yang (2022).

From these expressions, the instantaneous drifts of $[x, \mathcal{V}^{\mathbb{Q}}]_t$ under $\mathbb{P}$ and $\mathbb{Q}$ are therefore given by:

$$
\begin{aligned}
L_t^{\mathbb{P}} &= (\eta_t^{\sigma} + \mathtt{v}^{\mathbb{Q}}\eta_t^{\phi})\sigma_t + \int_{\mathbb{R}^3} x(y + \mathtt{v}^{\mathbb{Q}}z)\phi_t^{\mathbb{P}} F^{\mathbb{P}}(x,y,z)dxdydz, \\
L_t^{\mathbb{Q}} &= (\eta_t^{\sigma} + \mathtt{v}^{\mathbb{Q}}\eta_t^{\phi})\sigma_t + \int_{\mathbb{R}^3} x(y + \mathtt{v}^{\mathbb{Q}}z)\phi_t^{\mathbb{Q}} F^{\mathbb{Q}}(x,y,z)dxdydz.
\end{aligned}
\tag{12}
$$

The first terms, which are identical across the two expressions, arise from diffusive comovements between the price and the volatility. The second terms, which differ between $L_t^{\mathbb{P}}$ and $L_t^{\mathbb{Q}}$, stem from price-volatility co-jumps. Accordingly, we will refer to $L_t^{\mathbb{P}}$ and $L_t^{\mathbb{Q}}$ as the instantaneous leverage effect under $\mathbb{P}$ and $\mathbb{Q}$, respectively. In the next section, we will develop a novel model-free method for identifying $L_t^{\mathbb{Q}}$ from short-dated options, while, as discussed in Section 6, inference concerning $L_t^{\mathbb{P}}$ can be made from the use of high-frequency returns.

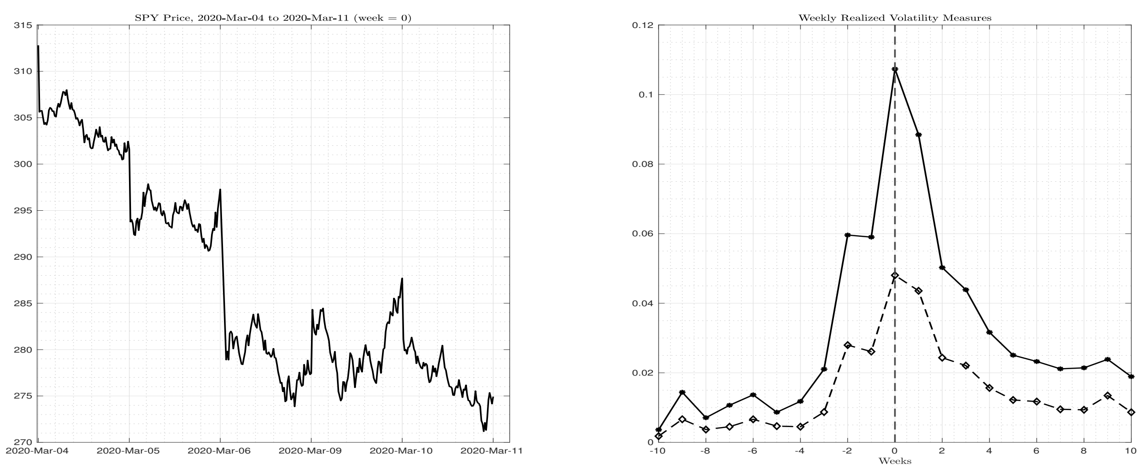

Figure 1: The left panel shows the SPY at a 5-minute intraday frequency for the week of March 4 - 11, 2020. The right plot shows the total weekly realized variation based on the summation of the squared 5-minute returns (solid line) and the realized jump variation (dashed line) constructed as the difference between the total variation and the bipower variation. The week of March 4 - 11, 2020 corresponds to week 0. Both of the variation measures are reported in annualized volatility units.

To help fix ideas and directly illustrate the practical importance of leverage effects through jumps, the left panel in Figure 1 shows the intraday prices for the S&P 500 market index at a 5-minute frequency for the week of March 4 - 11, 2020. Due to heightened fears about the global pandemic, the index dropped by more than 10% for that week as a whole. Meanwhile, even though the single large overnight decline on March 6 clearly stands out, there were also *many* other smaller intraday price jumps during that week. Consistent with the jump leverage effect, most of these smaller price jumps are clearly followed by heightened volatility.

This is further corroborated by the dynamics of the weekly realized volatility $\int_t^{t+T} \sigma_s^2 ds + \int_t^{t+T} \int_{\mathbb{R}^3} x^2 \mu(ds, dx, dy, dz)$ (solid line) and its jump component $\int_t^{t+T} \int_{\mathbb{R}^3} x^2 \mu(ds, dx, dy, dz)$ (dashed line) plotted in the right panel in Figure 1. While the realized jump risk for the week as a whole (indicated by the dashed line at week 0) did indeed increase compared to the week before (indicated by the dashed line at week -1), so did the total realized volatility inclusive of the diffusive price risk (indicated by the solid line).[16] In other words, negative jumps in the market index tend to induce positive jumps in the aggregate market volatility *and* the intensity of future jump arrivals.[17]

Further supporting the prominence of the jump leverage effect, the term structure of the unconditional skewness of high-frequency returns also exhibit a strong downward sloping pattern. In particular, while the skewness of the one-minute S&P 500 returns over our full 2007-2020 sample period is slightly positive at 0.26, the skewness of the daily returns is decidedly negative at -0.53. Put differently, the jump leverage effect hardly affects the skewness of returns over ultra short horizons, such as one-minute, but it clearly affects the skewness of the cumulative intraday returns over longer, say daily, horizons.

Before turning to the new expansions that we use for more formally identifying the jump leverage effect, a few observations are in order. By using $\mathcal{V}^{\mathbb{Q}}$ in our definition of the instantaneous leverage effect and the risks observed over very short horizons, the volatility due to jumps is assessed under the risk-neutral probability measure.[18] As shown below, this in turn allows for the identification of the $L_t^{\mathbb{Q}}$ jump measure from S&P 500 index options in a model-free manner without having to impose any restrictions on the form of the jumps. Relatedly, earlier parametric work has primarily been based on models with finite activity jumps, for which the jump part of $L_t^{\mathbb{P}}$ is naturally measured by $\frac{1}{\Delta} \lim_{\Delta \to 0} \mathbb{E}_t^{\mathbb{P}} \left(\sum_{\tau \in (t, t+\Delta)} \mathrm{Cov}_{\tau-}^{\mathbb{P}}(\Delta x_\tau, \Delta \mathcal{V}_\tau^{\mathbb{Q}}) \right)$, for τ being a stopping time associated with a jump, with an analogous expression for $L_t^{\mathbb{Q}}$. However, such measures are not well defined for models with infinite activity jumps covered by our general model-free theoretical framework and empirical analyses.

[16] The total weekly realized variation measures are based on the summation of the squared 5-minute returns over the week. The realized jump variation measures are constructed by subtracting the corresponding jump-robust bipower variation of Barndorff-Nielsen and Shephard (2004).

[17] These observations are also broadly consistent with the high-frequency-based parametric model estimates in Bates (2019), and the finding that large daily market moves typically represent the accumulation of a series of self-exciting intradaily volatility-price co-jumps; see also the more recent estimates in Ewald and Zou (2021).

[18] As previously noted, earlier nonparametric studies have similarly relied on risk-neutral variance measures in defining the leverage effect, see, e.g., Andersen et al. (2015a) and Kalnina and Xiu (2017).

3. Expansions of Risk-Neutral Variance Measures

In this section, we consider short-time expansions of two alternative risk-neutral variance measures. Namely, the $\mathcal{V}^{\mathbb{Q}}_{t,t+T}$ measure introduced in the previous section, and the standard conditional risk-neutral return variance. The difference between the two expansions in turn allows us to identify and nonparametrically estimate the risk-neutral jump leverage effect, $L^{\mathbb{Q}}_t$, from short-dated options.

The conditional risk-neutral variance of the log-price is naturally defined as

$$V^{\mathbb{Q}}_{t,t+T} \equiv \frac{1}{T}\text{Var}^{\mathbb{Q}}_t\left(x_{t+T} - x_t\right). \tag{13}$$

The instantaneous counterpart of $V^{\mathbb{Q}}_{t,t+T}$, say $V^{\mathbb{Q}}_t$, may alternatively be expressed in terms of the risk-neutral asset characteristics as

$$V^{\mathbb{Q}}_t = \sigma^2_t + v^{\mathbb{Q}}\phi^{\mathbb{Q}}_t, \qquad v^{\mathbb{Q}} = \int_{\mathbb{R}^3} x^2 F^{\mathbb{Q}}(x,y,z)dxdydz. \tag{14}$$

As the expressions in (9) and (14) make clear, the instantaneous variance measures $V^{\mathbb{Q}}_t$ and $\mathcal{V}^{\mathbb{Q}}_t$ only depend on the $\mathbb{Q}$ probability measure through their second jump components. As such, the difference between the two measures may be entirely explained by jumps,

$$V^{\mathbb{Q}}_t - \mathcal{V}^{\mathbb{Q}}_t = \int_{\mathbb{R}^3}[x^2 - 2(e^x - 1 - x)]F^{\mathbb{Q}}(x,y,z)dxdydz \approx -\frac{1}{3}\int_{\mathbb{R}^3} x^3 F^{\mathbb{Q}}(x,y,z)dxdydz, \tag{15}$$

that is the (scaled) $\mathbb{Q}$ third moment of market price jumps.

Over ultra short return horizons, or $T \approx 0$, the $V^{\mathbb{Q}}_{t,t+T}$ and $\mathcal{V}^{\mathbb{Q}}_{t,t+T}$ variance measures may naturally be used as proxies for the instantaneous variance measures $V^{\mathbb{Q}}_t$ and $\mathcal{V}^{\mathbb{Q}}_t$, respectively. However, for longer return horizons, or $T > 0$, mean reversion in volatility and jump intensity will both contribute nontrivially to $V^{\mathbb{Q}}_{t,t+T}$ and $\mathcal{V}^{\mathbb{Q}}_{t,t+T}$, thus rendering the difference between the two measures more difficult to interpret.[19] In order to formally account for this, it is useful to define the instantaneous drift term of any arbitrary process z evaluated under $\mathbb{Q}$ as

$$m^{\mathbb{Q}}_t(z) \equiv \lim_{\Delta\to 0}\frac{\mathbb{E}^{\mathbb{Q}}_t(z_{t+\Delta} - z_t)}{\Delta}. \tag{16}$$

Of course, if the process z is stationary, then $\mathbb{E}^{\mathbb{Q}}(m^{\mathbb{Q}}_t(z)) = 0$. Meanwhile, any risk premium for variation in z will generally imply that $\mathbb{E}^{\mathbb{P}}(m^{\mathbb{Q}}_t(z)) \neq 0$, with the sign being positive

[19] The connection between the $V^{\mathbb{Q}}_{t,t+T}$ and $\mathcal{V}^{\mathbb{Q}}_{t,t+T}$ variance measures has previously been used by Du and Kapadia (2012) in the construction of an options-based jump index, while Bondarenko (2014a) has previously analyzed the connection between the two measures in regards to variance trading and replication strategies.

(negative) if the specific variation in z is disliked (liked) by investors.

Now, utilizing the above notation, it is possible to show that

$$\mathcal{V}^{\mathbb{Q}}_{t,t+T} = \mathcal{V}^{\mathbb{Q}}_t + \frac{T}{2} \times m^{\mathbb{Q}}_t(\mathcal{V}^{\mathbb{Q}}_t) + T^2 \times C_t + O_p(T^3), \qquad T \downarrow 0, \tag{17}$$

where C_t denotes some $\mathcal{F}_t$-adapted random variable.[20] Notably, the leading term in the expansion, which does not depend upon T, is given by the instantaneous risk-neutral variance $\mathcal{V}^{\mathbb{Q}}_t$, while the slope of the expansion, when viewed as a function of T, is solely determined by the mean reversion of $\mathcal{V}^{\mathbb{Q}}_t$. The random variable C_t that dictates the T^2 term depends in a complicated way on various features of the volatility and jump intensity dynamics. This term is numerically small and we treat it as a nuisance parameter in what follows. The analogous expansion for the $V^{\mathbb{Q}}_{t,t+T}$ variance measure is given by:

$$V^{\mathbb{Q}}_{t,t+T} = V^{\mathbb{Q}}_t + \frac{T}{2} \times \left(m^{\mathbb{Q}}_t(V^{\mathbb{Q}}_t) - L^{\mathbb{Q}}_t\right) + T^2 \times C_t + O_p(T^3), \qquad T \downarrow 0, \tag{18}$$

where again C_t denotes some numerically small $\mathcal{F}_t$-adapted random variable.

The expressions for $\mathcal{V}^{\mathbb{Q}}_{t,t+T}$ and $V^{\mathbb{Q}}_{t,t+T}$ in (17) and (18) are obviously very similar except for one important distinction, namely the presence of the $L^{\mathbb{Q}}_t$ leverage term formally defined in (12) in the slope of the latter expansion. There are at least two intuitive reasons for this important "extra" term, stemming from the co-movements between the conditional mean and the martingale component of the log-price. Firstly, $V^{\mathbb{Q}}_{t,t+T}$ is defined as a centered second moment, while $\mathcal{V}^{\mathbb{Q}}_{t,t+T}$ is not. Secondly, $L^{\mathbb{Q}}_t$ is computed under the $\mathbb{Q}$ measure, and as discussed more formally in Appendix Appendix B, considerations of no-arbitrage explicitly restricts the drift term of the log-price under the risk-neutral measure to be a function of the jump intensity and the diffusive volatility. By comparison, there is no such restriction under the $\mathbb{P}$ measure.

Meanwhile, since the leverage effect and the dependence between price and variance innovations is generally found to be negative, the $L^{\mathbb{Q}}_t$ term will tend to heighten the term structure slope of $V^{\mathbb{Q}}_{t,t+T}$ compared to the slope of $\mathcal{V}^{\mathbb{Q}}_{t,t+T}$. In fact, since $m^{\mathbb{Q}}_t(V^{\mathbb{Q}}_t) - m^{\mathbb{Q}}_t(\mathcal{V}^{\mathbb{Q}}_t)$ is likely to be small in absolute value, as it depends on the third moment of jumps, the term structure slope of $V^{\mathbb{Q}}_{t,t+T} - \mathcal{V}^{\mathbb{Q}}_{t,t+T}$ will predominantly be determined by the $-L^{\mathbb{Q}}_t$ leverage term.

To more directly illustrate these features, Figure 2 plots the (normalized to unity at the shortest horizon) term structures of the risk-neutral variance ($\mathcal{V}^{\mathbb{Q}}_{t,t+T}$) and risk-neutral third moment ($3(V^{\mathbb{Q}}_{t,t+T} - \mathcal{V}^{\mathbb{Q}}_{t,t+T})$) calculated from short-dated S&P 500 options on two select

[20] Formal proofs of this expansion and the expansion presented below are provided in Appendix Appendix B.

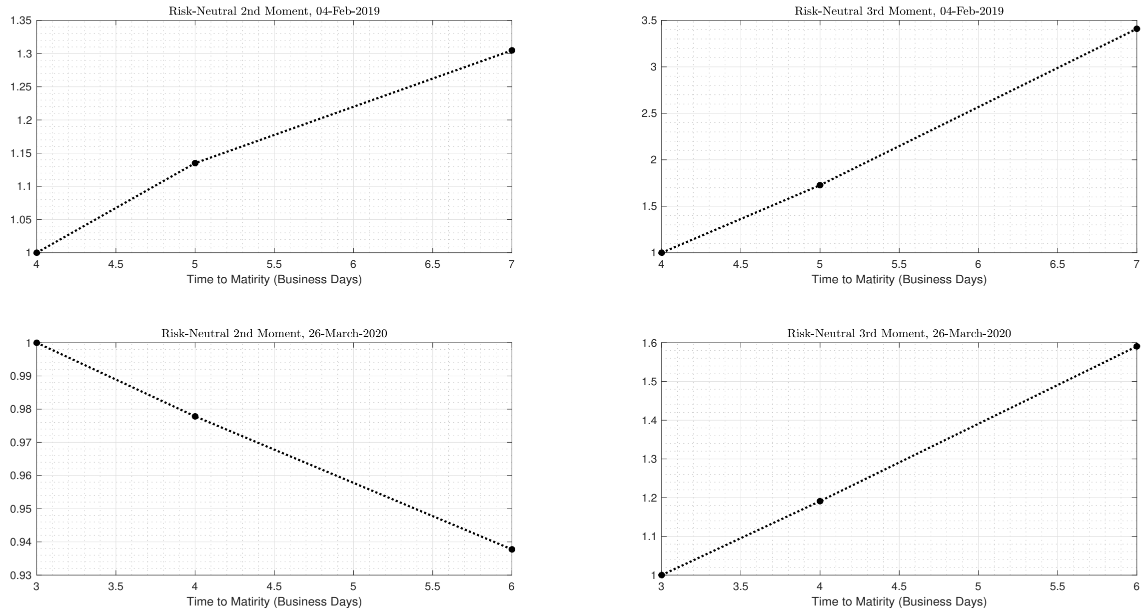

Figure 2: The left two panels show the second risk neutral moments calculated from short-dated S&P 500 options for different horizons on February 4, 2019 (top) and March 26, 2020 (bottom). The right two panels show the third risk neutral moments on the same two days. For easy of comparisons, the moments at the shortest horizons are normalized to unity.

days: February 4, 2019, a day with relatively low volatility, and March 26, 2020, a day with relatively high volatility.[21] Since volatility is well-known to be mean reverting, the risk-neutral variance term structure is naturally upward sloping on the low volatility day and downward sloping on the high volatility day due to $m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}})$ being positive and negative, respectively. By contrast, the term structure for the risk-neutral third moment is strongly upward sloping on *both* days. Looking across other high and low volatility days reveal the same general pattern: the risk-neutral third moment term structure is almost always upward sloping. This finding is difficult to reconcile with mean reversion in state variables, including the volatility and/or jump intensity, as one would expect these effects to manifest in opposite directions on high and low volatility days. Instead, the systematic upward sloping term structure for the risk-neutral third moment again underscores the importance of the jump leverage effect and the $-L_t^{\mathbb{Q}}$ term.[22]

[21] The estimates for the variance and the third moment shown in the figure are based on the expressions for $\widehat{\mathcal{V}}_{t,t+T}^{\mathbb{Q}}$ and $3(\widehat{V}_{t,t+T}^{\mathbb{Q}} - \widehat{\mathcal{V}}_{t,t+T}^{\mathbb{Q}})$ formally defined in equations (41) and (42) below.

[22] Relatedly, Bakshi et al. (2003), Kozhan et al. (2013) and Schneider et al. (2020) have previously emphasized the nontrivial pricing of skewness and coskewness risk embedded in options. The risk-neutral third moment is, of course, also formally affected by the total leverage effect; i.e., the covariation between price

Before considering our formal definitions of the jump and leverage risk premiums, to help further intuit the above results, it is instructive to more explicitly consider how the different features of the price process manifest in the new asymptotic expansions.[23] Specifically, denote the martingale component of the log-price process by

$$M_t = \int_0^t \sigma_s dW_s^{\mathbb{Q}} + \int_0^t \int_{\mathbb{R}^3} x(\mu - \nu^{\mathbb{Q}})(ds, dx, dy, dz). \tag{19}$$

Given the dynamics of x under $\mathbb{Q}$, and recalling our expressions for the two spot variance measures $V_t^{\mathbb{Q}}$ and $\mathcal{V}_t^{\mathbb{Q}}$ in (9), it follows by an application of Itô's formula that

$$x_t = x_0 - \frac{1}{2}\int_0^t \mathcal{V}_s^{\mathbb{Q}} ds + M_t. \tag{20}$$

From there, it follows that

$$\begin{aligned} T \times V_{t,t+T}^{\mathbb{Q}} = \mathbb{E}_t^{\mathbb{Q}} \left(\int_t^{t+T} \sigma_s^2 ds + \int_t^{t+T} \int_{\mathbb{R}^3} x^2 \nu^{\mathbb{Q}}(ds, dx, dy, dz) \right) \\ - \operatorname{Cov}_t^{\mathbb{Q}} \left(M_{t+T} - M_t, \int_t^{t+T} \mathcal{V}_s^{\mathbb{Q}} ds \right) + \frac{1}{4} \operatorname{Var}_t^{\mathbb{Q}} \left(\int_t^{t+T} \mathcal{V}_s^{\mathbb{Q}} ds \right). \end{aligned} \tag{21}$$

Comparing the above expression with that for $T \times \mathcal{V}_{t,t+T}^{\mathbb{Q}}$ in equation (7), reveals two "new" terms (the second and third) on the right-hand side of the expression for $T \times V_{t,t+T}^{\mathbb{Q}}$. The third term is small, as shown in the technical Appendix Appendix B. The second term accounts for the conditional covariance between the martingale increment of the log-price and an integrated measure of variance, i.e., a leverage effect. It is possible to expand this term by pretending that volatility and jump intensity remain constant over the short time

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and volatility shocks in general. Meanwhile, as argued below it appears impossible to reconcile the empirically very steep short-term slope observed in Figure 2 without explicitly allowing for pricing of the jump leverage effect. The additional results for the parametric double-jump stochastic volatility model discussed in Appendix Appendix D also further corroborate this.

[23] At the same time, it is important to stress, that the new expansion results are essentially model-free in the sense that they only require regularity type conditions for the $\mathbb{P}$ and $\mathbb{Q}$ dynamics, and correspondingly for the pricing kernel. These regularity type conditions are automatically satisfied if the price is embedded in a SDE of potentially larger dimension, as is the case for, e.g., the popular parametric affine jump-diffusion class of models of Duffie et al. (2000) discussed further in Appendix Appendix D below.

interval $[t, t+T]$, leading to:

$$\begin{aligned}\mathrm{Cov}_t^{\mathbb{Q}}\left(M_{t+T}-M_t, \int_t^{t+T} \mathcal{V}_s^{\mathbb{Q}} ds\right) &= \int_t^{t+T} \mathrm{Cov}_t^{\mathbb{Q}}\left(M_{t+s}-M_t, \mathcal{V}_s^{\mathbb{Q}}\right) ds \\ &= \int_t^{t+T} s L_t^{\mathbb{Q}} ds + C_t \times T^3 = \frac{T}{2} L_t^{\mathbb{Q}} + C_t \times T^3.\end{aligned} \tag{22}$$

Accordingly, the three different terms on the right-hand-side in (21) may be expressed as

$$\frac{1}{T}\mathbb{E}_t^{\mathbb{Q}}\left(\int_t^{t+T} \sigma_s^2 ds + \int_t^{t+T}\int_{\mathbb{R}^3} x(\mu-\nu^{\mathbb{Q}})(ds,dx,dy,dz)\right) = V_t^{\mathbb{Q}} + \frac{T}{2}\times m_t^{\mathbb{Q}}(V_t^{\mathbb{Q}}) + T^2 \times C_t + O_p(T^3),$$

$$\frac{1}{T}\mathrm{Cov}_t^{\mathbb{Q}}\left(M_{t+T}-M_t, \int_t^{t+T} \mathcal{V}_s^{\mathbb{Q}} ds\right) = \frac{T}{2} L_t^{\mathbb{Q}} + T^2 \times C_t + O_p(T^3),$$

$$\frac{1}{T}\mathrm{Var}_t^{\mathbb{Q}}\left(\int_t^{t+T} \mathcal{V}_s^{\mathbb{Q}} ds\right) = T^2 \times C_t + O_p(T^3),$$

where C_t is a nuisance $\mathcal{F}_t$-adapted random variable that can differ across the above equations. Combining these three expansions, in turn provides the expression for $V_{t,t+T}^{\mathbb{Q}}$ in (18).

4. Market Jump and Leverage Risk Premiums

The risk-neutral measures defined in connection with the expansions discussed in the previous section naturally suggest the definition of corresponding risk premiums.[24]

To begin, the instantaneous variance risk premiums for the two different variance measures may straightforwardly be defined by:

$$IVRP_t \equiv V_t^{\mathbb{Q}} - V_t^{\mathbb{P}}, \qquad \mathcal{IVRP}_t \equiv \mathcal{V}_t^{\mathbb{Q}} - \mathcal{V}_t^{\mathbb{P}}, \tag{23}$$

where $V_t^{\mathbb{P}}$ and $\mathcal{V}_t^{\mathbb{P}}$ denote the $\mathbb{P}$ counterparts to the $V_t^{\mathbb{Q}}$ and $\mathcal{V}_t^{\mathbb{Q}}$ instantaneous variances. Since the diffusive spot volatility σ_t^2 must be identical under the $\mathbb{P}$ and $\mathbb{Q}$ measures in order to prevent arbitrage opportunities, the $IVRP_t$ and $\mathcal{IVRP}_t$ risk premiums are solely determined by the pricing of jumps. However, that is not the case for the corresponding variance risk premiums defined over non-trivial time intervals $T > 0$,

$$VRP_{t,t+T} \equiv V_{t,t+T}^{\mathbb{Q}} - V_{t,t+T}^{\mathbb{P}}, \qquad \mathcal{VRP}_{t,t+T} \equiv \mathcal{V}_{t,t+T}^{\mathbb{Q}} - \mathcal{V}_{t,t+T}^{\mathbb{P}}. \tag{24}$$

The second of these two measures, in particular, has been extensively studied empirically in

[24] To help fix ideas, Appendix Appendix D further discusses how the different risk premiums defined below manifest in the context of the parametric double-jump stochastic volatility model of Duffie et al. (2000).

the recent literature, typically with T set to one-month mimicking the horizon of the popular VIX index used in place of $\mathcal{V}^{\mathbb{Q}}_{t,t+T}$.[25]

To help more formally assess what drives the $VRP_{t,t+T}$ and $\mathcal{VRP}_{t,t+T}$ measures over short horizons T, it is instructive to employ the expansions developed in the previous section. Doing so, we obtain the following two decompositions:

$$VRP_{t,t+T} = IVRP_t + \frac{T}{2} \times \left(m_t^{\mathbb{Q}}(V_t^{\mathbb{Q}}) - m_t^{\mathbb{P}}(V_t^{\mathbb{Q}})\right) - \frac{T}{2} \times \left(L_t^{\mathbb{Q}} - L_t^{\mathbb{P}}\right) + O_p(T^2), \tag{25}$$

and

$$\mathcal{VRP}_{t,t+T} = \mathcal{IVRP}_t + \frac{T}{2} \times \left(m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}}) - m_t^{\mathbb{P}}(\mathcal{V}_t^{\mathbb{Q}})\right) + O_p(T^2). \tag{26}$$

As previously noted, the leading component in $VRP_{t,t+T}$ (resp. $\mathcal{VRP}_{t,t+T}$), that is the instantaneous variance risk premium $IVRP_t$ (resp. $\mathcal{IVRP}_t$), is solely determined by asset price jumps. On the other hand, the second $m_t^{\mathbb{Q}}(V_t^{\mathbb{Q}}) - m_t^{\mathbb{P}}(V_t^{\mathbb{Q}})$ (resp. $m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}}) - m_t^{\mathbb{P}}(\mathcal{V}_t^{\mathbb{Q}})$) term, which depends linearly on the horizon T, reflects compensation demanded by investors for any changes in the investment opportunity set. In the context of the underlying general Itô semimartingale in (1), and its risk-neutral counterpart in (6), that is temporal variation in σ_t^2 and/or changes in the intensity of the jumps. More specifically, the volatility drift term under the true probability measure is given by:[26]

$$m_t^{\mathbb{P}}(\mathcal{V}_t^{\mathbb{Q}}) = b_t^{\sigma} + \mathtt{v}^{\mathbb{Q}} b_t^{\phi}. \tag{27}$$

By comparison, following the derivations in Appendix Appendix A, the pricing kernel implies the following volatility drift under the risk-neutral probability measure:

$$\begin{aligned} m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}}) = {} & m_t^{\mathbb{P}}(\mathcal{V}_t^{\mathbb{Q}}) + \lambda_t(\eta_t^{\sigma} + \mathtt{v}^{\mathbb{Q}}\eta_t^{\phi}) + \widetilde{\lambda}_t(\widetilde{\eta}_t^{\sigma} + \mathtt{v}^{\mathbb{Q}}\widetilde{\eta}_t^{\phi}) \\ & + \int_{\mathbb{R}^3} (y + \mathtt{v}^{\mathbb{Q}} z)(\phi_t^{\mathbb{Q}} F^{\mathbb{Q}}(x,y,z) - \phi_t^{\mathbb{P}} F^{\mathbb{P}}(x,y,z)) dxdydz \\ & + \int_{\mathbb{R}^2} (y + \mathtt{v}^{\mathbb{Q}} z)(\widetilde{\phi}_t^{\mathbb{Q}} \widetilde{F}^{\mathbb{Q}}(y,z) - \widetilde{\phi}_t^{\mathbb{P}} \widetilde{F}^{\mathbb{P}}(y,z)) dydz. \end{aligned} \tag{28}$$

Unlike the instantaneous variance risk premium in (23), which solely depends on price jumps, the difference $m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}}) - m_t^{\mathbb{P}}(\mathcal{V}_t^{\mathbb{Q}})$ manifests the pricing of all the different types of shocks to volatility and jump intensity. The first two terms in the expression in the first line in equation (28), in particular, stem from the pricing of diffusive risk in diffusive volatility and jump intensity. This includes shocks that directly affect the price (in the form of W_t) as well

[25] See, e.g., Bollerslev et al. (2009), Drechsler and Yaron (2011) and Bekaert and Hoerova (2014), along with many subsequent studies.

[26] Note that to ensure stationarity of the volatility process, it must be the case that $\mathbb{E}^{\mathbb{P}}(b_t^{\sigma}) = \mathbb{E}^{\mathbb{P}}(b_t^{\phi}) = 0$.

as shocks that do not (in the form of $\widetilde{W}_t$). The terms in the last two lines in (28) are due to the pricing of jumps in volatility and jump intensity, which either arrive together with the price jumps (second line) or independently (last line).

The decomposition for $VRP_{t,t+T}$ contains an additional linear-in-T component specifically due to the compensation for leverage risk. We will refer to this "extra" term as the instantaneous leverage risk premium in the sequel, or

$$ILRP_t \equiv L_t^{\mathbb{Q}} - L_t^{\mathbb{P}}. \tag{29}$$

In parallel to the instantaneous variance risk premiums, which are solely determined by jumps and the pricing thereof, $ILRP_t$ is solely determined by the jump leverage effect and the pricing thereof, that is the risk premium for co-jumps between the log-price x and $\mathcal{V}^{\mathbb{Q}}$. Indeed, given the definitions of $L_t^{\mathbb{Q}}$ and $L_t^{\mathbb{P}}$ in Section 2, we have

$$L_t^{\mathbb{Q}} - L_t^{\mathbb{P}} = \int_{\mathbb{R}^3} x(y + \mathtt{v}^{\mathbb{Q}} z)(\phi_t^{\mathbb{Q}} F^{\mathbb{Q}}(x,y,z) - \phi_t^{\mathbb{P}} F^{\mathbb{P}}(x,y,z))dxdydz. \tag{30}$$

Empirically, we would expect $ILRP_t$ to be negative and thus contribute positively to the variance risk premium $VRP_{t,t+T}$.

It is useful to compare the part of the $m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}}) - m_t^{\mathbb{P}}(\mathcal{V}_t^{\mathbb{Q}})$ risk premium attributed to the pricing of jumps with the instantaneous leverage risk premium. Even though the two expressions look similar, the mechanisms underlying the pricing of jumps are very different. In particular, while $m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}}) - m_t^{\mathbb{P}}(\mathcal{V}_t^{\mathbb{Q}})$ is generated by the level of jump risk in diffusive volatility and jump intensity, $L_t^{\mathbb{Q}} - L_t^{\mathbb{P}}$ is generated by comovements between the price and the volatility and/or jump intensity. Hence, in the absence of price and volatility co-jumps, as is the case in many asset pricing models as discussed further below, the instantaneous leverage risk premium will be identically equal to zero. At the same time, individual jumps in the volatility and/or the jump intensity may give rise to non-trivial volatility risk premiums, i.e., the third line in (28) might be different from zero. Of course, if $\int_{\mathbb{R}^3} x(y+\mathtt{v}^{\mathbb{Q}}z)(\phi_t^{\mathbb{Q}} F^{\mathbb{Q}}(x,y,z) - \phi_t^{\mathbb{P}} F^{\mathbb{P}}(x,y,z))dxdydz \neq 0$, then typically $\int_{\mathbb{R}^3} (y+\mathtt{v}^{\mathbb{Q}}z)(\phi_t^{\mathbb{Q}} F^{\mathbb{Q}}(x,y,z) - \phi_t^{\mathbb{P}} F^{\mathbb{P}}(x,y,z))dxdydz \neq 0$ as well. That is, price-volatility cojumps will contribute to the $m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}}) - m_t^{\mathbb{P}}(\mathcal{V}_t^{\mathbb{Q}})$ risk premium.

In addition to the above expressions for the instantaneous variance and leverage risk premiums, as shown in Appendix Appendix A, the pricing kernel also naturally implies the following expression for the instantaneous equity risk premium:

$$\alpha_t = -\lambda_t \sigma_t + \int_{\mathbb{R}} (e^x - 1)(\phi_t^{\mathbb{P}} F^{\mathbb{P}}(x,y,z) - \phi_t^{\mathbb{Q}} F^{\mathbb{Q}}(x,y,z))dxdydz. \tag{31}$$

As this expression makes clear, in contrast to the instantaneous variance and leverage risk premiums, both of which are solely driven by jumps, diffusive and jump risks both contribute to the instantaneous equity risk premium. Intuitively, diffusive risks are of second order importance for higher order moments, like variances and covariances, over short horizons. At the same time, the above expression also makes it clear that the instantaneous equity, leverage, and variance risk premiums are all related through compensation for price jump risk. Ceteris paribus a higher (lower) instantaneous leverage risk premium should naturally result in a higher (lower) equity risk premium. Similarly, a higher (lower) instantaneous variance risk premium should naturally result in a higher (lower) equity risk premium.

More specifically, suppose that $\phi_t^{\mathbb{P}} = \phi_t^{\mathbb{Q}}$, as is the case, for example, in equilibrium models with a representative agent equipped with Epstein-Zin preferences. In this situation, the components of the instantaneous equity, leverage and variance risk premiums due to the compensation for jump risk are all multiples of $\phi_t^{\mathbb{P}}$, leading to perfect linear dependence between the three components. A similar type of perfect dependence also occurs if $F^{\mathbb{P}} = F^{\mathbb{Q}}$, leading to the components of the instantaneous equity, leverage, and variance risk premiums due to the compensation for jump risk all being multiples of $\phi_t^{\mathbb{P}} - \phi_t^{\mathbb{Q}}$. We will briefly return to these connections in some of our empirical analyses below. At the same time, the connection between $m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}}) - m_t^{\mathbb{P}}(\mathcal{V}_t^{\mathbb{Q}})$ and the instantaneous equity risk premium is less clear, as the former includes the potential compensation for diffusive and jump volatility risks that are independent from the diffusive and jump price risks in (31).

5. Market Jump and Leverage Risk Premiums in Equilibrium Settings

The expressions derived in the previous section are completely general and model-free. Endowing the general setup with more specific economic structure, this section illustrates how jump leverage risk can be generated endogenously within the context of certain consumption-based equilibrium asset pricing models, while other equilibrium-based models imply that the jump leverage effect is nonexistent. To keep the presentation manageable, we restrict our attention to models that have previously been used in the literature to explain option prices. At the same time, we caution that consumption-based equilibrium models have generally not been developed to speak to empirical effects over the frequencies studied here.

We begin by considering the broad class of continuous-time jump-models proposed by Wachter (2013), Seo and Wachter (2019), and Eraker and Yang (2022); see also Eraker and Shaliastovich (2008) and Drechsler and Yaron (2011) who similarly consider equilibrium models with jumps in the consumption dynamics. In all of these models, the representative agent is endowed with Epstein-Zin preferences and faces the following consumption dynamics

$$\begin{aligned} \frac{dC_t}{C_{t-}} &= \mu dt + \sigma_t^c dB_t^c + \int_{R^2} (e^x - 1)\mu(dt, dx, dy), \\ d(\sigma_t^c)^2 &= \kappa_v(\theta_v - (\sigma_t^c)^2)dt + \sigma_v \sigma_t^c dB_t^v, \\ d\lambda_t^c &= \kappa_\lambda(\theta_\lambda - \lambda_t^c)dt + \sigma_\lambda \lambda_t^c dB_t^\lambda, \end{aligned} \tag{32}$$

where B_t^c, B_t^v and B_t^λ denote three independent Brownian motions, and μ is an integer-valued measure that counts the jumps in consumption and its volatility with compensator $\lambda_t^c dt\nu(dx, dy)$ for some function ν that captures the distribution of the jumps. This consumption dynamics obviously comprises several structural shocks: diffusive or jump shocks to the level of consumption, diffusive or jump shocks to consumption volatility, and diffusive shocks to the intensity of jumps in consumption and/or consumption volatility. However as a distinguishing feature, the model of Eraker and Yang (2022) explicitly rules out jumps in the level of consumption, while the models studied by Wachter (2013) and Seo and Wachter (2019) fix σ_t^c to be constant and thus rule out jumps in consumption volatility.

Meanwhile, all of the different shocks in (32) may in theory affect the equilibrium asset price X_t, while only shocks to consumption volatility and the jump intensity affect the stochastic volatility $\mathcal{V}_t^{\mathbb{Q}}$. That is

$$X_t = F(\ln C_t, \sigma_t^C, \lambda_t^C), \qquad \mathcal{V}_t^{\mathbb{Q}} = G(\sigma_t^C, \lambda_t^C), \tag{33}$$

for some functions F and G that depend on the model parameters. Correspondingly, the jumps in the equilibrium price and the spot risk-neutral volatility may be expressed succinctly as:

$$\begin{aligned} \Delta X_t &= F\left(\ln(C_{t-} + \Delta C_t), \sigma_{t-}^c + \Delta\sigma_t, \lambda_{t-}^c\right) - F\left(\ln C_{t-}, \sigma_{t-}^c, \lambda_{t-}^c\right), \\ \Delta\mathcal{V}_t^{\mathbb{Q}} &= G(\sigma_{t-}^c + \Delta\sigma_t, \lambda_{t-}^c) - G(\sigma_{t-}^c, \lambda_{t-}^c). \end{aligned} \tag{34}$$

In other words, jumps in the equilibrium price level X_t are either due to jumps in the level of consumption, or jumps in the consumption volatility, while jumps in the risk-neutral spot volatility measure $\mathcal{V}_t^{\mathbb{Q}}$ are only triggered by jumps in the consumption volatility. In particular, jumps in the level of consumption do not trigger jumps in the stochastic volatility of the price. Accordingly, in this setting the jump leverage effect can only be generated by consumption volatility jumps. As a result, while the model of Eraker and Yang (2022) is in theory consistent with the jump leverage effect, the earlier models of Wachter (2013) and Seo and Wachter (2019) do not accommodate this effect.

As a different class of consumption-based equilibrium models, we next describe the implications for jump leverage risk in a setting in which the representative agent has external-habit type preferences. This class of models was originally introduced and analyzed by Campbell and Cochrane (1999) and Menzly et al. (2004). However, we more closely follow the setup of Du (2011), who has previously used a habit-based model in the context of option pricing. The aggregate consumption in that model is determined by:

$$\frac{dC_t}{C_{t-}} = \mu dt + \sigma^c dB_t^c + \int_R (e^x - 1)\mu(dt, dx), \tag{35}$$

where μ is a standard Poisson measure. In contrast to the dynamic consumption process defined in (32), the process in (35) assumes that consumption growth is i.i.d. Accordingly, the temporal variation in the model-implied risk premiums is instead generated by temporal variation in the representative agent's degree of risk-aversion induced by the time-varying habits. The resulting dynamics for the agent's instantaneous risk-aversion coefficient is described by:

$$\frac{d\gamma_t}{\gamma_{t-}} = \mu^\gamma(\gamma_{t-})dt + \sigma^\gamma(\gamma_{t-})dB_t^c + \int_R h(\gamma_{t-}, x)\mu(dt, dx), \tag{36}$$

for some functions μ^γ, σ^γ and h. In other words, shocks to risk aversion are driven by shocks to consumption. With the two state variables in the model being the level of consumption and the level of risk aversion, the equilibrium price and its volatility may be written as:

$$X_t = F(\ln C_t, \gamma_t) \qquad \mathcal{V}_t^{\mathbb{Q}} = G(\gamma_t), \tag{37}$$

for some functions F and G, the exact forms of which again depend on the specific model parameters. The jumps in the price and the volatility may thus be succinctly expressed as:

$$\begin{aligned} \Delta X_t &= F\left(\ln(C_{t-} + \Delta C_t), \gamma_{t-} + \Delta\gamma_t\right) - F\left(\ln C_{t-}, \gamma_{t-}\right), \\ \Delta \mathcal{V}_t^{\mathbb{Q}} &= G(\gamma_{t-} + \Delta\gamma_t) - G(\gamma_{t-}). \end{aligned} \tag{38}$$

Since jumps in risk-aversion are triggered by jumps in consumption, the model therefore naturally accounts for jump leverage effects.

Intuitively, even though the jump structure for the consumption dynamics in (35) is similar to that of the models of Wachter (2013) and Seo and Wachter (2019), in the sense that jumps only manifest in the level of consumption and not in its volatility, the temporal variation in the degree of risk-aversion induced by the habit formation implies that jumps not only affect the price level, but also the spot price volatility. In that sense the model mirrors the model of Eraker and Yang (2022). However, the economic mechanisms underlying the jump leverage effect is still very different between the two types of models: in the model of Eraker and Yang (2022) the jump leverage effect arises directly from jumps in consumption volatility, while in the model of Du (2011) the effect occurs implicitly because of jumps in the level of risk aversion. It would be interesting to further calibrate the models and quantify the magnitudes of these different channels to help precise the mechanisms that drive the empirically "large" jump leverage risk premium documented below. We leave further work along these lines for future research. Instead, we turn next to a discussion of our model-free estimation of the different risk premiums.

6. Feasible Measures

Our feasible counterparts to the risk premium quantities defined in Section 4 rely on portfolios of short-dated options to estimate the $\mathbb{Q}$ risk-neutral quantities and high-frequency returns to proxy the corresponding $\mathbb{P}$ risk measures.

6.1. Option-Based Risk Measures

Let $O_{t,T}(K)$ denote the time t price of a European-style out-of-the-money option expiring at time $t+T$ with strike K. With the risk-free rate and the dividend yield both set to zero, the general results in Bakshi and Madan (2000) and Carr and Madan (2001) then provide the following two option spanning results:

$$\mathbb{E}_t^{\mathbb{Q}}(x_{t+T}-x_t)^2 = 2\int_0^\infty \left(1-\log\left(\frac{K}{X_t}\right)\right)\frac{O_{t,T}(K)}{K^2}dK, \tag{39}$$

and

$$\mathbb{E}_t^{\mathbb{Q}}(x_{t+T}-x_t) = -\int_0^\infty \frac{O_{t,T}(K)}{K^2}dK. \tag{40}$$

In practice, of course, we do not observe options on a continuum of strikes. Instead let the discrete grid of $N_{t,T}$ option prices observed at time t be denoted by $K_1 < .... < K_{N_{t,T}}$, where for simplicity we suppress the dependence of the strike grid on the pair (t,T). The actually observed option prices $\widehat{O}_{t,T}(K_j)$ are also subject to pricing errors, say $\widehat{O}_{t,T}(K_j) = O_{t,T}(K_j) + \epsilon_{t,T}(j)$ for $j = 1,...,N_{t,T}$. Following standard practice in the option pricing literature, we will assume that the $\epsilon_{t,T}(j)$ observation errors exhibit only weak spatial and temporal dependencies, and hence can be "averaged out" in the estimation.

Using the observed option prices and the spanning results in (39) and (40), we construct the following estimates for $V_{t,t+T}^{\mathbb{Q}}$ and $\mathcal{V}_{t,t+T}^{\mathbb{Q}}$:

$$\widehat{V}_{t,t+T}^{\mathbb{Q}} = \frac{2}{T}\sum_{j=2}^{N_{t,T}}\left(1-\log\left(\frac{K_{j-1}}{X_t}\right)\right)\frac{\widehat{O}_{t,T}(K_{j-1})}{K_{j-1}^2}(K_j-K_{j-1}) - \frac{T}{4}\left(\widehat{\mathcal{V}}_{t,t+T}^{\mathbb{Q}}\right)^2, \tag{41}$$

$$\widehat{\mathcal{V}}_{t,t+T}^{\mathbb{Q}} = \frac{2}{T}\sum_{j=2}^{N_{t,T}}\frac{\widehat{O}_{t,T}(K_{j-1})}{K_{j-1}^2}(K_j-K_{j-1}). \tag{42}$$

Guided by the expansions for the risk-neutral variance measures in (17) and (18), we then run the following linear regressions at each point time using all of the available tenors:

$$\widehat{V}_{t,t+T_j}^{\mathbb{Q}} = b_{t,0} + b_{t,1}T_j + b_{t,2}T_j^2 + \epsilon_{t,T_j}, \qquad \widehat{\mathcal{V}}_{t,t+T_j}^{\mathbb{Q}} = \beta_{t,0} + \beta_{t,1}T_j + \beta_{t,2}T_j^2 + \varepsilon_{t,T_j}. \tag{43}$$

Denoting the resulting OLS estimates by $\widehat{b}_{t,i}$ and $\widehat{\beta}_{t,i}$, respectively, our risk-neutral variance estimates are then simply defined by:

$$\widehat{V}_t^{\mathbb{Q}} = \widehat{b}_{t,0}, \qquad \widehat{\mathcal{V}}_t^{\mathbb{Q}} = \widehat{\beta}_{t,0}. \tag{44}$$

In lieu of (17) and (18), $L_t^{\mathbb{Q}}$ may seemingly be estimated by twice the difference in the estimated slopes from the two regressions in (43). However, this estimator will be biased due to the presence of the risk-neutral instantaneous drift $m_t^{\mathbb{Q}}(V_t^{\mathbb{Q}} - \mathcal{V}_t^{\mathbb{Q}})$ and the mean reversion in $\int_{\mathbb{R}}(z^2 - 2(e^z - 1 - z))\nu_t^{\mathbb{Q}}(dz)$. Even though this bias will typically be much smaller than $\int_{\mathbb{R}} z^2\nu_t^{\mathbb{Q}}(dz)$, if the mean reversion in the jump intensity and the diffusive volatility manifest in the same direction, we can easily correct for the bias. In particular, utilizing

$$\widehat{s}_t = \frac{\sum_{i=1}^{k_n} |\widehat{V}_{t-i/n}^{\mathbb{Q}} - \widehat{V}_{t-(i+1)/n}^{\mathbb{Q}}|}{\sum_{i=1}^{k_n} |\widehat{\mathcal{V}}_{t-i/n}^{\mathbb{Q}} - \widehat{\mathcal{V}}_{t-(i+1)/n}^{\mathbb{Q}}|}, \tag{45}$$

based on a "small" window of k_n variance increments, we obtain the following simple bias-corrected leverage estimator:

$$\widehat{L}_t^{\mathbb{Q}} = -2\left(\widehat{b}_{t,1} - \widehat{s}_t\widehat{\beta}_{t,1}\right). \tag{46}$$

The Monte Carlo simulation results reported in Appendix Appendix E underscore the accuracy of the resulting $\widehat{V}_t^{\mathbb{Q}}$, $\widehat{\mathcal{V}}_t^{\mathbb{Q}}$ and $\widehat{L}_t^{\mathbb{Q}}$ risk-neutral risk estimators in an empirically realistic setting, including the presence of $\epsilon_{t,T}(j)$ option price observation errors subject to spatial dependencies, as well as rounding and randomly missing observations.

We turn next to a discussion of our corresponding return-based risk estimates.

6.2. Return-Based Risk Measures

We assume that market prices and options data are sampled n times during the unit time interval. Relying on a window consisting of k_n return and variance increments prior to time t, we define our realized variance and leverage estimators as

$$\widehat{RV}_t = \frac{n}{k_n}\sum_{i=1}^{k_n}(x_{t-i/n} - x_{t-(i+1)/n})^2, \tag{47}$$

and

$$\widehat{RL}_t = \frac{n}{k_n}\sum_{i=1}^{k_n}[(x_{t-i/n} - x_{t-(i+1)/n})(\widehat{\mathcal{V}}_{t-i/n}^{\mathbb{Q}} - \widehat{\mathcal{V}}_{t-(i+1)/n}^{\mathbb{Q}})], \tag{48}$$

respectively. Under standard conditions, $\widehat{RV}_t$ and $\widehat{RL}_t$ consistently, for $n \to \infty$, estimate $QV_{t-k_n/n,t}$ and $QL_{t-k_n/n,t}$, formally defined as the quadratic variation of x and the quadratic covariation of x and $\mathcal{V}^{\mathbb{Q}}$, that is

$$QV_{t-k_n/n,t} \equiv \frac{n}{k_n}([x,x]_t - [x,x]_{t-k_n/n}), \tag{49}$$

and

$$QL_{t-k_n/n,t} \equiv \frac{n}{k_n}([x, \mathcal{V}^{\mathbb{Q}}]_t - [x, \mathcal{V}^{\mathbb{Q}}]_{t-k_n/n}), \tag{50}$$

respectively. Moreover, if $k_n \to \infty$ and $k_n/n \to 0$, the $QV_{t-k_n/n,t}$ and $QL_{t-k_n/n,t}$ estimands naturally converge to their spot counterparts.

6.3. Feasible Realized Risk Premium Estimates

Armed with the above risk-neutral and realized risk estimates, our feasible versions of $IVRP_t$ and $ILRP_t$, defined in (23) and (29), are simply obtained as

$$\widehat{IVRP}_t \equiv \widehat{V}_t^{\mathbb{Q}} - \widehat{RV}_t, \tag{51}$$

and

$$\widehat{ILRP}_t \equiv \widehat{L}_t^{\mathbb{Q}} - \widehat{RL}_t, \tag{52}$$

respectively. These estimators of the instantaneous variance and leverage risk premiums purposely rely on the easy-to-calculate model-free realized $\widehat{RV}_t$ and $\widehat{RL}_t$ measures, rather than their conditional expectations, $\mathcal{V}_t^{\mathbb{P}}$ and $L_t^{\mathbb{P}}$. Practical estimation of the conditional expectations would necessitate additional modeling assumptions pertaining to the dynamics of $\widehat{RV}_t$ and $\widehat{RL}_t$. Importantly, however, relying on the realized values instead of their conditional expectations, does not alter any of our main conclusions and average risk premium estimates. In particular, ignoring the (negligible) contribution stemming from estimation error, it readily follows that

$$\widehat{IVRP}_t = IVRP_t + \mathbb{P}\text{-martingale}, \qquad \widehat{ILRP}_t = ILRP_t + \mathbb{P}\text{-martingale},$$

where the $\mathbb{P}$-martingale terms reflect the differences between the respective realizations of the discrete-time stochastic processes and their conditional expectations. Although these $\mathbb{P}$-martingale terms are not necessarily "small" numerically, they are by definition mean-zero and unpredictable, and hence do not affect the expected values, nor the dynamic dependencies, of the estimated risk premiums.

7. Empirical Evidence for Market Jump and Leverage Risk Premiums

We begin the discussion of our main empirical findings with an account of the data underlying our results. We then provide a summary of the daily risk estimates, before finally considering the implications of our model-free estimates for the pricing of jumps and jump leverage risk in particular.

7.1. Data

Our empirical analyses is based on high-frequency returns and option data for the S&P 500 index spanning the period 2007-2020. The options are European style and are traded on the CBOE. We obtained the option data from the CBOE Data Shop. We rely on high-frequency price records of the SPY exchange traded fund (ETF) designed to track the S&P 500 index. The SPY data is obtained from the TAQ database. We record the SPY and options prices at a 5-minute frequency during the trading day, starting at 9.35 EST and ending at 15.55 EST, resulting in a total of 77 price records per day.

We apply standard filters and cleaning procedures to the data. In so doing, we remove any days for which the number of SPY zero returns exceeds twenty percent of the total daily number of high-frequency returns. This mostly eliminates trading days around holidays. We also remove the four 15-minute periods on March 9, 12, 16 and 18, 2020, where trading was halted due to market-wide circuit breakers. We take the option mid-quotes as our option price observations. We remove any options with zero bids and options for which the ratio of the ask relative to the bid exceeds ten. We also remove date and maturity pairs for which the minimum of the ratio of the out-of-the-money option price relative to its strike exceeds five percent of the maximum of this ratio. We further remove date and maturity pairs for which the maximum strike gap around the money exceeds twenty. Finally, to avoid anomalous results associated with large event risks, we also exclude any options for which the horizons span the dates of the 2016 Brexit and 2020 U.S. Presidential elections.

For the actual estimation, at each point in time we use the first two available shortest maturity options with at least three business days to expiration. We then keep adding options with tenors up until twelve business days. If on a given day the number of available tenors is just two, or if the gap between the shortest and longest available tenor is less than six business days, we drop the squared tenor term in the regressions in (43) in order to avoid any issues with multicollinearity (this mostly happens in the early part of the sample). Also, if at a given point in time, any of the estimated intercept terms in (43) are negative, we simply remove those observations from the analysis (again this rarely happens, and when it does, it is almost always occur around holidays).

The above choice of option tenors aims at striking a balance between bias and variance in the estimation. Since the expansion results in (17) and (18) explicitly pertain to small T, to reduce the bias ideally we would like to only use the shortest possible tenors on a given day. However, restricting the estimation to only the very shortest maturities will obviously limit the number of options, in turn resulting in noisier estimates. Motivated by the Monte Carlo simulation results discussed in Appendix Appendix E, and the finding that even for very high levels of mean reversion, the biases in the estimation for tenors up to around twelve business days appear quite small, we deliberately include options with maturities up to that horizon.

7.2. Risk Estimates

We begin by computing $\widehat{V}^{\mathbb{Q}}_{t,t+T}$ and $\widehat{\mathcal{V}}^{\mathbb{Q}}_{t,t+T}$ at each observation time, for all the included tenors, based on the expressions in (41) and (42), respectively. Following the extant literature, we measure time in business days; i.e., the length of time from the end of trading on one day to end of trading on the following business day is set to 1/252. In addition, to account for the well-known intraday pattern in volatility, we compute time-to-maturity during a trading day on a "business time scale," with the length of the within day windows set to equalize the contribution to the intraday volatility. To allow for possible dynamic changes in this intraday pattern, we further calculate the pattern on a one-year rolling basis from the intraday and overnight realized volatilities over the past year. Using the regressions in (43), we then calculate the instantaneous variance estimates $\widehat{V}_t^{\mathbb{Q}}$ and $\widehat{\mathcal{V}}_t^{\mathbb{Q}}$ as defined in (44). We rely on (46) for our computation of $\widehat{L}_t^{\mathbb{Q}}$, with the $\widehat{s}_t$ adjustment term in (45) calculated from the high-frequency return and option observations on the specific trading day. For each trading day, we then average all the high-frequency estimates and henceforth rely on these as our daily estimates. Our realized risk measures $\widehat{RV}_t$ and $\widehat{RL}_t$ are similarly computed over windows of one trading day. To allow for comparison with the risk-neutral measures, we further normalize the high-frequency return measures using an overnight adjustment factor, based on the ratio of the intraday to the overnight realized volatilities computed on a one-year rolling basis.

Table 1 reports the full-sample means for the resulting daily $\mathbb{P}$ and $\mathbb{Q}$ estimates, together with robust standard errors in parentheses.[27] Since all the distributions are heavily right skewed, we also report the corresponding quantiles. Looking first at the volatility estimates in the top-portion of the table, the $\widehat{V}_t^{\mathbb{Q}}$ spot estimates are naturally lower than the short and long maturity $\widehat{V}_{t,t+T}^{\mathbb{Q}}$ estimates. Meanwhile, it is noteworthy that the relative difference between $\widehat{V}_{t,t+T}^{\mathbb{Q}}$ and $\widehat{V}_t^{\mathbb{Q}}$ is the smallest for the highest quantile. This may be explained by the fact that mean reversion in volatility and the leverage effect impact the term structure of $\widehat{V}_{t,t+T}^{\mathbb{Q}}$ in opposite directions, thus diminishing the relative difference between $\widehat{V}_{t,t+T}^{\mathbb{Q}}$ and $\widehat{V}_t^{\mathbb{Q}}$ when volatility is high. Also, even though there is obviously a large gap between $\widehat{V}_t^{\mathbb{Q}}$ and $\widehat{RV}_t$, consistent with a positive $IVRP_t$ and the widely documented large variance risk premium over longer horizons, the gap between $\widehat{V}_t^{\mathbb{Q}}$ and $\widehat{V}_{t,t+T}^{\mathbb{Q}}$ for short T is also fairly large, pointing to a sizable risk premium for shocks to the instantaneous risk-neutral variance.

Turning next to the $3(\widehat{V}_{t,t+T}^{\mathbb{Q}} - \widehat{\mathcal{V}}_{t,t+T}^{\mathbb{Q}})$ estimates, and measures of asymmetry in the risk-neutral return distribution, reported in the mid-portion of the table, the values seemingly depend strongly on the horizon T. Recall from the discussion in Section 3, that the leading term in an asymptotic expansion of $3(V_{t,t+T}^{\mathbb{Q}} - \mathcal{V}_{t,t+T}^{\mathbb{Q}})$ for small T is $3\int_{\mathbb{R}}(z^2 - 2(e^z - 1 - z))\,\nu_t^{\mathbb{Q}}(dz)$. Meanwhile, the second-order term in said expansion that depends on the horizon T may be traced to mean-reversion in volatility and jump intensity, as well as the leverage effect. The pairwise differences in the sample means and the various quantiles of $3(\widehat{V}_{t,t+T}^{\mathbb{Q}} - \widehat{\mathcal{V}}_{t,t+T}^{\mathbb{Q}})$ for short and long tenors thus directly underscore the importance of these latter effects. Further supporting this conjecture, the mean and the median of the instantaneous $3(\widehat{V}_t^{\mathbb{Q}} - \widehat{\mathcal{V}}_t^{\mathbb{Q}})$

[27] We purposely report the 1% truncated means here and throughout to avoid the full-sample values being unduly influenced by a few very large "crisis" observations.

Table 1: **Risk Measures**

Statistic	Mean	Quantiles		
		Q_{25}	Q_{50}	Q_{75}
$\widehat{V}^{\mathbb{Q}}_{t,t+T}$, short T	0.0395 (0.0074)	0.0130	0.0224	0.0429
$\widehat{V}^{\mathbb{Q}}_{t,t+T}$, long T	0.0418 (0.0078)	0.0151	0.0248	0.0478
$\widehat{V}^{\mathbb{Q}}_{t}$	0.0378 (0.0068)	0.0111	0.0201	0.0396
$\widehat{RV}_t$	0.0282 (0.0044)	0.0064	0.0127	0.0278
$3(\widehat{V}^{\mathbb{Q}}_{t,t+T} - \widehat{\mathcal{V}}^{\mathbb{Q}}_{t,t+T})$, short T	0.0028 (0.0007)	0.0005	0.0010	0.0021
$3(\widehat{V}^{\mathbb{Q}}_{t,t+T} - \widehat{\mathcal{V}}^{\mathbb{Q}}_{t,t+T})$, long T	0.0060 (0.0016)	0.0011	0.0019	0.0051
$3(\widehat{V}^{\mathbb{Q}}_{t} - \widehat{\mathcal{V}}^{\mathbb{Q}}_{t})$	0.0011 (0.0003)	−0.0001	0.0002	0.0008
$-\widehat{L}^{\mathbb{Q}}_{t}$	0.0579 (0.0106)	0.0089	0.0203	0.0464
$-\widehat{RL}_t$	0.0331 (0.0074)	0.0019	0.0056	0.0179

Note: The table reports truncated at 1% sample means, with Newey-West robust standard errors in parentheses, and quantiles for the different daily risk estimates defined in Sections 6.1 and 6.2. The estimates are constructed from short-dated S&P 500 index options and high-frequency SPY returns spanning 2007-2020.

estimates are also substantially lower than those of $3(\widehat{V}^{\mathbb{Q}}_{t,t+T} - \widehat{\mathcal{V}}^{\mathbb{Q}}_{t,t+T})$ for short T. Looking at the $3(\widehat{V}^{\mathbb{Q}}_{t} - \widehat{\mathcal{V}}^{\mathbb{Q}}_{t})$ estimates, it is also worth noting that with $3(V^{\mathbb{Q}}_{t} - \mathcal{V}^{\mathbb{Q}}_{t})$ being approximately equal to the risk-neutral third moment of the jumps (recall equation (15)), the numbers in the table point to very little asymmetry in the implied $\mathbb{Q}$ jump size distribution.

7.3. Leverage Risk and Pricing

The final two rows in Table 1 summarize the results for our leverage risk estimates. Consistent with the idea of a nontrivially sized instantaneous leverage risk premium $ILRP_t$, the sample mean of the risk-neutral $-\widehat{L}^{\mathbb{Q}}_{t}$ exceeds that of the realized $-\widehat{RL}_t$ by more than twice of what the sample mean of the risk neutral spot variance $\widehat{V}^{\mathbb{Q}}_{t}$ exceeds that of the realized variance $\widehat{RV}_t$. The different quantiles for $-\widehat{L}^{\mathbb{Q}}_{t}$ are also all substantially higher than the corresponding quantiles for $-\widehat{RL}_t$.

To further help visualize the pricing of the jump leverage effect, and how it differs from the pricing of variance risk, Figure 3 plots the time series of $\widehat{V}_t^{\mathbb{Q}}$ and $\widehat{RV}_t$, while Figure 4 plots $-\widehat{L}_t^{\mathbb{Q}}$ and $-\widehat{RL}_t$. All of the measures obviously varied quite substantially over the 2007-2020 sample period, generally increasing during periods of crisis. As is to be expected, the realized measures, $\widehat{RV}_t$ and $-\widehat{RL}_t$, also typically exceeded their risk-neutral counterparts, $\widehat{V}_t^{\mathbb{Q}}$ and $-\widehat{L}_t^{\mathbb{Q}}$, at the onset of a crisis, with that ordering reversed in the aftermath of a crisis. As a case in point, during the financial crisis in the Fall of 2008, $\widehat{RV}_t$ exceeded $\widehat{V}_t^{\mathbb{Q}}$ for the month of September and beginning of October, while $\widehat{V}_t^{\mathbb{Q}}$ exceeded $\widehat{RV}_t$ by quite a wide margin for more than a year thereafter. This same general pattern is also evident for the $-\widehat{L}_t^{\mathbb{Q}}$ and $-\widehat{RL}_t$ leverage risk measures.

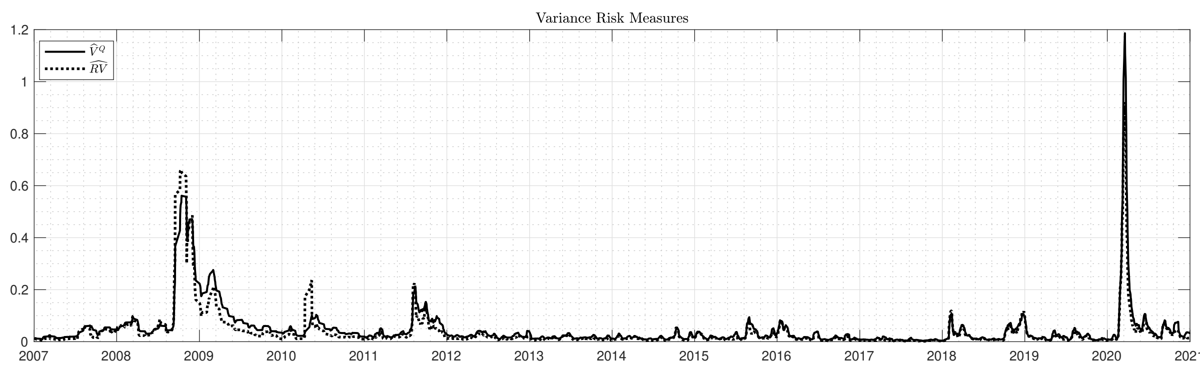

Figure 3: The figure plots the ten-day moving averages of the daily $\widehat{V}_t^{\mathbb{Q}}$ and $\widehat{RV}_t$ variance risk estimates.

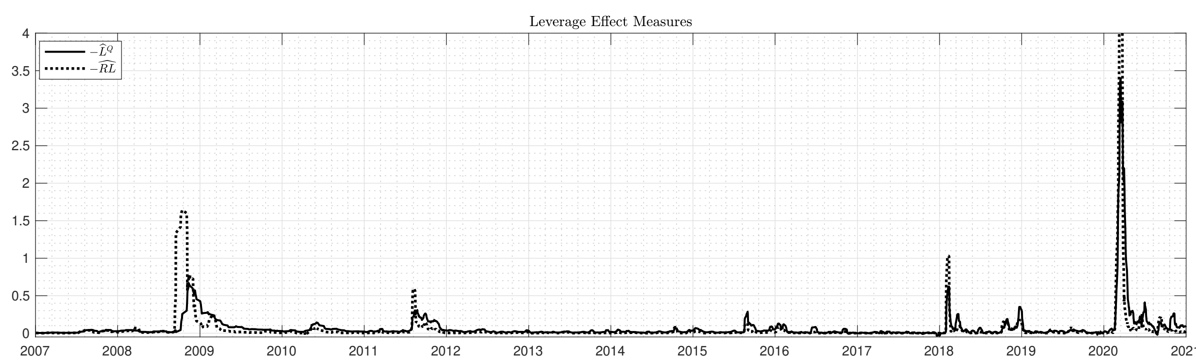

Figure 4: The figure plots the ten-day moving averages of the daily $-\widehat{L}_t^{\mathbb{Q}}$ and $-\widehat{RL}_t$ leverage risk estimates.

Meanwhile, the by far largest increase in all of the risk measures occurred in the Spring of 2020 coincident with the start of the COVID-19 pandemic. At that time $\widehat{V}_t^{\mathbb{Q}}$ reached a peak of roughly doubled that of its peak observed during the financial crisis of 2008. The heightened values of $\widehat{V}_t^{\mathbb{Q}}$ in the Spring of 2020 was, however, noticeable shorter-lived than the highs attained during the 2008 financial crisis. While these same general features manifest in $-\widehat{L}_t^{\mathbb{Q}}$ as well, there are also some noticeable differences.[28] In particular, $-\widehat{L}_t^{\mathbb{Q}}$ increased even more dramatically during the recent pandemic, reaching a high of almost five-fold its high observed during the 2008 financial crises. In other words, it appears as if leverage risk has become an even bigger concern to investors more recently than it has been historically.

To more directly illustrate the behavior of the volatility and leverage risks during periods of market turmoil, Figure 5 plots the SPY price and the instantaneous risk-neutral volatility measure $\widehat{V}_t^{\mathbb{Q}}$ at a five-minute frequency for the three weeks in the sample when the market fell by more than 10%, namely the week of November 5-12, 2008 (displayed in the first panel), the week of August 3-10, 2011 (displayed in the second panel), and the week of March 4-11, 2020 (displayed in the third panel).[29] Figure 6 plots the SPY and the leverage risk measure $-\widehat{L}_t^{\mathbb{Q}}$ for the same three weeks. Looking first at Figure 5, there is obviously a strong negative correlation between the price changes and the changes in the $\widehat{V}_t^{\mathbb{Q}}$ spot volatility estimates during each of these three turbulent weeks. These large realized leverage effects also clearly manifest in the $-\widehat{L}_t^{\mathbb{Q}}$ leverage risk estimates displayed in Figure 6.

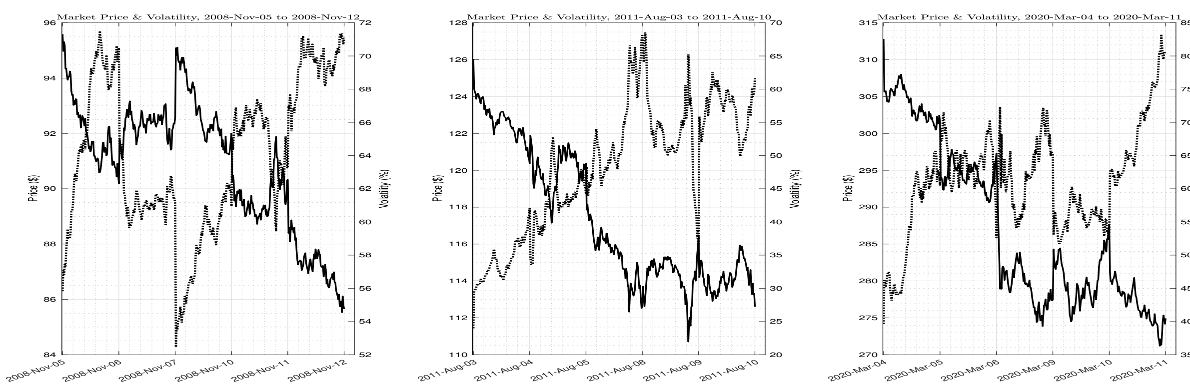

Figure 5: The figure shows the price of the SPY at a five-minute frequency (solid line) together with $\widehat{V}_t^{\mathbb{Q}}$ in annualized percentage form (dashed line) for November 5-12, 2008 (first panel), August 3-10, 2011 (second panel), and March 4-11, 2020 (third panel).

Although both of the two risk measures generally appear to be strongly negatively correlated with the price, there are also some important differences between the three episodes.

[28]Incidently, these differences in the time series behavior of the variance and leverage risk measures also indicate that the jump risks have a much more complicated structure than portrayed by most parametric models hitherto employed in the literature, the popular double-jump stochastic volatility model discussed in Appendix Appendix D included.

[29]To allow for easier interpretation, the volatility measures in Figure 5 are displayed in annualized volatility units, or $(252 \times \widehat{V}_t^{\mathbb{Q}})^{1/2}$. The market also fell by more than 10% the first week of October 2008. However, due to issues with missing observations and quality of the options data, we omit that week from the display.

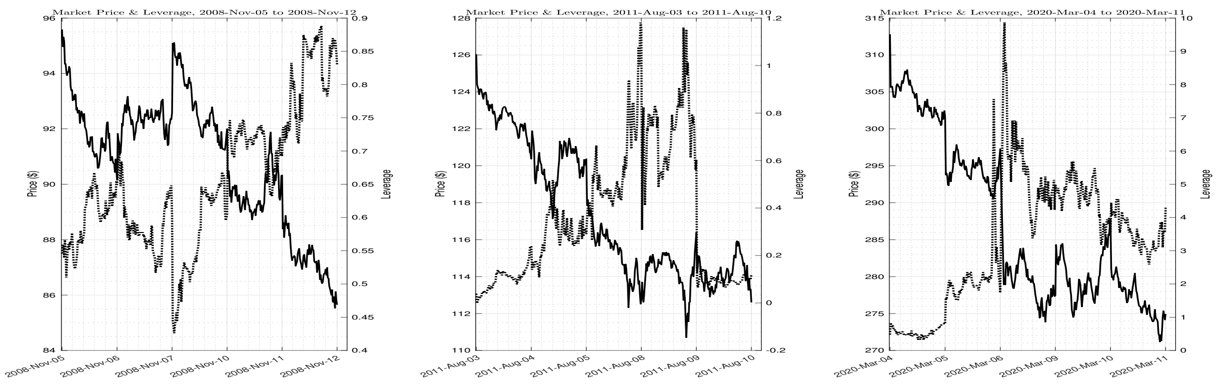

Figure 6: The figure shows the price of the SPY at a five-minute frequency (solid line) together with $-\widehat{L}_t^{\mathbb{Q}}$ (dashed line) for November 5-12, 2008 (first panel), August 3-10, 2011 (second panel), and March 4-11, 2020 (third panel).

In particular, while the average values of $\widehat{V}_t^{\mathbb{Q}}$ for the first week of November 2008 and the second week of March 2020 were very similar, at 64.0% and 61.2%, respectively, the volatility stayed within a relatively narrow range of around $55\% - 70\%$ for the first of the two weeks, while it fluctuated much more widely from a low of around 40% to a high of around 85% for the second of the two weeks. The volatility mean reversion and the leverage effect also both manifested very differently for these two different episodes. Specifically, while the average value of $(252 \times \widehat{\mathcal{V}}_{t,t+30}^{\mathbb{Q}})^{1/2}$, as proxied by the VIX, equalled 59.2% for the first week of November 2008, it was "only" equal to 43.1% for the second of March 2020. Recalling again the short-time expansions for the risk-neutral volatility measures in Section 3, this noticeably lower average value of the VIX for the second of the two weeks may naturally be attributed to stronger mean reversion during that week. As previously noted, this strong volatility mean reversion was also accompanied by more pronounced leverage effects during the pandemic. Indeed, as Figure 6 shows, the estimates for $-\widehat{L}_t^{\mathbb{Q}}$ are an order of magnitude larger for the second week of March 2020 compared to he first week of November 2008.

7.4. Leverage Risk and Equity and Variance Risk Premiums

The empirical evidence presented above shows that the instantaneous leverage risk premium $ILRP_t$ is both time-varying and strongly persistent. Our theoretical analyses in Section 4 further highlight that variations in the instantaneous leverage and equity risk premiums are naturally connected. These two observations in turn combine to suggest that the jump leverage risk premium should be able to predict future market returns. The same connection should also hold between the instantaneous variance risk premium and future market returns.

To empirically investigate this conjecture, we estimate a series of return predictability regressions for the aggregate market. For ease of interpretation, we restrict our analysis to simple univariate regressions of the form:

$$x_{t+h} - x_t = a_0 + a_1 z_t + \epsilon_{t+h}, \tag{53}$$

with return horizons h ranging from one month to twelve months, and the predictor variable z_t denoting the 20-day moving average of various predictors.[30] The predictors that we consider are: $\widehat{ILRP}_t$, $\widehat{IVRP}_t$, $\widehat{ILRP}_t^{\perp}$, $\widehat{RVRP}_t$, $\widehat{L}_t^{\mathbb{Q}}$ and $-\widehat{RL}_t$, where $\widehat{ILRP}_t^{\perp}$ denotes the residual from a linear projection of $\widehat{ILRP}_t$ on $\widehat{IVRP}_t$ (i.e., the part of $\widehat{ILRP}_t$ that is not in the linear span of $\widehat{IVRP}_t$), and following the extant literature (see, e.g., Bollerslev et al. (2009) and Drechsler and Yaron (2011)) $\widehat{RVRP}_t$ is computed as the difference between the square of the VIX index and the realized variance over the previous month. Due to limited availability of short-dated options for the early part of our sample, and nontrivial gaps in our option-based estimates prior to 2011, we restrict the estimation of all the regressions to the 2011-2020 period.

The results reported in Table 2 show that $\widehat{ILRP}_t$ and $\widehat{IVRP}_t$ both provide strong predictive signals for future market returns. This, of course, is consistent with the existence of the jump leverage effect and time-variation of the compensation for price jump risk, as discussed in Section 4, see equations (15), (23), (29)-(31). Moreover, the lack of significance of $\widehat{ILRP}_t^{\perp}$ indicate that the predictive contents of $\widehat{ILRP}_t$ and $\widehat{IVRP}_t$ for future market returns are similar. This again is in line with the discussion in Section 4, as $ILRP_t$ and $IVRP_t$ are both related to price jumps and hence to the equity risk premium.

Turning next to the $\widehat{L}_t^{\mathbb{Q}}$ and $-\widehat{RL}_t$ components of $\widehat{ILRP}_t$, the results show that both strongly predict future market returns across all horizons. Recall that both of these measures include diffusive leverage effects (which are the same under the $\mathbb{P}$ and $\mathbb{Q}$ measures), so that this effect may potentially also help in predicting the returns. Meanwhile, the signal-to-noise ratios for the $\widehat{L}_t^{\mathbb{Q}}$ and $-\widehat{RL}_t$ estimates are both much higher than the ratio for $\widehat{ILRP}_t$, possibly explaining the slightly stronger predictive results for $\widehat{L}_t^{\mathbb{Q}}$ and $-\widehat{RL}_t$, compared to those for $\widehat{ILRP}_t$.

In contrast to earlier empirical evidence, the monthly variance risk premium $\widehat{RVRP}_t$, as traditionally defined, does not help predict aggregate market return over the somewhat limited 2011-2020 time period.[31] Again following the discussion in Section 4, the monthly variance risk premium is attributable to several different sources of risks, not all of which are formally connected with the equity risk premium (e.g., volatility risk that is unrelated

[30]To avoid the estimates being driven by a few very large influential observations, following standard practice we winsorize z_t at 1%.

[31]This is also broadly consistent with the regression results for the S&P 500 reported in Heston and Todorov (2023) based on data over a similar more recent time period.

Table 2: **Market Return Predictability Regressions**

	$\widehat{ILRP}$	$\widehat{IVRP}$	$\widehat{IVRP}^{\perp}$	$-\widehat{L}^{\mathbb{Q}}$	$-\widehat{RL}$	$\widehat{VRP}$
			One Month Horizon			
$\widehat{a}_1$	1.56	11.88	0.15	1.00	0.76	-0.95
$t(\widehat{a}_1)$	(4.58)	(6.24)	(0.12)	(8.80)	(8.15)	(-0.40)
R^2	0.04	0.07	0.00	0.07	0.08	0.00
			Three Month Horizon			
$\widehat{a}_1$	1.14	8.89	0.08	0.78	0.56	-0.30
$t(\widehat{a}_1)$	(4.34)	(8.25)	(0.10)	(7.18)	(5.28)	(-0.15)
R^2	0.07	0.15	0.00	0.15	0.17	0.00
			Six Month Horizon			
$\widehat{a}_1$	0.74	5.49	0.11	0.52	0.38	0.25
$t(\widehat{a}_1)$	(3.98)	(7.46)	(0.20)	(5.68)	(4.68)	(0.14)
R^2	0.07	0.14	0.00	0.16	0.18	0.00
			Nine Month Horizon			
$\widehat{a}_1$	0.77	5.29	0.22	0.45	0.35	0.24
$t(\widehat{a}_1)$	(4.67)	(8.82)	(0.40)	(7.82)	(6.50)	(0.13)
R^2	0.11	0.18	0.01	0.17	0.21	0.00
			Twelve Month Horizon			
$\widehat{a}_1$	0.74	5.31	0.16	0.46	0.35	0.27
$t(\widehat{a}_1)$	(4.61)	(8.20)	(0.29)	(8.33)	(6.11)	(0.15)
R^2	0.11	0.19	0.00	0.19	0.24	0.00

Notes: The table reports the $\widehat{a}_1$ estimated slope coefficients, t-statistics, and R^2s, from predictive return regressions for the Fama-French market portfolio, for return horizons ranging from one month to a year. All of the predictor variables, indicated in the top row of the table, are define in the main text. Standard errors are computed using the Equal-Weighted Cosine estimator of the long-run variance with fixed-b critical values proposed by Lazarus et al. (2018) based on $0.4 \times T^{2/3}$ degrees of freedom, where T refers to the number of time series observations.

to equity risk). Instead, the more nuanced decomposition of the variance risk premium into potentially different priced risk components afforded by our new model-free procedures, including jump leverage risk, allows for much stronger predictive results.

To further underscore the nontrivial pricing of jump leverage risk, and the economic mechanisms at work, it follows that shocks in the form of jumps to the stochastic volatility and/or the jump intensity should be priced. Accordingly, we should expect the mean of the instantaneous drift $m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}})$ to be positive.[32] Consistent with that thesis and the expansion

[32]Ideally, we would like to have an estimate of the $m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}}) - m_t^{\mathbb{P}}(\mathcal{V}_t^{\mathbb{Q}})$ risk premium. However, estimating the drift of the volatility process $\mathcal{V}_t^{\mathbb{Q}}$ is impossible in a general nonparametric way.

in (17), the full-sample mean of the estimates for $\beta_{t,1}$ from the regression in (43) equals 0.0636, corresponding to an estimate of 0.1272 for $m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}})$. We can similarly estimate the forward quantity, say $\mathbb{E}_t^{\mathbb{Q}}(m_{t+T}^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))$ for some $T > 0$, by employing options recorded at time t expiring shortly after time $t + T$. Doing so with options expiring shortly after two weeks, the full-sample mean estimate of $\mathbb{E}_t^{\mathbb{Q}}(m_{t+T}^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))$ only equals 0.0050. A formal test for the null hypothesis that the mean of $m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}})$ equals that of $\mathbb{E}_t^{\mathbb{Q}}(m_{t+T}^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))$ is strongly rejected with a t-statistic of 4.34. This much lower estimate for $\mathbb{E}_t^{\mathbb{Q}}(m_{t+T}^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))$ compared to the estimate for $m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}})$ thus again suggests that the risk premium associated with the variation in $\mathcal{V}_t^{\mathbb{Q}}$ is mostly due to the compensation for short-lived spikes, or jumps.

7.5. Jump Leverage Risk Premiums Embedded in VIX Options

All of the key empirical findings discussed above are based on the pricing of S&P 500 index options. In this section we provide independent verification of the importance of the jump leverage risk premium through the use of options written on the VIX index and the pricing thereof. Intuitively, if the jump leverage effect constitutes an important source of risk that requires compensation by investors, then some of the jumps to volatility, namely the ones directly related to price jumps, should also be priced. In fact, based on existing empirical evidence (see, e.g., Jacod and Todorov (2010), Todorov and Tauchen (2011) and Caporin et al. (2017), among others), one would naturally expect many volatility jumps to be accompanied by simultaneous price jumps in the opposite direction.[33]

The VIX index computed by the CBOE is formally based on the $\widehat{\mathcal{V}}_{t,t+T}^{\mathbb{Q}}$ risk-neutral variance measure defined in (7), together with the two available tenors T closest to 30 calendar days.[34] We obtain end-of-day prices for VIX options spanning the 2007-2020 period from OptionMetrics. We further collect 5-minute price records for the VIX index itself from the CBOE Data Shop. We apply standard cleaning procedures and similar filters to the ones discussed in Section 7.1 used for processing the S&P 500 price and options data.[35] However, since the number of available short-tenor VIX index options is substantially less than the number of short-dated S&P 500 options, especially for the early part of the sample, we retain

[33] This strong negative dependence between jumps in the market and the volatility of the market also naturally arises in many equilibrium models, the previously discussed model by Eraker and Yang (2022) included. Counter to the empirical evidence, however, this particular theoretical model, as well as other exponentially-affine models, cannot account for negative volatility jumps; see also the discussion in Amengual and Xiu (2018).

[34] The CBOE uses a calendar day convention in their computation of the VIX, while as previously noted we rely on a business day convention in our high-frequency calculations of $\widehat{\mathcal{V}}_{t,t+T}^{\mathbb{Q}}$. However, since we are not actually modeling the connection between the VIX and our $\widehat{\mathcal{V}}_{t,t+T}^{\mathbb{Q}}$ measure, this difference is immaterial.

[35] We also further apply bounce-back filters similar to the ones proposed by Barndorff-Nielsen et al. (2009) to remove rare large intraday price swings caused in part by the rules applied by the exchange for the inclusion of deep out-of-the-money options in the computation of the VIX; see Andersen et al. (2015a) for a more detailed discussion of this issue.

a somewhat wider set of VIX index options with maturities ranging from three up to thirty business days.

Using the VIX index option data and the expressions in (41) and (42), we begin by computing the two separate estimates for the risk-neutral variances of the VIX. We then run the regressions in (43), omitting the T^2 terms due to the more limited availability of different tenors. We refer to the resulting estimated intercepts, and instantaneous risk-neutral variance estimates of the log(VIX) index, by $\widehat{VV}_t^{\mathbb{Q}}$ and $\widehat{\mathcal{VV}}_t^{\mathbb{Q}}$, respectively. In parallel to our analysis of the S&P 500 discussed above, we then compare these option-implied volatilities for the VIX with their daily realized counterparts,

$$\widehat{RVV}_t = \frac{n}{k_n}\sum_{i=1}^{k_n}\left(\Delta_{t,i}^n VIX\right)^2, \qquad \widehat{\mathcal{RVV}}_t = \frac{2n}{k_n}\sum_{i=1}^{k_n}\left(e^{\Delta_{t,i}^n VIX} - 1 - \Delta_{t,i}^n VIX\right), \tag{54}$$

where $\Delta_{t,i}^n VIX \equiv \log(VIX_{t-i/n}) - \log(VIX_{t-(i-1)/n})$ denote the intraday high-frequency logarithmic returns on the VIX.

Given the aforementioned strong empirical evidence for negative dependence between market price jumps and jumps in the VIX, together with the strong empirical evidence for the pricing of market price jumps, one would naturally expect volatility jump risk to be priced with the opposite sign to that of market price jumps. In particular, $3(\widehat{VV}_t^{\mathbb{Q}} - \widehat{\mathcal{VV}}_t^{\mathbb{Q}})$, which by the discussion in Section 3 provides a proxy for the third risk-neutral moment of the jumps in the log-VIX index, would naturally be expected to be less than the corresponding realized measure $3(\widehat{RVV}_t - \widehat{\mathcal{RVV}}_t)$, with the gap stemming from the pricing of volatility jump risk. Consistent with that conjecture, the full-sample mean of $3(\widehat{VV}_t^{\mathbb{Q}} - \widehat{\mathcal{VV}}_t^{\mathbb{Q}})$ equals -0.1102, while that of $3(\widehat{RVV}_t - \widehat{\mathcal{RVV}}_t)$ only equals -0.0007. The t-statistic of 19.16 for testing whether the risk-neutral and realized third moments of the jumps to the market volatility are the same is also highly significant. In other words, jumps to the state variables that drive the stochastic volatility and risk-neutral jump intensity of the market are obviously priced by investors, further corroborating the practical importance of the jump leverage effect and the sizeable magnitude of the corresponding jump leverage risk premium.

8. Concluding Remarks

Jumps in asset prices trigger two types of risks: (i) risk stemming from abrupt changes in the price level, and (ii) risk stemming from changes in the future investment opportunity set due to jumps in the diffusive volatility and/or jump intensity. We provide new empirical evidence in support of the nontrivial pricing of the latter type of jump risk, and so-called jump leverage risk in particular. Our analysis is distinctly model-free, relying on novel short-time risk-neutral variance expansions, together with short-dated S&P 500 index options and high-frequency S&P 500 returns for non-parametrically estimating different risk measures and quantifying the corresponding risk premiums. We further clarify the key economic mechanisms and types of shocks underlying the nontrivial pricing of jump leverage risk that we document, including possible equilibrium-based explanations. We leave more detailed analysis and estimation/calibration of specific parametric models designed to match the empirically "large" jump leverage risk premium for future research.

Appendix A. Pricing Kernel Derivations

Using Lemma III.5.18 in Jacod and Shiryaev (2013), we have that

$$W_t^{\mathbb{Q}} = W_t - \int_0^t \lambda_s ds, \ \widetilde{W}_t^{\mathbb{Q}} = \widetilde{W}_t - \int_0^t \widetilde{\lambda}_s ds,$$

are independent Brownian motions under the risk-neutral measure while the jump compensator of μ under $\mathbb{Q}$ is given by

$$\nu^{\mathbb{Q}}(dt, dx, dy, dz) = [\phi_t^{\mathbb{Q}} dt F^{\mathbb{Q}}(x, y, z) dx dy dz + \widetilde{\phi}_t^{\mathbb{Q}} dt \widetilde{F}^{\mathbb{Q}}(y, z) \epsilon_0(dx) dy dz].$$

This implies the following dynamics of X under $\mathbb{Q}$

$$\frac{dX_t}{X_{t-}} = \widetilde{\alpha}_t dt + \sigma_t dW_t^{\mathbb{Q}} + \int_{R^3} (e^x - 1)(\mu - \nu^{\mathbb{Q}})(dt, dx, dy, dz),$$

where

$$\widetilde{\alpha}_t = \alpha_t + \lambda_t \sigma_t - \int_{\mathbb{R}} (e^x - 1)(\phi_t^{\mathbb{P}} F^{\mathbb{P}}(x, y, z) - \phi_t^{\mathbb{Q}} F^{\mathbb{Q}}(x, y, z)) dx dy dz.$$

Since the process X is local martingale under $\mathbb{Q}$ (recall that we have set for simplicity the risk-free rate and the dividend yield to zero), we need $\widetilde{\alpha}_t = 0$. From here, the expression for α_t in (31) follows.

Next, given the prices for diffusive and jump risk, we have that the risk-neutral dynamics of σ_t^2 and $\phi_t^{\mathbb{Q}}$ is given by

$$\begin{aligned} d\sigma_t^2 = b_t^{\sigma} dt + \eta_t^{\sigma} dW_t^{\mathbb{Q}} + \widetilde{\eta}_t^{\sigma} d\widetilde{W}_t^{\mathbb{Q}} + \int_{R^3} y(\mu - \nu^{\mathbb{Q}})(dt, dx, dy, dz) \\ + \lambda_t \eta_t^{\sigma} dt + \widetilde{\lambda}_t \widetilde{\eta}_t^{\sigma} dt + \int_{R^3} y(\nu^{\mathbb{Q}} - \nu^{\mathbb{P}})(dt, dx, dy, dz), \end{aligned}$$

and

$$d\phi_t^{\mathbb{Q}} = b_t^{\phi}dt + \eta_t^{\phi}dW_t + \widetilde{\eta}_t^{\phi}d\widetilde{W}_t + \int_{R^3} z(\mu - \nu^{\mathbb{P}})(dt, dx, dz)$$
$$+ \lambda_t\eta_t^{\phi}dt + \widetilde{\lambda}_t\widetilde{\eta}_t^{\phi}dt + \int_{R^3} z(\nu^{\mathbb{Q}} - \nu^{\mathbb{P}})(dt, dx, dy, dz).$$

From here, the expression for $m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}})$ in (28) follows when taking into account the expressions for the $\mathbb{P}$ and $\mathbb{Q}$ jump compensators.

Finally, given the dynamics of the processes X, σ^2 and $\phi^{\mathbb{Q}}$ under the two probability measures, we have the following expressions for the quadratic covariation between the log-price and σ^2 and between the log-price and $\phi^{\mathbb{Q}}$:

$$\begin{aligned}
[x, \sigma^2]_t &= \int_0^t \sigma_s\eta_s^{\sigma}ds + \int_{\mathbb{R}^3} xy\mu(dt, dx, dy, dz) \\
&= \int_0^t \sigma_s\eta_s^{\sigma}ds + \int_{\mathbb{R}^3} xy\nu^{\mathbb{P}}(dt, dx, dy, dz) + \mathbb{P}\text{-martingale} \\
&= \int_0^t \sigma_s\eta_s^{\sigma}ds + \int_{\mathbb{R}^3} xy\nu^{\mathbb{Q}}(dt, dx, dy, dz) + \mathbb{Q}\text{-martingale},
\end{aligned}$$

and

$$\begin{aligned}
[x, \phi_t^{\mathbb{Q}}]_t &= \int_0^t \sigma_s\eta_s^{\phi}ds + \int_{\mathbb{R}^3} xz\mu(dt, dx, dy, dz) \\
&= \int_0^t \sigma_s\eta_s^{\phi}ds + \int_{\mathbb{R}^3} xz\nu^{\mathbb{P}}(dt, dx, dy, dz) + \mathbb{P}\text{-martingale} \\
&= \int_0^t \sigma_s\eta_s^{\phi}ds + \int_{\mathbb{R}^3} xz\nu^{\mathbb{Q}}(dt, dx, dy, dz) + \mathbb{Q}\text{-martingale}.
\end{aligned}$$

From here, the expressions for $L_t^{\mathbb{P}}$ and $L_t^{\mathbb{Q}}$ in (12) follow.

Appendix B. Short-Term Variance Expansions

Appendix B.1. Assumptions

We start with stating the assumptions needed for the short-time expansions of the variance measures. For this derivations, we need only the dynamics of X in (6) in which the risk-free rate and the dividend yield are set to zero and assumptions about the risk-neutral dynamics that we make here. Throughout this section, for simplicity, we set $x_0 = 0$. Further, as in the main text, we denote the martingale component of x with

$$M_T = \int_0^T \sigma_s dW_s^{\mathbb{Q}} + \int_0^T \int_{\mathbb{R}^3} x(\mu - \nu^{\mathbb{Q}})(ds, dx, dy, dz). \tag{B.1}$$

To derive the expansions, we assume that the $\mathbb{Q}$-dynamics of z, for z being one of the processes $\mathcal{V}^{\mathbb{Q}}$, $V^{\mathbb{Q}}$, $m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$, $m^{\mathbb{Q}}(V^{\mathbb{Q}})$, $m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))$, $L^{\mathbb{Q}}$, $m^{\mathbb{Q}}(L^{\mathbb{Q}})$, $m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})M$, $m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})M)$, $[\mathcal{V}^{\mathbb{Q}}, \mathcal{V}^{\mathbb{Q}}]$ and $m^{\mathbb{Q}}([\mathcal{V}^{\mathbb{Q}}, \mathcal{V}^{\mathbb{Q}}])$, is of the form

$$z_t = z_0 + \int_0^t m_s^{\mathbb{Q}}(z) ds + \mathbb{Q} - \text{martingale}, \tag{B.2}$$

with the above martingale being $\mathcal{F}_0$-conditionally square-integrable, $m_t^{\mathbb{Q}}$ being a process with càdlàg paths, and $\mathbb{E}_0^{\mathbb{Q}}(m_t^{\mathbb{Q}})^2 < \infty$. We further denote the martingale part of $\mathcal{V}^{\mathbb{Q}}$ by $Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$. We assume that the above decomposition also holds for the processes $m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$, $m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))$ and $m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$.

The above assumptions are rather weak and generally satisfied for the continuous-time asset pricing models used in finance, including standard reduced-form models, as well as equilibrium-based models. In particular, the assumptions hold when the asset price dynamics is embedded in a general stochastic differential equation (SDE), and the coefficients of the SDE have finite conditional moments of certain order.

Appendix B.2. Proofs

Given the decomposition of x in (20) and an application of Itô's formula for the product of two processes, we have

$$\begin{aligned} T \times V_{0,T}^{\mathbb{Q}} &= \mathbb{E}_0^{\mathbb{Q}} \left(-\frac{1}{2} \int_0^T (\mathcal{V}_s^{\mathbb{Q}} - \mathbb{E}_0^{\mathbb{Q}}(\mathcal{V}_s^{\mathbb{Q}})) ds + M_T \right)^2 \\ &= \frac{1}{4} \text{var}_0^{\mathbb{Q}} \left(\int_0^T \mathcal{V}_s^{\mathbb{Q}} ds \right) + \mathbb{E}_0^{\mathbb{Q}}(M_T^2) - \mathbb{E}_0^{\mathbb{Q}} \left(\int_0^T V_s^{\mathbb{Q}} ds M_T \right) \\ &= \frac{1}{4} \text{var}_0^{\mathbb{Q}} \left(\int_0^T \mathcal{V}_s^{\mathbb{Q}} ds \right) + \int_0^T \mathbb{E}_0^{\mathbb{Q}}(V_s^{\mathbb{Q}}) ds - \int_0^T \text{cov}_0^{\mathbb{Q}}(\mathcal{V}_s^{\mathbb{Q}}, M_s) ds, \end{aligned} \tag{B.3}$$

and

$$T \times \mathcal{V}_{0,T}^{\mathbb{Q}} = \int_0^T \mathbb{E}_0^{\mathbb{Q}}(\mathcal{V}_s^{\mathbb{Q}}) ds. \tag{B.4}$$

Using our assumption for $\mathcal{V}_s^{\mathbb{Q}}$, $m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$ and $m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))$, we further have

$$\begin{aligned}\mathbb{E}_0^{\mathbb{Q}}(\mathcal{V}_s^{\mathbb{Q}}) &= \int_0^s \mathbb{E}_0^{\mathbb{Q}}(m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))du = \int_0^s \mathbb{E}_0^{\mathbb{Q}}\left(m_0^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) + \int_0^u m_v^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))dv\right)du \\ &= \int_0^s \mathbb{E}_0^{\mathbb{Q}}\left(m_0^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) + u \times m_0^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})) + \int_0^u \int_0^v m_z^{\mathbb{Q}}(m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})))dzdv\right)du \\ &= s \times m_0^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) + s^2 \times C_0 + \widetilde{C}_0(s),\end{aligned} \tag{B.5}$$

for some $\mathcal{F}_0$-adapted random variable C_0 and $\mathcal{F}_0$-adapted random function $\widetilde{C}_0(s)$ satisfying $|\widetilde{C}_0(s)| \leq s^3 \times \widetilde{C}_0'$, where $\widetilde{C}_0'$ is another $\mathcal{F}_0$-adapted random variable. A similar expansion for $\mathbb{E}_0^{\mathbb{Q}}(V_s^{\mathbb{Q}})$ also readily obtains. Next, applying Itô's formula and using the definition of the processes M and $L^{\mathbb{Q}}$ (note in particular that $M_0 = 0$), as well as our assumptions for the dynamics of $L^{\mathbb{Q}}$, $m^{\mathbb{Q}}(L^{\mathbb{Q}})$, $m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})M$ and $m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})M)$, we obtain

$$\begin{aligned}\mathrm{cov}_0^{\mathbb{Q}}(\mathcal{V}_s^{\mathbb{Q}}, M_s) &= \mathbb{E}_0^{\mathbb{Q}}\left(\int_0^s m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})duM_s\right) + \mathbb{E}_0^{\mathbb{Q}}(Z_s^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})M_s) \\ &= \mathbb{E}_0^{\mathbb{Q}}\left(\int_0^s m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})M_u du\right) + \mathbb{E}_0^{\mathbb{Q}}\left(\int_0^s L_u^{\mathbb{Q}}du\right) \\ &= \mathbb{E}_0^{\mathbb{Q}}\left(\int_0^s \int_0^u m_v^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})M)dvdu\right) + \mathbb{E}_0^{\mathbb{Q}}\left(\int_0^s \left(L_0^{\mathbb{Q}} + \int_0^u m_v^{\mathbb{Q}}(L^{\mathbb{Q}})dv\right)du\right) \\ &= s \times L_0^{\mathbb{Q}} + s^2 \times C_0 + \widetilde{C}_0(s),\end{aligned} \tag{B.6}$$

where C_0 and $\widetilde{C}_0(s)$ satisfy the same properties as above. Finally, direct expansion and the fact that $Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$ is a martingale leads to:

$$\begin{aligned}&\mathrm{var}_0^{\mathbb{Q}}\left(\int_0^T \mathcal{V}_s^{\mathbb{Q}}ds\right) = \mathbb{E}_0^{\mathbb{Q}}\left(\int_0^T \int_0^s \left(m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) - \mathbb{E}_0^{\mathbb{Q}}(m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))\right)duds\right)^2 \\ &+ \mathbb{E}_0\left(\int_0^T \int_0^T Z_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})Z_v^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})dudv\right) \\ &+ 2\mathbb{E}_0^{\mathbb{Q}}\left(\int_0^T \int_0^s \left(m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) - \mathbb{E}_0^{\mathbb{Q}}(m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))\right)duds \int_0^T Z_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))du\right).\end{aligned} \tag{B.7}$$

Using the definition of quadratic variation and the fact that $Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$ is a martingale, we may write:

$$\mathbb{E}_0\left(\int_0^T \int_0^T Z_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})Z_v^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})dudv\right) = 2\int_0^T (T-s)\mathbb{E}_0^{\mathbb{Q}}([\mathcal{V}^{\mathbb{Q}}, \mathcal{V}^{\mathbb{Q}}]_s)ds.$$

Next, using again the fact that $Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$ is a martingale,

$$
\begin{aligned}
&\mathbb{E}_0^{\mathbb{Q}}\left(\int_0^T \int_0^s \left(m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) - \mathbb{E}_0^{\mathbb{Q}}(m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))\right) du ds \int_0^T Z_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) du\right) \\
&= \mathbb{E}_0^{\mathbb{Q}}\left(\int_0^T (T-s) \int_0^s \left(m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) - \mathbb{E}_0^{\mathbb{Q}}(m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))\right) du Z_s^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) ds\right) \\
&\quad + \mathbb{E}_0^{\mathbb{Q}}\left(\int_0^T \int_0^s \left(m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) - \mathbb{E}_0^{\mathbb{Q}}(m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))\right) du \int_0^s Z_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) du ds\right) \\
&= 2\mathbb{E}_0^{\mathbb{Q}}\left(\int_0^T (T-s) m_s^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) Z_s^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})\right) ds \\
&\quad + \mathbb{E}_0^{\mathbb{Q}}\left(\int_0^T \int_0^s (m_s^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) - m_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})) Z_u^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}) du ds\right).
\end{aligned}
$$

From here, by making use of our assumptions for $[\mathcal{V}^{\mathbb{Q}}, \mathcal{V}^{\mathbb{Q}}]$, $m^{\mathbb{Q}}([\mathcal{V}^{\mathbb{Q}}, \mathcal{V}^{\mathbb{Q}}])$, $m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$, $m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))$, $m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))$ and $m^{\mathbb{Q}}(m^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}}))Z^{\mathbb{Q}}(\mathcal{V}^{\mathbb{Q}})$, it now follows that

$$
\operatorname{var}_0^{\mathbb{Q}}\left(\int_0^T \mathcal{V}_s^{\mathbb{Q}} ds\right) = T^3 \times C_0 + \widetilde{C}_0(T), \tag{B.8}
$$

where C_0 is some $\mathcal{F}_0$-adapted random variable, and the $\mathcal{F}_0$-adapted random function $\widetilde{C}_0(T)$ satisfies $|\widetilde{C}_0(T)| \leq T^4 \times \widetilde{C}_0'$ for some some $\mathcal{F}_0$-adapted random variable $\widetilde{C}_0'$.

Appendix C. Nonzero Interest Rate and Dividend Yield

All of our expansions and empirical calculations are based on the simplifying assumption that the risk-free rate and the dividend yield are identically equal to zero. In this appendix, we show how the empirical results are practically unaffected by this assumption.

We denote the constant continuously-compounded risk-free rate and dividend yield by r and d, respectively. We further denote the time t price of a futures contract written on the market expiring at time $t+T$ by $F_{t,T}$, with the corresponding log price denoted by $f_{t,T}$. By standard no-arbitrage pricing arguments, $F_{t,T} = e^{(r-d)T} X_t$. With this additional notation, the definitions of our two risk-neutral variance measures previously defined in (13) and (7) now naturally become

$$
V_{t,t+T}^{\mathbb{Q}} = \frac{1}{T}\operatorname{Var}_t^{\mathbb{Q}}(x_{t+T} - f_{t,T}), \qquad \mathcal{V}_{t,t+T}^{\mathbb{Q}} = -\frac{2}{T}\mathbb{E}_t^{\mathbb{Q}}(x_{t+T} - f_{t,T}). \tag{C.1}
$$

Note that since $f_{t,T}$ is adapted to $\mathcal{F}_t$, the definition of $V_{t,t+T}^{\mathbb{Q}}$ remains unchanged compared to the case with $r = d = 0$, while the centering term in $\mathcal{V}_{t,t+T}^{\mathbb{Q}}$ gets slightly modified. With this adjustment, the key expansion results in (17) and (18) both continue to hold.

Correspondingly, the option spanning results needed to construct the estimates of $\mathcal{V}^{\mathbb{Q}}_{t,t+T}$ and $V^{\mathbb{Q}}_{t,t+T}$ now take the form

$$\mathbb{E}^{\mathbb{Q}}_t(x_{t+T} - f_{t,T})^2 = 2\int_0^\infty \left(1 - \log\left(\frac{K}{X_t}\right)\right) \frac{e^{rT} O_{t,T}(K)}{K^2} dK, \tag{C.2}$$

and

$$\mathbb{E}^{\mathbb{Q}}_t(x_{t+T} - f_{t,T}) = -\int_0^\infty \frac{e^{rT} O_{t,T}(K)}{K^2} dK. \tag{C.3}$$

Compared to the previous expressions in (39) and (40), the only difference is that we now consider the future values of the option prices. Using these modified spanning results, $\widehat{\mathcal{V}}^{\mathbb{Q}}_{t,t+T}$ and $\widehat{V}^{\mathbb{Q}}_{t,t+T}$ may again be constructed exactly as previously done in (41) and (42).

Importantly, the above analysis further implies that if nonzero interest rates and dividend yields are ignored, then, up to a higher order term, the OLS estimates of $b_{t,0}$, $b_{t,1}$, $\beta_{t,0}$ and $\beta_{t,1}$ from the regressions in (43) will simply be scaled down by the same interest rate factor e^{-rT} relative to their counterparts where nonzero r and d are explicitly accounted for. Obviously, $e^{-rT} = 1 - rT + O(T^2)$ as $T \downarrow 0$. Hence, not only will this correction similarly affect all the coefficient estimates of interest, given the invariably low value of the risk-free interest rate, it will also be numerically very small.

Appendix D. Jumps and Leverage Risk in a Parametric Model

This Appendix demonstrates how the risk premium measures defined in Section 4 manifest in the popular parametric double-jump stochastic volatility model of Duffie et al. (2000). This particular model has been extensively used in the empirical asset pricing literature.

The model postulates the following asset dynamics under $\mathbb{P}$

$$\frac{dX_t}{X_{t-}} = \alpha_t dt + \sigma_t dW^{\mathbb{P}}_t + \int_{\mathbb{R}^2} (e^z - 1)(\mu - \nu^{\mathbb{P}})(ds, dz, dy), \tag{D.1}$$

$$d\sigma^2_t = \kappa^{\mathbb{P}}(\theta^{\mathbb{P}} - \sigma^2_t)dt + \eta\sigma_t dB^{\mathbb{P}}_t + \int_{\mathbb{R}^2} y(\mu - \nu^{\mathbb{P}})(ds, dz, dy), \tag{D.2}$$

where α_t is a linear function of σ^2_t, $(W^{\mathbb{P}}_t, B^{\mathbb{P}}_t)$ is a bivariate Brownian motion with correlation ρ, and the jump compensator takes the form

$$\nu^{\mathbb{P}}(ds, dz, dy) = (\lambda^{\mathbb{P}}_0 + \lambda^{\mathbb{P}}_1 \sigma^2_t) \frac{e^{-\frac{(z-\mu^{\mathbb{P}}_z)^2}{2v^{\mathbb{P}}_z}}}{\sqrt{2\pi v^{\mathbb{P}}_z}} \frac{e^{-y/\mu^{\mathbb{P}}_y}}{\mu^{\mathbb{P}}_y} 1_{\{y>0\}} ds dz dy. \tag{D.3}$$

This specification in turn accounts for a number of key empirically relevant features: volatility is time-varying, price and volatility can both jump, and the jumps typically arrive together, the intensity of the jumps is an affine function of σ_t^2 and therefore time-varying. We note that we have slightly constrained the original specification of Duffie et al. (2000) by assuming that the size of the price and volatility jumps are independent of each other, even though they arrive together. This simplification of the double-jump volatility model is only for ease of exposition and has no bearing on our analysis of jump leverage.

Per the discussion in the main text, assume for simplicity that the risk-free interest rate and the dividend yield are both zero. The dynamics of X under the $\mathbb{P}$ and $\mathbb{Q}$ measures then formally coincide, although the pricing of various risks may cause the values of the risk-neutral parameters, superscripted with $\mathbb{Q}$ in the following, to differ from the corresponding $\mathbb{P}$ superscripted parameters. Importantly, however, the ρ and η parameters do not change under this equivalent change of measure.

To begin, consider the instantaneous variance risk premium. Using the shorthand notation $\widetilde{v}_z^{\mathbb{Q}} = 2\left(e^{\mu_z^{\mathbb{Q}}+v_z^{\mathbb{Q}}/2} - 1 - \mu_z^{\mathbb{Q}}\right)$, the two differently defined risk-neutral instantaneous variances may be expressed as

$$V_t^{\mathbb{Q}} = \sigma_t^2 + (\lambda_0^{\mathbb{Q}} + \lambda_1^{\mathbb{Q}}\sigma_t^2)((\mu_z^{\mathbb{Q}})^2 + v_z^{\mathbb{Q}}), \qquad \mathcal{V}_t^{\mathbb{Q}} = \sigma_t^2 + (\lambda_0^{\mathbb{Q}} + \lambda_1^{\mathbb{Q}}\sigma_t^2)\widetilde{v}_z^{\mathbb{Q}}. \tag{D.4}$$

The same expressions also obtain for $V_t^{\mathbb{P}}$ and $\mathcal{V}_t^{\mathbb{P}}$, except for the $\mathbb{Q}$-parameters being replaced by their $\mathbb{P}$ counterparts. Correspondingly, the $IVRP_t$ instantaneous variance risk premium takes the specific form

$$IVRP_t = (\lambda_0^{\mathbb{Q}} + \lambda_1^{\mathbb{Q}}\sigma_t^2)((\mu_z^{\mathbb{Q}})^2 + v_z^{\mathbb{Q}}) - (\lambda_0^{\mathbb{P}} + \lambda_1^{\mathbb{P}}\sigma_t^2)((\mu_z^{\mathbb{P}})^2 + v_z^{\mathbb{P}}). \tag{D.5}$$

As previously noted, $IVRP_t$ generally only reflects the pricing of price jump risk, which in the context of the double-jump stochastic volatility model can arise from either different jump intensities, or different jump distributions under the $\mathbb{P}$ and $\mathbb{Q}$ measures.

Turning next to the leverage risk, we have

$$\frac{1}{T}\left([x, \mathcal{V}^{\mathbb{Q}}]_{t+T} - [x, \mathcal{V}^{\mathbb{Q}}]_t\right) = \frac{1}{T}(1 + \lambda_1^{\mathbb{Q}}\widetilde{v}_z^{\mathbb{Q}})\left(\rho\eta\int_t^{t+T}\sigma_s^2 ds + \sum_{s\in[t,t+T]}\Delta x_s \Delta\sigma_s^2\right). \tag{D.6}$$

The dependence of the leverage risk on the $\mathbb{Q}$-parameters stems from the fact that this risk is defined as a co-movement between x and $\mathcal{V}^{\mathbb{Q}}$, which depends on the $\mathbb{Q}$-parameters because of jumps in the price and the pricing thereof. Meanwhile, the first term in the parentheses above stems from the dependence between the Brownian motions driving the price and volatility, while the second term is due to the price-volatility co-jumps. Taking conditional expectations and letting $T \downarrow 0$, it further follows that

$$L_t^{\mathbb{Q}} = (1 + \lambda_1^{\mathbb{Q}} \widetilde{v}_z^{\mathbb{Q}}) \left[\rho\eta\sigma_t^2 + \mu_z^{\mathbb{Q}} \mu_y^{\mathbb{Q}} (\lambda_0^{\mathbb{Q}} + \lambda_1^{\mathbb{Q}} \sigma_t^2) \right], \tag{D.7}$$

and

$$L_t^{\mathbb{P}} = (1 + \lambda_1^{\mathbb{Q}} \widetilde{v}_z^{\mathbb{Q}}) \left[\rho\eta\sigma_t^2 + \mu_z^{\mathbb{P}} \mu_y^{\mathbb{P}} (\lambda_0^{\mathbb{P}} + \lambda_1^{\mathbb{P}} \sigma_t^2) \right]. \tag{D.8}$$

These expressions for the instantaneous leverage effects in turn implies the following instantaneous leverage risk premium

$$ILRP_t = (1 + \lambda_1^{\mathbb{Q}} \widetilde{v}_z^{\mathbb{Q}}) \left[\mu_z^{\mathbb{P}} \mu_y^{\mathbb{P}} (\lambda_0^{\mathbb{P}} + \lambda_1^{\mathbb{P}} \sigma_t^2) - \mu_z^{\mathbb{Q}} \mu_y^{\mathbb{Q}} (\lambda_0^{\mathbb{Q}} + \lambda_1^{\mathbb{Q}} \sigma_t^2) \right]. \tag{D.9}$$

Since the terms in $L_t^{\mathbb{Q}}$ and $L_t^{\mathbb{P}}$ due to the continuous leverage effect coincide, these terms cancel in the $ILRP_t$ risk premium, which therefore only depends on price-volatility co-jumps. As such, $ILRP_t$ is effectively a difference of, re-scaled by constants, $\mathbb{P}$ and $\mathbb{Q}$ jump intensities. This also directly parallels the expression for $IVRP_t$. Correspondingly, $IVRP_t$ and $ILRP_t$ are both affine functions of σ_t^2, and thus exhibit the same dynamics. This result is a particular feature of the double-jump volatility model. If the intensity of the price jumps that arrive together with the volatility jumps differed from those that do not, this would naturally cause the dynamics of $IVRP_t$ and $ILRP_t$ to differ.

Finally, it is easy to see that

$$m_t^{\mathbb{P}}(\mathcal{V}_t^{\mathbb{Q}}) = (1 + \lambda_1^{\mathbb{Q}} \widetilde{v}_z^{\mathbb{Q}}) \kappa^{\mathbb{P}} (\theta^{\mathbb{P}} - \sigma_t^2), \qquad m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}}) = (1 + \lambda_1^{\mathbb{Q}} \widetilde{v}_z^{\mathbb{Q}}) \kappa^{\mathbb{Q}} (\theta^{\mathbb{Q}} - \sigma_t^2), \tag{D.10}$$

which implies the following change in the instantaneous drift term of $\mathcal{V}_t^{\mathbb{Q}}$

$$m_t^{\mathbb{Q}}(\mathcal{V}_t^{\mathbb{Q}}) - m_t^{\mathbb{P}}(\mathcal{V}_t^{\mathbb{Q}}) = (1 + \lambda_1^{\mathbb{Q}} \widetilde{v}_z^{\mathbb{Q}}) [\kappa^{\mathbb{Q}} (\theta^{\mathbb{Q}} - \sigma_t^2) - \kappa^{\mathbb{P}} (\theta^{\mathbb{P}} - \sigma_t^2)]. \tag{D.11}$$

This in turn implies the following compensation for jumps risk

$$(1 + \lambda_1^{\mathbb{Q}} \widetilde{v}_z^{\mathbb{Q}}) \int_{\mathbb{R}^2} y (\nu_t^{\mathbb{Q}}(dz, dy) - \nu_t^{\mathbb{P}}(dz, dy)) = (1 + \lambda_1^{\mathbb{Q}} \widetilde{v}_z^{\mathbb{Q}}) \left[\mu_y^{\mathbb{Q}} (\lambda_0^{\mathbb{Q}} + \lambda_1^{\mathbb{Q}} \sigma_t^2) - \mu_y^{\mathbb{P}} (\lambda_0^{\mathbb{P}} + \lambda_1^{\mathbb{P}} \sigma_t^2) \right]. \tag{D.12}$$

This expression, not surprisingly, looks very similar to the expression for $ILRP_t$ in (D.9) above, as they both reflect the compensation for the simultaneously arriving jumps in price and volatility.

To more concretely illustrate the important role played by price-volatility co-jumps in the pricing of jump risk, we finish our discussion of the parametric double-jump stochastic volatility model by showing the model-implied term structure of the second and third risk-neutral moments with and without volatility jumps. To help more clearly illustrate the effect, we further plot the different term structures obtained for initial low and high levels of volatility, as defined relative to the unconditional model-implied volatility. We rely on the same parameter values listed in Table E.3 that we use in our Monte Carlo simulations discussed in Appendix Appendix E.

Looking first at the left panels in Figures D.7 and D.8 with and without volatility jumps, respectively, the general features of the term structures for the second moments appear largely unaffected by volatility jumps. By comparison, the term structures of the third risk-neutral moments clearly differ across the two figures, and are obviously much steeper when volatility jumps are allowed for. In fact, the slope of the term structure appears almost flat in the high volatility regime in the absence of volatility jumps, while the slopes are highly significant and positive regardless of whether the initial volatility is low or high when volatility jumps are included. Importantly, the patterns evident in Figure D.7 also fairly closely mirror the corresponding actual empirical illustration in Figure 2, thus indirectly underscoring the importance of price-volatility co-jumps, and the jump leverage effect, from a practical pricing perspective.

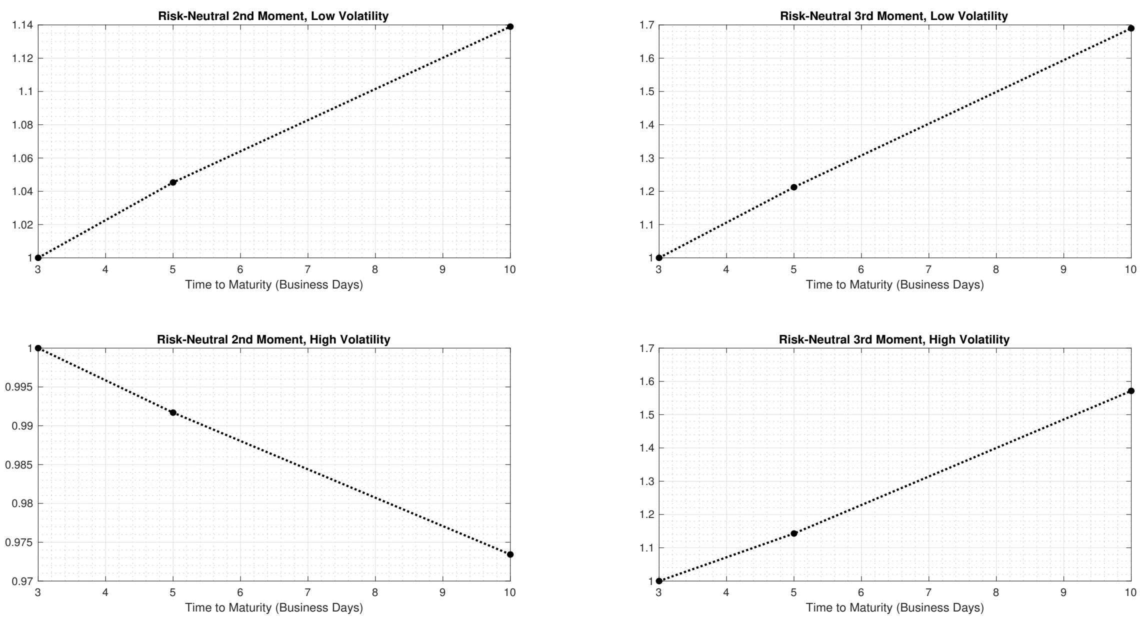

Figure D.7: The figure plots the model-implied term structure of the risk-neutral second and third moments allowing for volatility jumps. The risk-neutral moments are computed from the double-jump stochastic volatility model with the parameters given in Table E.3. The low and high volatility regimes corresponds to $V_0 = 0.017$ and $V_0 = 0.035$, respectively. The moments at the shortest horizons in each of the panels are normalized to unity.

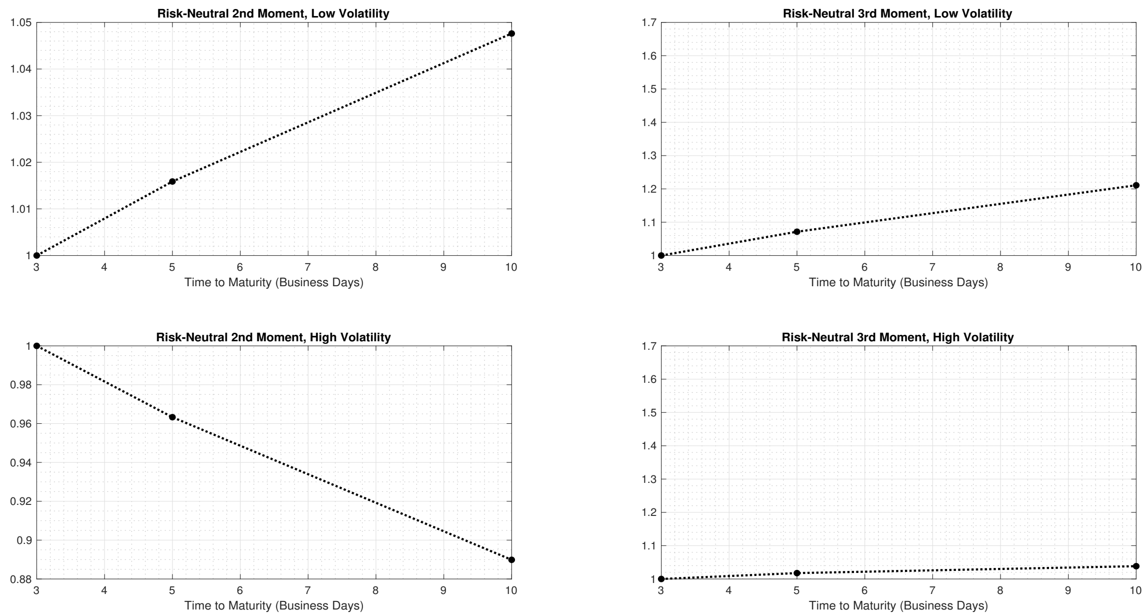

Figure D.8: The figure plots the model-implied term structure of the risk-neutral second and third moments not allowing for volatility jumps. The risk-neutral moments are computed from the double-jump stochastic volatility model with parameters given in Table E.3, except for $\mu_y^{\mathbb{Q}}$ which is fixed at zero. The low and high volatility regimes corresponds to $V_0 = 0.017$ and $V_0 = 0.035$, respectively. The moments at the shortest horizons in each of the panels are normalized to unity.

Appendix E. Monte Carlo

This appendix evaluates the accuracy of the new $\widehat{V}_t^{\mathbb{Q}}$, $\widehat{\mathcal{V}}_t^{\mathbb{Q}}$ and $\widehat{L}_t^{\mathbb{Q}}$ feasible risk-neutral measures discussed in Section 6.1. We rely on the double-jump stochastic volatility model discussed in Appendix Appendix D to generate the true option prices. The specific values of the model parameters used in the simulations are reported in Table E.3.

The parameter values imply that $\mathbb{E}^{\mathbb{Q}}(\sigma_t^2) = 0.0257$, together with a relatively fast mean reversion of volatility with a half-life of around 8 days only. The jump parameters are set in a way so that $\int_{\mathbb{R}^2} z^2 \nu_t(dz, dy) = \sigma_t^2$, implying that jumps contribute half of the risk-neutral jump variation at any point in time. We also allow for nontrivial volatility jumps, with a mean value equal to 0.0234, very close to that of the risk-neutral mean of σ_t^2.

The observed option prices used in the estimation are contaminated by the following errors

$$\widehat{O}_{t,T}(K_j) = O_{t,T}(K_j)(1 + 0.025 \times z_{t,T}(j)), \quad j = 1, ..., N_{t,T},$$

where

$$z_{t,T}(j) = \frac{1}{\sqrt{1 + 0.5^2}} \left[\epsilon_{t,T}(j) + 0.5\epsilon_{t,T}(j-1)\right], \quad j = 2, ..., N_{t,T},$$

Table E.3: **Parameter Values**

Parameter	Value	Parameter	Value	Parameter	Value
$\kappa^{\mathbb{Q}}$	30	$\theta^{\mathbb{Q}}$	0.018	η	0.2
ρ	-0.9	$\lambda_0^{\mathbb{Q}}$	0	$\lambda_1^{\mathbb{Q}}$	385
$\mu_z^{\mathbb{Q}}$	-0.05	$\mu_y^{\mathbb{Q}}$	0.0234	$\sqrt{v_z^{\mathbb{Q}}}$	0.01

Note: The table reports risk-neutral parameter values for the double-jump stochastic volatility model in (D.1)-(D.3) in Appendix Appendix D used in the simulations.

and $\{\epsilon_{t,T}(j)\}_{j=1}^{N_{t,T}}$ are sequences of *i.i.d.* standard normal independent random variables. The size of the observation error is calibrated to roughly match the bid-ask spreads of the S&P 500 index options used in our empirical analyses. In the above setting, following nonparametric evidence reported in Andersen et al. (2021), we allow for spatial dependence in the observation error of MA(1) type. Finally, we round the generated option prices with error to a multiple of 0.025 and perform computations on the rounded prices. Our rounding mimics that in observed SPX option prices.

We initialize the simulations by setting the time-t value of the spot variance to a low, average and a high value of 0.0170, 0.0204 and 0.0267, corresponding to the 25-th, 50-th and 75-th quantiles, respectively, of the unconditional distribution of σ_t^2. The initial level of the underlying stock price is set to $4,500$. For each (t, T) pair the strike grid is equidistant with gaps between strikes of 5. Starting from the at-the-money strike of $4,500$, the strikes are then extended on both sides until the true out-of-the-money option price falls below 0.075. We employ three different tenors in the estimation, namely $T_1 = 3/252$, $T_2 = 5/252$ and $T_3 = 10/252$, corresponding to 3, 5 and 10 business days to expiration, respectively. Again, this specification of the strike grids and the choice of tenors mimics those of the S&P 500 index options used in the actual estimation. Finally, the $\widehat{s}_t$ bias-adjustment term for $\widehat{L}_t^{\mathbb{Q}}$ is computed from 80 high-frequency return and option observations, approximately corresponding to a day worth of observations when sampling at the 5-minute frequency. To mimic missing observations in the data, we randomly remove 6 out of the 80 option observations from the computations. The locations of the missing observations are drawn uniformly from the observation times.

Table E.4 reports the quantiles of the resulting $\widehat{V}_t^{\mathbb{Q}}$, $\widehat{\mathcal{V}}_t^{\mathbb{Q}}$ and $\widehat{L}_t^{\mathbb{Q}}$ risk estimates based on 1,000 Monte Carlo replications. As the table shows, the new estimates generally perform admirably, with the medians almost exactly equal to the true values, together with fairly narrow interquartile ranges.

Table E.4: **Monte Carlo Simulation Results**

Estimand	True Value	Estimates		
		Q_{25}	Q_{50}	Q_{75}
	Initial Volatility $\sigma_t^2 = 0.0170$			
$V_t^{\mathbb{Q}}$	0.0340	0.0335	0.0340	0.0345
$\mathcal{V}_t^{\mathbb{Q}}$	0.0337	0.0332	0.0337	0.0342
$-L_t^{\mathbb{Q}}$	0.0214	0.0210	0.0223	0.0237
	Initial Volatility $\sigma_t^2 = 0.0204$			
$V_t^{\mathbb{Q}}$	0.0408	0.0402	0.0408	0.0413
$\mathcal{V}_t^{\mathbb{Q}}$	0.0404	0.0399	0.0404	0.0410
$-L_t^{\mathbb{Q}}$	0.0257	0.0241	0.0256	0.0271
	Initial Volatility $\sigma_t^2 = 0.0267$			
$V_t^{\mathbb{Q}}$	0.0534	0.0527	0.0533	0.0540
$\mathcal{V}_t^{\mathbb{Q}}$	0.0529	0.0522	0.0529	0.0536
$-L_t^{\mathbb{Q}}$	0.0336	0.0300	0.0317	0.0336

Note: The table reports the simulated quantiles of the $V_t^{\mathbb{Q}}$, $\mathcal{V}_t^{\mathbb{Q}}$, and $-L_t^{\mathbb{Q}}$ risk estimates for different initial volatility levels based on 1,000 Monte Carlo replications.

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