Homework Assignment 1: Answers

1) Net present value calculations. These calculations should look very familiar from Finance I. If they do not, you should review your notes from Finance I immediately. You want to make sure you are comfortable with the basics.

A) The first year cashflow is 110. The second year cashflow is 10% larger, it is 121.0. The cashflow in the tenth year is 259.4. Discounting these 10 numbers at 15%, yields a NPV of $789.5. This calculation can be done with a spreadsheet or by using a financial calculator. Use which ever you find easier.

B) This calculation is harder to do in a spreadsheet, so I resorted to a formula. From Brealey and Myers or your Fin 1 notes, the NPV of a growing annuity is:

$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \dots$$

$$= \frac{C_1}{(1+r)^1} + \frac{C_1\ (1+g)^1}{(1+r)^2} + \frac{C_1\ (1+g)^2}{(1+r)^3} + \dots$$

$$= \sum_{t=0}^{\text{infinity}} \frac{C_1\ (1+g)^{t-1}}{(1+r)^t} = \frac{C_1}{r - g}$$

$$= \frac{110}{.15 - .10} = 2200$$

The cashflow the first year (C_1) is $110. The cashflows are growing at 10% per year. The correct discount rate is 15%. Thus the present value of this growing perpetuity is 2200. Notice that a significant fraction of the value is based on cashflows after year 10. In fact only 35% of the value of this perpetuity comes from cashflows in years 1 to 10.

2) Junk Bond Valuation

A) If the bonds never default, interest and principal payments should be discounted at the same rate as long government bonds. Long government bonds have a zero probability of default. Discounting the cashflows to the junk bond at 6.9% yields a bond price of $1261.8.

$$P_{Bond} = \sum_{t=1}^{6} \frac{123.75}{(1 + .069)^t} + \frac{1000}{(1 + .069)^6} = 1261.8$$

The bond sells for more than face value (1000) because the expected rate (which is also the coupon rate in this case: 12.375%) is greater than the discount rate (6.9%).

B) The probability of the bond defaulting, conditional on not defaulting yet, is p. Thus the probability that the bond will pay off its promised payment at the end of the first year is (1-p). The bond defaults with probability p. The bond does not default with probability (1-p). For the bond to make its promised payment at the end of 1992, it must not default in 1991 and then not default again in 1992. The probability of not defaulting twice is $(1-p)^2$.

C) To value a bond you discount the expected cashflows at the appropriate discount rate. Write the present value of the bond=s expected cashflow as a function of p, set your expression equal to the market value of the bond ($750), and solve for p. The expected cashflow the first year is:

$$E[\text{ Cashflow}_{1991}] = 123.75\ (1 - p) + 0\ p = 123.75\ (1-p)$$

Using the same method, you can write the expected cashflows for years 2 through 6. The following expression needs to be solved:

$$750 = \sum_{t=1}^{6} \frac{123.75\ (1-p)^t}{(1 + .124)^t} + \frac{1000\ (1-p)^6}{(1 + .124)^6}$$

Solving such an expression can be done in a spreadsheet by guessing until you have the correct answer. Alternatively, many spreadsheets now have the capacity to solve for a single unknown. The probability of default is 6.26 percent. Using this answer, the probability that the bond makes all its interest and principal payments is $(1-.0626)^6$ or 68%. There is a 32% probability that the bond will default before maturity. This is high. However, these are high yield -- read high risk -- bonds.

D) Using the same method as in C) the present value of the bond can be written as:

$$870 = \sum_{t=1}^{6} \frac{123.75\ (1-p)^t}{(1 + .124)^t} + \frac{1000\ (1-p)^6}{(1 + .124)^6}$$

The implied probability of default is now only 3.05 percent per year. The bondholders are better off because the probability that they will be repaid has risen dramatically. The lower probability of default is why the market price of the bonds jumps $120 per bond. Now the probability that the bonds will default by maturity is only 17%. In reality, a junk bond's probability of default is not constant across time. Older junk bonds are significantly more likely to default than younger ones. The probability of default starts at almost zero and grows over time. The cumulative probability of default rises to approximately a after ten years.

3) Stone Container.

A) The beta that measures the risk of Stone Container's assets is its asset beta. Since a firm's assets are a portfolio of the firm's equity and debt, we can use the following equation.

$$\text{if } A = D + E \text{ then}$$

$$\beta_A = \beta_D \frac{D}{D+E} + \beta_E \frac{E}{D+E}$$

Since the market value of the debt is $2M and the market value of the equity is $10M,the firm is 16.7%

debt. The asset beta is:

$$\beta_{\text{Stone Container's Assets}} = 0.167\ (0.20) + 0.833\ (1.40) = 1.20$$

B) The appropriate discount rate measures compensation for both the time value of money (the risk free rate) and compensation for risk (the risk premium). The 3 percent discount rate is not appropriate, since it measure only compensation for the time value of money. The project we are considering has priced (systematic) risk, and thus a discount rate above the risk free rate is appropriate.

C) We can use the CAPM equation to derive the appropriate discount rate once we have the project beta. Since Stone Container is expanding capacity, its asset beta may seem like a good estimate of the project beta. However, Stone Container has two types of assets. It has a T-bill portfolio, whose beta is 0, and a cardboard production and sales facilities. It is the beta of the latter assets which we seek. Since Stone Container's assets are a portfolio of these two types of assets, we can make the following calculation.

$$\begin{aligned}\beta_{\text{SC's Assets}} = 1.2 &= \frac{3}{12}\,\beta_{\text{T-bill Assets}} + \frac{9}{12}\,\beta_{\text{Cardboard Assets}} \\ &= 0.25\ 0 + 0.75\ \beta_{\text{Cardboard Assets}} \\ &\rightarrow \beta_{\text{Cardboard Assets}} = 1.60\end{aligned}$$

Given an asset beta of 1.60 for the cardboard expansion project, the discount rate for this project is:

$$\begin{aligned}r_{\text{Cardboard assets}} &= r_{\text{risk free}} + \beta_{\text{Cardboard assets}}\, E[\, r_{\text{market}} - r_{\text{risk free}}\,] \\ &= 3.00 + 1.60\ (8.40) = 16.44\end{aligned}$$

D) The equity beta will change when the firm's capital structure or assets change.

$$\beta_E = \beta_A + \frac{D}{E}\,(\,\beta_A - \beta_D\,)$$

When Stone Container invested in the extra production and sales capacity for its cardboard boxes, it did not change its capital structure. It did, however, change its assets. It sold low risk assets (Tbills) and purchased higher risk assets (cardboard assets). The asset beta of Stone Container will therefore rise. It will increase from 1.2 to 1.47. The increase in the asset beta will cause an increase in the equity beta. You should therefore expect the new estimate of the equity beta to be above 1.4. If the debt beta remains unchanged at 0.2, then the equity beta will rise to:

$$\beta_E = 1.47 + \frac{2}{10}\,(\,1.47 - 0.20\,) = 1.72$$

In practice, the debt beta will probably rise slightly. Remember the assets are now riskier and the debt security is a claim on the assets. Thus the equity beta will probably be slightly below 1.72.