

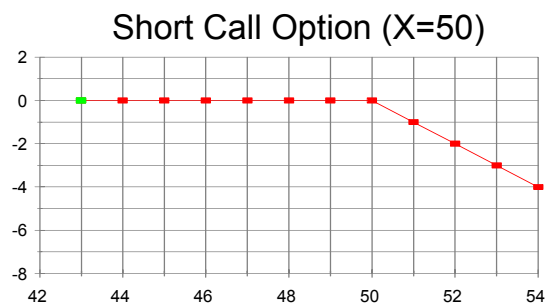
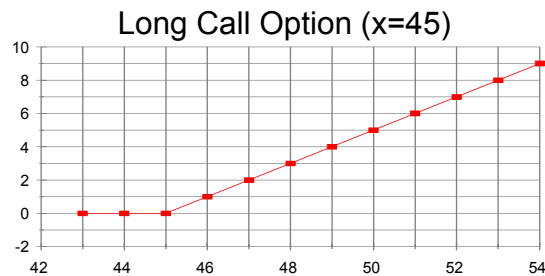
Homework Assignment 3: Answers

1. This portfolio is referred to as a bull spread. It is a bullish in the sense that you will make more if the value of the underlying rises than if it falls. Notice, however, that the upside is limited in exchange for limiting your initial investment.

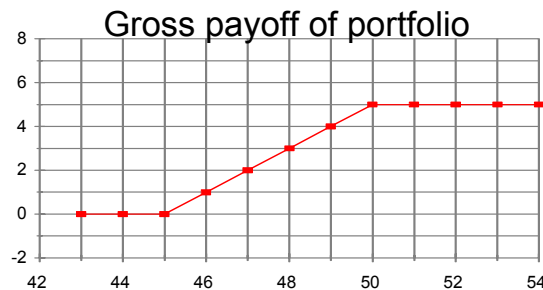
(A) The payoff and net profit of this portfolio are as follows.

Position	$S_T < 45$	$45 < S_T < 50$	$S_T > 50$
Buy call (X=45)	0	$(S_T - 45)$	$(S_T - 45)$
Sell call (X=50)	0	0	$-(S_T - 50)$
Total payoff less initial investment	0 -4.25	$(S_T - 45)$ -4.25	5 -4.25
Total net profit	-4.25	$S_T - 49.25$	0.75
Percentage return	-100%	-100 to 17.6%	17.6%

You were asked to draw the gross payoff. You should become comfortable with this method. First draw the payoff diagram for the two pieces: long a call with a strike price of 45 and short a call with a strike price of 50.



Now add the two portfolios together. Given there are two kinks in our original pictures -- one at 45 and one at 50 -- there will be no more than two kinks -- one at 45 and one at 50 -- in our gross payoff diagram. If you want the net payoff diagram, subtract off the cost of the portfolio. This will move the entire picture down by \$4.25.



Notice that the maximum upside (17.6%) is small relative to the maximum downside (-100%). This must imply that the **market** believes that the probability of the stock being 50 or greater is significantly higher than the probability of the stock being 45 or less.

(B) As the name implies this is a bullish bet. Investors which put a greater probability weight on the underlying's price rising a little, compared to the market's probability weight, will find this portfolio attractive. This is the kind of bet an individual with a little good news would take. This is a good bet to make if you think the stock price is going to rise to 50 by next year. If an investor felt the stock price was going to increase dramatically (i.e. above 50), then it wouldn't make sense selling the call with a strike price of 50. This lowers our initial investment, but at the cost of selling off any upside above 50.

2. This portfolio is a butterfly spread.

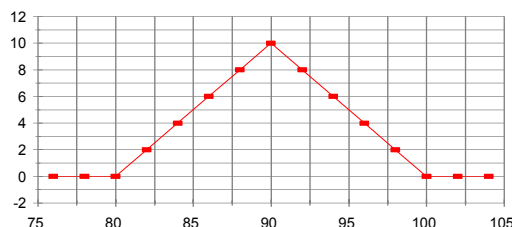
(A) First the payoff and net profit of this portfolio. The payoff and net profit diagrams, which you were asked to draw, are reproduced below. I have drawn the payoff for long an option with a strike price of 80, long 2 options with a strike price of 90 (notice the slope is twice as steep), and long an option with a strike price of 100. The portfolio (drawn with a line in the diagram) is constructed by adding the first ($X=80$), subtracting the second ($X=90$), and adding the third ($X=100$). Subtracting a portfolio which is long two options with a strike price of 90 is identical to adding a portfolio which is short two options with a strike price of 90.

Position	$S_T < 80$	$80 < S_T < 90$	$90 < S_T < 100$	$S_T > 100$
Buy call (X=80)	0	(S_T-80)	(S_T-80)	(S_T-80)
Sell two calls (X=90)	0	0	$-2(S_T-90)$	$-2(S_T-90)$
Buy call (X=100)	0	0	0	(S_T-100)
Total payoff	0	(S_T-80)	$(100-S_T)$	0
less initial investment (in $t=T$ dollars)	-2.12	-2.12	-2.12	-2.12
Total net profit	-2.12	$S_T-82.12$	$97.88-S_T$	-2.12
Percentage return	-100%	-100 to 395%	395 to -100%	-100%

Percentage returns are obtained by dividing the gross payoff by the initial investment (in $t=T$ dollars), subtracting 1 and multiplying by 100. For example, if the stock price at expiration is \$90, the gross payoff is \$10. Dividing this by the initial investment of \$2.02 ($=2.12/1.05$) gives 4.95. Subtracting 1 and multiplying by 100 gives the percentage return of 395%.

Now add the three payoffs together to get the gross payoff of the portfolio. The gross payoff is graphed at right. To get the net payoff lower each value by \$2.12.

Gross payoff of portfolio



(B) This portfolio is a bet on volatility. An investor who believes that the market has overestimated the volatility of this asset will find this portfolio attractive. If the value of the asset doesn't change too much, then the portfolio has a high payoff (i.e. if the value of the underlying stays close to 90). However, if the value of the underlying rises above 100 or falls below 80, the investor loses her entire investment.

Options, unlike investments in the underlying asset itself, allow you to speculate on both the volatility and the mean of the underlying's future value.

One last comment. Notice that investments in options are very leveraged investments. One way to see this is to calculate the percentage returns on the option portfolio and compare it to the percentage return on the underlying asset. For the butterfly spread portfolio, the percentage returns range from -100 percent (when the stock price is below 80 or above 100 at expiration) to almost 400% (when the stock price equals 90 at expiration). This range of percentage returns occurs over a very small range of returns on the underlying asset. The stock return is 0% when the stock price

equals 90 and 11.1% when the stock price equals 100.