

Government Finance in the Wake of Currency Crises

Technical Appendix

Craig Burnside* Martin Eichenbaum† Sergio Rebelo‡

April 2005

1. A Discrete Time Approximation to the Model

In this section we describe a discrete time approximation to our model. We divide time into small intervals each of length n years. So, for example, $n = 1/12$ would imply that time was being measured in months. The variable t is used here to index these time intervals. All flow variables defined in the main text are measured as flows per small interval but are expressed at annual rates.

1.1. The Household's Problem

Define the dollar prices of nontraded and traded goods:

$$p_t^N = P_t^N / S_t \tag{1.1}$$

$$p_t^T = P_t^T / S_t = (\bar{P}_t + \delta P_t^N) / S_t = 1 + \delta p_t^N. \tag{1.2}$$

Also define the CPI measured in dollars as

$$p_t = P_t / S_t = (p_t^T)^\omega (p_t^N)^{1-\omega}. \tag{1.3}$$

The household's disposable income is

$$y_t^D = y_t + rB/S_t + v_t - \tau_t. \tag{1.4}$$

where

$$y_t = y_t^T + y_t^N p_t^N, \tag{1.5}$$

$$v_t = \hat{v}_t p_t + \tilde{v}_t, \text{ and} \tag{1.6}$$

$$\tau_t = \tau^y y_t + \tau_t^L. \tag{1.7}$$

*Duke University and NBER

†Northwestern University, NBER and Federal Reserve Bank of Chicago

‡Northwestern University, NBER and CEPR

The representative household's flow budget constraint for $t \geq 0$ is given by:

$$f_t = (1 + nr_{t-1})f_{t-1} + Z_t - (M_t - M_{t-1})/S_t, \quad (1.8)$$

where

$$Z_t = n (y_t^D - p_t^T c_t^T - p_t^N c_t^N). \quad (1.9)$$

Here Z_t/n represents household saving, measured at annual rates. Iterating on the flow budget constraint starting at time 0, we have:

$$\begin{aligned} f_{-1} &= (1 + nr_{-1})^{-1} f_0 - (1 + nr_{-1})^{-1} [Z_0 - (M_0 - M_{-1})/S_0] \\ &= (1 + nr_{-1})^{-1} (1 + nr_0)^{-1} f_1 - (1 + nr_{-1})^{-1} (1 + nr_0)^{-1} [Z_1 - (M_1 - M_0)/S_1] - \\ &\quad (1 + nr_{-1})^{-1} [Z_0 - (M_0 - M_{-1})/S_0] \\ &\quad \vdots \\ &= \Xi_{t-1} f_{t-1} - \sum_{j=0}^{t-1} \Xi_j [Z_j - (M_j - M_{j-1})/S_j]. \end{aligned}$$

where

$$\Xi_t = \prod_{j=0}^t (1 + nr_{j-1})^{-1} = (1 + nr_{-1})^{-1} (1 + nr_0)^{-1} \cdots (1 + nr_{t-1})^{-1}. \quad (1.10)$$

When the interest rate is constant (as in the primary example explored in the paper) Ξ_t reduces to $(1 + nr)^{-(t+1)}$. Imposing the condition $\lim_{t \rightarrow \infty} \Xi_{t-1} f_{t-1} = 0$, we obtain the intertemporal budget constraint:

$$f_{-1} + \sum_{t=0}^{\infty} \Xi_t Z_t = \sum_{t=0}^{\infty} \Xi_t (M_t - M_{t-1})/S_t. \quad (1.11)$$

The cash-in-advance constraint is $\eta(P_t^T c_t^T + P_t^N c_t^N) \leq M_t$ which we assumed is binding and rewrite as

$$\eta(p_t^T c_t^T + p_t^N c_t^N) = M_t/S_t. \quad (1.12)$$

The household's problem is to choose $\{M_t, f_t, c_t^T, c_t^N\}_{t=0}^{\infty}$ to maximize:

$$\sum_{t=0}^{\infty} (1 + \rho n)^{-t} n \frac{c_t^{1-\sigma} - 1}{1 - \sigma}, \quad (1.13)$$

where

$$c_t = (c_t^T)^\omega (c_t^N)^{1-\omega}, \quad (1.14)$$

subject to its lifetime budget constraint, (1.11), and the cash-in-advance constraint, (1.12). The household takes the initial values of f_{-1} , B and M_{-1} , the paths for r_t , y_t^T , y_t^N , \hat{v}_t , \tilde{v}_t , τ_t^L , p_t^T , p_t^N , p_t and S_t and the value of τ^y as given.

1.2. Solving the Household's Problem

The Lagrangian for the household's problem is

$$\begin{aligned}
L = & \sum_{t=0}^{\infty} (1 + \rho n)^{-t} n \frac{[(c_t^T)^\omega (c_t^N)^{1-\omega}]^{1-\sigma} - 1}{1 - \sigma} + \\
& + \sum_{t=0}^{\infty} \Lambda_t [(1 + nr_{t-1})f_{t-1} + Z_t - (M_t - M_{t-1})/S_t - f_t] \\
& + \sum_{t=0}^{\infty} \Theta_t [M_t/S_t - \eta(p_t^T c_t^T + p_t^N c_t^N)]
\end{aligned}$$

subject to the definition of Z_t , (1.9).

The first-order conditions, other than the constraints, are

$$\begin{aligned}
c_t^T & : (1 + \rho n)^{-t} n [(c_t^T)^\omega (c_t^N)^{1-\omega}]^{1-\sigma} \omega (c_t^T)^{-1} = (\Lambda_t n + \Theta_t \eta) p_t^T, \\
c_t^N & : (1 + \rho n)^{-t} n [(c_t^T)^\omega (c_t^N)^{1-\omega}]^{1-\sigma} (1 - \omega) (c_t^N)^{-1} = (\Lambda_t n + \Theta_t \eta) p_t^N, \\
f_t & : \Lambda_t = \Lambda_{t+1} (1 + nr_t), \\
M_t & : \Lambda_t = \Lambda_{t+1} S_t / S_{t+1} + \Theta_t.
\end{aligned}$$

It is convenient to define $\lambda_t = \Lambda_t (1 + \rho n)^t$ and $\theta_t = \Theta_t (1 + \rho n)^t$. We then have:

$$n [(c_t^T)^\omega (c_t^N)^{1-\omega}]^{1-\sigma} \omega (c_t^T)^{-1} = (\lambda_t n + \theta_t \eta) p_t^T, \quad (1.15)$$

$$n [(c_t^T)^\omega (c_t^N)^{1-\omega}]^{1-\sigma} (1 - \omega) (c_t^N)^{-1} = (\lambda_t n + \theta_t \eta) p_t^N, \quad (1.16)$$

$$\lambda_t = \lambda_{t+1} (1 + nr_t) / (1 + \rho n), \quad (1.17)$$

$$\lambda_t = (1 + \rho n)^{-1} \lambda_{t+1} S_t / S_{t+1} + \theta_t. \quad (1.18)$$

1.3. The Government Budget Constraint

The government's flow budget constraint is:

$$b_t = (1 + nr_{t-1})b_{t-1} + X_t - (M_t - M_{t-1})/S_t, \quad (1.19)$$

where

$$X_t = n (g_t^T + p_t^N g_t^N + p_t \hat{v}_t + \tilde{v}_t - \tau_t + rB/S_t). \quad (1.20)$$

Here X_t/n represents the government's primary deficit. The government's lifetime budget constraint at time 0 is given by

$$b_{-1} + \sum_{t=0}^{\infty} \Xi_t X_t = \sum_{t=0}^{\infty} \Xi_t (M_t - M_{t-1})/S_t. \quad (1.21)$$

1.4. Aggregate Resource Constraints

The equilibrium condition for nontradables is given by:

$$y_t^N = c_t^N + \delta c_t^T + g_t^N. \quad (1.22)$$

If we aggregate (1.8) and (1.19) we get a flow resource constraint for tradables:

$$f_t - b_t = (1 + nr_{t-1})(f_{t-1} - b_{t-1}) + Z_t - X_t,$$

or

$$a_t = (1 + nr_{t-1})a_{t-1} + Z_t - X_t, \quad (1.23)$$

where $a_t = f_t - b_t$ is the country's net foreign asset position. Notice that, using (1.4), (1.5), (1.9), (1.20), (1.22), and the definitions of p_t^N and p_t^T , we can show that $Z_t - X_t = n(y_t^T - g_t^T - c_t^T)$. From (1.23), the lifetime resource constraint for tradables is $a_{-1} + \sum_{t=0}^{\infty} \Xi_t(Z_t - X_t) = 0$.

2. Equilibrium

Given the initial values of b_{-1} , B , f_{-1} , and M_{-1} , the paths of y_t^T , y_t^N , g_t^T , g_t^N , \hat{v}_t , \tilde{v}_t , and τ_t^L and the value of τ^y , a *competitive equilibrium* consists of paths for b_t , M_t , f_t , c_t^T , c_t^N , p_t^T , p_t^N , p_t and S_t such that:

- (a) the paths for M_t , f_t , c_t^T , and c_t^N solve the household's problem,
- (b) the government's flow and lifetime budget constraints are satisfied,
- (c) the market clearing condition for nontraded goods is satisfied,¹
- (d) the paths of M_t and S_t are consistent with the government's threshold rule for abandoning the fixed exchange rate regime.

3. A Sustainable Fixed Exchange Rate Regime

It is straightforward to characterize an equilibrium in which the fixed exchange rate regime is sustained indefinitely. For our purposes it is convenient to characterize an equilibrium of this type that corresponds to a steady state for the economy. We assume that $r_t = r = \rho$, $y_t^T = y^T$, $y_t^N = y^N$, $g_t^T = g^T$, $g_t^N = g^N$, $\hat{v}_t = \hat{v}$, $\tilde{v}_t = \tilde{v}$, and $\tau_t^L = \tau^L$ for all t , and assume values for τ^y , b_{-1} , B , f_{-1} and M_{-1} . Since the exchange rate is fixed (by assumption), $S_t = S$ for all t , for some arbitrary S . We conjecture that the competitive equilibrium has the following properties: $b_t = b = b_{-1}$, $f_t = f = f_{-1}$, $M_t = M = M_{-1}$, $c_t^N = c^N$, $c_t^T = c^T$, $p_t^N = p^N$, $p_t^T = p^T = 1 + \delta p^N$ and $p_t = p = (p^T)^\omega (p^N)^{1-\omega}$ for all t .

Given our conjectures, the resource constraint for tradables, (1.23), implies

$$c^T = ra_{-1} + y^T - g^T, \quad (3.1)$$

where, recall, $a_t = f_t - b_t$. From (1.22) the consumption of nontradables is given by

$$c^N = y^N - g^N - \delta c^T. \quad (3.2)$$

¹The lifetime resource constraint for traded goods is not a separate constraint, as it holds as long as the household obeys its lifetime budget constraint and the government satisfies its lifetime budget constraint.

From these it is convenient to solve for

$$c = (c^T)^\omega (c^N)^{1-\omega}. \quad (3.3)$$

Next consider the household's problem. Notice that the household's first-order condition for f_t implies $\lambda_t = \lambda$ for all t , when $r_t = \rho$ for all t . From this result and the first-order condition for M_t we have $\theta_t = \theta = \rho n \lambda / (1 + \rho n)$ for all t . Eliminating θ , the household's first-order conditions for c^T and c^N can be rewritten as:

$$\begin{aligned} n c^{1-\sigma} \omega / c^T &= \lambda \left(n + \frac{\rho n}{1 + \rho n} \eta \right) (1 + \delta p^N), \\ n c^{1-\sigma} (1 - \omega) / c^N &= \lambda \left(n + \frac{\rho n}{1 + \rho n} \eta \right) p^N. \end{aligned}$$

Given our previous results, the unknowns in these two equations are λ and p^N . Taken together the two equations imply:

$$p^N = \frac{(1 - \omega) c^T}{\omega c^N - (1 - \omega) c^T \delta}. \quad (3.4)$$

Hence

$$p^T = 1 + \delta p^N. \quad (3.5)$$

and $p = (p^T)^\omega (p^N)^{1-\omega}$.

When we calibrate the model we choose the arbitrary normalization $y^T = y^N = 1$ and set the relative price of nontradables to a value p^N consistent with the share of nontradables in GDP in the model, $p^N / (1 + p^N)$, being equal to the corresponding value in our data set. We set g^T and g^N consistent with the data. Notice that this implies that (3.2), (3.1), and (3.4) then form a system of three equations which can be solved for three unknowns: c^T , c^N , and a_{-1} .

Our results thus far can be used to determine the steady state values of

$$\lambda = \frac{n(1 - \omega)}{\left(n + \frac{\rho n}{1 + \rho n} \eta \right) p^N c^N} c^{1-\sigma}, \quad (3.6)$$

and

$$\theta = \rho n \lambda / (1 + \rho n). \quad (3.7)$$

Notice that at this point we can pin down the money supply from the cash-in-advance constraint:

$$M = \eta S (p^T c^T + p^N c^N). \quad (3.8)$$

We have $\tau_t = \tau = \tau^y (y^T + p^N y^N) + \tau^L$, $g_t = g^T + p^N g^N$, and $v_t = v = \hat{v} p + \tilde{v}$. Hence, the government's lifetime budget constraint implies that

$$\tau = r b_{-1} + g + v + r B / S. \quad (3.9)$$

4. Calibration

Our calibration of the model is also discussed in the main text. Here we describe some of the details useful in understanding our programs for solving the model. The particular parameter values are justified in the main text.

We set the steady state value of $r = 0.055$ and let $\rho = r$. We set $\sigma = 1$, $\delta = 0.5$, $\omega = 0.5$, $S = 1$, $y^T = 1$, and $y^N = 1$. We assume that the share of tradables output in GDP is $s^T = 0.358$. Since GDP is $y = y^T + p^N y^N$ this implies that $y^T = s^T(y^T + p^N y^N)$ or

$$p^N = \frac{(1 - s^T)y^T}{s^T y^N} = \frac{1 - s^T}{s^T}.$$

Hence

$$p^T = 1 + \delta p^N.$$

Also $P^N = p^N S$, and $y = y^T + p^N y^N$. We set the steady state value of total government spending $g = 0.154y$. Then we set $g^T = 0.132g$ and $g^N = (g - g^T)/p^N$. Equations (3.4), (3.1) and (3.2) imply

$$a_{-1} = \frac{\omega p^N (y^N - g^N) - (1 - \omega + \delta p^N) (y^T - g^T)}{(1 - \omega + \delta p^N) r}.$$

We set $b_{-1} = -0.057y$, implying $f_{-1} = a_{-1} + b_{-1}$. We have c^T and c^N from (3.1) and (3.2) and c from (3.3). We set $M_{-1} = 0.067Sy$ and set η consistent with (1.12) in the steady state: $\eta = M/[S(p^T c^T + p^N c^N)]$. We set $v = 0.043y$, and, when we incorporate indexed transfers we choose $\hat{v}p^N = 0.027y$ (in the program we define $V = 0.027yS$, so this V is equivalent to $\hat{v}P^N$). The remaining dollar transfers, in the steady state, are $\tilde{v} = v - \hat{v}p^N$. When we have nominal debt we set $B = 0.075Sy$.

Our previous assumptions allow us to determine λ and θ in the steady state using (3.6) and (3.7). We set steady state taxes, τ , consistent with the lifetime budget constraint, 3.9. We set $\tau^y = 0.216$ and set $\tau^L = \tau - \tau^y y$. We set the size of the banking sector bailout to $\phi = 0.135y$ and $M_T = 1.12$. We set $t^* = 0.35/n$ and $T = 0.5/n$, where $1/n$ is the number of periods in a year.

5. A Competitive Equilibrium with a Crisis

We assume that the crisis involves an increase in the present value of transfers. In particular, we assume that

$$\begin{cases} \tilde{v}_t = \tilde{v} & \text{for } 0 \leq t < T', \\ \tilde{v}_t > \tilde{v} & \text{for } t \geq T', \end{cases}$$

and that

$$\sum_{t=0}^{\infty} \Xi_t n \tilde{v}_t = \sum_{t=0}^{\infty} \Xi_t n \tilde{v} + \phi. \quad (5.1)$$

Here ϕ represents the total cost of a bailout of the banking sector.

Once the information about the increase in prospective deficits arrives at time 0, the government initially adjusts monetary policy to defend the fixed exchange rate regime. We

will denote the level of the money supply to which the government adjusts at date 0 as M_0 , but this will, of course, be endogenously determined. The government uses a threshold rule for abandoning the fixed exchange rate regime. Let date t^* be the first date at which, with the exchange rate still fixed at S , the demand for money falls to the level $M_0(1 - \chi)$, with $\chi > 0$. In the following period the government abandons the fixed exchange rate regime and leaves the money supply unchanged until date T . The government sets the money supply equal to $M_t = M_T(1 + \mu n)^{t-T}$ for $t \geq T$.

We assume that $y_t^T = y^T$, $y_t^N = y^N$, $g_t^T = g^T$, $g_t^N = g^N$, $\hat{v}_t = \hat{v}$ and $\tau_t^L = \tau^L$ for all t , and assume values for τ^y , b_{-1} , B , f_{-1} and M_{-1} . For simplicity, we will assume that $r_t = r$ for $t < t^*$ and for $t \geq T_r > t^*$, but we will allow for a more general process for r_t at intervening dates, $t^* \leq t < T_r$. We treat t^* as a parameter to be calibrated and determine χ endogenously, across experiments, rather than vice versa. By definition we set $S_t = S$, an arbitrary value, for $0 \leq t \leq t^*$. We choose calibrated values for T and M_T .

We must solve for 11 paths b_t , M_t , f_t , c_t^T , c_t^N , p_t^T , p_t^N , p_t , S_t , λ_t and θ_t such that the household's first order conditions, (1.15)–(1.18), its budget constraint, (1.8), the cash-in-advance constraint, (1.12), the equations defining p_t^T and p_t , (1.2) and (1.3), the resource constraint for nontraded goods, (1.22), and the government budget constraint, (1.19), are satisfied. These represent 10 equations, suggesting that the 11 paths might be underdetermined. However, the exchange rate, S_t , is exogenous for $0 \leq t \leq t^*$. Also, given a value of μ , the process for M_t is exogenous for $t \geq T$. Finally, we have $M_t = M_{t-1}$ for $t^* < t < T$. These restrictions represent, in a sense, an additional equation.

As with any system of dynamic first order conditions, given a conjectured value of μ there will be infinitely many paths for our 11 variables satisfying our 11 equations. The trick is to find the unique path for which the government and household lifetime budget constraints hold. This involves iterating over μ and the long-run level of tradables consumption until we find a path along which these constraints are satisfied. We will also indicate, below, how we solve for χ .

5.1. The Post-Crisis Steady State: $t \geq \bar{T}$

Define $\bar{T} = \max\{T_r, T\}$. Our assumptions about r_t mean that, from (1.17), $\lambda_t = \bar{\lambda}$ for $t \geq T_r$ and, therefore, $\lambda_t = \bar{\lambda}$ for $t \geq \bar{T}$. We do not yet solve for $\bar{\lambda}$. Instead we conjecture a solution in which $c_t^T = \bar{c}^T$ for $t \geq \bar{T} + 1$. Our algorithm for solving the model begins with an outer loop in which we guess the value of \bar{c}^T .

It follows from (1.22) that

$$c_t^N = \bar{c}^N \equiv y^N - g^N - \delta \bar{c}^T, \text{ for } t \geq \bar{T} + 1. \quad (5.2)$$

It follows that $c_t = \bar{c} \equiv (\bar{c}^T)^\omega (\bar{c}^N)^{1-\omega}$ for $t \geq \bar{T} + 1$. We conjecture that $S_{t+1}/S_t = 1 + \mu n$ for $t \geq \bar{T} + 1$. Defining $N \equiv (1 + \rho n)(1 + \mu n)$, we then have, from (1.18),

$$\theta_t = \bar{\theta} = \bar{\lambda} \frac{N - 1}{N}, \text{ for } t \geq \bar{T} + 1. \quad (5.3)$$

The first-order conditions for c_t^T and c_t^N , (1.15) and (1.16), combined with (5.3), imply that p_t^N is constant and equal to \bar{p}^N for $t \geq \bar{T} + 1$ and that \bar{p}^N satisfies:

$$n\bar{c}^{1-\sigma}\omega/\bar{c}^T = \bar{\lambda} \left(n + \frac{N - 1}{N}\eta \right) (1 + \delta\bar{p}^N) \quad (5.4)$$

$$n\bar{c}^{1-\sigma}(1-\omega)/\bar{c}^N = \bar{\lambda} \left(n + \frac{N-1}{N}\eta \right) \bar{p}^N. \quad (5.5)$$

Equations (5.4) and (5.5) can be solved for $\bar{\lambda}$ and \bar{p}^N :

$$\bar{p}^N = \frac{(1-\omega)\bar{c}^T}{\omega\bar{c}^N - \delta(1-\omega)\bar{c}^T}, \quad (5.6)$$

and

$$\bar{\lambda} = \frac{nN\bar{c}^{1-\sigma}(1-\omega)}{[nN + (N-1)\eta]\bar{p}^N\bar{c}^N}. \quad (5.7)$$

It follows that $\bar{\theta}$ can then be determined using (5.3). The cash-in-advance constraint, (1.12), implies $M_t/S_t = \bar{m}$ for $t \geq \bar{T} + 1$, where

$$\bar{m} = \eta[(1 + \delta\bar{p}^N)\bar{c}^T + \bar{p}^N\bar{c}^N]. \quad (5.8)$$

Hence $S_t = M_t/\bar{m}$ for $t \geq \bar{T} + 1$. This, of course, verifies our guess that $S_{t+1}/S_t = 1 + \mu n$ for $t \geq \bar{T} + 1$, since $M_{t+1}/M_t = 1 + \mu n$ for $t \geq T$ and $T \leq \bar{T}$.

We also note, now, that the solution for $\bar{\lambda}$ allows us to compute λ_t for all t using (1.17). The recursion: $\lambda_{t-1} = \lambda_t(1 + nr_{t-1})/(1 + \rho n)$ can be used to generate the sequence $\lambda_{\bar{T}-1}, \lambda_{\bar{T}-2}, \dots, \lambda_{t^*}$, using the initial condition $\lambda_{\bar{T}} = \bar{\lambda}$. We use the notation $\underline{\lambda}$ to denote the solution for λ_{t^*} . Since $r_t = \rho$ for $0 \leq t < t^*$, we have $\lambda_t = \underline{\lambda}$ for $0 \leq t < t^*$.

5.2. The Period Between T and $\bar{T} + 1$

This interval is just the point T if $T = \bar{T}$. On the other hand, if $T_r > T$, implying that $\bar{T} = T_r > T$ this interval corresponds to more than one point. We have already used (1.17) to solve for the entire $\{\lambda_t\}$ sequence. We also know that $M_t = M_T(1 + \mu n)^{t-T}$ in this interval. Hence we can use (1.15), (1.16), (1.18), (1.22) and (1.12) to solve recursively for $c_t^T, c_t^N, p_t^N, \theta_t$ and S_t starting at $t = \bar{T}$ and working backward in time until we get to date T .

Let x denote the unknown value of c_t^T . Given a conjectured value of x , from (1.22) we have

$$c_t^N(x) = y^N - g^N - \delta x. \quad (5.9)$$

We can then solve for $p_t^N(x)$ using (1.15) and (1.16)

$$p_t^N(x) = \frac{(1-\omega)x}{\omega c_t^N(x) - \delta(1-\omega)x}. \quad (5.10)$$

From (1.16) this implies

$$\theta_t(x) = \frac{n}{\eta} \left[\frac{[x^\omega c_t^N(x)^{1-\omega}]^{1-\sigma}(1-\omega)}{p_t^N(x)c_t^N(x)} - \lambda_t \right]. \quad (5.11)$$

From (1.18) we then have

$$S_t(x) = (1 + \rho n)[\lambda_t - \theta_t(x)]S_{t+1}/\lambda_{t+1}. \quad (5.12)$$

Finally, given (1.12) we get

$$c_t^T(x) = \frac{M_t/S_t(x) - \eta p_t^N(x)c_t^N(x)}{\eta[1 + \delta p_t^N(x)]}. \quad (5.13)$$

We solve the equation $c_t^T(x) = x$ to find the value of c_t^T . Once we have c_t^T we have the values of all the other variables. We can proceed with this scheme until we have solved for all variables for $T \leq t \leq \bar{T}$. In our code, we define a function `FlexTTB` which computes $c_t^T(x) - x$ for a given x . We find the zero of this function using `fzero`.

5.3. The Pre-Crisis Period: $0 \leq t < t^*$

We have assumed that $S_t = S$ for $0 \leq t \leq t^*$. Hence we have

$$\theta_t = \underline{\theta} = \underline{\lambda} \frac{\rho n}{1 + \rho n} \text{ for } 0 \leq t < t^*. \quad (5.14)$$

The conditions (1.15), (1.16) and (1.22) imply that c_t^T , c_t^N and p_t^N are constant and equal to \underline{c}^T , \underline{c}^N and \underline{p}^N , respectively, for $0 \leq t < t^*$. Let x denote the unknown value of \underline{c}^T . We can write \underline{c}^N as $\underline{c}^N(x)$ using (1.22):

$$\underline{c}^N(x) = y^N - g^N - \delta x \quad (5.15)$$

and then use (1.15) and (1.16) to write \underline{p}^N as:

$$\underline{p}^N(x) = \frac{(1 - \omega)x}{\omega \underline{c}^N(x) - \delta(1 - \omega)x} \quad (5.16)$$

We can then use (1.15) to define a nonlinear equation in x :

$$n [x^\omega \underline{c}^N(x)^{1-\omega}]^{1-\sigma} \omega/x = (\underline{\lambda}n + \underline{\theta}\eta)[1 + \delta \underline{p}^N(x)]. \quad (5.17)$$

We define a MATLAB function `Flexlow` as the difference between the two sides of (5.17) and find its zero using the routine `fzero`. Once we have the solution for x this is the value of \underline{c}^T and it is then straightforward to again use (5.15) and (5.16) to solve for \underline{c}^N and \underline{p}^N .

The cash-in-advance constraint, (1.12) implies that

$$M_t = \underline{M} \equiv \eta [(1 + \delta \underline{p}^N) \underline{c}^T + \underline{p}^N \underline{c}^N] S, \text{ for } 0 \leq t < t^*. \quad (5.18)$$

5.4. The Transition Period: $t^* \leq t < T$

By construction $M_t = \underline{M}(1 - \chi)$ for $t^* \leq t < T$ so the money supply path in the transition period is fully determined given a value of χ . We recursively generate S_t starting from t^* , where we have assumed $S_{t^*} = S$. We then work forward, allowing us to take S_t as given when solving for the other variables. Starting from $t = t^*$, we solve for θ_t , c_t^T , c_t^N , p_t^N , and from these determine the next value of S_t .

We use (1.12), (1.15) (1.16) and (1.22), to solve for c_t^T , c_t^N , p_t^N , and θ_t . Let x denote the unknown value of c_t^T . Equation (1.22) implies that c_t^N is given by (5.9). We also know

that (1.15) and (1.16) imply that p_t^N is given by (5.10). But p_t^N is also given by (1.12): $(\eta^{-1}m_t - x)/[\delta x + c_t^N(x)]$ where $m_t = M_t/S_t$ is real balances. When these equations are combined we find that x is the solution to the quadratic equation

$$\delta x^2 - (y^N - g^N + \delta\eta^{-1}m_t)x + \omega\eta^{-1}m_t(y^N - g^N) = 0. \quad (5.19)$$

When $\delta = 0$ we have the simple solution $x = \omega m_t/\eta$. When $\delta > 0$ there are two solutions to the quadratic equation:

$$x = \frac{(y^N - g^N + \delta\eta^{-1}m_t) \pm \sqrt{(y^N - g^N + \delta\eta^{-1}m_t)^2 - 4\delta(y^N - g^N)\omega\eta^{-1}m_t}}{2\delta}$$

It is easy to establish that since $\omega < 1$ the larger root is greater than $(y^N - g^N)/\delta$, violating feasibility.² The smaller root is bounded below by 0 and above by $(1 + \omega)\eta^{-1}m_t/2$.³ Clearly the only possibility is to use the smaller root.

Once we have c_t^T we generate c_t^N , p_t^N and θ_t from:

$$c_t^N = y^N - g^N - \delta c_t^T, \quad (5.20)$$

$$p_t^N = \frac{M_t/S_t - \eta c_t^T}{\eta(\delta c_t^T + c_t^N)}, \quad (5.21)$$

$$\theta_t = \frac{n}{\eta} \left(\frac{c_t^{1-\sigma}(1-\omega)}{p_t^N c_t^N} - \lambda_t \right), \quad (5.22)$$

where $c_t = (c_t^T)^\omega (c_t^N)^{1-\omega}$. This allows us to generate S_{t+1} using (1.18):

$$S_{t+1} = \frac{\lambda_{t+1}}{\lambda_t - \theta_t} \frac{S_t}{1 + \rho n}. \quad (5.23)$$

5.5. Iterating on χ

When we are done generating the data for the “transition period” we will have a sequence $\{S_t\}$ for $t^* < t \leq T$. Notice that the S_T generated this way, $S_T^{(2)}$ may not match the S_T we generated in the “period between T and \bar{T} ,” denoted $S_T^{(1)}$. If it does not match, then the value of χ must be changed. If $S_T^{(2)} > S_T^{(1)}$ we have found that in practice we must make χ smaller to find a fixed point.

5.6. Iterating on \bar{c}^T

The lifetime budget constraint of the household, (1.11), must be satisfied. We have assumed that $\sum_{t=0}^{\infty} \Xi_t n \tilde{v}_t = \sum_{t=0}^{\infty} \Xi_t n \tilde{v} + \phi$. Using (1.4)–(1.7) and (1.9) we can rewrite the lifetime budget constraint, (1.11), as

$$f_{-1} + \phi + \sum_{t=0}^{\infty} \Xi_t n \left\{ (1 - \tau^y)y^T - (1 + \delta p_t^N)c_t^T + p_t^N [(1 - \tau^y)y^N - c_t^N] - \tau_t^L + \hat{v}p_t + \tilde{v} + \frac{rB}{S_t} \right\}$$

²To see this one can set $\omega = 1$ making the larger root as *small* as possible. In this case, one obtains $x = (y^N - g^N)/\delta$.

³To see this one can set $\omega = 0$ making the smaller root as *small* as possible. In this case, one obtains $x = 0$. On the other hand it is easy to show that the smaller root is less than $(1 + \omega)(\delta\eta^{-1}m_t)/(2\delta)$ and that it is also less than

$$= \sum_{t=0}^{\infty} \Xi_t \frac{M_t - M_{t-1}}{S_t}. \quad (5.24)$$

It will be useful to note that given the r_t sequence we can define the Ξ_t sequence ahead of time as

$$\Xi_t = \begin{cases} (1 + nr)^{-(t+1)} & \text{for } 0 \leq t \leq t^* \\ \Xi_{t^*} (1 + nr_{t^*})^{-1} \cdots (1 + nr_{t-1})^{-1} & \text{for } t^* < t \leq T_r \\ \Xi_{T_r} (1 + nr)^{-(t-T_r)} & \text{for } t > T_r. \end{cases}$$

The right hand side of (5.24), which I will denote by RHS , is seigniorage. It equals

$$\begin{aligned} RHS &= \Xi_0 \frac{M - M}{S} - \Xi_{t^*} \frac{M\chi}{S} + \Xi_T \frac{M_T - M(1 - \chi)}{S_T} + \\ &\quad \sum_{t=T+1}^{\bar{T}} \Xi_t \frac{M_t - M_{t-1}}{S_t} + \Xi_{\bar{T}} \frac{\mu \bar{m}}{(1 + \mu n)r}. \end{aligned} \quad (5.25)$$

We can compute the relevant pieces of the left-hand side as follows. Define $LHS_1 \equiv \sum_{t=0}^{\infty} \Xi_t Z_{1t}$, where

$$Z_{1t} = n \{ (1 - \tau^y) y^T - (1 + \delta p_t^N) c_t^T + p_t^N [(1 - \tau^y) y^N - c_t^N] - \tau^L \}.$$

Since

$$\begin{aligned} Z_{1t} &= \begin{cases} \underline{Z}_1 & \text{for } 0 \leq t < t^* \\ \bar{Z}_1 & \text{for } t \geq \bar{T} + 1 \end{cases}, \\ \underline{Z}_1 &= n \{ (1 - \tau^y) y^T - (1 + \delta \underline{p}^N) \underline{c}^T + \underline{p}^N [(1 - \tau^y) y^N - \underline{c}^N] - \tau^L \}, \\ \bar{Z}_1 &= n \{ (1 - \tau^y) y^T - (1 + \delta \bar{p}^N) \bar{c}^T + \bar{p}^N [(1 - \tau^y) y^N - \bar{c}^N] - \tau^L \}, \end{aligned}$$

we have

$$LHS_1 = (1 - \Xi_{t^*-1}) \frac{\underline{Z}_1}{nr} + \sum_{t=t^*}^{\bar{T}} \Xi_t Z_{1t} + \frac{\Xi_{\bar{T}}}{nr} \bar{Z}_1. \quad (5.26)$$

We also have $LHS_2 \equiv \sum_{t=0}^{\infty} \Xi_t nrB/S_t$ so that

$$LHS_2 = \frac{B}{S} (1 - \Xi_{t^*}) + nrB \sum_{t=t^*+1}^{\bar{T}} \Xi_t / S_t + nrB \frac{\Xi_{\bar{T}}}{(N-1)M_{\bar{T}}/\bar{m}}. \quad (5.27)$$

We have $LHS_3 \equiv \sum_{t=0}^{\infty} \Xi_t n(\hat{v}p_t + \tilde{v})$ so that

$$LHS_3 = (1 - \Xi_{t^*-1}) \frac{\hat{v}p + \tilde{v}}{r} + n \sum_{t=t^*}^{\bar{T}} \Xi_t (\hat{v}p_t + \tilde{v}) + \frac{\Xi_{\bar{T}}}{r} (\hat{v}\bar{p} + \tilde{v}), \quad (5.28)$$

where $\underline{p} = (1 + \delta \underline{p}^N)^\omega (\underline{p}^N)^{1-\omega}$ and $\bar{p} = (1 + \delta \bar{p}^N)^\omega (\bar{p}^N)^{1-\omega}$.

If we find that the household's lifetime budget constraint is not satisfied, we change our guess for \bar{c}^T .

5.7. Iterating on μ

Using (3.9), (5.1), (5.25), (5.27), (5.28), we can rewrite the government's lifetime budget constraint, (1.21), as

$$\phi + LHS_1^G + LHS_2^G + LHS_2 + LHS_3 - \frac{1}{r}(g - \tau + rB/S + v) = RHS,$$

where $LHS_1^G \equiv \sum_{t=0}^{\infty} \Xi_t n (g^T + p_t^N g^N)$ and $LHS_2^G \equiv -\sum_{t=0}^{\infty} \Xi_t n [\tau^y (y^T + y^N p_t^N) + \tau_t^L]$. We can show that

$$LHS_1^G = (1 - \Xi_{t^*-1}) \frac{g}{r} + \sum_{t=t^*}^{\bar{T}} \Xi_t n (g^T + p_t^N g^N) + \Xi_{\bar{T}} \frac{\bar{g}}{r}$$

where $\underline{g} = g^T + \underline{p}^N g^N$ and $\bar{g} = g^T + \bar{p}^N g^N$, and

$$LHS_2^G = - \left\{ (1 - \Xi_{t^*-1}) \frac{\underline{\tau}}{r} + \sum_{t=t^*}^{\bar{T}} \Xi_t n [\tau^y (y^T + y^N p_t^N) + \tau_t^L] + \Xi_{\bar{T}} \frac{\bar{\tau}}{r} \right\}$$

where $\underline{\tau} = \tau^y (y^T + y^N \underline{p}^N) + \tau^L$ and $\bar{\tau} = \tau^y (y^T + y^N \bar{p}^N) + \tau^L$. If the government's lifetime budget constraint is not satisfied, we adjust our guess for μ .

6. Sticky Prices

We make the same assumptions regarding $y_t^T, y_t^N, g_t^T, g_t^N, \hat{v}_t, \tau_t^L, \tau^y, b_{-1}, B, f_{-1}, M_{-1}$ and r_t as in the case of the competitive equilibrium. When nontradable prices are sticky they do not clear the market for nontradables after the crisis. There will either be excess demand, in which case nontradables are rationed to households (whose first-order conditions for c_t^N don't hold), or there will be excess supply, in which case resources are wasted (the market clearing condition for nontraded goods is slack).

We must solve for 10 paths $b_t, M_t, f_t, c_t^T, c_t^N, p_t^T, p_t, S_t, \lambda_t$ and θ_t such that the household's first order conditions, (1.15), (1.17) and (1.18), its budget constraint, (1.8), the cash-in-advance constraint, (1.12), the equations defining p_t^T and p_t , (1.2) and (1.3) and the government budget constraint, (1.19), are satisfied. Additionally, the resource constraint for nontraded goods, (1.22), must be satisfied if (1.16) is not satisfied, while (1.16) must be satisfied if (1.22) is slack.

We add the following condition to capture price stickiness in the nontraded goods sector: $P_t^N = P^N$ for $0 \leq t \leq T_p$. For $t > T_p$ we assume that $P_t^N = P^N (1 + \mu n)^{t - T_p}$. We define $\bar{T} = \max\{T_r, T, T_p - 1\}$ so that if $T_p > \max\{T_r, T\}$, $T_p = \bar{T} + 1$.

6.1. The Post-Crisis Steady State: $t \geq \bar{T}$

As in the competitive equilibrium, (1.17) implies $\lambda_t = \bar{\lambda}$ for $t \geq \bar{T}$. We conjecture a solution in which $c_t^T = \bar{c}^T, c_t^N = \bar{c}^N, \theta_t = \bar{\theta}, S_{t+1} = (1 + \mu n)S_t$, and $p_t^N = \bar{p}^N$ for $t \geq \bar{T} + 1$. Generally speaking nontraded goods will be underpriced in the long-run steady state (\bar{p}^N will be "too low" due to price stickiness), so we will impose (1.22) and relax (1.16). Given a value for \bar{c}^T , (1.22) implies

$$\bar{c}^N = y^N - g^N - \delta \bar{c}^T,$$

while (1.12) implies

$$S_{\bar{T}+1} = \frac{M_{\bar{T}+1} - \eta (\delta \bar{c}^T + \bar{c}^N) P_{\bar{T}+1}^N}{\eta \bar{c}^T}.$$

Here $M_{\bar{T}+1} = M_T(1 + \mu n)^{\bar{T}+1-T}$ while $P_{\bar{T}+1}^N = P_N(1 + \mu n)^{\bar{T}+1-T_p}$. We then have $\bar{p}^N = P_{\bar{T}+1}^N/S_{\bar{T}+1}$. Then (1.15) and (1.18) can be solved for $\bar{\lambda}$ and $\bar{\theta}$:

$$\bar{\lambda} = \frac{nN\bar{c}^{1-\sigma}\omega/\bar{c}^T}{[Nn + (N-1)\eta](1 + \delta\bar{p}^N)} \quad (6.1)$$

and

$$\bar{\theta} = \bar{\lambda}(N-1)/N. \quad (6.2)$$

6.2. The Period Between T and $\bar{T} + 1$

Once we obtain $\bar{\lambda}$ we can again use (1.17) to solve for the entire $\{\lambda_t\}$ sequence. We also know that $M_t = M_T(1 + \mu n)^{t-T}$ in this interval and P_t^N is either P^N if $t \leq T_p$ or $P_N(1 + \mu n)^{t-T_p}$ if $t > T_p$. Generally speaking nontraded goods will be underpriced so we will impose (1.22) and relax (1.16). We can use (1.15), (1.18), (1.22) and (1.12) to solve recursively for c_t^T , c_t^N , θ_t and S_t starting at $t = \bar{T}$ and working backward in time until we get to date T . Let x denote the unknown value of c_t^T . Given a conjectured value of x , (1.22) implies

$$c_t^N(x) = y^N - g^N - \delta x.$$

Equation (1.12) then implies that

$$S_t(x) = \frac{M_t - \eta[\delta x + c_t^N(x)]P_t^N}{\eta x}.$$

We can then use (1.18) to solve for θ_t

$$\theta_t(x) = \lambda_t - (1 + \rho n)^{-1} \lambda_{t+1} S_t(x) / S_{t+1}.$$

If our conjecture is correct then from (1.15):

$$n [x^\omega c_t^N(x)^{1-\omega}]^{1-\sigma} \omega/x - [\lambda_t n + \theta_t(x) \eta] [1 + \delta P_t^N / S_t(x)] = 0. \quad (6.3)$$

In our code, we define a function `StickyTTB` which is the left-hand side of (6.3) for a given x . We find the zero of this function using `fzero`. Once we have c_t^T we have the values of all the other variables. We can proceed with this scheme until we have solved for all variables for $T \leq t \leq \bar{T}$.

6.3. The Pre-Crisis Period: $0 \leq t < t^*$

We have assumed that $S_t = S$ for $0 \leq t \leq t^*$. As with flexible prices, the fact that $\lambda_t = \underline{\lambda}$ for $t \leq t^*$ implies that $\theta_t = \underline{\theta}$ as given in (5.14). Since $P_t^N = P^N$ for $0 \leq t \leq t^*$, we have $p_t^N = \underline{p}^N = p^N = P^N/S$ for $0 \leq t \leq t^*$. We conjecture that $c_t^T = \underline{c}^T$ and $c_t^N = \underline{c}^N$ for $0 \leq t \leq t^*$.

To solve for \underline{c}^T and \underline{c}^N we first assume that (1.15) and (1.16) hold and that (1.22) does not hold with equality. Solving (1.15) and (1.16) for \underline{c}^T and \underline{c}^N we get:

$$\underline{c}^T = \left[\frac{(\lambda n + \underline{\theta}\eta)(1 + \delta \underline{p}^N)}{n\omega} \right]^{\frac{(\omega-1)\sigma-\omega}{\sigma}} \left[\frac{n(1-\omega)}{(\lambda n + \underline{\theta}\eta)\underline{p}^N} \right]^{(\omega-1)\frac{\sigma-1}{\sigma}}, \quad (6.4)$$

$$\underline{c}^N = \left[\frac{n(\underline{c}^T)^{\omega(1-\sigma)}(1-\omega)}{(\lambda n + \underline{\theta}\eta)\underline{p}^N} \right]^{\frac{1}{\omega+(1-\omega)\sigma}}. \quad (6.5)$$

Using these solutions we check whether $y^N - g^N - \delta \underline{c}^T - \underline{c}^N \geq 0$. If it is we stop.

If the condition we checked in the previous paragraph is violated, then we abandon the household's first order condition for c_t^N , (1.16), and impose the resource constraint, (1.22), instead. Letting x be the unknown value of \underline{c}^T we use (1.22) to determine

$$\underline{c}^N(x) = y^N - g^N - \delta x$$

and use (1.15) to get the following nonlinear equation in x :

$$n\omega [x^\omega \underline{c}^N(x)^{1-\omega}]^{1-\sigma} = (\lambda n + \underline{\theta}\eta)(1 + \delta \underline{p}^N)x. \quad (6.6)$$

We define a MATLAB function `StickyLow` as the difference between the two sides of (6.6) and find its zero using the routine `fzero`. The solution, x , is \underline{c}^T . We solve for $M_t = \underline{M}$ using (5.18).

6.4. The Transition Period: $t^* \leq t < T$

When prices are sticky we will use (1.12), (1.15), and either (1.16) or (1.22) to solve for θ_t , c_t^T , and c_t^N . We always know the current value of S_t , so we also know $p_t^N = P_t^N/S_t$. We also know that $M_t = \underline{M}(1 - \chi)$.

$$n [x^\omega (c_t^N)^{1-\omega}]^{1-\sigma} \omega = (\lambda_t n + \theta_t \eta)(1 + \delta p_t^N)x, \quad (6.7)$$

$$c_t^N = \frac{1-\omega}{\omega} \frac{1 + \delta p_t^N}{p_t^N} c_t^T, \quad (6.8)$$

$$\eta p_t^N c_t^N + \eta(1 + \delta p_t^N) c_t^T = M_t/S_t$$

Assuming that (1.22) is the equation that does not hold with equality, we can combine (1.15) and (1.16) to get:

$$c_t^N = \frac{1-\omega}{\omega} \frac{1 + \delta p_t^N}{p_t^N} c_t^T$$

Using (1.12), this implies

$$c_t^T = \frac{\omega M_t/S_t}{\eta(1 + \delta p_t^N)}$$

We can then use (1.16) to obtain θ_t :

$$\theta_t = \frac{n}{\eta} \left[\frac{c_t^{1-\sigma}(1-\omega)}{c_t^N p_t^N} - \lambda_t \right],$$

where $c_t = (c_t^T)^\omega (c_t^N)^{1-\omega}$.

If at any point (1.22) is violated, in that c_t^N is too large, we must let the first-order condition for c_t^N be violated. In this case we solve (1.12), (1.15), and (1.22) for θ_t , c_t^T , and c_t^N . Since (1.12) and (1.22) are linear in c_t^T and c_t^N , and do not involve θ_t we easily obtain

$$\begin{aligned} c_t^T &= \eta^{-1} M_t / S_t - \eta p_t^N (y^N - g^N) \\ c_t^N &= (1 + \delta p_t^N) (y^N - g^N) - \delta \eta^{-1} M_t / S_t. \end{aligned}$$

We then solve for θ_t using (1.15):

$$\theta_t = \frac{n}{\eta} \left[\frac{c_t^{1-\sigma} \omega}{(1 + \delta p_t^N) c_t^T} - \lambda_t \right],$$

where $c_t = (c_t^T)^\omega (c_t^N)^{1-\omega}$. Once we have θ_t , c_t^T and c_t^N we can generate S_{t+1} using (5.23).

7. The Model without Nontraded Goods

We also conduct some experiments in which there is only a single tradable good. For simplicity, in these experiments we assume that $r_t = r = \rho$ for all t . PPP holds at the level of the CPI, since all goods are traded and there are no distribution costs. So $P_t = S_t$. The household's disposable income is still given by (1.4). Now there is no distinction between indexed transfers and lump-sum transfers, nor between proportional taxes and lump-sum taxes. The representative household's flow budget constraint for $t \geq 0$ is given by (1.8), but now $Z_t = n (y_t^D - c_t)$. The cash-in-advance constraint is just $\eta c_t \leq M_t / S_t$. The household's problem is to choose $\{M_t, f_t, c_t\}_{t=0}^\infty$ to maximize (1.13), subject to its lifetime budget constraint, (1.11), and the cash-in-advance constraint. The household takes the initial values of f_{-1} , B and M_{-1} and the paths for y_t , v_t , τ_t and S_t as given.

The first-order condition for consumption is

$$\begin{aligned} c_t &: (1 + \rho n)^{-t} n c_t^{-\sigma} = \Lambda_t n + \Theta_t \eta, \\ f_t &: \Lambda_t = \Lambda_{t+1} (1 + nr), \\ M_t &: \Lambda_t = \Lambda_{t+1} S_t / S_{t+1} + \Theta_t. \end{aligned}$$

The first-order conditions for f_t and M_t are the same as before. If we again define $\lambda_t = \Lambda_t (1 + \rho n)^t$ and $\theta_t = \Theta_t (1 + \rho n)^t$ we get $\lambda_t = \lambda$ for all t ,

$$n c_t^{-\sigma} = \lambda n + \theta_t \eta, \tag{7.1}$$

and (1.18).

The government's flow budget constraint is still (1.19) but with $X_t = n (g_t + v_t - \tau_t + rB/S_t)$. The government's lifetime budget constraint is still given by (1.21).

The flow resource constraint is still (1.23).

7.1. The Sustainable Fixed Exchange Rate Regime

The resource constraint implies $c = ra_{-1} + y - g$. As before, $\theta_t = \theta = \rho n \lambda / (1 + \rho n)$ for all t . Eliminating θ from (7.1) we get $\lambda = c^{-\sigma} / [1 + \eta \rho / (1 + \rho n)]$. We set $S = 1$, $y = 1$, and g and v consistent with their shares of output in the data. The money supply must be $M = \eta S c$. The government's lifetime budget constraint implies that $\tau = rb_{-1} + g + v + rB/S$.

7.2. The Post-Crisis Steady State: $t \geq T$

Our assumptions imply $\lambda_t = \lambda$ for all t . We do not yet solve for λ . Instead we conjecture a solution in which $c_t = \bar{c}$ for $t \geq T$. Our algorithm for solving the model begins with an outer loop in which we guess the value of \bar{c} . Given this guess, and the conjecture that $S_{t+1}/S_t = 1 + \mu n$ for $t \geq T$, we can solve (7.1) and (1.18) to get $\lambda = nN\bar{c}^{-\sigma}/[nN + (N-1)\eta]$, and $\theta_t = \bar{\theta} = \lambda(N-1)/N$, for $t \geq T$, where $N = (1 + \rho n)(1 + \mu n)$. The cash-in-advance constraint implies $M_t/S_t = \bar{m} = \eta\bar{c}$ for $t \geq T$, where $\bar{m} = \eta\bar{c}$. Hence $S_t = M_t/\bar{m}$ for $t \geq T$.

7.3. The Pre-Crisis Period: $0 \leq t < t^*$

We have assumed that $S_t = S$ for $0 \leq t \leq t^*$. Hence we have, $\theta_t = \underline{\theta} = \lambda\rho n/(1 + \rho n)$. So $c_t = \underline{c} = [(\lambda n + \underline{\theta}\eta)/n]^{-1/\sigma}$. The cash-in-advance constraint implies that $M_t = \underline{M} = \eta\underline{c}S$, for $0 \leq t < t^*$.

7.4. The Transition Period: $t^* \leq t < T$

By construction $M_t = \underline{M}(1 - \chi)$ for $t^* \leq t < T$ so the money supply path in the transition period is fully determined given a value of χ . We recursively generate S_t starting from t^* , where we have assumed $S_{t^*} = S$. We then work forward, allowing us to take S_t as given when solving for the other variables. Starting from $t = t^*$, we solve for c_t using the cash-in-advance constraint: $c_t = M_t/(\eta S_t)$. Then, from (7.1), we get: $\theta_t = (n/\eta)(c_t^{-\sigma} - \lambda)$. Finally, from (1.18), we have $S_{t+1} = S_t\lambda/[(\lambda - \theta_t)(1 + \rho n)]$.

7.5. Iterating to Find the Equilibrium

Iterating over χ works exactly as in the previous cases. To find the equilibrium value of \bar{c} , we require that the lifetime budget constraint of the household is satisfied. We can rewrite it as

$$f_{-1} + \phi + \sum_{t=0}^{\infty} \Xi_t n \left(y - c_t - \tau + v + \frac{rB}{S_t} \right) = \sum_{t=0}^{\infty} \Xi_t \frac{M_t - M_{t-1}}{S_t}. \quad (7.2)$$

The right hand side of (7.2) is

$$RHS = \Xi_0 \frac{M - M}{S} - \Xi_{t^*} \frac{M\chi}{S} + \Xi_T \frac{M_T - \underline{M}(1 - \chi)}{S_T} + \Xi_T \frac{\mu\bar{m}}{(1 + \mu n)r},$$

but now we have the much simpler expression $\Xi_t = (1 + nr)^{-(t+1)}$ for all t . The left-hand side is just $f_{-1} + \phi + (y - \tau + v)/r + LHS_1 + LHS_2$ where $LHS_1 \equiv -\sum_{t=0}^{\infty} \Xi_t n c_t$ and $LHS_2 = \sum_{t=0}^{\infty} \Xi_t n r B/S_t$. We have $c_t = \underline{c}$, for $0 \leq t < t^*$, and $c_t = \bar{c}$ for $t \geq T + 1$, so

$$LHS_1 = -(1 - \Xi_{t^*-1}) \frac{\underline{c}}{r} - \sum_{t=t^*}^T \Xi_t n c_t - \frac{\Xi_T}{r} \bar{c}.$$

We also have

$$LHS_2 = \frac{B}{S}(1 - \Xi_{t^*}) + nrB \sum_{t=t^*+1}^T \Xi_t/S_t + nrB \frac{\Xi_T}{(N-1)M_T/\bar{m}}.$$

If we find that the household's lifetime budget constraint is not satisfied, we change our guess for \bar{c} .

To find the equilibrium value of μ we require that the government's lifetime budget constraint, be satisfied. It can be written as $\phi + LHS_2 - B/S = RHS$. If the government's lifetime budget constraint is not satisfied, we adjust our guess for μ .