

Trading Down and the Business Cycle*

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Abstract

We document two facts. First, during the Great Recession, consumers traded down in the quality of the goods and services they consumed. Second, the production of low-quality goods is less labor intensive than that of high-quality goods. When households traded down, labor demand fell, increasing the recession's severity. We find that the trading-down phenomenon accounts for a substantial fraction of the decline in U.S. employment in the recent recession. We show that embedding quality choice in a business-cycle model improves the model's amplification and comovement properties.

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1 Introduction

A classic macroeconomics research area is the study of how households make consumption choices and how these choices impact the economy. The large empirical literature on this topic goes as far back as the work of Burns and Mitchell (1946). This literature has received renewed attention after the Great Recession.¹

In this paper, we contribute to this line of research as follows. First, we show that during the Great Recession, consumers traded down in the quality of the goods and services they consumed. Second, we show that the production of low-quality goods is generally less labor intensive than that of high-quality goods. These two facts imply that when households traded down, labor demand fell, increasing the severity of the Great Recession.

To quantify the implications of “trading down” for employment during the Great Recession, we combine various data sources to construct a data set with firm-level measures of product quality, labor intensity, and market share.

For most of our analysis, we use prices as a proxy for quality. Our assumption is that if consumers are willing to pay more for an item they perceive it to be of higher quality. We corroborate the plausibility of this assumption using data with independent measures of quality and price.²

We obtain price measures from two sources: data scraped from Yelp!, a website where consumers post review information about different goods and services, and the confidential micro data set used to construct the Producer Price Index (PPI). We merge these data with Compustat data to measure labor intensity and market share for each firm in our sample. We estimate that about half of the decline in employment in the Great Recession is accounted for by consumers trading down in the quality of the goods

¹Recent contributions to this literature include Aguiar, Hurst, and Karabarbounis (2013), Huo and Rios-Rull (2015), Kaplan and Menzio (2015), Nevo and Wong (2015), and Stroebel and Vavra (2018).

²Chen and Juvenal (2016) provide evidence of trading down in the wine market during the Great Recession using wine ratings as quality measures.

and services they purchased.

To study the effects of trading down from a theoretical perspective, we embed quality choice into an otherwise standard business-cycle model. We find that the presence of quality choice magnifies the response of these economies to shocks, generating larger booms and deeper recessions. To understand this amplification mechanism, consider a negative shock that reduces household income. The fall in income leads the household to trade down in the quality of the goods it consumes. Because lower-quality goods are produced with less labor, trading down reduces labor demand.

The quality-augmented model has two other interesting properties. First, it can generate comovement between employment in the consumption and investment sectors, a property that is absent in most business-cycle models (see Christiano and Fitzgerald (1998)). Second, the model produces an endogenous, countercyclical labor wedge. As Shimer (2009) emphasizes, this type of wedge is necessary to reconcile business-cycle models with the empirical behavior of hours worked.

Our paper is organized as follows. In Section 2, we describe our data and present our empirical results. In Section 3, we study the quantitative properties of two models with quality choice. The first model has a representative agent, while the second has heterogeneous agents. Section 4 concludes.

2 Empirical findings

In this section, we analyze the impact of trading down on labor demand. By trading down we mean shifts in the composition of consumption across firms *within* narrowly defined sectors, toward lower quality goods. To study the effect of trading down on employment, we estimate the labor intensity of different quality levels within each sector. We then study how shifts in the market share of the different quality levels affect employment. This within-sector trading-down channel differs from other channels through which demand can affect employment, such as shifts in spending *across* sectors

(e.g., from restaurants to the grocery sector) or declines in total spending.³

Our empirical approach to study the effects of trading down is as follows. We denote by M the number of sectors in the economy. Total aggregate employment across these sectors is:

$$H_t = \sum_{m=1}^M H_{m,t}, \quad (1)$$

where $H_{m,t}$ denotes employment at time t in sector m .

In each sector, goods can belong to one of J levels of quality. The market share of quality tier j in sector m ($S_{j,m,t}$) is the ratio of sales of goods in quality tier j ($Y_{j,m,t}$) to total sales in sector m ($Y_{m,t}$):

$$S_{j,m,t} = \frac{Y_{j,m,t}}{Y_{m,t}}.$$

The measure of labor intensity ($LI_{j,m,t}$) we construct is the ratio of employees ($H_{j,m,t}$) to sales ($Y_{j,m,t}$):

$$LI_{j,m,t} = \frac{H_{j,m,t}}{Y_{j,m,t}}.$$

Using this notation, we can write total employment in sector m in period t as:

$$H_{m,t} = Y_{m,t} \sum_{j=1}^J S_{j,m,t} LI_{j,m,t}, \quad (2)$$

where $Y_{m,t}$ denotes total sales in sector m , $Y_{m,t} = \sum_{j=1}^J Y_{j,m,t}$. Combining equations (1) and (2), we can write aggregate employment as:

$$H_t = Y_t \sum_{m=1}^M \frac{Y_{m,t}}{Y_t} \sum_{j=1}^J S_{j,m,t} LI_{j,m,t}, \quad (3)$$

where Y_t denotes aggregate sales across the M sectors. Using equation (3), we can write

³See Section 2.5 for a discussion of shifts in spending across sectors.

the log-percentage change in employment, $\log(H_{t+1}/H_t)$ as:

$$\begin{aligned} \log(H_{t+1}/H_t) &= \log(Y_{t+1}/Y_t) + \log\left(\sum_{m=1}^M \frac{Y_{m,t+1}}{Y_{t+1}} \sum_{j=1}^J S_{j,m,t+1} LI_{j,m,t+1}\right) \\ &\quad - \log\left(\sum_{m=1}^M \frac{Y_{m,t}}{Y_t} \sum_{j=1}^J S_{j,m,t} LI_{j,m,t}\right). \end{aligned} \quad (4)$$

In order to quantify the effect of trading down on employment, we calculate a counterfactual value for employment in period $t + 1$:

$$H_{t+1}^{CF} = Y_{t+1} \sum_{m=1}^M \left(\frac{Y_{m,t+1}}{Y_{t+1}} \sum_{j=1}^J S_{j,m,t} LI_{j,m,t+1} \right). \quad (5)$$

This counterfactual level of employment that which would have occurred in the absence of trading down—that is, if the market shares of each quality tier were the same at time t and $t + 1$.

In the absence of trading down, the percentage change in employment would have been:

$$\begin{aligned} \log(H_{t+1}^{CF}/H_t) &= \log(Y_{t+1}/Y_t) + \log\left(\sum_{m=1}^M \frac{Y_{m,t+1}}{Y_{t+1}} \sum_{j=1}^J S_{j,m,t} LI_{j,m,t+1}\right) \\ &\quad - \log\left(\sum_{m=1}^M \frac{Y_{m,t}}{Y_t} \sum_{j=1}^J S_{j,m,t} LI_{j,m,t}\right). \end{aligned} \quad (6)$$

The difference between the actual and counterfactual changes in employment gives us an estimate of the importance of trading down as a driver of changes in employment. Below, we produce this estimate using our data.

2.1 Empirical measures

We start by using a data set that merges information from Yelp!, the Census of Retail Trade, and Compustat. We then extend our analysis to the manufacturing sector by

using the micro data gathered by the BLS to construct the PPI. Finally, we consider several other data sets.

We define the relative quality of goods and services as including anything for which consumers are willing to pay. This notion of quality encompasses both the item itself, and the service or convenience of the product. For example, for restaurants, it includes both the food and the ambience. For grocery stores, it is not only the items sold but also the convenience and service.

We measure quality using the price distribution of firms within each sector. This measure is the most comprehensive in terms of sectoral coverage, making our analysis consistent across all the sectors considered.

2.2 Results obtained with Yelp! and Census of Retail Trade data

Here, we discuss the results we obtain using data from Yelp! and the Census of Retail Trade. The combined data set covers five North American Industry Classification System (NAICS) sectors: apparel, grocery stores, restaurants, home furnishings, and general merchandise. These sectors represent 17 percent of private non-farm employment. As we discuss below, our analysis includes the effects of trading down on the demand for intermediate inputs from all sectors in the economy, computed using the BLS input-output data.

We focus our analysis on two main time periods. Our recession period goes from 2007 to 2012. Our pre-recession period goes from 2004 to 2007.⁴ We use data for the period 2004-2007 to control for trends in trading down, to try to isolate the cyclical component of trading down associated with the Great Recession.

⁴Even though the NBER determined that the recession ended in June 2009, average and median household incomes continued to fall until 2012. In addition, employment recovered very slowly: in December 2012, employment was still 3 percent below its December 2007 level. In Appendix E, we report results when we use 2007-2009 as the recession period.

Yelp! data

For sectors other than General Merchandise, we collect information on prices by scraping data from Yelp!, a website where consumers share reviews of goods and services. For each store and location pair, Yelp! asks users to classify the price of the goods and services they purchased into one of four categories: \$ (low), \$\$ (middle), \$\$\$ (high), and \$\$\$\$ (very high). Because there are few observations in the very-high category, we merge the last two categories into a single high-price category.

We construct the Yelp! data set as follows (see Appendix A for more details). We first associate each firm (for example, Cost Plus, Inc.) with its brand names and retail chains (for example, Cost Plus owns the retail chain World Market). We find the Yelp! profile for each retail chain and brand in the 18 largest U.S. cities, and collect the first match (for example, the first match for World Market in Chicago is the store on 1623 N. Sheffield Av.). We then compute the average price category across the first match for each of the 18 cities (to compute this average, we assign 1 to the low-price category, 2 to middle and 3 to high).⁵

Consistent with the assumption that prices proxy for quality, we find a positive correlation between the average rating and the average price in the Yelp! data.

U.S. Census of Retail Trade data

For General Merchandise, the U.S. Census of Retail Trade splits firms into three price tiers that correspond to three levels of quality: non-discount stores (high quality), discount department stores (middle quality), and other general merchandise stores including family dollar stores (low quality). For each of these tiers, the Census provides information about employment and sales. We use this information to construct labor intensity measures and market shares. The Census categorization in the general merchandise store sector aligns with the Yelp! categorization of Compustat firms into

⁵The dispersion in price categories across cities is relatively small; it is rare for firms to be included in different price categories in different cities.

price categories, highlighting the consistency of our quality categorization across the different data sources.

Compustat data

We merge the price information for each firm in our Yelp! data set with data from Compustat on number of employees, sales, operating expenses, and cost of goods sold. Our measure of labor intensity is the ratio of employees to sales. We focus on this measure because of its availability. The U.S. Securities and Exchange Commission requires that firms report the number of employees and, as a result, these data are available for all companies in Compustat. In contrast, fewer than one quarter of the firms in Compustat report labor costs.⁶

Findings

Tables 1 and 2 present our estimates of labor intensity and market share by quality tier. We use these estimates as inputs into equations (4) and (6). Labor intensity increases with quality in all five sectors. For example, in 2007 the number of employees per million dollars of sales is 6.33, 9.23 and 10.86 for low-, middle-, and high-quality apparel stores, respectively (Table 1). Overall, middle-quality producers use 46 percent more workers per million dollar of sales than low-quality producers. Similarly, high-quality producers use 18 percent more workers per million dollars of sales than middle-quality producers. All other things being equal, a shift of one million dollars of sales from middle- to low-quality stores reduces employment by roughly three jobs.

Between 2007 and 2012, firms that produce middle- and high-quality items lost

⁶For the sample of firms that report the labor share in cost, the correlation between labor share and the labor intensity measure of employees/sales is 0.94. The correlation between the labor share in cost and employees/gross margin is 0.97. As a robustness check, we also use the ratio of employees to gross margin as a measure of labor intensity (Appendix D). Gross margin, which is sales minus cost of goods sold, is a measure that is close to value added. Value added is equal to the gross margin minus energy and services purchased. We cannot compute value added because Compustat does not report data on energy and services purchased. The correlation between employees/gross margin and employees/sales is 0.72.

market share relative to firms that produce low-quality items. In 2007, the low-, middle- and high-price categories accounted for 42, 52 and 6 percent of sales, respectively. In contrast, by 2012, high-quality producers lost about 0.5 percentage points in market share, middle-quality producers lost 6.5 percentage points, and low-quality producers gained 7 percentage points. This pattern is present in all the sectors we consider, with one exception: the market share of high-quality grocery stores increased.⁷ This exception is driven by an outlier: WholeFoods, a high-quality supermarket that gained market share despite the recession.

With the information in Tables 1 and 2, we can implement the empirical approach described by equations (4) and (6). Overall employment in the sectors included in our data set fell by 3.39 percent. Using equation (6), we find that in the absence of trading down, employment would have fallen by only 0.39 percent. This result implies that trading down accounts for $(3.39 - 0.39)/3.39 = 88$ percent of the decline in employment.⁸

Trend versus Cycle

Trading down was occurring before the recession, it has both trend and cycle components. In order to disentangle these components, we proceed as follows. We compute market share by quality tier for each sector for the period 2004-2007. We use the change in these shares over this period to linearly extrapolate what the market shares of different quality tiers would have been in 2012. Using these market shares, we construct

⁷Using the PPI data discussed below, we find that there is no correlation between the changes in prices that occurred during the recession and the quality tier of the firm. This fact suggests that changes in market shares are driven mostly by changes in quantities rather than by changes in prices or markups. This inference is consistent with the finding in Anderson, Rebelo and Wong (2018) that markups in the retail sector remained relatively stable during the Great Recession.

⁸Our calculations are based on Census estimates of sector expenditures. We are implicitly assuming that the market shares and labor intensities in our data are representative of each sector as a whole.

Table 1: Market shares and Labor Intensity: 2007

Industry	\$m Sales	Labor intensity			Market share		
		Low	Middle	High	Low	Middle	High
Home furnishings and appliances	654'535	4.4	5.0	7.1	1%	95%	5%
Grocery stores	547'837	3.4	4.7	7.6	39%	59%	2%
Food services and drinking places	444'551	15.6	24.0	22.4	52%	41%	7%
Clothing stores	221'205	7.6	9.4	16.5	11%	78%	11%
General merchandise stores	578'582	3.7	6.9	7.2	64%	23%	13%
Total	2'446'710	6.3	9.2	10.9	35%	58%	7%

Note: This table depicts the 2007 total sales, labor intensity, and market share for different retail sectors. The last row is a sales-weighted measure. Sales are from the U.S. Census of Retail Trade, and the labor intensities and market shares are from Compustat. Labor intensity is defined as the number of employees per million dollars of sales, and the market share is the share of sales for each price tier within each sector. Price tiers are denoted by low, middle and high, and are based on Yelp! classifications of prices \$, \$\$, and \$\$\$, respectively. See text for more information.

Table 2: Market shares and Labor Intensity: 2012

Industry	\$m Sales	Labor intensity			Market share		
		Low	Middle	High	Low	Middle	High
Home furnishings and appliances	609'323	3.5	4.9	5.9	1%	94%	5%
Grocery stores	631'486	1.9	4.1	6.1	43%	53%	5%
Food services and drinking places	524'892	13.4	19.5	22.4	61%	33%	6%
Clothing stores	241'386	6.5	9.2	15.1	15%	77%	7%
General merchandise stores	649'754	3.7	6.9	7.2	72%	18%	10%
Total	2'656'841	5.4	8.5	10.4	42%	52%	6%

Note: This table depicts the 2012 total sales, labor intensity, and market share for different retail sectors. The last row is a sales-weighted measure. Sales are from the US Census of Retail Trade, and the labor intensities and market shares are from Compustat. Labor intensity is defined as the number of employees per million dollars of sales, and the market share is the share of sales for each price tier within each sector. Price tiers are denoted by low, middle and high, and are based on Yelp! classifications of prices \$, \$\$, and \$\$\$, respectively. See text for more information.

the 2012 employment implied by the extrapolated market shares:

$$N_{2012}^{CF2} = Y_{2012} \sum_{m=1}^5 \left(\frac{Y_{m,2012}}{Y_{2012}} \sum_{j=1}^3 \left[S_{j,m,2007} + \frac{S_{j,m,2007} - S_{j,m,2004}}{(2007 - 2004)} \times (2012 - 2007) \right] [LI_{j,m,2012}] \right). \quad (7)$$

This counterfactual measure of employment is 2.38 percent lower than the level of employment in 2007. We conclude that $(3.39 - 2.38)/3.39 = 30$ percent of the decline in employment is due to trend factors. Recall that trading down accounted for a 3 percentage point fall in employment, so the part of trading down associated with cyclical factors is: $[3 - (3.39 - 2.38)] = 1.99$. In other words, $1.99/3.39 = 58$ percent of the fall in employment is due to trading down associated with cyclical factors.

Because of data limitations, our analysis covers only a subset of the sectors in the economy, excluding sectors such as education, healthcare, government, finance, and construction. Using our results, we can compute a lower bound on the importance of trading down for employment by assuming that there was no trading down in the sectors excluded from our analysis.⁹ This lower bound is 21 percent of the decrease in aggregate employment. So trading down accounts for at least 21 percent of the cyclical decline in employment.

2.3 PPI data

In order to extend the analysis to the manufacturing sector, we use the confidential micro data collected by the Bureau of Labor Statistics (BLS) to construct the PPI.¹⁰ As with the Yelp! data, we merge the PPI data with Compustat to obtain price, labor intensity, and market share for each firm. This combined data set has 62,000 monthly observations for the period 2007-2012. Overall, the sectors covered by the merged PPI and Compustat data account for 15 percent of private non-farm employment.

⁹It is of course possible that there was trading up in the excluded sectors. But we view this possibility as implausible.

¹⁰Examples of other papers that use these data include Nakamura and Steinsson (2008), Gilchrist et al (2014), Gorodnichenko and Weber (2014), and Weber (2015).

We focus on the 2-digit NAICS manufacturing sectors 31, 32, and 33 because in these sectors we are able to merge the PPI and Compustat data for over 10 firms per sector and span a range of quality tiers.¹¹

Our quality measure for a product of a given firm is based on its price relative to the median price of that product across firms. We refer the reader to Appendix B for more details. Our analysis is based on products defined at a six-digit industry code level. For reporting purposes, we aggregate the results to the two-digit level using shipment revenue.

One challenge with using the PPI data is that firms in the same industry report prices that correspond to different units of measurement (e.g., some firms report price per pound, others price per dozen). To address this problem we convert prices into a common metric whenever possible (for example, converting ounces into pounds). The PPI provides information on the unit of measure for each item, which we use to ensure that prices in our sample refer to the same unit of measurement (e.g., pounds). Unfortunately, a large number of observations on the unit of measure are missing before 2007. This limitation restricts our ability to account for the “pre-Great Recession” trend.

To make our results comparable to those obtained with Yelp! data, we proceed as follows. Once we rank establishments by their relative price, we assign the top 7 percent to the high-quality category, the middle 58 percent to the middle-quality category, and the bottom 35 percent to the low-quality category. Recall that this is the distribution of firms across quality tiers that characterizes firms included in the Yelp! data set.

We aggregate the establishment quality tier assignment to the firm level by taking a shipment-value weighted average of the quality tier and rounding to the closest qual-

¹¹The three-digit sectors included in our data are: 311 Food manufacturing, 312 Beverage & Tobacco Product Manufacturing, 315 Apparel Manufacturing, 321 Wood Product Manufacturing, 321 Wood Product Manufacturing, 325 Chemical Manufacturing, 326 Plastics & Rubber Products Manufacturing, 331 Primary Metal Manufacturing, 333 Machinery Manufacturing, 334 Computer & Electronic Product Manufacturing, 336 Transportation Equipment Manufacturing, 337 Furniture & Related Product Manufacturing, and 339 Miscellaneous Manufacturing.

ity tier. Finally, we merge the firm-level quality tier assignment from the PPI with the Compustat sample of firms.¹² This merged data set enables us to compute labor intensity by quality tier.¹³

Tables 3 and 4 shows that our two key facts hold in the PPI data. First, low-quality firms gained market share between 2007 and 2012 at the expense of middle and high-quality firms. Second, quality is correlated with labor intensity. High-quality producers have higher labor intensity than middle-quality producers, and middle-quality producers have higher labor intensity than low-quality producers.¹⁴

We now use the PPI data to implement our empirical approach. Overall employment in the sectors included in our data fell by approximately 8.6 percent. The counterfactual decline in employment that would have occurred without trading down is 3.9 percent. Hence, trading down accounts for 54 percent of the decrease in employment.

In sum, our results using the PPI data are consistent with those obtained with Yelp! and Census of Retail Trade data. Stores with higher prices, which are generally more labor intensive, lost market share during the recent recession. This loss of market share accounts for about half of the overall decline in employment.

2.4 NPD data

In this subsection, we discuss results obtained using data on the evolution of market shares in restaurants of different quality levels. This data set collected by the NPD Group (a marketing consulting firm) includes restaurant traffic (number of meals served) and consumer spending in restaurants broken into four categories of service: quick-

¹²The aggregation of establishments up to firm level uses the matching done by Gorodnichenko and Weber (2014), who shared their code with us. In their work, they manually matched the names of establishments to the name of the firm. They also searched for names of subsidiaries and checked for any name changes of firms within the Compustat data set. See Gorodnichenko and Weber (2014) for more detail. A similar exercise of matching establishments to firms is used in Gilchrist et al (2014).

¹³We use the entire sample of establishments within the PPI to rank the establishments, not only those that we are able to match with Compustat.

¹⁴We do not have any firms within the high-quality tier for Sector 31 (defined based on the PPI data) that could be merged with the Compustat data.

Table 3: PPI Sectors 2007

Industry	\$m Expenditure in 2007	Labor Intensity			Market share		
		Low	Middle	High	Low	Middle	High
31	811,751	0.74	3.41	n.a.	23%	77%	n.a.
32	1,434,885	2.73	2.99	4.62	26%	45%	29%
33	2,457,336	2.04	2.60	4.05	31%	63%	6%
Total	4,703,972	2.03	2.86	3.53	28%	58%	14%

Note: This table depicts the 2007 total sales, labor intensity and market share for different manufacturing sectors. The last row is a sales-weighted measure. Sales are from Census, and the labor intensities and market shares are from Compustat. Labor intensity is defined as the number of employees per million dollars of sales and the market share are the share of sales for each price tier within each sector. Price tiers are denoted by low, middle and high, and are based on firm-level producer price data. See text for more information.

Table 4: PPI Sectors 2012

Industry	\$m Expenditure in 2012	Labor Intensity			Market share		
		Low	Middle	High	Low	Middle	High
31	956,083	0.40	3.41	n.a.	34%	66%	n.a.
32	1,461,253	2.69	2.85	4.59	27%	47%	26%
33	2,494,959	1.40	2.41	3.32	38%	57%	5%
Total	4,912,295	1.59	2.74	3.06	33%	54%	13%

Note: This table depicts the 2012 total sales, labor intensity and market share for different manufacturing sectors. The last row is a sales-weighted measure. Sales are from Census, and the labor intensities and market shares are from Compustat. Labor intensity is defined as the number of employees per million dollars of sales and the market share are the share of sales for each price tier within each sector. Price tiers are denoted by low, middle and high, and are based on firm-level producer price data. See text for more information.

service restaurants, midscale restaurants, casual dining, and fine dining/upscale hotel. These categories are designed to represent different levels of quality.

These data can shed light on the appropriateness of our assumption that the price of a good or service is a good proxy for its quality. If we sort firms using the average price of a meal as a proxy for quality, we obtain a sorting by quality tiers similar to NPD's.¹⁵ The average price of dinner (lunch) is \$6.50 (\$5.80) in quick-service restaurants, \$11.20 (\$9.20) in midscale restaurants, and \$14.90 (\$11.70) in casual dining.¹⁶

We find clear evidence of trading down in the NPD data. Consider first the number of meals served. Table 5 shows that the percentage of meals served by quick-service restaurants increased from 76.1 percent in 2007 to 78.2 percent in 2012. At the same time, the fraction of meals served declined in all the other segments: midscale, casual and fine dining.¹⁷ Table 6 reports results for market share. We see that over the period 2007-2012, the market share of quick-service restaurants rose from 57.7 percent to 60 percent. At the same time, the market shares declined for all other segments.¹⁸

Unfortunately, we cannot do our accounting calculations directly with these data because we do not have the breakdown of labor intensities for the restaurant categories used by NPD. However, we can use the labor intensity estimates for Food services and drinking places reported in Tables 1 and 2, which are based on Yelp! data, as a proxy for the labor intensity in the NPD categories.¹⁹ To do so, we equate the low-, middle-

¹⁵There is a literature on the role of search frictions in generating price dispersion. While search frictions are likely to be important, the price differences across categories in our dataset are clearly too large to be accounted for by these frictions alone. Aguiar and Hurst (2006) estimate that doubling of shopping frequency lowers the price paid for a given good by 7 to 10 percent. The price differences across different categories in our data are almost an order of magnitude larger than these estimates.

¹⁶These price data were collected in March 2013. We do not have average meal prices for fine-dining restaurants.

¹⁷There is also some evidence in the NDP data that consumers traded down in terms of the meal they choose to eat at restaurants, eating out for breakfast and lunch instead of for dinner.

¹⁸Tables 5 and 6 show that after the worst of the recession was over in 2010, fine dining started to recover. But overall, the fraction of meals served and market share of fine dining are still lower in 2012 than in 2007.

¹⁹NPD provided us a partial list of restaurants classified according to the NPD categories. Using this list, we concluded that their classification is almost identical to the one we obtain using Yelp!

Table 5: Percentages of Restaurant Traffic by Year and Quality Segment

Quality segment	2007	2008	2009	2010	2011	2012
Quick service restaurants	76.1	76.4	76.8	77.2	77.8	78.2
Midscale	11.4	11.1	11.0	10.7	10.3	10.0
Casual dining	11.2	11.1	11.1	10.9	10.7	10.4
Fine dining/upscale hotel	1.4	1.4	1.2	1.2	1.3	1.3

Note: The data are from NPD Group.

and high-quality categories in Yelp! to Quick Service, Midscale plus Casual Dining, and Fine Dining categories in NPD, respectively. We find that trading down accounts for 92 percent of the decline in employment. These results are similar to the total effect of trading down (including both trend and cycle) that we estimated using the Yelp! data (88 percent). Because we only have data from NPD since 2007, we cannot separate trend effects from cyclical effects.

2.5 Substitution across categories

In our analysis, we focus on the implications of trading down for employment. We also studied the employment implications of substitution across categories, for example from luxuries to necessities. Our analysis is based on the Consumer Expenditure Survey (CEX) and the National Income and Products Account (NIPA) personal consumer expenditure (PCE) data (see Appendix C for more detail on the calculations).

It is well known that different categories of expenditure have different income elasticities and hence different cyclical properties. For example, expenditure on food away

Table 6: Restaurant Market Share by Year and Quality Segment

Quality segment	2007	2008	2009	2010	2011	2012
Quick service restaurants	57.7	58.0	58.7	59.0	59.4	60.0
Midscale	15.4	15.2	15.1	14.8	14.5	14.1
Casual dining	21.5	21.4	21.4	21.3	20.9	20.5
Fine dining/upscale hotel	5.5	5.3	4.9	5.0	5.2	5.4

Note: The data are from NPD Group.

from home falls during recessions by much more than expenditure on personal care.

We find that substitution across categories has a negligible effect on employment. This result is driven by the low correlation between the income elasticities of different categories and labor intensity. For example, expenditures on both food away from home and vehicle purchases fall during recessions. But food away from home has high labor intensity, while vehicle purchases has low labor intensity. We summarize our results in Appendix C.

3 Quality choice in business-cycle models

In this section, we use several business-cycle models to show that the presence of quality choice amplifies the response to shocks. We first derive some theoretical results in a partial-equilibrium setting. We then study a representative-agent general-equilibrium model and show that this model can help resolve some long-lasting challenges in the business-cycle literature. Finally, we extend our model to include heterogeneity in con-

sumer quality choices. This extension enables us to forge a tighter connection between the model and our empirical work.

We focus our discussion on flexible-price version models. We show in Appendix F that the same mechanism that amplifies real shocks also amplifies nominal shocks. We do so by embedding quality choice in a model with Calvo (1983)-style sticky prices.

3.1 A partial-equilibrium model

3.1.1 Production

We begin by describing the production side of the economy, which is the same across the models we discuss. Consumer goods are produced by a continuum of measure one of competitive firms according to the following production function:

$$C_t = A_t \left[\alpha \left(\frac{H_t^C}{q_t} \right)^\rho + (1 - \alpha) (K_t^C)^\rho \right]^{\frac{1}{\rho}}, \quad (8)$$

where q_t denotes quality and H_t^C and K_t^C denote labor and capital employed in the consumption sector, respectively.

The consumption firms' problem is to maximize:

$$\max_{H_t^C, K_t^C} P_{q,t} C_t - W_t H_t^C - R_t K_t^C, \quad (9)$$

where W_t and R_t denote the wage and rental rate, respectively. The solution to this problem implies that the equilibrium price of a consumption good with quality q_t is given by:

$$P_{q,t} = \frac{1}{A_t} \left[\alpha^{\frac{1}{1-\rho}} (q_t W_t)^{\frac{\rho}{\rho-1}} + (1 - \alpha)^{\frac{1}{1-\rho}} R_t^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}}. \quad (10)$$

We assume that $\rho < 0$, so there is less substitution between capital and labor than in a Cobb-Douglas production function. It is easy to show that this assumption is necessary so that, consistent with our empirical results, higher-quality goods are more labor intensive. The assumption that $\rho < 0$ also implies that the price of a

consumption good is an increasing function of its quality. This property is consistent with our empirical approach, which uses price as a proxy for quality.

The optimal factor intensity for consumption producers is:

$$\frac{H_t^C}{K_t^C} = q_t^{\frac{\rho}{\rho-1}} \left(\frac{\alpha}{1-\alpha} \frac{R_t}{W_t} \right)^{\frac{1}{1-\rho}}. \quad (11)$$

Since $\rho < 0$, a decline in the quality chosen by the household reduces, *ceteris paribus*, the demand for hours worked. This effect amplifies the response of the economy to shocks.

3.1.2 Household

On the household side, we consider two different models. The utility function is non-homothetic in consumption in both models. As we discuss below, this condition is necessary so that the household's optimal level of quality increases with income.²⁰ We first consider a model in which households choose the quality of the single good they consume. We then consider a model in which households consume two goods with different quality levels.

Variable quality model The household derives utility (U) from both the quantity (C) and quality (q) of consumption, and solves the following maximization problem:

$$\max U(C_t, q_t), \quad (12)$$

s.t.

$$P_{q,t} C_t = \xi_t, \quad (13)$$

where ξ_t is household income. The first-order conditions for consumption and quality imply that:

$$\frac{U_2(C_t, q_t)}{U_1(C_t, q_t)} = \frac{P'_{q,t} C_t}{P_{q,t}}. \quad (14)$$

²⁰See Faber and Fally (2018) for evidence that higher-income households consume higher-quality goods.

We assume that:

$$U_1(C, q) > 0, U_2(C, q) > 0. \quad (15)$$

The requirement that quality be a normal good, so that higher-income consumers choose to consume higher-quality goods, imposes restrictions on the utility function. Equation (14) implies that if U is homogeneous in C , quality is independent of income. So, in order for quality to be a normal good, U must be non-homothetic in C .

With this requirement in mind, we adopt the following parametric form for the utility function:

$$U(C_t, q_t) = \frac{q_t^{1-\theta}}{1-\theta} \log(C_t). \quad (16)$$

We compare the response of labor demand to exogenous changes in household income, for constant values of W and R in versions of the model with and without quality choice.²¹ In the model without quality choice, q_t is not time varying and the household budget constraint is simply $C_t = \xi_t$. The following proposition, proved in Appendix H, summarizes the key result.

Proposition 1. *For given levels of W and R , the decline in the total number of hours employed by firms in response to an exogenous drop in household income is always higher in the model with quality choice.*

The intuition for this result is as follows. Consider first the model without quality choice. A drop in household income leads to a reduction in the quantity of consumption goods demanded. Because the prices of the two production factors are constant, the firm reduces the demand for labor and capital in the same proportion (see equation (11) with a constant value of q_t). Consider now the model with quality choice. A decline in consumer income is associated with a drop in the quantity and quality consumed. The decrease in quality leads firms to contract more the demand for labor than the

²¹In the model with quality choice, the price of consumption changes in response to changes in the optimal quality chosen by the household.

demand for capital (again, see equation (11)). As a result, hours worked fall more in the economy with quality choice.

Stone-Geary model In this model, households consume two goods with different quality levels. These goods enter the utility function according to a Stone-Geary utility function:²²

$$Max_{C_1, C_2} \sigma_1 \log(C_1 - \phi_1) + \sigma_2 \log(C_2), \quad (17)$$

subject to

$$\sum_{j=1}^2 P_j C_j = \xi. \quad (18)$$

Good 1 is an inferior good in the sense that its demand income elasticity is lower than one. Good 2 is a superior good in the sense that its demand income elasticity exceeds one. Because we want quality consumed to be an increasing function of income, we assume that $q_2 > q_1$. Both consumption goods are produced according to:

$$Y_j = C_j = A \left[\alpha \left(\frac{H_j}{q_j} \right)^\rho + (1 - \alpha) (K_j)^\rho \right]^{\frac{1}{\rho}}. \quad (19)$$

The expenditure shares of good 1 (s_{C1}) and 2 (s_{C2}) are given by

$$s_{C1} = \frac{\sigma_1}{\sigma_1 + \sigma_2} + \frac{\sigma_2}{\sigma_1 + \sigma_2} \frac{P_1 \phi_1}{\xi}, \quad (20)$$

$$s_{C2} = \frac{\sigma_2}{\sigma_1 + \sigma_2} \left[1 - \frac{P_1 \phi_1}{\xi} \right]. \quad (21)$$

The following proposition, proved in Appendix H, summarizes this model's key property.

²²This form of utility is used widely in the literature on structural change, see for example, Echevarria (1997), Kongsamut, Rebelo and Xie (2001), and Herrendorf, Rogerson, and Valentinyi (2014).

Proposition 2. *For given levels of W and R , the decline in the total number of hours employed by firms in response to an exogenous decrease in household income is always higher when the superior good is more labor intensive than the inferior good.*

The intuition for this result is simple. The share of the inferior good increases in response to a negative income shock. If this sector has a lower labor intensity, labor demand falls.

3.2 A general-equilibrium model

In this section, we solve a general equilibrium version of the model with variable quality discussed above.²³ Our goal is to assess the qualitative and quantitative implications of quality choice when prices of goods and factors of production are endogenous. We first study the properties of a representative-agent version of the model. Then, we discuss the properties of a heterogenous-agent version of the model.²⁴

The household chooses quantity consumed, quality consumed, and hours worked to maximize:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{q_t^{1-\theta}}{1-\theta} \log(C_t) - \phi \frac{H_{s,t}^{1+\nu}}{1+\nu} \right], \quad (22)$$

subject to

$$P_{q,t}C_t + I_t = W_tH_t + R_tK_t, \quad (23)$$

$$K_{t+1} = I_t + (1 - \delta)K_t, \quad (24)$$

where E_0 is the conditional expectation operator. As above, W_t is the wage rate, R_t is the rental rate of capital, and $P_{q,t}$ is the price of one unit of consumption with quality q . I_t denotes the level of investment.

²³We focus on the variable-quality model because it performs better, for the parameters we consider, than the Stone-Geary model, both in terms of amplification and comovement.

²⁴We abstract from technical progress in both the consumption and investment sectors. See Appendix I for a version of the model with labor-augmenting technical progress consistent with balanced growth.

We assume that the disutility of work is separable from the utility of consumption. An advantage of this assumption is that it enables us to nest the usual separable logarithmic utility in consumption and power disutility in hours worked as a special case. This property simplifies the comparison of versions of the model with and without quality choice.

Consumption goods are produced by a continuum of measure one of competitive firms with the following production function:

$$C_t = A_t \left[\alpha \left(\frac{H_t^C}{q_t} \right)^\rho + (1 - \alpha) (K_t^C)^\rho \right]^{\frac{1}{\rho}}. \quad (25)$$

Investment goods are produced by a continuum of measure one of competitive firms with the following production function:

$$I_t = A_t \left[\alpha \left(\frac{H_t^I}{q^I} \right)^\rho + (1 - \alpha) (K_t^I)^\rho \right]^{\frac{1}{\rho}}. \quad (26)$$

Because our evidence about trading down is only for consumption goods, we assume that investment goods have a constant quality level, q^I . We set this investment quality level equal to the steady state level of quality in the consumption goods sector. This assumption ensures that differences in steady-state levels of labor intensity in the consumption and investment sectors play no role in our results.

We choose the investment good as the numeraire, so its price is equal to one. The problem of a producer of investment goods is:

$$\max I_t - W_t H_t^I - R_t K_t^I. \quad (27)$$

where H_t^I and K_t^I denote labor and capital employed in the investment sector, respectively.

This investment production function is the same as the one used in the consumption sector, but the level of quality is constant.

Household first-order conditions The first-order condition for C_t is:

$$\frac{q_t^{1-\theta}}{1-\theta} \frac{1}{P_{q,t} C_t} = \lambda_t, \quad (28)$$

where λ_t denotes the Lagrange multiplier associated with the budget constraint of the household in period t .

The first-order condition for q_t is:

$$q_t^{-\theta} \log(C_t) = \lambda_t C_t P'_{q,t}. \quad (29)$$

Combining the last two equations, we see that the quantity consumed comoves with the elasticity of the price with respect to quality:

$$\log(C_t) = \frac{1}{1-\theta} \frac{P'_{q,t} q_t}{P_{q,t}}. \quad (30)$$

The first-order conditions for hours worked and capital accumulation have the following familiar form:

$$\phi H_{s,t}^\nu = \lambda_t W_t, \quad (31)$$

$$\lambda_t = \beta E_t \lambda_{t+1} (R_{t+1} + 1 - \delta). \quad (32)$$

Equilibrium The definition of equilibrium is as follows. Households maximize utility, taking the wage rate, rental rate of capital, and price-quality schedule as given. Because households are identical, they all choose the same level of quality, and only this quality level is produced in equilibrium. Firms maximize profits, taking the wage rate and rental rate of capital as given. The labor and capital market clear, so total demand for labor and capital equals their supply:

$$K_t^C + K_t^I = K_t, \quad (33)$$

$$H_t^C + H_t^I = H_{s,t}. \quad (34)$$

Real output (Y_t) in the economy is given by:

$$Y_t = P_{q,t} C_t + I_t. \quad (35)$$

This expression assumes that real output is computed using hedonic adjustments. When the price of consumption rises, the statistical authorities recognize that this rise is due solely to an increase in the quality of the goods consumed.

Simulation We solve the model numerically by linearizing its equilibrium conditions around the steady state. Our parameter choices are described in Table 7. There are three parameters that cannot be directly calibrated from the existing literature: α , ρ , and θ . We choose the first two parameters to match the consumption share in output (80%) and the labor share in income (68%). To facilitate comparison across models, we use these parameters in all of the models we discuss below. We evaluate the properties of the representative-agent model for three different values of θ : 0.33, 0.5, and 0.7.

Table 7: Calibration

Parameter	Moment/Description	Value
β	Quarterly discount rate	0.985
ν	Inverse of Frisch elasticity	0.001
ϕ	Match steady state H	5.31
δ	Depreciation rate	0.025
κ	AR(1) coefficient of TFP	0.95
α	Production function share	0.1
ρ	EOS between K and N: $\frac{1}{1-\rho}$	-0.5
θ	Elasticity of utility to quality	0.5

To study the model’s business-cycle implications, we simulate quarterly versions of the models with and without quality choice model driven by an AR(1) total factor productivity shock. We HP-filter both the U.S. data and the time series generated by the models, with a smoothing parameter of 1600.

Table 8 reports the ratio of the standard deviation of different variables to the standard deviation of aggregate output (denoted by σ_X/σ_{GDP}). This table also reports the correlation of aggregate output (denoted by $Cor^{X,GDP}$) with the following variables: consumption, investment, total hours worked, hours worked in the consumption sector, hours worked in the investment sector, the labor wedge, and the marginal utility of consumption. Columns 1-2, 3-4, and 5-6 in Panel A present the results for U.S. data, our representative agent model with quality choice, and the model without quality choice, respectively. The latter model is a benchmark one-sector Real Business Cycle (RBC) model with a CES production function. For future reference, columns 7-8 in Panel A present the results for the heterogeneous agent model with quality choice. Panel B displays the second-moment implications of the representative agent model for different calibrations of θ .

Table 8: Second Moments

Panel A:								
Variable	Data		RA Quality: $\theta = 0.5$		No Quality		Het Agents with Quality	
	$\frac{\sigma_X}{\sigma_{GDP}}$	$Cor^{X,GDP}$	$\frac{\sigma_X}{\sigma_{GDP}}$	$Cor^{X,GDP}$	$\frac{\sigma_X}{\sigma_{GDP}}$	$Cor^{X,GDP}$	$\frac{\sigma_X}{\sigma_{GDP}}$	$Cor^{X,GDP}$
Total Hours	1.1	0.78	0.84	0.99	0.55	0.97	0.81	0.99
Hours in C	0.80	0.48	0.41	0.93	0.10	-0.76	0.19	0.80
Hours in I	2.48	0.86	2.86	0.94	3.17	0.95	2.64	0.96
Consumption	0.80	0.85	0.56	0.93	0.42	0.90	0.40	0.883
Investment	3.16	0.87	3.05	0.95	3.59	0.98	2.88	0.99
Labor Wedge	1.1	-0.69	0.40	-0.98	NA	NA	NA	NA
λ	NA	NA	0.18	-0.71	0.42	-0.90	NA	NA

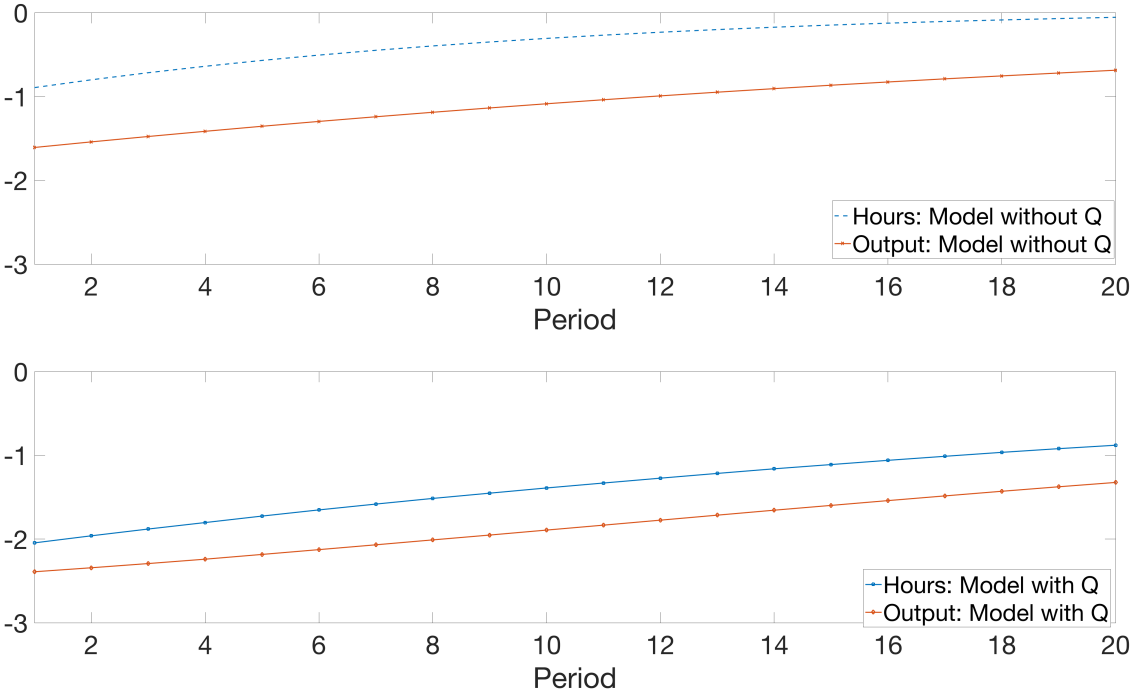
Panel B:				
Variable	RA with Quality: $\theta = 0.33$		RA with Quality: $\theta = 0.7$	
	$\frac{\sigma_X}{\sigma_{GDP}}$	$Cor^{X,GDP}$	$\frac{\sigma_X}{\sigma_{GDP}}$	$Cor^{X,GDP}$
Total Hours	0.98	0.99	0.70	0.99
Hours in C	0.60	0.48	0.23	0.95
Hours in I	4.18	0.86	2.72	0.95
Consumption	0.60	0.85	0.53	0.95
Investment	4.21	0.87	3.05	0.97
Labor Wedge	0.53	-0.73	0.21	-0.95
λ	0.14	0.29	0.33	-0.88

Amplification

Consider first Panel A of Table 8. This panel shows that the volatility of hours is higher in the version of the model with quality choice. This amplification is visible in Figure 1. This figure shows the impulse response of labor and output to the same

negative TFP shock in the model with and without quality choice.

Figure 1: Impulse response of hours and output for a decline in A (% deviations from the steady state)



The interaction between the endogeneity of the labor supply and the choice of quality gives rise to an additional source of amplification: income effects on the labor supply are much weaker in the model with quality choice than in the model without quality choice. The intuition for this property is as follows. In the model without quality choice, agents have only one margin to adjust their consumption in response to an exogenous income shock, which is to reduce the quantity they consume. In the model with quality choice agents can use two margins of adjustment: change the quantity and/or the quality of their consumption. As a result, the marginal utility of consumption, λ , responds less to an income shock in the version of the model with quality choice. In our calibrated model, the volatility of λ_t is 33 percent lower in the model with quality choice (see

Table 8). These weaker income effects lead to larger movements in hours worked (see equation (31)). Panel B of Table 8 shows that the magnitude of amplification depends on the value of θ , the parameter that controls the curvature of the utility from quality. Lower values of θ imply that quality is more volatile and hence that the quality channel is more important, providing greater amplification.

Comovement

The results reported in Table 8 show another interesting difference between the models with and without quality choice. As emphasized by Christiano and Fitzgerald (1998), hours worked in the consumption sector are procyclical in the data but countercyclical in the standard RBC model.²⁵ The model with quality choice generates procyclical hours worked in the consumption sector.

To understand this property, it is useful to write the first-order condition for labor choice for a standard RBC model with a Cobb-Douglas production function:

$$\phi(H_t^C + H_t^I)^\nu = \frac{\alpha}{H_t^C}.$$

It is clear that H_t^I and H_t^C are negatively correlated, so that H_t^I and H_t^C cannot be both positively correlated with aggregate output. Using a CES production function changes the form of the first-order condition but does not help generate comovement.

Consider the first-order condition for labor choice in the model with quality choice:

$$\phi(H_t^C + H_t^I)^\nu = \frac{q_t^{1-\theta-\rho}}{1-\theta} \frac{\alpha}{(H_t^C)^{1-\rho}} \left(\frac{A_t}{C_t}\right)^\rho.$$

This equation shows that H_t^C and H_t^I can be positively correlated, because quality is procyclical. The rise (fall) in quality increases (decreases) the demand for labor in the consumption sector, contributing to the comovement between H_t^C and H_t^I .

²⁵To classify the sectors into “consumption” and “investment,” we follow standard practice. We use the BEA’s 2002 benchmark I/O “use tables.” To compute the share of sectoral output used for private consumption versus private investment, we assign a sector to the consumption (investment) sector if most of its final output is used for consumption (investment). For the hours/sectors data we use the Current Employment Statistics 1964:Q1 - 2012:Q4.

An endogenous labor wedge Shimer (2009) modifies the standard Euler equation for labor to allow for a “labor wedge,” τ_t , that acts like a tax on the labor supply:

$$\phi H_t^\nu = (1 - \tau_t) \frac{1}{C_t} w_t. \quad (36)$$

The labor wedge can be computed using empirical measures of H_t , C_t , and w_t . Shimer (2009) finds that τ_t is volatile and counter-cyclical: workers behave as if they face higher taxes on labor income in recessions than in expansions.

The analogue of equation (36) in our model is:

$$\phi H_t^\nu = \frac{q_t^{1-\theta}}{1-\theta} \frac{1}{C_t} \frac{w_t}{P_{q,t}}. \quad (37)$$

Because the quality consumed, q_t , is procyclical (see Table 8), our model generates a counter-cyclical labor wedge.

Summary We find that introducing quality choice into an otherwise standard model amplifies the response to real shocks, giving rise to higher fluctuations in hours worked. This property enables the model to match the overall relative variability of hours to output that is observed in U.S. data. Moreover, the model can also account for the sectoral comovement in hours worked in the consumption and investment sectors, and generate a counter-cyclical labor wedge.

3.3 A heterogeneous-agent model

In the representative-agent version of the variable-quality model, there is a single level of quality consumed in equilibrium. In this subsection, we consider an extension of the model in which there are multiple quality levels produced in equilibrium. This distribution of quality facilitates comparison of the model implications and our empirical findings. We show that the key properties of the representative-agent model are preserved in the presence of heterogeneity.

We assume that individuals are endowed with different levels of efficiency units of labor, ϵ , distributed according to the cumulative distribution function, $\Gamma(\epsilon)$. To reduce the dimension of the state space and make the problem more tractable, we model household decisions as made by families that contain a representative sample of the skill distribution in the economy.

The family's objective is to maximize:

$$V_\epsilon = \text{Max}_{\{C_{\epsilon,t}, q_{\epsilon,t}, H_{s,\epsilon,t}, K_{s,\epsilon,t+1}, I_{\epsilon,t}\}} \sum_{t=0}^{\infty} \beta^t \int_{\epsilon} \omega_{\epsilon} \left\{ \frac{q_{\epsilon,t}^{1-\theta}}{1-\theta} \log(C_{q_{\epsilon,t}}) - \phi \frac{H_{s,\epsilon,t}^{1+\nu}}{1+\nu} \right\} \Gamma'(\epsilon) d\epsilon. \quad (38)$$

The variable ω_{ϵ} denotes the weight attached by the family to an individual of ability ϵ . The variables $q_{\epsilon,t}$, $C_{q_{\epsilon,t}}$, and $H_{s,\epsilon,t}$ denote the quality and quantity of the good consumed and the labor supplied by an individual with ability ϵ in period t , respectively.

The family's budget constraint is:

$$K_{t+1} = \int_{\epsilon} [W_t \epsilon H_{\epsilon,t} - P_{q_{\epsilon,t}} C_{q_{\epsilon,t}}] \Gamma'(\epsilon) d\epsilon + R_t K_t + (1 - \delta) K_t, \quad (39)$$

where $P_{q_{\epsilon,t}}$ is the price of one unit of consumption of the quality consumed by an individual with ability ϵ in period t . The variable K_t denotes the family's stock of capital.

The first-order conditions for the quantity and quality of consumption and hours worked for each family member are the same those of the representative-agent model (equations (28)-(30)).

The goods consumed by agents with ability ϵ are produced by perfectly competitive producers according to the following CES production function:

$$Y_{\epsilon,t} = A_t \left[\alpha \left(\frac{H_{d,\epsilon,t}}{q_{\epsilon,t}} \right)^{\rho} + (1 - \alpha) (K_{d,\epsilon,t})^{\rho} \right]^{\frac{1}{\rho}}, \quad (40)$$

The variables $H_{d,\epsilon,t}$ and $K_{d,\epsilon,t}$ denote the labor and capital employed by the producer, respectively. As in our representative-agent model, we assume that $\rho < 0$.

The problem of a producer who sells its goods to consumers with ability ϵ is given by:

$$\max P_{q_{\epsilon,t}} Y_{\epsilon,t} - W_t H_{d,\epsilon,t} - R_t K_{d,\epsilon,t}. \quad (41)$$

This solution to this problem implies that the price schedule, $P_{q_{\epsilon,t}}$, is:

$$P_{q_{\epsilon,t}} = \frac{1}{A_t} \left[\alpha^{\frac{1}{1-\rho}} (q_{\epsilon,t} W_t)^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}} R_t^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}}, \quad (42)$$

and that firm's optimal labor-capital ratio is

$$\frac{H_{d,\epsilon,t}}{K_{d,\epsilon,t}} = q_{\epsilon,t}^{\frac{\rho}{\rho-1}} \left(\frac{\alpha}{1-\alpha} \frac{R_t}{W_t} \right)^{\frac{1}{1-\rho}}. \quad (43)$$

Investment goods are produced by a continuum of measure one of competitive firms according to:

$$I_t = A_t \left[\alpha \left(\frac{H_{d,inv,t}}{q_{inv}} \right)^{\rho} + (1-\alpha) K_{d,inv,t}^{\rho} \right]^{\frac{1}{\rho}}, \quad (44)$$

where $H_{d,inv,t}$ and $K_{d,inv,t}$ denote labor and capital employed in the investment sector, respectively. As in the representative-agent model, we assume that there is no quality choice in the investment sector.

Equilibrium The equilibrium definition is as follows. Households maximize utility taking the wage rate per efficiency unit of labor, the rental rate of capital, and the price-quality schedule as given. Firms maximize profits taking the wage rate per efficiency unit of labor and the rental rate of capital as given. The labor market clears, so total demand for labor equals total supply:

$$\int_{\epsilon} H_{d,\epsilon,t} d\epsilon + H_{d,inv,t} = \int_{\epsilon} H_{s,\epsilon,t} \epsilon \Gamma'(\epsilon) d(\epsilon). \quad (45)$$

The capital market clears, so total demand for capital equals total supply:

$$\int_{\epsilon} K_{d,\epsilon,t} d\epsilon + K_{d,inv,t} = K_t. \quad (46)$$

The goods market clears so, for each quality level, production equals consumption:

$$Y_{\epsilon,t} = \Gamma'(\epsilon)C_{q_{\epsilon,t}}. \quad (47)$$

Using investment as the numeraire, real output, Y_t , is given by:

$$Y_t = I_t + \int_{\epsilon} P_{q_{\epsilon,t}} C_{q_{\epsilon,t}} d\epsilon. \quad (48)$$

3.3.1 Calibration

Diamond and Saez (2011) argue that U.S. income follows approximately a Pareto distribution. Motivated by this observation, we assume that ability follows a Pareto distribution.

To solve the model numerically, we discretize the Pareto distribution using a support with n ability levels. The value of n must be large enough that there are enough agents near the lower bounds of the high- and medium-quality bins. It is the trading down by these agents in response to a negative shock that leads to changes in the market shares of the three quality categories. At the same time, because we solve for the optimal quality and quantity of the consumption good for each ability, we are limited in the number of types we can consider. We discretize the support of the ϵ distribution with $n = 100$ grid points.

We set the shape parameter of the Pareto distribution to 1.5, which is the value estimated by Diamond and Saez (2011). The ratio of the upper and lower bounds of the support of the ϵ distribution, $\epsilon_{max}/\epsilon_{min}$, is chosen to match the income ratio of the top 5 percent to the second income quintile between 2010-2014. This ratio is equal to 4.9.²⁶ We choose the weights attached by the family to a worker of skill ϵ to be the ratio of ϵ to the average skill, $\epsilon/E(\epsilon)$.

For comparability, we set ρ and α to be equal to the values reported in Table 7. We find that the resulting consumption share and labor share in this economy are close

²⁶Our data source is <http://www.taxpolicycenter.org/statistics/historical-income-distribution-all-households>

to those in the representative-agent model (75% and 61%, respectively). As in the representative-agent model, we assume that the quality parameter in the investment production function is constant. One of our calibration targets is to equate the quality parameter in the investment sector with the revenue-weighted quality values in the consumption sector in the steady state.²⁷

Interestingly, while not being a target, the labor intensities in the model come quite close to their empirical counterparts. We calculate the logarithm of the ratio of the labor intensity in the “middle-quality” market to the labor intensity in the “low-quality” market in 2007, and the logarithm of the ratio of the labor intensity in the “high-quality” market to the labor intensity in the “middle-quality” market in 2007. These values are 0.31 and 0.16 in the model and 0.38 and 0.16 in the data, respectively. We discuss the details of this calculation in Appendix G.

3.4 Results

Steady State Figure 2 illustrates some key properties of the steady state that are consistent with our empirical facts. The first panel shows that higher-skilled individuals (those with a higher ϵ) consume more quantity and higher quality. The second panel shows that higher-quality goods have higher prices and are produced with higher labor intensities.

The Effect of Trading Down Recall that about half of the observed decline in employment was due to cyclical trading down. We follow the empirical approach described in Section 2, summarized in equations (4)-(6), to compute the impact of trading down on employment in our model.

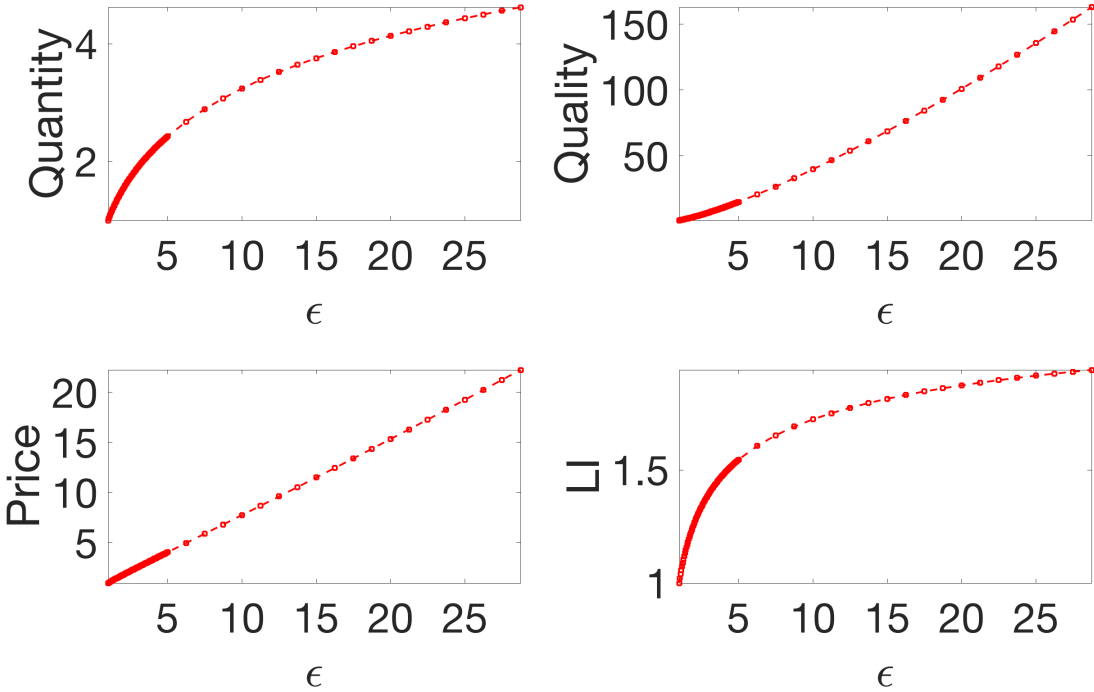
We subject the economy to a TFP shock that generates the same 7 percent increase in the share of the low-quality segment we estimate with Yelp! data. We calculate the market share of the low-, middle- and high-quality markets, and the labor intensity

²⁷For this experiment we set $\theta = 0.7$ because the model did not converge for lower values of θ .

in each of these three segments in the period when the shock occurs. We use this information to compute the counterfactual change in hours worked that would have occurred in the absence of trading down.

Overall, we find that hours worked in the consumption sector fall on impact by 4.8 percent. We find that, absent trading down, hours worked would have fallen by only 2.7 percent. These results imply that trading down accounts for about 43 percent of the observed changes in employment, which is roughly consistent with our empirical estimates.

Figure 2: Optimal Choices and Implications



Business Cycle Moments The last two columns in Panel A in Table 8 report the cyclical properties of the heterogeneous-agent model. We see that the business-cycle

properties of this model are very similar to those of the representative-agent model. The heterogeneous-agent model generates amplification in hours worked, comovements of hours worked in the consumption and investment sectors, and a countercyclical labor wedge.

4 Conclusion

We document two facts. First, during the Great Recession consumers traded down in the quality of the goods and services they consumed. Second, lower-quality products are generally less labor intensive, so trading down reduces the demand for labor. Our calculations suggest that trading down accounts for about half of the decline in employment during the Great Recession.

We study a general equilibrium model in which consumers choose both the quantity and quality of consumption. We show that in response to a decline in TFP the model generates trading down in quality and declines in employment that are broadly consistent with our empirical findings.

The presence of quality choice improves the model's performance along two dimensions. First, cyclical changes in the quality of what is consumed amplify the effects of shocks to the economy, resulting in higher variability in hours worked. Second, the model generates comovement between labor employed in the consumption and investment good sectors.

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Appendix

In this appendix, we accomplish four main tasks. First, we provide more details about the construction of the Yelp! and PPI data set. Second, we report additional robustness checks on the estimates presented in the main text regarding the effects of trading down on employment. Third, we discuss the algorithm used to calibrate the heterogeneous-agent model. Finally, we discuss a version of the representative model with Calvo-style price stickiness.

A Yelp! data

We scraped data for Yelp in April 2014. For firms that own more than one brand, we compute the average price category for each brand and then compute the average price category for the firm, weighting each brand by their sales volume. One concern about this procedure is that we might be averaging high-quality and low-quality brands. In practice, this situation is rare: 73 percent of the firms in our sample have a single brand. For multi-brand firms, 54 percent have all their brand in the same price category. For example, the firm Yum! Brands owns three brands (Taco Bell, KFC, and Pizza Hut), but they are all in the same price category (low price). For robustness, we redid our analysis including only firms that either have a single brand or have all their brands in the same price category. We obtain results that are very similar to those we obtain for the whole universe of firms.

In merging the data with Compustat for companies with operations outside of the U.S., we use the information on sales by business region to compute U.S. sales. We also use the break down of employment by business region to compute labor intensity in the U.S. We exclude from our sample manufacturing firms for which this breakdown is not available. For retail firms, foreign operations are generally small, so we include companies with foreign operations in our sample. As a robustness check, we redo our analysis excluding these companies. The results are similar to those we obtain for the

full sample.

Table 9 presents some description of the data used to analyze quality shifts in expenditure in five retail sectors. It describes the data source (column I), the number of firms covered in the sample in 2007 (II), the average annual firm sales revenue (III), and the percent of the overall sector sales that our sample covers (IV).

Table 9: Data Sample Description

Sectors	Data Source (I)	Number of Firms (II)	2007 Annual Sales	
			of Average Firm (\$m) (III)	% of U.S. Sector (IV)
Apparel	Compustat and company reports	54	1,648	41%
Grocery stores	Compustat and company reports	9	34,348	56%
Restaurants	Compustat and company reports	74	1,012	19%
Home furnishings	Compustat	41	4,750	39%
General Merchandise	U.S. Census	n.a.	n.a.	100%

Note: This table describes for each sector the data source used (I), the number of firms within the sample (II), and the average annual sales of each firm (III). (IV) reports the share of the sales of the entire sector that our data set covers.

B PPI

Using the PPI data presents two challenges. First, firms in the same industry report prices that correspond to different units of measurement, e.g. some firms report price per pound, others price per dozen. To circumvent this problem, we first convert prices into a common metric whenever possible (for example, converting ounces into pounds). We then compute the modal unit of measurement for each 6-digit NAICS category and

restrict the sample to the firms that report prices for this model unit. This filtering procedure preserves 2/3 of the original data, which is comprised of 16,491 establishments out of a sample of about 25,000 establishment surveyed by the PPI. Some establishments are excluded because we only include items that are recorded at the modal unit of measure within the 6-digit product category.

Second, some of the firms included in the PPI data offshore their production, so their reported employment does not generally include production workers. It includes primarily head-office workers and sales force in the U.S. Using information in the firms' annual reports, we exclude firms that have most of their production offshore. The resulting data set preserves over half of the merged PPI/Compustat data.

In order to construct a quality measure for each firm, we proceed as follows. For each product k that establishment e sells in year t , we calculate its price, p_{ket} , relative to the median price in the industry for product k in year t , \bar{p}_{kt} :

$$R_{ket} = p_{ket}/\bar{p}_{kt}.$$

Our analysis is based on products defined at a six-digit industry code level and then further disaggregated by product type. Therefore, although the results are presented at a 2-digit level, all relative prices are defined at a narrow 6-digit level for comparability. For details of this disaggregation, see Table 11 of the BLS PPI Detailed Report. The variable \bar{p}_{kt} is a shipment-value weighted average within the product category. For reporting purposes, we aggregate the results to the two-digit level. The aggregation is based on shipment revenue.

For single-product establishments, we use this relative price as the measure of the quality of the product produced by establishment e . For multi-product establishments, we compute the establishment's relative price as a weighted average of the relative price

of different products, weighted by shipment revenue in the base year (w_{ke}):²⁸

$$R_{et} = \sum_{k \in \Omega} w_{ke} R_{ket}.$$

where Ω denotes the set of all products in the PPI data set that we examined.

C Substitution across categories

Our main analysis focuses on trading-down behavior within categories. We also examined substitution across consumption categories and the effect on employment. We use data from the CEX and NIPA’s PCE. We consider 31 different consumption categories (see Figure 3).

We examine the effect of across-category substitution on employment in two steps. First, we construct the labor intensity for each consumption category. We match CEX consumption categories with the NIPA PCE commodity definitions. This matching allows us to use the Input/Output commodity-level data to construct labor intensity measures for each consumption category. Second, we compute the change in budget share for each consumption category over 2007-2012 for the average household. To isolate the cyclical component of the budget reallocation across consumption categories, we estimate elasticities of the category budget shares to total household expenditure. We then multiply the elasticities by the actual change in household expenditure to obtain the change in budget allocation for each category.

We derive shifts in expenditure associated with the drop in household income that occurs during the recession by estimating the following Engel-curve elasticities:

$$w_{ht}^k = \alpha^k + \beta^k \ln(X_{ht}) + \sum_j \gamma_j \ln(P_{jt}) + \theta_{ht}^k \cdot Z_{ht} + \epsilon_{ht}^k, \quad (49)$$

²⁸This approach for constructing firm-level price indices is similar to that used by Gorodnichenko and Weber (2014), and Gilchrist et al (2014). However, we compute relative prices using a much finer product definition than these authors. We refer the reader to Section II in Gorodnichenko and Weber (2014) for a discussion of how the BLS samples products and firms.

where w_{ht}^k is the budget share allocated to category k by household h at time t ; X_{ht} is total household expenditure; and P_{jt} is the price index of each expenditure category j at time t . The variable Z_{ht} is a vector of household demographics variables, including the age and square of the age of the head of household, dummies based on the number of earners (<2,2+), and household size (<2,3-4,5+). The error term is denoted by ϵ_{ht}^k . We estimate equation (49) using household sample weights given in the CEX data based on the 1980-2012 waves of the CEX Surveys. The coefficient β^k gives the fraction change in budget share allocation to expenditure category k , given a 100 percentage point change in total household expenditure.²⁹

²⁹There are potential problems with estimating equation (49). For instance, mismeasurement of individual goods may be cumulated into total expenditure, which would bias the estimated coefficients. See Aguiar and Bils (2015) for a discussion of these and other measurement issues. For robustness, we also use the standard approach of instrumenting total expenditure with total income reported by the household. The estimated elasticities yield similar results to our base estimation without instrumenting.

Figure 3: Scatter plot of labor intensity and elasticities across sectors

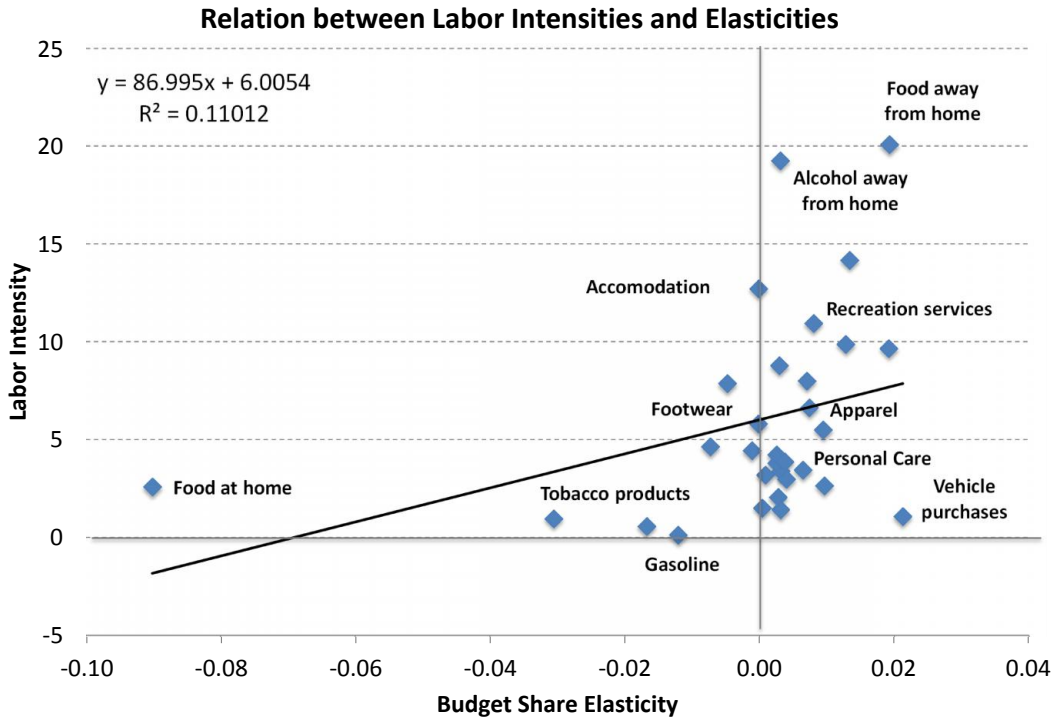


Figure 3 shows that there is a low positive correlation between a category's expenditure elasticity and its labor intensity. To examine the effect of across-category substitution on employment, we perform a similar exercise as our trading-down calculation. We compute the change in the number of employed workers between 2007 and 2012 due to changes in the shares of the expenditures categories, holding fixed the measured labor intensity.³⁰ This exercise yields a negligible effect of the substitution across categories on aggregate employment, in contrast to our findings of large effects of quality trading-down within categories.

³⁰We also did the computation using actual observed changes in expenditure allocations in the NIPA PCE data and found similar results.

D Results based on alternative labor intensities

As a robustness check, Tables 10 and Tables 11 report our calculation using our second measure of labor intensity, employment/gross margin. For the period 2007-12, the change in employment accounted for by trading down represents 37 percent of the fall in employment. For the period 2007-09, this fraction represents 28 percent of the fall in employment.

Table 10: Market shares and Labor Intensity: 2007

Industry	\$m Sales - COGS	Labor intensity			Market share		
		Low	Middle	High	Low	Middle	High
Home furnishing and appliances	248,751	14.05	15.27	27.90	1%	95%	4%
Grocery stores	353,472	16.23	18.06	21.22	16%	80%	4%
Food services and drinking places	270,390	68.08	131.75	93.70	58%	36%	6%
Clothing stores	119,579	27.34	24.34	29.59	8%	76%	16%
General merchandise stores	425,163	16.23	24.26	19.49	56%	25%	19%
Total	1,417,355	26.68	41.65	36.41	33%	58%	10%

Note: This table depicts the 2007 sales less cost of goods sold, labor intensity and market share for different retail sectors. The last row is a value-added-weighted measure. Sales and cost of goods sold are from the US Census of Retail Trade, and the labor intensities and market shares are from Compustat. Labor intensity is defined as the number of employees (thousands) per sales less cost of goods sold (millions), and the market share are the share of sales less cost of goods sold for each price tier within each sector. Price tiers are denoted by low, middle and high, and are based on Yelp! classifications of prices \$, \$\$, and \$\$\$, respectively. See text for more information.

Table 11: Market shares and Labor Intensity: 2012

Industry	\$m Sales - COGS	Labor intensity			Market share		
		Low	Middle	High	Low	Middle	High
Home furnishing and appliances	212,650	8.25	14.68	15.57	1%	93%	5%
Grocery stores	410,500	14.62	16.61	16.33	21%	72%	7%
Food services and drinking places	313,782	53.53	85.08	125.80	70%	26%	4%
Clothing stores	129,444	21.20	21.62	26.68	11%	79%	10%
General merchandise stores	473,441	16.23	24.26	19.49	65%	21%	14%
Total	1,539,817	22.72	33.07	40.37	41%	51%	8%

Note: This table depicts the 2012 sales less cost of goods sold, labor intensity and market share for different retail sectors. The last row is a value-added-weighted measure. Sales and cost of goods sold are from the US Census of Retail Trade, and the labor intensities and market shares are from Compustat. Labor intensity is defined the ratio the number of employees (thousands) per sales less cost of goods sold (millions), and the market share are the share of sales less cost of goods sold for each price tier within each sector. Price tiers are denoted by low, middle and high, and are based on Yelp! classifications of prices \$, \$\$, and \$\$\$, respectively. See text for more information.

E Results based on alternative definition of the recession

As a robustness check, Table 12 and 13 report our calculation using the period 2007-09 for the retail sector and manufacturing sectors, respectively. For the period 2007-09, the change in employment accounted for by trading down represents 20 percent and 16 percent of the fall in employment for the retail sector and manufacturing sectors, respectively.

Table 12: Market shares and Labor Intensity: 2009

Industry	\$m Sales in 2009	Labor intensity			Market share		
		Low	Middle	High	Low	Middle	High
Grocery stores	558,337	n.a.	4.40	4.83	n.a.	95%	5%
Food services and drinking places	690,225	1.83	4.01	5.91	44%	51%	5%
Clothing stores	621,902	12.34	21.67	21.97	43%	47%	10%
General merchandise stores	255,052	6.69	8.40	9.27	19%	71%	10%
Total	673,729	3.61	5.74	7.24	76%	16%	9%

Note: This table depicts the 2009 total sales, labor intensity and market share for different retail sectors. The last row is a sales-weighted measure. Sales are from the US Census of Retail Trade, and the labor intensities and market shares are from Compustat. Labor intensity is defined as the number of employees per million dollars of sales and the market share are the share of sales for each price tier within each sector. Price tiers are denoted by low, middle and high, and are based on Yelp! classifications of prices \$, \$\$, and \$\$\$, respectively. See text for more information.

Table 13: Market shares and Labor Intensity: 2009

Industry	\$m Sales in 2009	Labor Intensity			Market share		
		Low	Middle	High	Low	Middle	High
31	824,269	0.48	3.27	n.a.	34%	66%	n.a.
32	1,201,015	3.10	3.36	4.74	27%	47%	26%
33	1,902,289	1.98	2.96	3.86	38%	57%	5%
Total	3,927,573	2.01	3.15	3.32	33%	54%	13%

Note: This table depicts the 2009 total sales, labor intensity and market share for different manufacturing sectors. The last row is a sales-weighted measure. Sales are from Census, and the labor intensities and market shares are from Compustat. Labor intensity is defined as the number of employees per million dollars of sales and the market share are the share of sales for each price tier within each sector. Price tiers are denoted by low, middle and high, and are based on firm-level producer price data. See text for more information.

F A sticky-price model

In this appendix, we show that the same mechanism that amplifies real shocks also amplifies nominal shocks. We do so by embedding quality choice in a model with Calvo-style sticky prices. To highlight the role of quality choice in a parsimonious way, we abstract from capital accumulation.

F.1 The household problem

The representative household maximizes expected life-time utility defined in equation (22). The two constraints of the household problem are:

$$P_{q,t}C_t + B_{t+1} = B_t(1 + R_t) + W_tH_t, \quad (50)$$

and

$$E_0 \lim_{t \rightarrow \infty} B_{t+1} / [(1 + R_0)(1 + R_1) \dots (1 + R_t)] \geq 0.$$

Here, B_{t+1} denotes the number of one-period nominal bonds purchased at time t , and R_t is the one-period nominal interest rate.

The first-order conditions for the household are two equations associated with the static model (equations (28) and (29)), together with the following additional condition:

$$\lambda_t = E_t \beta \lambda_{t+1} (1 + R_{t+1}), \quad (51)$$

where λ_t is the Lagrange multiplier associated with the budget constraint (50).

Final good firms

The final good is produced by competitive firms using a continuum of intermediate goods, $Y_t^i(q_t)$:

$$Y(q_t) = \left(\int_0^1 [Y_t^i(q_t)]^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1. \quad (52)$$

We assume that producing a final good of quality q_t requires that all intermediate inputs have quality q_t .

The problem of firms in the final-goods sector is:

$$\max P(q_t)Y(q_t) - \int_0^1 P_t^i(q_t) Y_t^i(q_t) di,$$

where $P_t^i(q_t)$ is the price of intermediate good i . The first-order conditions of the firms' problem imply:

$$P_t^i(q_t) = P(q_t) \left[\frac{Y(q_t)}{Y_t^i(q_t)} \right]^{\frac{1}{\varepsilon}}, \quad (53)$$

where P_t is the price of the homogeneous final good. Using the first-order conditions of the firms' problem we can express this price as:

$$P(q_t) = \left(\int_0^1 P_t^i(q_t)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}.$$

Intermediate Good Firms

The i th intermediate good is produced by a monopolist using a technology that is the limiting case of the flexible price model without capital:

$$Y_t^i(q_t) = \frac{A_t}{q_t} H_t^i(q_t). \quad (54)$$

Here, $H_t^i(q_t)$ denotes the labor employed by the i^{th} monopolist who is producing a product of quality q_t . If prices were flexible, the optimal price for the i th monopolist would be given by the usual mark-up formula:

$$P_t^i(q_t) = \frac{\varepsilon}{\varepsilon - 1} \frac{W_t}{A_t} q_t.$$

However, producers are subject to Calvo-style pricing frictions. We assume that monopolists post a pricing schedule that is linear in q_t :

$$P_t^i(q_t) = \mu_t^i q_t.$$

The monopolist can re-optimize the slope of the pricing schedule, μ_t^i , with probability $1 - \xi$. With probability ξ , the firm has to post the same price schedule as in the previous period:

$$P_t^i(q_t) = \mu_{t-1}^i q_t.$$

We denote by $\tilde{\mu}_t^i$ the optimal price-quality schedule for firms that have the opportunity to re-optimize μ_t^i at time t . Since only a fraction $1 - \xi$ of the firms have this opportunity, the aggregate price level is given by:

$$P(q_t) = \mu_t q_t, \tag{55}$$

where

$$\mu_t = \left[(1 - \xi) (\tilde{\mu}_t^i)^{1-\varepsilon} + \xi \mu_{t-1}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{56}$$

Firm i chooses $\tilde{\mu}_t^i$ to maximize its discounted profits, given by:

$$E_t \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} \left[P_t^i(q_{t+j}) Y_t^i(q_{t+j}) - W_{t+j} H_t^i(q_{t+j}) \right], \tag{57}$$

subject to the Calvo price-setting friction, the production function, and the demand function for $Y_t^i(q_t)$.

Given that the price schedule is linear in quality, the demand function for $Y_t^i(q_t)$ can be written as:

$$\tilde{\mu}_t^i = \frac{P(q_t)}{q_t} \left[\frac{Y(q_t)}{Y^i(q_t)} \right]^{\frac{1}{\varepsilon}}. \tag{58}$$

Since the price schedule chosen in period t is only relevant along paths in which the firm cannot reoptimize its schedule, the firm's problem is given by:

$$E_t \sum_{j=0}^{\infty} \beta^j \xi^j \lambda_{t+j} q_{t+j}^{1-\varepsilon} \left[P(q_{t+j}) \right]^\varepsilon Y(q_{t+j}) \left[(\tilde{\mu}_t^i)^{1-\varepsilon} - W_{t+j} \frac{(\tilde{\mu}_t^i)^{-\varepsilon}}{A_{t+j}} \right].$$

Following the usual procedure to solve Calvo-style models, we obtain the following modified Phillips Curve:

$$\hat{\pi}_t = \frac{(1 - \beta\xi)(1 - \xi)}{\xi} \theta \hat{H}_t + \beta \hat{\pi}_{t+1}, \tag{59}$$

and the following intertemporal Euler condition:

$$\hat{H}_{t+1} - \hat{H}_t = \frac{-rr + R_{t+1} - \hat{\pi}_{t+1}}{\theta}. \quad (60)$$

It is useful to compare these two equations with those associated with a version of the model with no quality choice:

$$\hat{\pi}_t = \frac{(1 - \beta\xi)(1 - \xi)}{\xi} \hat{H}_t + \beta\hat{\pi}_{t+1}, \quad (61)$$

$$\hat{H}_{t+1} - \hat{H}_t = -rr + R_{t+1} - \hat{\pi}_{t+1}. \quad (62)$$

Comparing these two sets of equations we see that, since $\theta < 1$, the model with quality choice produces a higher response to monetary shocks than the standard model. This difference in amplification is illustrated in Figure 4.³¹

To understand this difference, it is useful to consider first a flexible-price version of the model without quality choice. In this model, if the central bank raises the nominal interest rate, the price level falls and expected inflation rises, leaving the real interest rate unchanged. As a result, the change in the nominal interest rates has no effect on real variables.

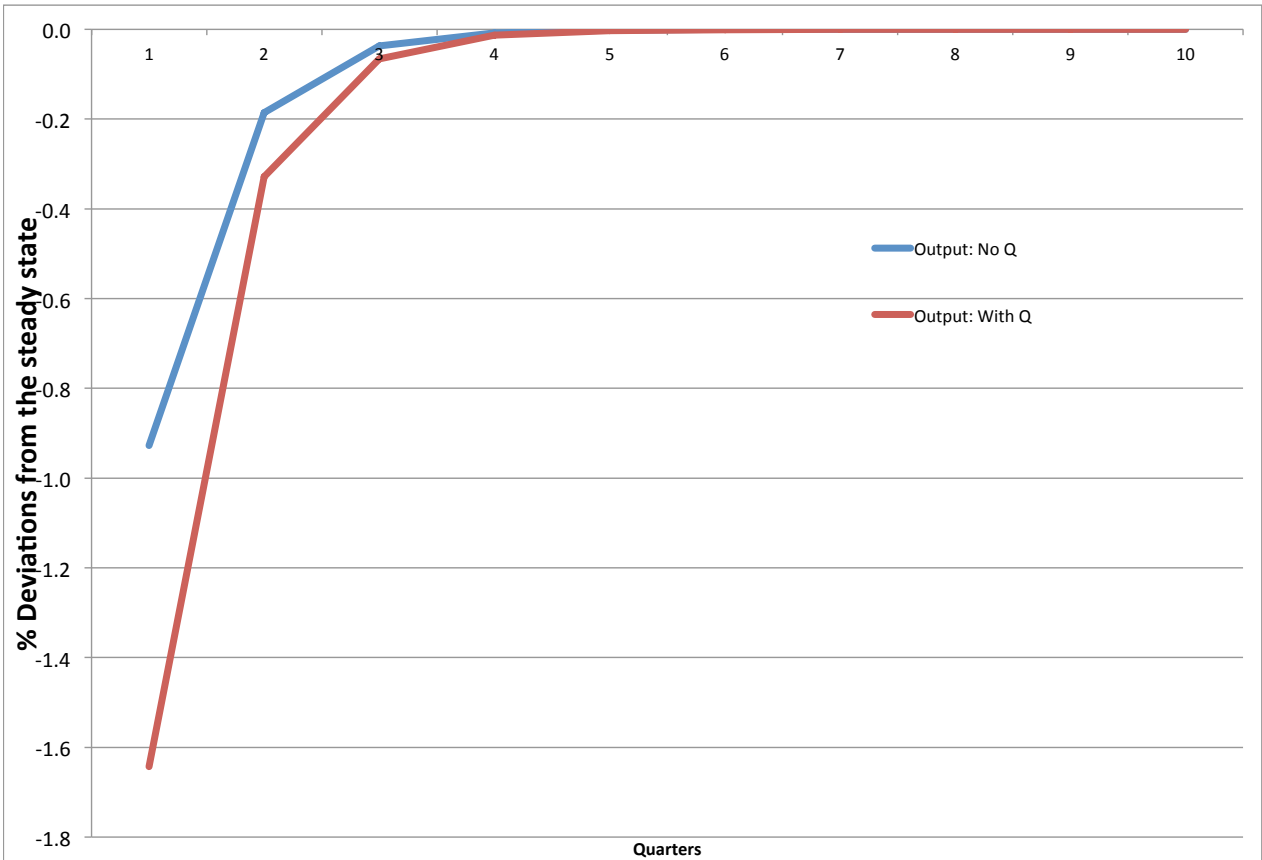
Now consider the model with sticky prices but no quality choice. When the central bank raises the nominal interest rate, only a few firms can lower prices and so the price decline is spread over time. As a result, the real interest rate rises. This rise in the real interest rate makes households want to consume less today and more in the future. The current demand for consumption falls and, since employment is demand determined, hours fall.

The key difference between the model with and without quality choice is that the former exhibits a stronger response of the labor supply to shocks. As a consequence, the wage rate has to fall more to clear the labor market than in the standard model.

³¹We use the same parameter values for β , θ , and ν as in the flexible price model. We set $\varepsilon = 0.75$ so firms optimize their price schedule on average every four quarters. We use a Taylor rule with a 1.5 coefficient on inflation.

The rate of inflation becomes more negative than in the standard model, as firms lower prices in response to the lower labor costs. This higher rate of deflation implies that the real interest rate is higher in the model with quality choice than in the standard model. This higher real interest rate induces households to postpone consumption, generating a larger fall in consumption than in the standard model. As a result, employment also falls by more in the quality-choice model than in the standard model.

Figure 4: Impulse Response Functions for a Negative Monetary Shock



G Calibration of Heterogeneous Agent Model

In this appendix we discuss the calculations of the labor intensities in the model. Given the parameter values, we solve for the optimal conditions of the households and the firms, and check whether the market clearing condition for hours worked and capital (equations (45) and (46)) hold. Once we clear the two factor markets, we find the quality threshold, q_{low} , below which we have 35 percent of total consumption, i.e.

$$\sum_{j=q_{min}}^{q_{low}} \frac{P_{j\epsilon} Y_j}{\sum_{z=q_{min}}^{q_{max}} P_{z\epsilon} Y_z} = 0.35.$$

Similarly, we find the quality level q_{middle} such that consumption in the middle quality segment accounts for 58 percent of total consumption, i.e.,

$$\sum_{j=q_{low}}^{q_{middle}} \frac{P_{j\epsilon} Y_j}{\sum_{z=q_{min}}^{q_{max}} P_{z\epsilon} Y_z} = 0.58.$$

The remaining 7 percent correspond to the high-quality segment. These values are the market shares in our data. As in our empirical analysis, we then calculate the revenue-weighted average of the labor intensity within each of the three quality categories, i.e.

$$\begin{aligned} LI_{low} &= \sum_{j=q_{min}}^{q_{low}} \left(\frac{H_j}{P_{j\epsilon} Y_j} \right) \left(\frac{P_{j\epsilon} Y_j}{\sum_{z=q_{min}}^{q_{low}} P_{z\epsilon} Y_z} \right), \\ LI_{middle} &= \sum_{j=q_{low}}^{q_{middle}} \left(\frac{H_j}{P_{j\epsilon} Y_j} \right) \left(\frac{P_{j\epsilon} Y_j}{\sum_{z=q_{low}}^{q_{middle}} P_{z\epsilon} Y_z} \right), \\ LI_{high} &= \sum_{j=q_{middle}}^{q_{max}} \left(\frac{H_j}{P_{j\epsilon} Y_j} \right) \left(\frac{P_{j\epsilon} Y_j}{\sum_{z=q_{middle}}^{q_{max}} P_{z\epsilon} Y_z} \right). \end{aligned}$$

H Proofs

H.1 Proposition 1

Consider first the model without quality choice. The production function is given by

$$C = [\alpha (H)^\rho + (1 - \alpha) K^\rho]^\frac{1}{\rho} \tag{63}$$

The producer's problem implies that the ratio of the optimal demand for hours worked and capital is given by

$$\frac{H}{K} = \left(\frac{\alpha}{1-\alpha} \frac{r}{W} \right)^{\frac{1}{1-\rho}} \quad (64)$$

and the optimal price is given by

$$P = \left[\alpha^{\frac{1}{1-\rho}} (W)^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}} (r)^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}} \quad (65)$$

Income is spent solely on consumption so

$$\xi = P \times C$$

Since this is a partial-equilibrium setting, where the rental price and the wage are constant, the price of the consumption good is also constant. As a result, consumption moves one-to-one with income. Substituting equation (64) into equation (63) we obtain

$$C = K \left[\alpha \left(\left(\frac{\alpha}{1-\alpha} \frac{r}{W} \right)^{\frac{1}{1-\rho}} \right)^{\rho} + (1-\alpha) \right]^{\frac{1}{\rho}}.$$

We proceed by log-linearizing the equilibrium conditions. Denoting the percentage deviations of a variable from steady state by a circumflex, it follows that

$$\hat{C} = \hat{K} = \hat{\xi}.$$

From equation (64) it follows that hours worked move one-to-one with consumption and thus with income, i.e.

$$\hat{H} = \hat{\xi}.$$

Thus, in the model without quality, the elasticity of hours worked with respect to income is 1. In what follows we show that this elasticity in the model with quality choice is always greater than 1.

Recall that the production function in the model with quality is given by

$$C = \left[\alpha \left(\frac{H}{q} \right)^{\rho} + (1-\alpha) K^{\rho} \right]^{\frac{1}{\rho}} \quad (66)$$

and that the resulting factor maximization implies that the ratio of the optimal demand for hours worked and capital is given by

$$\frac{H}{K} = q^{\frac{\rho}{\rho-1}} \left(\frac{\alpha}{1-\alpha} \frac{r}{W} \right)^{\frac{1}{1-\rho}} \quad (67)$$

The resulting optimal price is given by

$$P = \left[\alpha^{\frac{1}{1-\rho}} (qW)^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}} (r)^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}} \quad (68)$$

For future reference it is useful to write this expression as

$$P = \left[(r)^{\frac{\rho}{\rho-1}} \left\{ \alpha^{\frac{1}{1-\rho}} \left(\frac{qW}{r} \right)^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}} \right\} \right]^{\frac{\rho-1}{\rho}}$$

$$P = r \left[\alpha^{\frac{1}{1-\rho}} \left(\frac{qW}{r} \right)^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}} \right]^{\frac{\rho-1}{\rho}} \quad (69)$$

From the household's first-order conditions for quality and quantity it follows that

$$C = \exp \left(\frac{1}{1-\theta} \frac{qP'_q}{P} \right) \quad (70)$$

The price elasticity is given by

$$\frac{P'_q q}{P} = \eta_{P,q} = \frac{\alpha^{\frac{1}{1-\rho}} (qW)^{\frac{\rho}{\rho-1}}}{\left[\alpha^{\frac{1}{1-\rho}} (qW)^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}} (r)^{\frac{\rho}{\rho-1}} \right]} \quad (71)$$

Linearizing the above equilibrium condition results in the following five equations.

First, the linearized budget constraint

$$\hat{\xi} = \hat{P} + \hat{C}. \quad (72)$$

Second, a linearized demand for the factors of production

$$\hat{H} = \frac{\rho}{\rho-1} \hat{q} + \hat{K}. \quad (73)$$

Third, a linearization of the first-order condition for quality and quantity combined with the expression for the linearized price elasticity,

$$\hat{C} = \left(\frac{1}{1-\theta} \right) \eta_{P,q} \widehat{\eta_{P,q}} = \left(\frac{1}{1-\theta} \right) \frac{\alpha^{\frac{1}{1-\rho}} \left(\frac{qW}{r} \right)^{\frac{\rho}{\rho-1}} (1-\alpha)^{\frac{1}{1-\rho}}}{\left[\alpha^{\frac{1}{1-\rho}} \left(\frac{qW}{r} \right)^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}} \right]^2} \frac{\rho}{\rho-1} (\hat{q}). \quad (74)$$

Fourth, a linearized production function

$$\hat{C} = \frac{\alpha \left(\frac{H}{q} \right)^\rho (\hat{H} - \hat{q}) + (1-\alpha) K^\rho \hat{K}}{\alpha \left(\frac{H}{q} \right)^\rho + (1-\alpha) K^\rho}. \quad (75)$$

And finally, a linearized price function

$$\hat{P} = \frac{\alpha^{\frac{1}{1-\rho}} \left(\frac{qW}{r} \right)^{\frac{\rho}{\rho-1}}}{\left[\alpha^{\frac{1}{1-\rho}} \left(\frac{qW}{r} \right)^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}} \right]} \hat{q}. \quad (76)$$

Combining equations (72), (74), and (76), we can relate changes in income to the optimal change in quality;

$$\hat{q} = \frac{1}{\frac{\alpha^{\frac{1}{1-\rho}} \left(\frac{qW}{r} \right)^{\frac{\rho}{\rho-1}}}{\left[\alpha^{\frac{1}{1-\rho}} \left(\frac{qW}{r} \right)^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}} \right]} \left(1 + \frac{\left(\frac{1}{1-\theta} \right) (1-\alpha)^{\frac{1}{1-\rho}}}{\left[\alpha^{\frac{1}{1-\rho}} \left(\frac{qW}{r} \right)^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}} \right]} \frac{\rho}{\rho-1} \right)} \widehat{\xi} \quad (77)$$

Equation (77) relates the exogenous change in income to a change in quality. Note that all the coefficients are positive; i.e. a positive (negative) change in income leads to quality increasing (decreasing).

Combining the remaining equations and doing some algebraic manipulations we obtain the following relation between the optimal number of hours demanded by the firm and the optimal quality choice of the consumer

$$\hat{H} = \frac{\hat{q} \left(\frac{\alpha^{\frac{1}{1-\rho}} \left(\frac{qW}{r} \right)^{\frac{\rho}{\rho-1}} (1-\alpha)^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{\rho}{\rho-1}}}{(1-\alpha)^{\frac{\rho}{\rho-1}}} + \left(\frac{1}{1-\theta} \right) \frac{\alpha^{\frac{1}{1-\rho}} \left(\frac{qW}{r} \right)^{\frac{\rho}{\rho-1}} (1-\alpha)^{\frac{1}{1-\rho}}}{\left[\alpha^{\frac{1}{1-\rho}} \left(\frac{qW}{r} \right)^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}} \right]} \frac{\rho}{\rho-1} \right)}{\left(\alpha^{\frac{1}{1-\rho}} \left(\frac{qW}{r} \right)^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}} \right)} \quad (78)$$

Combining equation (77) and equation (78) results in the following relation between hours worked and income changes

$$\hat{H} = \hat{\xi} \left(1 + \frac{\left(\frac{(1-\alpha)^{\frac{1}{1-\rho}}}{\alpha^{\frac{1}{1-\rho}} \left(\frac{gW}{r} \right)^{\frac{\rho}{\rho-1}} \frac{\rho}{\rho-1}} \right)}{\left(1 + \frac{\left(\frac{1}{1-\theta} \right) (1-\alpha)^{\frac{1}{1-\rho}}}{\left[\alpha^{\frac{1}{1-\rho}} \left(\frac{gW}{r} \right)^{\frac{\rho}{\rho-1}} + (1-\alpha)^{\frac{1}{1-\rho}} \right] \frac{\rho}{\rho-1}} \right)} \right) \quad (79)$$

Recall that $\rho < 0$. This condition implies that the coefficient on the right hand side is always greater than 1. QED.

H.2 Proposition 2

As in the variable-quality model, income is spent solely on consumption:

$$\xi = P_1 C_1 + P_2 C_2.$$

Log-linearizing this equation, we obtain

$$\hat{\xi} = s_{c1} \hat{C}_1 + (1 - s_{c1}) \hat{C}_2, \quad (80)$$

where s_{c1} denotes the share of expenditure of good 1. We follow the same steps used in the proposition 1:

$$C_1 = K_1 \times X_1,$$

$$C_2 = K_2 \times X_2,$$

where X_1, X_2 are constant parameters. In equilibrium:

$$\hat{C}_1 = \hat{K}_1,$$

$$\hat{C}_2 = \hat{K}_2.$$

Linearizing the equation for the optimal use of the two production factors we find that the response of hours is

$$\hat{C}_1 = \hat{H}_1$$

$$\widehat{C}_2 = \widehat{H}_2$$

This result is independent of the form of the utility function. Where the preferences matter is in the response of consumption to the income shock.

Consider first the case of homothetic preferences. Then the share of expenditure of each good is constant, so from

$$P_1 C_1 = s_{c1} \xi \tag{81}$$

it follows that

$$\begin{aligned} \widehat{C}_1 &= \widehat{\xi}. \\ \widehat{C}_2 &= \widehat{\xi}. \end{aligned}$$

Using the fact that $H = H_1 + H_2$ we obtain

$$\widehat{H} = \frac{H_1}{H} \widehat{H}_1 + \frac{H_2}{H} \widehat{H}_2 = \frac{H_1}{H} \widehat{\xi} + \frac{H_2}{H} \widehat{\xi} = \widehat{\xi}. \tag{82}$$

So, the elasticity of hours worked with respect to income is one. In what follows we show that this elasticity always exceeds one in the Stone-Geary model.

Log-linearizing equation (81) we obtain

$$\widehat{C}_1 = \widehat{s}_{c1} + \widehat{\xi}.$$

We know from above that

$$\widehat{C}_1 = \widehat{s}_{c1} + \widehat{\xi} = \widehat{H}_1$$

and

$$\widehat{C}_2 = \left(\frac{-s_{c1}}{1-s_{c1}} \right) \widehat{s}_{c1} + \widehat{\xi} = \widehat{H}_2$$

Combining these two equations with equation (82) to obtain

$$\widehat{H} = \frac{H_1 \left(\widehat{s}_{c1} + \widehat{\xi} \right) + H_2 \left(\widehat{\xi} - \frac{s_{c1}}{1-s_{c1}} \widehat{s}_{c1} \right)}{H} = \widehat{\xi} + \frac{\widehat{s}_{c1}}{H} \left(H_1 - \frac{s_{c1}}{1-s_{c1}} H_2 \right) = \widehat{\xi} + \frac{\widehat{s}_{c1}}{H} (H_1 - s_{c1} H) \tag{83}$$

Since good 1 is the inferior good we know that in response to an increase in income $\widehat{s_{c1}} < 0$. Hence if $H_1 - s_{c1}H < 0$ then hours would increase more in this model than in the model of homothetic preferences. To see the conditions under which this case holds we define

$$x = 1 + \frac{s_{c2}}{s_{c1}} \times \frac{LI_2}{LI_1} = 1 + \frac{P_2C_2}{P_1C_1} \times \frac{LI_2}{LI_1} = 1 + \frac{P_2C_2}{P_1C_1} \times \frac{\frac{H_2}{P_2C_2}}{\frac{H_1}{P_1C_1}} = 1 + \frac{H_2}{H_1} = \frac{H}{H_1}$$

where LI_1, LI_2 denote the labor intensity in the two consumption goods. Thus,

$$H_1 - s_{c1}H = H_1 - s_{c1}xH_1 = H_1(1 - s_{c1}x).$$

Then

$$s_{c1}x = s_{c1} + s_{c2} \times \frac{LI_2}{LI_1} > 1$$

since $\frac{LI_2}{LI_1} > 1$. It follows that $(1 - s_{c1}x) < 0$, implying that $H_1 - s_{c1}H < 0$. Hence, the two conditions necessary so that the elasticity of hours with respect to income exceeds one is that the preferences are nonhomothetic and that the inferior good is less labor intensive than the superior good. QED.

I Balanced growth

In this appendix, we show that a modified version of the flexible price model is consistent with balanced growth.

The economy's planner's problem is:

$$\max U = \sum_{t=0}^{\infty} \beta^t \left\{ \frac{q_t^{1-\theta}}{1-\theta} \frac{(C_t/X_t)^{1-\sigma} - 1}{1-\sigma} - X_t^{1-\theta} \phi \frac{N_t^{1+\nu}}{1+\nu} \right\},$$

s.t.

$$C_t = A_t \left[\alpha \left(\frac{X_t^2 N_t^C}{q_t} \right)^\rho + (1-\alpha) (K_t^C)^\rho \right]^{\frac{1}{\rho}}, \quad (84)$$

$$K_{t+1} = A_t \left[\alpha (X_t N_t^I)^\rho + (1-\alpha) (K_t^I)^\rho \right]^{\frac{1}{\rho}} + (1-\delta)K_t, \quad (85)$$

$$N_t^C + N_t^I = N_t,$$

$$K_t^C + K_t^I = K_t.$$

Making the utility function compatible with balanced growth requires two modifications. The first is to scale the disutility of labor by $X_t^{1-\theta}$. Without this modification, labor effort increases over time. We can interpret $X_t^{1-\theta}$ as representing technical progress in home production. The second modification is that $C_t^{1-\sigma}$ needs to be replaced with $(C_t/X_t)^{1-\sigma}$. This modification resembles Abel's (1990) external habit formulation.

In order for quantities to grow at a constant rate in the steady state we need, as usual, labor-augmenting technical progress. If the production function for consumption takes the form:

$$C_t = A_t \left[\alpha \left(\frac{X_t N_t^C}{q_t} \right)^\rho + (1 - \alpha) (K_t^C)^\rho \right]^{\frac{1}{\rho}},$$

C_t grows in the steady state at the same rate as X_t but the quality of the goods consumer, q_t , remains constant. In order for both C_t and q_t to grow at the same rate as X_t we need labor-augmenting technical progress to depend on X_t^2 as in equation (84).

It is easy to see that the resource constraints (84) and (85) are consistent with C_t , q_t , and K_t growing at the same rate as X_t .

To show that the modified model is consistent with balanced growth, we use the first-order conditions for the planner's problem. Combining the first-order condition for C_t and q_t we obtain:

$$\frac{(C_t/X_t)^{1-\sigma} - 1}{1 - \sigma} = \frac{1}{1 - \theta} \frac{q_t}{C_t} A_t \left[\alpha \left(\frac{N_t^C}{q_t/X_t} \right)^\rho + (1 - \alpha) (K_t^C/X_t)^\rho \right]^{\frac{1-\rho}{\rho}} \alpha (N_t^C)^\rho (X_t/q_t)^{1+\rho}.$$

Combining the first-order condition for N_t and C_t we obtain:

$$\left(\frac{X_t}{q_t} \right)^{1-\theta} \phi N_t^\nu = \frac{q^{1-\theta}}{1 - \theta} \left(\frac{C_t}{X_t} \right)^{-\sigma} A_t \left[\alpha \left(\frac{N_t^C}{q_t/X_t} \right)^\rho + (1 - \alpha) \left(\frac{K_t^C}{X_t} \right)^\rho \right]^{\frac{1-\rho}{\rho}} \alpha \left(\frac{1}{q_t/X_t} N_t^C \right)^\rho (N_t^C)^{-1}.$$

Both equations are consistent with C_t , q_t , and K_t growing at the same rate as X_t .