Aggregate Fertility and Household Savings:
A General Equilibrium Analysis Using Micro Data∗

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Abstract

This study uses micro data and an overlapping generations (OLG) model to show that general equilibrium (GE) forces are critical for understanding the relationship between aggregate fertility and household savings. First, we document that parents perceive children as an important source of old-age support and that, in partial equilibrium (PE), increased fertility lowers household savings. Then, we construct an OLG model that parametrically matches the PE empirical evidence. Finally, we extend the model to conduct a GE analysis and show that under standard assumptions and with the parameters implied by the data, GE forces can substantially offset the PE effects. Thus, focusing only on the PE can substantially overstate the effect of aggregate fertility on household savings. Keywords: Savings, Demographic Structure; JEL Keywords: J11, J13, O11, O4

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1 Introduction

The linkage between aggregate fertility change and economic performance is central to models of economic growth. A large literature has provided important evidence relating aggregate fertility change to growth (e.g., Romer, 1986; Kremer, 1993; Jones, 1999; Galor and Weil, 2000), growth and inequality (e.g., De La Croix and Doepke, 2003), culture (e.g., Fernandez and Fogli, 2006; Fernandez and Fogli, 2009), social security (e.g., Boldrin and Jones, 2002; Boldrin, De Nardi, and Jones, 2005; Song, Storesletten, Wang, and Zilibotti, 2012) and savings (e.g., Becker and Barro, 1988; Barro and Becker, 1989; Manuelli and Seshadri, 2009).

In particular, Modigliani and Cao (2004) argue that changes in aggregate fertility can also lead to significant changes in household savings through their effect on the dependency ratio and wage growth. The authors support their claim with descriptive time series data from China, where a substantial reduction in fertility during the 1970s and 1980s resulting from family planning policies was accompanied by a rapid rise in the savings rate.

Such time series correlations are difficult to interpret since aggregate fertility change is likely to coincide with other macroeconomic changes, such as changes in the returns to human capital or in the relative female wage. In the case of China, an additional concern is the possibility that the increase in savings and the reduction in fertility are both consequences of the massive economic reforms that took place. Moreover, fertility is likely to affect savings through mechanisms other than the pure aggregation channel proposed by Modigliani and Cao (2004). The recent literature has therefore taken advantage of more specific demographic shocks (e.g., the introduction of China’s family planning policies, the implementation of family policies in Bangladesh under the leadership of the International Centre for Diarrhea Disease Research, or the birth of twins) to empirically estimate the causal effect of fertility on savings. These studies find a large negative effect of fertility on savings (e.g., Ruthbah, 2007; Banerjee, Meng, and Qian, 2011; Ge, Yang, and Zhang, 2012; Choukhmane, Coeurdacier, and Jin, 2013).\footnote{This paper supersedes Banerjee, Meng, and Qian (2011).}

Studies such as Curtis, Lugauer, and Mark (2011) and Choukhmane, Coeurdacier, and Jin (2013) then use the evidence from micro data to calibrate partial equilibrium (PE)
overlapping generations (OLG) models to understand the quantitative effect of an aggregate fertility change on savings.

While these studies provide compelling evidence that fertility affects savings decisions, they are unlikely to identify the correct quantitative effect of a change in aggregate fertility on savings. This is because an aggregate change in fertility may alter other economic factors that affect savings, such as the interest rate and the rate of wage growth, through their effect on the capital-labor ratio (e.g., Barro and Becker, 1989; Galor and Weil, 1996). The quasi-experimental micro evidence, which relies on comparisons of households with different levels of fertility within the same economy, nets out such general equilibrium (GE) effects, without which we cannot obtain the correct full equilibrium effect of a change in fertility. In particular, the fact that higher fertility leads to higher future interest rates and to slower wage growth, both of which may lead to higher savings, has the potential to partly undo the negative PE effect of fertility on savings that is estimated in the micro empirical analyses.

The goal of this paper is to use a combination of parameter estimates from a natural experiment and other micro data and careful modeling to understand whether studies of the relationship between aggregate fertility and savings need to take GE effects seriously in drawing macro policy conclusions from micro empirical estimates. While the principle that GE effects matter is widely accepted (e.g., Heckman, Lochner, and Taber, 1998 and Acemoglu, 2010), concrete examples of their potential quantitative importance are scarce. To the best of our knowledge, we are the first study of the relationship between aggregate fertility and savings to provide such an example.

Our study proceeds in several steps. First, to motivate the study and obtain parameter values for calibrating the model later in the paper, we use nationally representative survey data to document that contemporary Chinese parents perceive children and especially sons as an important source of old-age support. At the time of this study, there were no data

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2The seminal study of Barro and Becker (1988, 1989) models children as consumption and introduces endogenous fertility and intergenerational transfers to optimal growth models. Becker and Barro (1988) uses an open economy framework, where interest rates are exogenous. Barro and Becker (1989) uses a closed economy framework where fertility increases the capital-labor ratio and interest rates. Note that the main difference between our framework and theirs is that we view children as an investment good. This is discussed in detail later in the introduction.
that contained both total fertility history and data on income and expenditures. Thus, we conducted a survey that provides this data and use it to establish that for individuals age 50–65, the shift in Chinese family planning policies from pro-natal to anti-natal reduced fertility and increased household savings, particularly for those who had only one daughter. Because in these cohorts there was no sex-selection we argue that this can be interpreted as the causal effect of the family being restricted to one daughter. Moreover both the data and our estimated parameter values suggest that child-related expenditures are similar for daughters and sons. Therefore the finding that a reduction in fertility increases savings more for those with only-child-daughters is evidence for models where parents take savings decisions based on how much financial or psychological old-age support they can expect to get from their children (e.g., Caldwell, 1978; Weil, 1997; Boldrin and Jones, 2002; Tertilt, 2005).³

Next, we characterize the savings decision in a parsimonious Diamond-style OLG model with the additional feature that parents anticipate have to spend resources on bringing up children but the expect transfers from children in old age and both of these influence their savings decisions. We calibrate the PE version of this model to match the empirical findings. Then, we introduce GE effects to our model by endogenizing wages and interest rates (e.g., Barro and Becker, 1989; Galor and Weil, 1996).⁴

We find that depending on parameter values, GE effects can either dampen or exacerbate the PE effects of an increase in fertility (or leave them unchanged). On the one hand, GE effects may be more muted because the rise in the interest rate and the fall in wage growth generated by an increase in fertility reduce the present value of future transfers from children and thus induces parents to save more. On the other hand, GE effects may be stronger because of the income effect from the rise in the interest rate. Using the parameter estimates

³Caldwell (1978) argues that children provide old-age security. Weil (1997) finds that intergenerational transfers occur in both directions – from parents to children and from children to parents. Boldrin and Jones (2002) uses a growth model to formalize the ideas of Caldwell (1978) and shows that it can account for demographic patterns in the data. Boldrin, De Nardi, and Jones (2005) goes further to argue that if children provide old-age security, then the observed cross-country differences in fertility rates can be explained by cross-countries differences in social security. Tertilt (2005) also considers children as a form of savings for parents and finds that banning polygyny is likely to decrease fertility and increase savings.

⁴That savings rates are associated with interest rates in practice is consistent with Horioka and Wan’s (2007) finding that, in China, average household savings rates at the province level is positively associated with real interest rates.
we obtain from the micro-empirical analysis, we find that the GE effect of increased fertility is only 30% of the PE effect estimated from micro data. This is true as long as the intertemporal elasticity of substitution is not too far below one, which is consistent with the data.

Since our estimates of the parameters are not always very precise and the match between our model and reality is, as always, inexact, we check the robustness of our GE estimates by varying the parameters over a wide range, and for the most part the results remain the same. We then consider a number of extensions of our model that bring in endogenous fertility, endogenous transfer rate and endogenous human capital investment. Our results are robust to these extensions as well.

The key contribution of our paper is to provide a concrete example of the importance of GE effects for understanding how a shift in aggregate fertility affects savings. Applying PE estimates to macro policy without interpreting the results with the appropriate model can therefore be very misleading. In a methodological sense this is similar to Weil (1994) who notes that aggregate savings is negatively associated with the size of the elderly population despite the lack of micro evidence that the elderly dis-save. To reconcile these patterns, he develops a model to demonstrate that in the absence of full annuitization, if the elderly saves for bequests, then the anticipation of income from bequests will cause children to save less. At the same time, our study illustrates the importance of obtaining reliable micro evidence since the quantitative effects are sensitive to parameter values. In this sense we address the general methodological concern that there is often a “discordance between the macro models used in policy evaluation and the microeconomic models used to generate the empirical evidence” (Browning, Hansen, and Heckman, 1999) and provide an example of a growth models that are “built up” from well-identified parameters estimated using experimental and quasi-experimental data as proposed by Banerjee and Duflo (2005) (for parallel exercise on the macro impact of microcredit see Buera, Kaboski, and Shin (2012)).

More generally this work is related to studies that explore the effects of aggregate fertility change going back to the classic work of Becker, Murphy, and Tamura (1994) and the important subsequent work by De La Croix and Doepke (2003), Galor and Weil (2000) and
Manuelli and Seshadri (2014). In emphasizing the macro effects of demographic changes in the contemporary Chinese context, our study is closely related to Song, Storesletten, Wang, and Zilibotti (2013), which shows that the demographic transition in China implies that pay-as-you-go pension systems have redistributive effects across generations.

Finally, our study adds to the previously discussed studies that explain Chinese savings rates with demographic factors. For example, Song and Yang (2010) elaborates the argument originally made by Modigliani and Cao’s (2004) and provides evidence that links the spike in aggregate savings, the growth rate and the flattening of experience profiles over time. Rosenzweig and Zhang (2014) explains the savings of the young with family size and transfers of housing from parents to children. Finally, in their innovative work, Wei and Zhang (2011) shows that savings rates for middle-aged parents today are partly driven by the anticipation of paying “bride prices” for sons in a future where there will be many more men than women in the marriage market.\(^5\)

For policy makers in China, our results suggest that abandoning family planning policies and allowing fertility to rise, if our model is to be believed, will have little effect on household savings.

This paper is organized as follows. Section 2 documents parents’ beliefs of the importance of children as a source of old-age support. Section 3 documents the relationship between fertility change and savings. Section 4 presents the results from the model, including the calibration of the parameters and the quantitative estimates. Section 5 offers concluding remarks.

\(^5\)There also a number of studies of Chinese savings that emphasize factors other than demography: Chamon and Prasad (2010) provides evidence that financial underdevelopment and the precautionary motive are important contributors to savings; similarly, a recent study by He, Huang, Liu, and Zhu (2014) finds precautionary saving and increased employment risk due to the downsizing of the state sector to be important determinants of household savings; and Horioka and Wan (2007) finds that savings are associated with lagged savings rate, the income growth rate, the real interest rate and the inflation rate.
2 Children as Old-Age Security

In this section, we briefly summarize we believe that the parents we study, who are 50 to 65 years old urban workers in 2008, anticipate that their children will be an important potential source of support in old age; and that typically, sons are likely to provide more support than daughters. This is certainly what one would have expected based on Confucian traditions, but in contemporary urban China, state-provided pensions are a very important source of old-age support (Song, Storesletten, Wang, and Zilibotti, 2012). However, since pensions have not been indexed to the high income growth rates of urban areas, the real value of pensions have tended to decline rapidly over time. In contrast, children’s earnings will (on average) increase with prices and thus remain a stable source of support for parents for long haul. Also, it is important to note that the elderly almost always live with their children (or other family members) when they are very old. There is very little old-age care in the form of nursing homes, etc., that we see in more developed economies.

Outside of pensions and children, urban households have access to few savings instruments. Bank interest rates are very low and though more recently access to financial markets has expanded, in 2007, almost all household savings (other than housing) in urban areas were in bank deposits (He and Cao, 2007).

Existing descriptive evidence support the conventional wisdom that elderly parents in urban China receive substantial old-age support from children, which is increasing in the number of children, whether a parent has a son and the age of the parent. Using data from the 2011 CHARLS survey, which samples a nationally representative set of households (see the Data Appendix for a discussion of all of the surveys used in this section), Figure 1 plots urban net household transfer income from children as a fraction of total household income against the age of the main respondent. The main respondent in the survey is typically the oldest male household member. The figure shows that total transfer income increases with age and the number of children (total fertility). For example, for those with one child, net transfer income increases to approximately 20% of total income by age 90. For those with two children, transfer income increase to almost 80% of total income by age 90. Since
the CHARLS only reports transfers for parents who do not live with their children, we also examine co-habitation patterns. For this, we can use the UHIES, which is a representative sample of urban households, of which we have data from nineteen cities in nine provinces. Using the information on cohabitation and the relationship to the household head, we are able to calculate the fraction of parents living with a child for the entire urban population. Figure 2 shows an inverse U-shape. The fraction of parents living with children decreases from almost 100% at age 40 to around 50% by age 65 and returns to 100% by age 90. Unfortunately, the UHIES does not allow us to distinguish whether parents live with their daughter or son. However, the CHARLS (2011) data on cohabitation reports that for urban households, approximately 20% of adult sons (age 45–64) and only 10% of adult daughters live with elderly parents (age 65 or older).

Finally, we examine the survey evidence on parents’ perceptions of old-age support. In the CHARLS (2011), respondents were asked “Whom do you think you can (most) rely on for old-age support?”. Figure 3 plots the answers given by urban households according to the age of the household head. It shows that, on average, less than 10% of respondents rely on “savings”, approximately 60% choose “pension or retirement salary” and slightly less than 30% choose “children” (the other possible responses to this question were “commercial pension insurance” and “other”). As parents age from 65, when most workers retire to 90, the percentage reporting pensions as the most important source of support declines from 65% to 40%, whereas those reporting children as the most important increases from 20% to 60%.

3 The PE Effects of Fertility on Savings

3.1 Family Planning Policies

In its early years, the communist government (1949–) had a pro-natal stance on fertility (Chang, Lee, McKibben, Poston, and Walther, 2005; Scharping, 2013). The government pursued policies that encouraged fertility such as conditioning food rations on the number

6Most famously, Ma Yinchu’s “New Population Theory”, which argued that a rapidly growing population would hinder economic development and that the government should implement population control policies, was officially discredited as being pro-Malthusian and anti-Socialist (Yang, 1986).
of family members and making access to contraceptives difficult until a certain number of children had already been born. Discussions about curbing population growth were confined to the top policy makers until the early 1970s. However in 1971, Mao Zedong and Zhou Enlai made a sudden public policy shift and announced that “population must be controlled”, which signaled a turning point in family planning policy practice in China. Efforts began in earnest in 1972 with measures to clarify the shift in family planning policy and energize the bureaucracy. By 1973, 23 provinces had established the necessary bureaucracies for implementing family planning related policies.

Our study focuses on the unanticipated and unprecedented initial shift in family planning policy from anti-natal to pro-natal that occurred in 1972, which encouraged birth spacing of three to four years. An unanticipated increase in birth spacing is likely to reduce total fertility since, for example, some mothers will become too old to have a second child after the required waiting period. In urban China, the reduction due to birth spacing was magnified by the subsequent introduction of the One Child Policy in 1980 (1979 in Shanghai), when the government took another unanticipated move of restricting fertility to only one child. When this occurred, parents who had their first child after 1976 (1975 in Shanghai) and were waiting to pass the required birth spacing to have their second child found that they would remain one-child families.

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7 On Feb. 15th, 1971, Zhou Enlai re-emphasized the importance of family planning when meeting with the provincial representatives at the National Planning Conference in Beijing: “It’s important to control population growth. Government should advocate late marriages and birth control, and vigorously publicize these policies from now on.” On July 8th, the State Council published “the Report on Doing Well in Family Planning.” The written instruction by the State Council on the document pointed out that “Family planning is an important issue that Chairman Mao has advocated for years. All levels of officials must treat the issue seriously.”

8 On January 17, 1972, provincial leaders attended a meeting organized by the Ministry of Public Health where the central government demanded that local governments publicize and enforce Mao’s instructions on family planning, and instructed all levels of government to establish or reinforce their bureaucracies for organizing or implementing family planning related tasks. In May of that year, the Ministry of Public Health organized a national workshop on family planning measures where all provinces had to participate. The details of family planning policy history is documented (in Chinese) by the China Population Information Network (POPIN), a branch of the China Population Development and Research Center (CPDR or CPIRC) on their website.

9 The One Child Policy (OCP) punished households that had more than one child with fines, job loss, and the loss of access to public goods, and rewarded those with only one child with bonuses. Family planning polices also became better defined over time. For example, in 1978, the state defined details on things such as what counted as late marriages and the bonuses and subsidies for workers and farmers if they go through sterilizing operations, etc. See “The Report on the State Councils Family Planning Groups First Meeting” (1978).
Similar policies were introduced in rural areas, but there was more flexibility across regions and over time.\textsuperscript{10}

3.2 Estimating the Effect of Fertility on Savings

We will use household-level data to first demonstrate two reduced form relationships: (i) family planning reduced fertility; (ii) family planning increased savings. These estimates, together, imply that reduced fertility increased savings. Since parents traditionally rely on sons more than daughters, “fertility” from the perspective of parents thinking about future transfers is a weighted sum of children, where daughters receive less weight than sons. Since we do not know the weights, we simply treat daughters and sons separately and estimate the following reduced-form equation


g_{ij} = \delta p_{ij} + \alpha m_{ij} + \zeta(p_{ij} \times m_{ij}) + \Delta X_{ij} + \theta_j + \varepsilon_{ij}. \tag{1}

\(g_{ij}\), for household \(i\) living in region \(j\), represents outcomes like the total number of children, savings, etc. We specify that it is a function of: a dummy variable for whether the first child was born in or after 1972, \(p_{ij}\); a dummy for whether the first child is male, \(m_{ij}\); the interaction term between \(p_{ij}\) and \(m_{ij}\); a vector of household-level controls, \(X_{ij}\); region fixed effects, \(\theta_j\); and a household-specific error term, \(\varepsilon_{ij}\). The standard errors are clustered at the sex (of the first child), year of birth (of the first child) and city level for all of our results.\textsuperscript{11} \(\delta\) is the effect of having a first child in 1972 or afterwards for households that have a daughter for the first child. \(\delta + \zeta\) is the effect of having a first child in 1972 or afterwards for households that have a son for the first child.

These estimates test the hypothesis that parents perceive children, especially the male first

\textsuperscript{10}The variation in the implementation of the One Child Policy in rural China can be seen in the \textit{China Health and Nutritional Survey}, which reports the relaxations of the policy that are allowed at the community and year level. In contrast, the data show very little variation in these variables across communities or over time for urban areas. These descriptive statistics are available upon request.

\textsuperscript{11}There are 131 clusters. We can alternatively cluster the standard errors at the sex and year of birth (of the first child) level and then correct for the small number of clusters by estimating wild bootstrapped standard errors. The first stage and reduced form estimates are very similar between these two levels of clustering. These estimates are available upon request. There is no correction for the small number of clusters for the 2SLS estimates.
child, as an important provider of old-age support. The claim that having one’s first child during or after 1972 decreased total fertility both when the first child is female and when he is male, translates into a test for whether both $\hat{\delta} < 0$ and $\hat{\delta} + \hat{\zeta} < 0$. Similarly, the claim that parents rely more on sons than daughters for old-age support, and therefore parents who gave birth in or after 1972 and had a first male child need to save less and can retire earlier compared to parents who gave birth in or after 1972 and have a first female child would imply, $\hat{\zeta} < 0$ in the savings equations. The vector $X_{ij}$ includes household-specific controls that we will discuss and motivate later as they become relevant.

For a sense of the implied magnitudes, we also estimate an instrumental variables specification, which assumes that the only thing that changed in 1972 for this population was the number of children they could have

$$y_{ij} = \delta n_{ij} + \alpha m_{ij} + \zeta (n_{ij} \times m_{ij}) + \Delta X_{ij} + \theta_j + \varepsilon_{ij}. \quad (2)$$

Here, $n_{ij}$ is the number of children the family eventually had and is instrumented by $p_{ij}$. Similarly, $n_{ij} \times m_{ij}$ is instrumented by $p_{ij} \times m_{ij}$.

The aim of our study is to assess whether GE effects should be taken seriously for understanding the relationship between aggregate fertility and savings. Thus, the instrumental variables estimates are used to illustrate the approximate magnitude of the relationship between fertility and savings and to provide parameter values that are useful for the calibration of our model in Section 4. We do not interpret the 2SLS literally as the causal effect of fertility on savings since there are potential violations of the exclusion restriction.$^{12}$

Note that there is little sex selection in our sample. Female infanticide rates in urban China are very low and we restrict our sample to households that bore children before sex-selective abortion became available in the 1980s. Consistent with no sex-selection, 50.3% of all children in our sample are male. Thus, we interpret the coefficient for the sex of the first child, $m_{ijt}$, as exogenous. Also, note that given the introduction of family planning policies,

$^{12}$It is possible, for example, that even if the actual number of children was unaffected, the option of having another child later in life might have independent effects.
we have many fewer observations for second or higher parity children than for first parity children. For this reason, our sample size is not large enough to examine the differential effects of male and female higher parity children.

There are two important caveats to our strategy. First, households in the control group will be older on average than those in the treatment group, which can affect savings patterns if parents of the two groups are at different parts in their life cycle (even within the narrow age band that we examine). One way to address this is to control for the age of the household head. However, while this controls for age, it can introduce selection bias if parents choose fertility timing based on factors that are correlated with savings later in life. We will discuss this in further detail when we interpret the results.

This raises a second difficulty. For example, parents that have children later in life may be more risk averse, which will, in turn, cause them to save more. To investigate this possibility, we directly examine the correlation between age at first birth and savings, controlling for the same baseline controls. We find no correlation.\textsuperscript{33}

3.3 Data

To document the relationship between fertility and household savings, we use the urban household portion of the larger survey that we collected in 2008, called the 2008 \textit{Rural-Urban Migration in China} (RUMiC). At the time of our study, these were the only data that allowed us to measure both the total number of children ever born and savings rate for a sufficient

\textsuperscript{33}There are several additional facts to keep in mind for interpreting the coefficients. First, the policies for population control gradually tightened over time. This means that the effect of family planning policies on total fertility is not uniform across households that have their first child in or after 1972; the later they have their first child, the fewer children they will have. This is important to keep in mind when interpreting the magnitude of the estimates, which give the average post-reform effect. Second, family planning policy is relatively uniform across urban areas (e.g., Ebenstein, 2010; Qian, 2009) and there are relatively few ethnic minority households (who get some exemption from the policy in most Chinese cities). Our empirical strategy estimates the average change after 1972.

Note that our identification strategy assumes that the shift to fertility control in the early 1970s was unanticipated. For example, if parents anticipated fertility control policies, those who desired more children may have had more children than otherwise in the years leading up the policy. This would cause an “Ashenfelter dip” and our strategy will overestimate the effect of the policy on reducing the number of children. If parents who intentionally had more children also had a lower propensity to save for reasons unrelated to fertility, this will also cause our strategy to overestimate the effect of the policy on increasing savings. The historical evidence discussed earlier suggests that it is very unlikely that there was anticipation. To the best of our knowledge, no existing study of family planning in China mentions this possibility.
number of households for statistical analysis.\textsuperscript{14} In this paper, we only use the urban data because family planning policies and access to savings instruments were relatively uniform in urban areas, and equally importantly, because there was little sex-selection. The data are organized as a household-level birth cohort panel according to the birth year of the first child. The empirical analysis focuses on households that had their first child five years before or after the policy shift in 1972, i.e., 1967–77, an arbitrary interval that we chose to be symmetric around the reform date. Figure 4a shows the kernel density plot for the distribution of the ages of firstborn children in our sample.

We restrict the sample to households headed by individuals who are 50 to 65 years of age to focus on a point in the lifecycle when individuals are most likely to be saving for their retirement. This is the period of the life cycle when children require relatively little expenditure from parents, when parents are still working, and when children are not yet making transfers to parents. Figure 4b is a kernel density plot for the distribution of the ages of the household heads in our sample.

This is also the age range when co-residence is very low in the urban population.\textsuperscript{15} This allays two concerns: (i) that the households we observe are headed by individuals who are selected on characteristics such as propensity to save; (ii) that the household savings decisions that we observe are influenced by expenditure on children. Indeed, there are very few households with children living at home in our sample.\textsuperscript{16} The narrow age band is also advantageous because individuals are likely to be on the same part of the life cycle and therefore comparable to each other. Note that this sample is younger than the sample of elderly parents ages 65 or older with which we examined transfers and cohabitation in Section 2.

The final sample contains 475 households in eighteen cities. Table 1 shows the descriptive statistics. Households in our sample on average have total incomes of 49,584 RMB and

\textsuperscript{14} See the Data Appendix for a detailed discussion of the RUMiC and other survey data from China.
\textsuperscript{15} A 0.1% sample of urban households in eighteen provinces (UHIES, see Data Appendix) show that less than 10% of individuals of this age range co-reside with children or parents. This is the lowest for any age group.
\textsuperscript{16} In our sample, there are only five households with any children eighteen or younger and only fifteen households with any children age 22 or younger. Figure 4c plots the kernel density plot of the distribution of the youngest children in our sample.
expenditures of 32,421 RMB. Savings, the difference between total income (except for transfer income) and total expenditures, are on average 17,162 RMB.\textsuperscript{17} The average savings rate is 26%. Figure 4d plots the kernel density of household savings in our sample. It is approximately normally distributed and takes negative as well as positive values. Figure 4e plots the kernel density of household savings rates in our sample.

The average household has approximately two children, 50.3% of which are male. On average, parents had their first child in 1973 and their youngest child in 1976. This means that when the survey was conducted in 2008, households in our sample on average had children aged 32–35 years. Our sample contains households headed by individuals 50–65 years of age. On average, household heads are approximately 61 years of age and have approximately ten years of education (i.e. one year of high school education). Approximately 42% of our sample is headed by women.\textsuperscript{18}

3.4 Results

The Effects of Family Planning on Fertility

Table 2 presents the estimated effects of the introduction of family planning on fertility. Column (1) shows a specification that only controls for city fixed effects. The estimates show that parents that gave birth to their first child in 1972 or afterwards had, on average, 0.6 fewer children. In column (4), we estimate equation (1) where we add controls for whether the first child is a son and the interaction of that term with whether the first child was born during 1972 or afterwards. The estimate for the uninteracted post-1972 term shows that parents who had their first daughter in or after 1972 had approximately one less child (-0.8). The sum of the uninteracted post-1972 term and its interaction with the first child being a son, shown at the bottom of the table along with its p-value, is also negative and significant. But it is smaller in magnitude than

\textsuperscript{17}These variables are defined in detail in the Data Appendix. In results not presented in this paper, we used several alternative definitions of expenditures, such as with or without including social security contributions (which can be viewed partly as a form of savings). These make little difference to our results and are not presented for brevity. They are available upon request.

\textsuperscript{18}This does not necessarily mean that these women had no male spouse – it could just be that the survey respondent was the oldest female in the household. To be cautious and to avoid the potentially confounding effects from having a female household head, we will control for this in our regressions.
the uninteracted term (-0.4).

The results mean that parents who had their first son in or after 1972 were also likely to have had fewer children than those who had their first son before 1972 but the reduction in the number of children was smaller in magnitude than for parents that had a first daughter. This is most likely driven by the boy-biased “stopping rule”: before 1972, some parents stopped having children once they had a son, with the result that males have on average fewer siblings than females. This can be seen in the negative coefficient on the dummy variable for whether the first child is a son.\textsuperscript{19} All of the coefficients discussed here are statistically significant at the 1\% level. In columns (2)–(3) and (5)–(8), we add controls which we will discuss in the next section.

The results in Table 2 confirm that the introduction of family planning reduced total fertility and that there is a prejudice in favor of sons. Both of these findings are important to keep in mind for interpreting our results later in the paper.

**The Effect of Family Planning on Savings** Next, we examine the effect of the introduction of family planning on savings. We estimate the same regressions as before, except that we replace the dependent variable with household savings rates. Table 3 shows the reduced form results. Column (1) presents the estimates when we only control for city fixed effects. On average, parents that had their first child after 1972 saved 6,175 RMB more in 2008. The estimate is statistically significant at the 1\% level. In column (2), we control for basic demographic characteristics of the parents: the age of the household head and its squared term, the educational attainment of the household head and its squared term. These are important since income and consumption patterns, and thus savings patterns, can differ by age (even in our limited age range). Similarly, educated parents may have a different propensity to save than less educated parents. Column (2) shows that including these controls has little effect on the estimated effect of having the first child in or after 1972.

As we discussed earlier, controlling for the age of the household head introduces a specific

\textsuperscript{19} Consistent with the stopping rule, on average, boys in our sample come from households with 1.7 children, while girls come from households with two children.
type of selection: it raises the question of whether parents that chose to have children at an earlier time in life will save less than parents that chose to have children later in life for reasons other than the difference in total fertility. To address this, we drop the two controls for the age of the household head in column (3). The estimate is only slightly smaller than the one in column (2) and is statistically different from zero at the 1% level. The estimates in columns (2) and (3) are not statistically different from each other.

In column (4), we introduce controls for the sex of the first child and its interaction with whether he/she is born in or after 1972. We return to a specification where we only control for city fixed effects. In column (5), we add the four controls for parental characteristics (age and its squared term, the years of education and its squared term). The estimate of the uninteracted effect of having a first child in or after 1972 shows that parents that have a daughter as a first child in or after 1972 save 12,613 RMB more than parents that have a first daughter prior to 1972. The interaction effect shows the differential effect for parents who have their first child in or after 1972 but who have a son. The sums of the uninteracted and interacted effects are shown at the bottom of the table. This coefficient, 1748, is positive, but small in magnitude and statistically insignificant.

Thus, parents that have their first child in or after 1972 save much more if their first child is a daughter and may save a little more if their first child is a son. Given the earlier results that parents who had their first child in or after 1972 also had fewer children on average, especially if the first child was a daughter, these results are consistent with parents saving more when they have fewer children.\footnote{Note that the uninteracted dummy variable for whether a first child is a son is large, positive and statistically significant. This variable, which reflects the effect of having a first child who is male prior to 1972 may partly reflect the fact that such households had fewer total children because of the stopping rule (recall Table 2 column (4) shows that the coefficient of the first child being on the total number of children is -0.455). Fewer children can, in turn, results in lower expenditures, and higher savings.}

In column (6), we remove the controls for the age of the household head and its squared term for the reasons that we discussed earlier. The estimates change little. In column (7), we add additional controls. The control for whether the head of the household is under 55 years of age addresses the possibility that being over the “mandatory” retirement age (from
public enterprises) increases unemployment probabilities and savings behavior. Controlling for the age of the youngest child addresses the possibility that having a young child increases consumption and affect savings. The dummy variable for whether the youngest child is under 22 years of age also addresses this point. Finally, we control for whether the mother is the household head in case this variable reflects intrahousehold bargaining power and, thereby, savings behavior. In column (8), we include all of the controls in column (7) except for the age of the household head and its squared term. The estimates are precisely estimated and statistically similar to the baseline in column (5).

To summarize, the estimates in Table 3 show that parents that had their first child in or after 1972, in particular those with daughters, save more. It is also interesting to note that the estimates change very little with the changing controls. This is consistent with our identification assumption that the introduction of fertility restrictions was “randomly” assigned.21

The Implied Effect of Fertility on Savings We use 2SLS to scale the reduced form estimates from Table 3 and interpret the instrumented estimates as a rough approximation of the effect of fertility. In Table 4 columns (1)–(3), we report the instrumental variables estimates. The absolute value of the instrumented estimates is roughly similar in magnitude to the reduced form estimate. Column (3) shows that an additional child reduces savings by approximately 18,570 RMB if the first child is a daughter. This is statistically significant at the 1% level. The interaction effect of the number of children with a dummy for first child being male is positive and significant at the 1% level. As before, this suggests that family size matters less if the first child is male. This is shown more formally by the sum of the uninteracted and interacted effects of the number of children, which is -7,518 RMB for the level of savings in column (3). The joint estimates are statistically insignificant (they and

21We also conduct a placebo experiment to examine the possibility that our post-1972 variable is picking up parents who prefer to have children later in life. We estimate an equation similar to equation (1), except that we replace the post-1972 dummy variable with the household head’s age at first birth (both by itself and interacted with a dummy for whether the first child is a son). If our main results were driven by selection, should find the coefficient for the interaction effect to be positive. We find no effect: the coefficient is 0.00187 and the standard error is 0.00777 (these results are not reported in tables).
their standard errors are not reported in the tables). We also see that the effect of the first child being male is strongly negative and significant, consistent with the theory that parents who have an oldest son expect that they will be taken care of.

Finally, we consider the alternative mechanism raised by Wei and Zhang (2011) that parents in regions with strong male-biased sex ratios and who have sons must save so that their sons can obtain brides in the future. We directly control for the interaction term of regional sex ratio and a dummy variable for whether the first child is a son (the uninteracted effect of regional sex ratio is already controlled for by the city fixed effects). Our prior is that this mechanism is less relevant for our study because there is little sex imbalance for these cohorts. Indeed, column (4) shows that our key results are very robust to the inclusion of this control.

22 The Effect of Fertility on Earnings Table 5 reports the instrumented effect of fertility on earnings. For brevity, we report the 2SLS estimates. Column (1) shows that an additional child results in 11,236 RMB less income in 2008 for parents if the first child is a daughter. Fertility has little effect on income for parents whose first child is a son (−11,236 + 8,636 = −2600). Columns (2)–(7) show that this is mainly driven by wage income. These results are consistent with the notion that parents with only one daughter work more in order to increase savings for old age.

We acknowledge that in inferring the stock of savings from savings in one year, we must assume that the two variables are positively correlated. For example, our interpretation would be misleading if parents with more children accumulated more assets than parents with fewer children and therefore had stopped saving before the age at which we observe them (50–65). We believe that this is unlikely since more children require more expenditures when the children are young. So, if anything, parents with more children would have accumulated less assets before age 50. In urban China, the two main savings vehicles are savings deposits

22 Regional sex ratio is measured as the fraction of males of those born during 1949–1975 in each city. We experimented with several alternative measures and always obtain similar results. Estimates using these other measures are available upon request.

23 Note that the uninteracted effect of whether the first is a son is no longer meaningful by itself since it captures the effect of having a son as the first child in regions where there are no males.
and housing. Since savings deposits generate interest income and real estate generates rental income, we can investigate this alternative explanation by examining interest income and rental income, which should scale with the stock of assets. Column (5) of Table 5 shows that there is no relationship between the instrumented fertility variable and interest and rental income.\footnote{In our data, we also observe households’ own durables such as refrigerators, motorcycles, and cars; and the imputed value of housing. We find suggestive evidence that parents with children (instrumented) have, if anything, more assets than parents with fewer children. The estimates are imprecise and are available upon request.}

**The Effect of Fertility on the Savings Rate** We recognize that fertility affects many aspects of people’s lives (e.g. it affects both level of savings and income). However, for the purpose of the calibration, it will be convenient to summarize the effect on fertility by a single variable, the savings rate. Since we wish to compare these results with a model where only the number of children changes, we focus on the instrumental variables estimate. These are reported in Table 4. Columns (5) and (6) show that each additional child reduces the savings rate by 11 percentage points. Column (7) shows that for parents with first daughters, additional children reduces the savings rate by 16 percentage points, while for those with sons, an additional child reduces savings rates by four percentage points ($-0.158 + 0.118 \approx -0.04$).

We note that the estimates on the savings rate are less precise than the estimates on savings levels. This is likely due to the fact that fertility and the sex of the eldest child also affect income. This is another reason to interpret the instrumented estimates on the savings rate as illustrative.

### 3.5 Interpretation

The main empirical finding is that the reduction in fertility caused by the introduction of family planning policies increased household savings, especially for parents with only one daughter. This is consistent with the descriptive evidence discussed in Section 2 that parents see children, and particularly sons, as an important source of old-age support.

Note that our results also suggest that the relationship between fertility and savings is
driven by anticipated transfers rather than child-related expenditures. For example, consider
the alternative explanation that daughters and sons provide the same level of transfers to par-
ants, but parents with only one daughter save more because daughters cost less to raise than
sons. This is inconsistent with the stopping rule that we see in the data (see Table 2) which
suggests that parents prefer to have sons. If sons and daughters provide the same level of
support and daughters cost less, then parents should instead prefer to have daughters. More-
over, we note that for the cohort of urban children that we are studying, major expenditures
related to child rearing (child care, housing, schooling, and even food) were state-provided.
Thus, there was little cost difference between male and female children.\textsuperscript{25}

4 A Model of Fertility and Savings

The empirical estimates showed that an additional child reduced savings for a household that
lives in an otherwise identical economic environment as a household that has one less child.
However, if we are interested in the effects of a change of aggregate fertility, the assumption of
an otherwise unchanged economic environment is unlikely to be right. A change in aggregate
fertility has an impact on the economic environment, for example, through its effect on factor
prices, which in turn affects savings.\textsuperscript{26} The micro empirical evidence cannot therefore be
directly used to predict the relationship between aggregate fertility and savings. In order to
address this concern, we now develop a simple overlapping generation model of savings that
helps us to interpret the empirical results. We begin with the simplest version of the model
to build intuition and then proceed to a more quantitative version.

\textsuperscript{25}For example, in the 1989 UHIES, total expenditure for urban households with at least one male child was
on average 1122 RMB and for households with at least one female child was on average 1129 RMB. The gap
is similarly small for other years (1990–2005).

As emphasized by Wei and Zhang (2011), the tendency in China in recent years has been towards a bride
price rather than a dowry, which would raise the cost of male children, though in this cohort, which predates
sex-selective abortions, this effect is probably not very important either way.

\textsuperscript{26}For example, Horioka and Wan (2007) finds limited evidence that, in China, average household savings
rates at the province level are associated with real interest rates.
4.1 The simplest OLG Model

The empirical findings that the policy-driven reduction in fertility increases household savings are consistent with the qualitative evidence that parents anticipate more transfers in expectation when they have more children. We therefore start from a variant of the classic Diamond OLG model with two additional features: (i) children transfer a fraction $\tau$ of their income to parents, (ii) parents pay a linear cost, a $\theta$ fraction of their income, to raise each child. We do not model the decision to have children, but assume that every household is endowed with an exogenous number of children $n_i$. This choice is due to the fact that we want to consider the effect of an exogenous change in fertility, as generated by the One Child Policy (or its relaxation), on savings. Endogenous fertility is nonetheless discussed in Subsection 4.5.1. We assume log utility, a Cobb-Douglas production function, full depreciation of capital within one generation (given that a generation is 25 years, this is not a restrictive assumption) and constant productivity growth at an exogenous rate $1 + g$. The assumption of log utility implies that income and substitution effects perfectly offset each other, so that a change in the interest rate does not have any direct effect on savings. We relax this assumption later. The economy is inhabited by a continuum of households with mass $\frac{1}{m}$. Households are identical except for the number of children. Household $i$, with children $n_i$, solves the following problem:

$$\max_{c_{i,t}, c_{i,t+1}} \log(c_{i,t}^Y) + \beta \log(c_{i,t+1}^O)$$

s.t.

$$c_{i,t} + \frac{c_{i,t+1}}{1 + r_{t+1}} \leq A_t w_t (1 - \tau - \theta n_i) + \frac{A_{t+1} w_{t+1}}{1 + r_{t+1}} \tau n_i.$$  \hspace{1cm} (3)

From the first order condition of this problem, we can find the household optimal savings rate, defined as $s_{i,t} \equiv \frac{A_t w_t (1 - \tau - \theta n_i) - c_{i,t}^Y}{A_t w_t}$:

$$s_{i,t} = \left[ \frac{\beta}{1 + \beta} \right] \left[ (1 - \tau - \theta n_i) - \frac{\tau n_i}{\beta (1 + r_{t+1})} \left( \frac{A_{t+1} w_{t+1}}{A_t w_t} \right) \right].$$  \hspace{1cm} (4)

From this formula, it is clear that the model predicts that households with more children
save less. More specifically, the number of children, \( n_i \), impacts the savings rate through two channels. First, if \( n_i \) increases, then parents have to spend more on children, so that their disposable income is reduced and consequently they save less. We name this the “expenditure channel.” An additional child decreases the savings rate by \( \left( \frac{\beta}{1+\beta} \right) \theta \) through the expenditure channel. Second, if \( n_i \) increases, then parents expect to receive more transfers in old age, their need to save for retirement is therefore not as acute and this causes them to save less. We call the latter mechanism the “transfer channel.” An additional child decreases the savings rate by \( \tau \left(1 + \beta \right) \left(1 + r_t + 1\right) \left( \frac{A_{t+1} w_{t+1}}{A_t w_t} \right) \) through the transfer channel.

As we have demonstrated, this PE model is able to account for the cross-households relationship between fertility and savings. However, a change in aggregate fertility has an impact on prices as well. In order to discuss how aggregate savings are affected, we therefore need to understand the aggregation and GE properties of the model.

4.1.1 \( \text{GE} \)

In order to find the GE solution, we need to show how the model aggregates. Defining \( n \) and \( s \) to be aggregate fertility and savings rate, the following relationships hold: \( n = \int n_i \, di \) and \( s = \int s_i \, di \). Aggregation is trivial due to the fact that households differ only with respect to the number of children, and savings rates are linear in \( n_i \).

The empirical results provide us with estimates of \( \frac{\partial s_i}{\partial n_i} \), while, as already pointed out, we would like to have estimates of \( \frac{\partial s}{\partial n} \) in order to understand the effect of the One Child Policy on Chinese savings rates. To this end, we need to solve the GE of the model.

We first focus on steady states. The standard law of motion of capital for the Diamond model applies to our setting and reads as

\[
k_{t+1} = (1 - \alpha) \frac{s_t k_{t}^{\alpha}}{(1 + g) \, n},
\]

from which we get the steady state interest rate

\[
1 + r = \frac{\alpha (1 + g) \, n}{(1 - \alpha) \, s}.
\]
We substitute the equilibrium interest rate into (4) and notice that, in steady state, \( w_{t+1} = w_t \). We find that
\[
s_i = \left( \frac{\beta}{1 + \beta} \right) \left[ (1 - \tau - \theta n_i) - \tau n_i s \left( \frac{1 - \alpha}{\alpha \beta} \right) \right]. \tag{5}
\]
Summing (5) over all households and using the fact that \( s = \int s_i \, di \) and \( n = \int n_i \, di \), we obtain an explicit expression for the equilibrium aggregate savings rate
\[
s = \frac{\alpha \beta (1 - \tau - \theta n)}{\alpha (1 + \beta) + (1 - \alpha) \tau}. \tag{6}
\]
Equations 5 and 6 allow us to clearly see the differences between the PE and GE effects of a change in fertility on savings.

The PE effect is the derivative \( \frac{\partial s}{\partial n_i} \) for fixed \( n \) and \( s \). It is given by
\[
\partial_{PE} \equiv \frac{\partial s_i}{\partial n_i} = -\left( \frac{\beta}{1 + \beta} \right) \left( \theta + \frac{\tau s (1 - \alpha)}{n \alpha \beta} \right).
\]
We can then substitute equation (6) to find \( \partial_{PE} \) evaluated at equilibrium, which we name \( \partial_{PE,EQ} \) and reads as
\[
\partial_{PE,EQ} = -\left( \frac{\beta}{1 + \beta} \right) \theta - \left( \frac{\beta}{1 + \beta} \right) \left( \frac{\tau}{n} \right) \left( \frac{(1 - \tau - \theta n)(1 - \alpha)}{\alpha (1 + \beta) + (1 - \alpha) \tau} \right). \tag{7}
\]
The GE effect is instead the derivative \( \frac{\partial s}{\partial n} \), which must be computed from the equilibrium savings rate given by equation (6). It is given by
\[
\partial_{GE,EQ} \equiv -\left( \frac{\beta}{1 + \beta} \right) \theta \frac{\alpha (1 + \beta)}{\alpha (1 + \beta) + (1 - \alpha) \tau}. \tag{8}
\]

Comparison of PE and GE effects

We now compare the PE and GE effects of an increase of fertility on savings rates. First we note that \( \partial_{PE,EQ} \) is made of two parts: (i) \( \partial_{PE,Expend} \equiv -\left( \frac{\beta}{1 + \beta} \right) \theta \) and (ii) \( \partial_{PE,Transf} \equiv -\left( \frac{\beta}{1 + \beta} \right) \left( \frac{\tau}{n} \right) \left( \frac{(1 - \tau - \theta n)(1 - \alpha)}{\alpha (1 + \beta) + (1 - \alpha) \tau} \right) \). Part (i) is the expenditure channel: an additional child decreases savings due to the fact that current income is reduced by direct expenses for child
support. Part (ii) is the transfer channel evaluated at the equilibrium interest and savings rates: an additional child increases the transfers received while retired so that households can afford to save less.\(^{27}\) The transfer channel, \(\partial_{PE,Transf}\), is equal to zero when \(\tau = 0\), while it is negative for all other admissible values of \(\tau\).

Second, notice that \(\partial_{GE,EQ}\) can be rewritten as

\[
\partial_{GE,EQ} = \partial_{PE,Expend} \varphi (\alpha, \beta, \tau),
\]

where \(\varphi (\alpha, \beta, \tau) \leq 1\) for all parameters and is equal to 1 only if \(\tau = 0\). From this last equation, we see that absent any transfer from children to parents (i.e., \(\tau = 0\)), we will get \(\partial_{GE,EQ} = \partial_{PE,EQ}\): the PE and GE effects of fertility on savings are identical because the expenditure channel is identical in PE and GE. In contrast, for any positive \(\tau\), \(\partial_{GE,EQ} > \partial_{PE,EQ}\), so that the effect of an additional child on saving is smaller in GE than in PE.

It is important to note that the extent to which GE and PE effects of fertility on savings are similar depends on the relative contributions of the expenditure and transfer channels in explaining the PE relationship between fertility and savings. If the PE relationship is generated only by the expenditure channel, as in the case in which \(\tau = 0\), then the PE and GE are identical. Instead if the PE relationship is generated only by the transfer channel, as in the case in which \(\theta = 0\), then the GE effect is muted, and thus very different from the PE one. In Section 4.3, we use the empirical estimates to assess the relative contributions of the expenditure and transfer channels and quantify the difference between PE and GE.

**Discussion**

PE and GE effects are different for two reasons: (i) in GE, the transfer channel is muted, so that \(\partial_{GE,Transf} = 0\); and (ii) in GE, the expenditure channel is smaller than in PE, which is

\(^{27}\)Note that an additional child provides a negative income shock through channel (i), while it provides a positive income shock through channel (ii). Our interpretation of the timing of this model is that the negative income shock happens when the household is saving while the positive income shock happens when the household is dissaving. Note that when we observe these households in the data, children are already adults and beyond the age when they major expenditures are needed. The interpretation of the expenditure effect therefore rests on the idea that households spent more on their children when their children were young, thus postponing other expenditures (house purchase, house repair, etc.) until they are older.
given by $\varphi(\alpha, \beta, \tau) \leq 1$.

We first discuss (i). An additional child provides a benefit in the future: parents need to save less today because they are expecting to receive more transfers from children after retirement. The present value of these future transfers is lower if the interest rate is higher. This is what Summers (1981) called a wealth effect, to distinguish it from the income effect of increasing the interest rate, which exactly offsets the substitution effect in this log utility case. In GE, an increase in aggregate fertility raises the interest rate and under the assumptions of log utility and full depreciation, this consequent reduction in the value of the transfer exactly offsets the direct impact of increased fertility on total transfers. As a consequence, the transfer channel is effectively turned off: $\partial_{GE,Transf} = 0$.

Next, we discuss (ii). The expenditure channel does not directly depend on the interest rate. This is because both the spending on children and the savings decision are made in the same period. However, in GE, the direct effect of an additional child on spending reduces aggregate savings, which then implies capital scarcity and higher interest rates. The resulting reduction in the value of future transfers leads, as before, to higher savings, which partly compensates for the reduction in savings coming from the expenditure channel. This is why we find that $\varphi(\alpha, \beta, \tau) \leq 1$. Obviously, when there are no transfers from children ($\tau = 0$), this effect is shut down and $\varphi(\alpha, \beta, \tau) = 1$.

**Out of Steady-State Dynamics**

So far, our focus has been on steady states. We now show that the previous results, and in particular the important role that GE forces have on the relationship between fertility, transfers and savings, hold on the transition path from one steady state to another. The only change that occurs when we consider the transition path is that there is a wage effect as well as an interest rate effect: wage growth is slowing down (relative to steady-state trend) and the interest rate is rising as the labor force grows (because of increased fertility). Both of these effects encourage parents to save more: the interest rate effect for reasons already discussed and the wage effect because lower children’s earnings imply lower transfers in the future.
More formally, we can substitute the equilibrium expression for interest rate, \(1 + r_{t+1} = \alpha k_{t+1}^{1-\alpha}\), and wage, \(w_t = (1 - \alpha) k_{t}^\alpha\), in the formula for the savings rate to obtain:

\[
s_{i,t} = \left( \frac{\beta}{1 + \beta} \right) \left[ (1 - \tau - \theta n_{i,t}) - \frac{\tau n_{t}}{\alpha \beta} (1 + g) \frac{k_{t+1}}{k_{t}^\alpha} \right].
\]

We can further manipulate this expression, substituting the law of motion of capital, which must hold even out of steady state, and summing over all households in order to solve for the aggregate savings rate on the transition path

\[
s_t = \frac{\alpha \beta (1 - \tau - \theta n_{t+1})}{\alpha (1 + \beta) + \tau (1 - \alpha)}.
\]

This formula mirrors the steady state formula in equation (6), such that \(\frac{\partial s_t}{\partial n_{t+1}} = \frac{\partial s}{\partial n}, \forall t\). In other words, in this example with full depreciation and log preferences, being on the path to a steady state is identical with being at the steady with respect to how fertility affects savings. This is because the smaller rise in interest rate along the transition path (because capital does not jump to its new steady state value) is compensated by the reduction in wage growth (which dissipates when we reach the new steady state).

4.2 Generalizing the model

In order to bring the model closer to the data, we enrich the set of demographic features and relax the assumption of log utility in favor of a CRRA utility function.

**Demographics** We introduce two new elements into the previous model: (i) we allow a household to include a father and a mother, both of whom transfer to their own parents; (ii) we distinguish between sons and daughters, to match the fact that parents rely more on sons than daughters for old-age support. We assume that males and females earn the same.\(^{28}\)

However, daughters transfer a fraction, \(\lambda < 1\), of what sons transfer to their parents.\(^{29}\)
Following the discussion in Section 3.5, we assume that the cost of raising children is the same whether they are a boy or a girl.

Accounting for these demographic characteristics, the budget constraint in equation (3) becomes

\[
c_{i,t} + \frac{c_{i,t+1}}{1 + r_{t+1}} \leq 2A_tw_t \left(1 - \tau (1 + \lambda) - \theta \left(n_{i}^m + n_{i}^f\right)\right) + \frac{A_{t+1}w_{t+1}}{1 + r_{t+1}} \tau \left(n_{i}^m + \lambda n_{i}^f\right),
\]

where \(n_{i}^m\) is the number of sons in household \(i\) and \(n_{i}^f\) is the number of daughters in household \(i\).

**CRRA Utility Function** To allow households to have an inter-temporal elasticity of substitution different than one, we use a CRRA utility function, \(u(x) = \frac{x^{1-\rho}}{1-\rho}\), where \(\frac{1}{\rho}\) is the inter temporal elasticity of substitution (IES). If \(\rho > 1\), then the IES is smaller than 1, which implies that an increase in the interest rate decreases savings because the substitution effect is weaker than the income effect. \(\rho = 1\) gives the log utility case already analyzed.

**4.2.1 Some Intuition for this Case**

We solve the first order conditions of the model with the new budget constraint and the CRRA utility to obtain the savings rate for household \(i\)

\[
s_{i,t} = \left[\frac{\beta^{\frac{1}{\rho}} (1 + r_{t+1})^{\frac{1-\rho}{\rho}}}{1 + \beta^{\frac{1}{\rho}} (1 + r_{t+1})^{\frac{1-\rho}{\rho}}} \left[\left(1 - \tau (1 + \lambda) - \theta \left(n_{i}^m + n_{i}^f\right)\right) - \frac{\tau \left(n_{i}^m + \lambda n_{i}^f\right)}{\beta^{\frac{1}{\rho}} (1 + r_{t+1})^{\frac{1}{\rho}}} \left(\frac{A_{t+1}w_{t+1}}{2A_tw_t}\right)\right] \right].
\]

(9)

To build some intuition, we sum equation (9) over all households and using the formula for the steady state interest rate, which is unchanged by the new assumptions, we obtain a
formula for the steady state aggregate savings rate

\[
s = \left[ \frac{\beta^{\frac{1}{\rho}} \left( \frac{(1+g)(n^m+n^f)}{(1-\alpha)s} \right)^{\frac{1-\rho}{\rho}}}{1 + \beta^{\frac{1}{\rho}} \left( \frac{(1+g)(n^m+n^f)}{(1-\alpha)s} \right)^{\frac{1-\rho}{\rho}}} \right] \left[ (1 - \tau(1 + \lambda) - \theta (n^m + n^f)) - \frac{\tau (n^m + \lambda n^f)}{\beta^{\frac{1}{\rho}} \left( \frac{(1+g)(n^m+n^f)}{(1-\alpha)s} \right)^{\frac{1-\rho}{\rho}}} \right],
\]

where \(n^m\) and \(n^f\) are the aggregate numbers of sons and daughters fertility, and \(s\) is the aggregate savings rate.

The steady state savings rate is the product of two square-bracketed terms. Within the second bracket, the first term is the cost of an extra child and the second term captures the fact that an extra child brings more future income and hence reduces savings. In GE, these two PE effects are augmented by two more effects, both operating through the denominator of the second term. The first is the wealth effect resulting from the increase in the interest rate caused by the increase in fertility. The second is the feedback from the increase in savings, which pushes the interest rate down and therefore mitigates the wealth effect.

The first square bracket captures the income and substitution effects resulting from the increase in the interest rate. Assuming that \(\rho > 1\) (we later argue that this is the interesting case), the increase in the interest rate induced by the increase in fertility must reduce the part of savings that is determined by the income and substitution effects. This reduction in savings in turn has a feedback effect which further raises the interest rate and further reduces savings. This positive feedback loop is the reason why the GE effect can be larger than the PE effect. We will provide some examples when we present the quantitative results.

### 4.3 Using the Micro Evidence to Identify Model Parameters

In this section, we use the micro empirical evidence from earlier to pin down some of the key parameters of the model and predict the GE relationship between fertility and savings.
In particular, as previously emphasized, we wish to use the empirical estimates in order to understand the relative weights that the expenditure and transfer channels have in explaining the estimated PE relationship between households savings and fertility.

The regressions from Section 3.4 give us two coefficients that are useful for identifying the relative magnitudes of the expenditure and transfer channels. The results in Table 4 column (7) show two relationships: (i) that households with only one son save on average approximately 10 percentage points less than households with only one daughter; and (ii) that households with two children save on average approximately 10 percentage points less than households with only one child.\footnote{The coefficients are the following: \# kids -0.158, \# kids times 1st is male 0.116, 1st is male -0.215. Ignoring the constant, fixed effects and controls in the regression, the predicted savings rates for households with different numbers and sexes of children are the following: 1 son -0.158 + 0.118 - 0.215 = -0.255, 1 daughter -0.158, 1 son + 1 other child 2(-0.158) + 2(0.118) - 0.215 = -0.295, 1 daughter + 1 other child 2(-0.158) = -0.316. Thus, the difference in savings rate between a household with only one son and only one daughter is -0.255 - (-0.158) \approx 0.1, and the difference between households with two children and households with one child is around 0.099, which is the average of -0.295 - (-0.255), -0.295 - (-0.158), -0.316 - (-0.255), -0.316 - (-0.158).} Admittedly, the coefficients are not very precisely estimated. Thus, in Section C in the Online Appendix, we conduct a robustness exercise to show how our results are sensitive to different parameter values. Finally, we note that the average savings rate in our sample, which allows us to pick the discount factor $\beta$, is 26 percentage points.

Empirical results (i) and (ii) identify the contributions of the expenditure and transfer channels to savings as a function of the parameter $\lambda$, which captures the relative transfers of a daughter as a function of those of a son. As an intermediate step, it is useful to redefine the expenditure and transfer channels in the complete model.\footnote{The introduction of CRRA utility slightly alters the formula for the expenditure and transfer channels, which now both depend on the values of the IES (Intertemporal Elasticity of Substitution) and the interest rate. Note that in PE, we can decompose the effect of an additional child on savings in the direct effect from higher immediate expenditures (the expenditure channel) and the indirect effect from expected future transfers (the transfer channel). Thus, we can still consider the formula under CRRA as capturing the expenditure and transfer effects. Taking GE effects into account will affect savings through both the channels.} We call the two channels $\tilde{\partial}_{PE,\text{Expend}}$ and $\tilde{\partial}_{PE,\text{Transf}}$ to distinguish them from the previous formula of the simplest model. They are given by

$$
\tilde{\partial}_{PE,\text{Expend}} \equiv \left[ \frac{\beta_{t+1} (1 + r_{t+1})^{-1}}{1 + \beta_{t+1} (1 + r_{t+1})^{-1}} \right] \theta,
$$

(10)
\[
\tilde{\partial}_{PE,Transf} \equiv \left[ \frac{\tilde{\beta}_{t+1}}{1 + \hat{\beta}_{t+1} (1 + r_{t+1})^{-1}} \right] \left( \frac{A_{t+1} w_{t+1}}{2 A_{t} w_{t}} \right) \tau, 
\]

(11)

where we have defined \( \tilde{\beta} \equiv \beta_{t+1}^{1} (1 + r_{t+1})^{\frac{1}{2}} \). In order to identify \( \tilde{\partial}_{PE,Transf} \), we use empirical result (i). According to the model, the difference in the savings rate between a household with only one daughter and a household with only one son is given by \((1 - \lambda) \tilde{\partial}_{PE,Transf}\). Hence, using the empirical evidence, we have

\[
0.10 = (1 - \lambda) \tilde{\partial}_{PE,Transf},
\]

(12)

which identifies \( \tilde{\partial}_{PE,Transf} \) as a function of \( \lambda \).

In order to identify \( \tilde{\partial}_{PE,Expend} \), we use empirical result (ii). According to the model, the difference in the savings rate between a household with one child and a household with two children is given by \( \tilde{\partial}_{PE,Expend} + \frac{1}{2} (1 + \lambda) \tilde{\partial}_{PE,Transf} \). Hence, using the empirical evidence, we have

\[
0.10 = \tilde{\partial}_{PE,Expend} + \frac{1}{2} (1 + \lambda) \tilde{\partial}_{PE,Transf}.
\]

(13)

Equations (12) and (13) can be solved to obtain values for the expenditure and transfer channels as a function of \( \lambda \):

\[
\tilde{\partial}_{PE,Expend} = 0.10 \left( 1 - \frac{1}{2} \frac{1 + \lambda}{1 - \lambda} \right),
\]

\[
\tilde{\partial}_{PE,Transf} = 0.10 \left( \frac{1}{1 - \lambda} \right).
\]

It is immediately obvious that \( \tilde{\partial}_{PE,Expend} \) is decreasing in \( \lambda \), while \( \tilde{\partial}_{PE,Transf} \) is increasing in \( \lambda \). Intuitively, if \( \lambda \) is close to one, parents expect similar transfers from daughters and sons. For parents with sons and daughters to have very different savings, the level of the transfers must be high enough to magnify the relatively small gender difference in transfer rates into large differences in transfers and hence savings, which implies that \( \tilde{\partial}_{PE,Transf} \) must be very large. If \( \tilde{\partial}_{PE,Transf} \) is large, all of the difference in savings between households with one and two children will be driven by the expectation of future transfers, and the expenditure channel
will thus be of limited relevance, which explains why $\tilde{\partial}_{PE,Expend}$ is instead decreasing in $\lambda$.

Since it is costly to raise children, we assume that $\tilde{\partial}_{PE,Expend} \geq 0$. This restriction implies that $\lambda \in [0, \frac{1}{3}]$, which is consistent with the stylized evidence from Section 2 that daughters transfer considerably less than sons. The range of $\lambda \in [0, \frac{1}{3}]$ corresponds to $\tilde{\partial}_{PE,Expend} \in \left[0, \frac{1}{2}\tilde{\partial}_{PE,Transf}\right]$ - i.e., the empirical evidence implies that the transfer channel dominates the expenditure channel.

Next, we want to solve for the primitive parameters $\theta$ and $\tau$. To do this, we need to pin down a few additional parameters. In particular, equations (13) and (12) show that we need to choose values for $\frac{A_{t+1}w_{t+1}}{A_{t}w_{t}}$, $1 + r$ and $\tilde{\beta}$. We calculate $\frac{A_{t+1}w_{t+1}}{A_{t}w_{t}}$, the growth rate of wage income, from the UHS data. We use the average real deposit rate in China as the value for $r$. Song, Storesletten, and Zilibotti (2013) reports that the average real deposit rate in China between 1998 and 2012 is equal to 0.91%. We use this estimate. We then notice that the average savings rate is strictly increasing in $\tilde{\beta}$ and we thus pick $\tilde{\beta}$ in order to match the average savings rate in our data, which is equal to 26%. In order to calculate the average savings rate, we need to pick a value for the average number of children. We use $n = 1.88$, which is the average number of children in the sample used for our regression analysis. Then, for a given value of $\lambda$, we can calculate the corresponding values of $\theta$, $\tau$.

In Table 6, we report the estimated parameter values for the two extreme cases of $\lambda = 0$ and $\lambda = \frac{1}{3}$. The value of $\tau$ implies that an adult male transfers between 8% and 15% of his income to his parents. This is consistent with the UHIES data, which report that total transfer expenditures are approximately 8% of total household income for the average household with a male household head between 25 to 40 years of age. Our estimated value of $\tau$ is thus consistent with the limited empirical evidence.\textsuperscript{32}

The value of $\theta$ is estimated to be no more than 10%, which implies that every child costs no more than 10% of household income. This is roughly consistent with the data reported by the China Health and Nutritional Survey, which shows that an urban household in 1989

\textsuperscript{32}We acknowledge that assessing the plausibility of the transfer rate is difficult. To the best of our knowledge, there is no reliable data on transfers to parents at the individual level. Moreover, the ability of children to insure old parents in bad states of the world and cohabitation during old age is likely to be very valuable to parents and is difficult to measure or monetarize.
spends approximately 8\% of total income on food, clothing and schooling for children.\textsuperscript{33}

The value of $\beta$ depends on the value of $\rho$, and thus varies for different calibrations. For our preferred estimates, $\rho = 1$, and $\beta$ is equal to 0.995. This means that to match the high savings rate in the data, individuals need to be quite patient.

4.4 Quantitative Results

With the estimates of the primitive parameters, we can now quantify how the GE effect relates to the PE effect. Before doing so, we need to discuss how we deal with the interest rate within the model. In the calibration exercise, we have used the market interest rate that households face on deposits. The model has instead a prediction for the marginal product of capital. The marginal product of capital implied by the model in the baseline equilibrium, the one with $s = 0.26$ and $n = 1.88$, is not equal to the observed returns on savings in China. We thus need to calibrate a last parameter – the wedge between the marginal product of capital and the interest rate that households face on savings. We call this wedge $\psi$, which solves $1 + r = \psi \left[ \left( \frac{\alpha}{1-\alpha} \right) \left( \frac{n}{s} \right) (1 + g) \right]$, where the left-hand side of the equation is the market interest rate in China, as previously discussed, and the right-hand side is the wedge multiplied by the marginal product of capital in equilibrium as a function of savings rate and fertility, evaluated at the baseline parameters of $n = 1.88$ and $s = 0.26$. We assume that $\psi$ is invariant to policies that affect fertility and we thus keep it constant throughout the counterfactual experiments, so that changes in $n$ and $s$ are going to be reflected into changes of the interest rate that households face. Given this setup, we can vary the exogenous level of fertility $n$, and solve for the endogenous savings rate $s$ that is predicted by the model.

Using this simple procedure, we compute the hypothetical aggregate savings rates that the model implies for values of $n$ between 1 and 3. We repeat the same procedure for different values of $\rho$ between 0.5 and 3. In Figure 5, we plot aggregate savings rates as a function of

\textsuperscript{33}This result must be interpreted with caution. The fact that our empirical results use a sample of individuals age 50 to 65 who spend less on children than younger parents who have younger children means that our results could underestimate the effect of the expenditure channel. In light of this, the benchmark exercise considers the case where parental expenditures on children, $\theta$, takes the maximum value (i.e., $\lambda = 0$). Appendix Section C further explores the sensitivity of our calibration results to alternative parameter values.
aggregate fertility for the case in which $\lambda = 0$. In Figure 6, we repeat the same exercise for the case in which $\lambda = \frac{1}{3}$. For comparison purposes, we include the PE relationship between fertility and savings in the figure, which is from the earlier empirical estimates.

Figure 5 is the case where $\lambda = 0$, such that daughters transfer nothing. The red line displays the PE relationship, which is the observed savings rates of households in the same economy with different numbers of children. The black solid line displays the GE savings rates that are implied by different levels of aggregate fertility when $\rho = 1$. It shows the savings rate that the model predicts for a hypothetical situation in which all households would change their fertility level. The black line is flatter than the red line. This implies that an increase in aggregate fertility has a smaller effect on savings than the one that we estimated comparing different households.

The difference between PE and GE is large: a household that has one additional child on average saves ten percentage points less; but if all households have one additional child, the aggregate savings rate decreases by only 3.3 percentage points. The additional lines in the figure show aggregate savings rates for different values of $\rho$. As noted earlier, if $\rho$ is larger than one, then the general and PE effects are more similar. For very high values of $\rho$, it is even possible for the GE effect to be stronger than the PE effect. For example, as shown in Figure 5, if $\rho = 3$, then due to a very strong income effect, the GE effect would be larger than the PE effect.

Figure 6 is identical to the previous one, but uses the parameters estimated assuming that $\lambda = \frac{1}{3}$. When $\lambda = \frac{1}{3}$, the expenditure channel is completely shut down (since $\lambda = \frac{1}{3}$ implies $\theta = 0$), which means that an increase in aggregate fertility has no effect on aggregate savings when $\rho = 1$. This is why the black GE line is flat at 26 percentage points in the figure. In general, increasing $\lambda$ magnifies the estimated difference between PE and GE effects. This is because for a high $\lambda$, the transfer channel (which is substantially different in PE and GE) plays a larger role in explaining the relationship between fertility and savings.

In summary, the quantitative analysis shows that extrapolating from the PE evidence to predict the effect of the removal of the One Child Policy is likely to significantly overestimate
the resulting reduction in savings. We note that the literature has not yet formed a consensus on the true value of $\rho$. A recent review of the literature, Attanasio and Weber (2010) argues that $\rho$ is likely to be around 1.5. Our results indicate that in this case, extrapolating from the PE evidence to predict the effect of an aggregate increase in fertility would overestimate the increase in savings rate by as much as 50%, even in the most conservative calibration with $\lambda = 0$.

4.5 Further Generalizations

Thus far, we have considered a model with the minimal amount of structure that is necessary for matching the empirical results and which allow us to conduct the GE counterfactual. We now explore the implications of an extended model where we endogenize fertility, human capital investment and transfer rates. We investigate how they change the difference in the PE and GE effects of fertility on savings. The discussion follows the baseline model from Section 4.1 and focuses on the key insights.\textsuperscript{34}

4.5.1 Endogenous Fertility

In the baseline model, we assumed that fertility was exogenous, which was appropriate since the PE empirical estimates relied on exogenous variation in fertility caused by family planning policies and also because the GE counterfactual aims to understand the effect of an exogenous increase in aggregate fertility caused by policy changes. Now, we consider the case where fertility is a choice.

If we assume instead that parents treat children as investment goods (e.g. Caldwell, 1982 and Boldrin and Jones, 1988), then the decision to have a child is an investment that has an immediate cost (from raising the child) and entails the future benefit of transfers that are received from the adult child.

Suppose that parents can invest in two assets, children and savings, and try to optimize their portfolio across these two assets.\textsuperscript{35} Because of the “lumpiness” of the number of children, \textsuperscript{34}Section B in the appendix provides a more formal and detailed description. \textsuperscript{35}We use the word “invest” to indicate actions that move wealth from one period to the next. In the context
not everyone will invest in the same number of children – i.e., at the optimum, otherwise identical families will choose different portfolios of children versus investment. In a cross-section of families, the correlation between savings and the number of children will be negative.

Now suppose a new regulation is introduced which restricts the preferred number of children to be below a certain cutoff. For the households for whom this constraint is binding, the number of children will go down and savings will go up in PE. This is very similar to our analysis of an exogenous change in the number of children.

However, in GE, there are two additional effects. First, wages will rise faster than productivity for some time, and this might induce some unconstrained households to increase their fertility. This will counteract the effect of the regulation. Second, interest rates will decline, making investment in children relatively more attractive. This again would push the unconstrained households to have more children. Thus, the PE and GE relationships between fertility and savings are still likely to be quite different with endogenous fertility.

4.5.2 Human Capital Investment

Here, we discuss the implications of allowing parents to choose the amount to invest in the human capital of their children. In the model, parents are willing to invest in their children’s human capital in anticipation of higher future transfers – i.e., investment in children’s human capital increases their future wages and, as a consequence, anticipated transfers. Children’s education is thus an investment, which requires an upfront cost but pays a benefit in the form of higher expected transfers.

We begin by assuming that there is no quantity-quality tradeoff in PE, such that parents’ investment in their children’s human capital is independent of the number of children. Nevertheless, a quantity-quality tradeoff emerges in GE: increased aggregate fertility causes a reduction in human capital investment per child. The reason is that higher aggregate fertility increases the interest rate, which reduces the value of transfers and thus the incentives for
parents to invest in children's education. The decrease in human capital investment, in turn, reduces expenditure per child, which will increase savings. Under the assumptions of the model in Section 4.1, the decrease in human capital caused by the increase in aggregate fertility will compensate for the increase in expenditure caused by having more children. Recall that the transfer channel is muted in GE (see Section 4.1.1). Thus, in GE, the relationship between fertility and savings will be significantly muted relative to the case in PE.

Next, we consider the presence of PE quantity-quality tradeoffs such that households with more children invest less in human capital per child. The PE quantity-quality tradeoff does not have a corresponding effect in the steady state of the GE economy because an increase in fertility also reduces the human capital of parents and thus the opportunity cost of raising children.

These two results together imply that the introduction of endogenous human capital investment makes the difference between PE and GE results even larger, and thus cannot overrule our main qualitative result that GE forces are important to take into account for understanding the effect of a change in aggregate fertility.

4.5.3 Endogenous Transfers

We now extend our model along the lines of Boldrin and Jones (2002) and assume that children make transfers because they care about their parents' well-being. If we allow the transfer rate to be endogenously determined, the PE and GE relationships between fertility and savings become even more different. When the transfer rate is endogenous, increasing the number of children reduces the transfer rate from each child. This occurs for three reasons. First, the increase in fertility decreases the incentive of each child to transfer to parents due to the strategic interactions among siblings. Second, it implies that young individuals must spend more on child rearing, and thus, they transfer less to their own parents. Third, an increase in aggregate fertility increases the interest rate, which reduces the value of transfers to parents, and thus reduces the incentives of altruistic children to make transfers.

The first two reasons are present both in PE and GE, while the third one emerges from
the effect of fertility on the interest rate – in GE, the negative relationship between the number of children and the transfer rate is stronger. As aggregate fertility increases, the total amount of transfers received from children increase less than proportionally because each child transfers less. Consequently, parents save more relative to the case with exogenous transfer rates. Allowing for endogenous transfer rates thus magnifies the difference between PE and GE results.

5 Conclusion

The goal of this paper is to illustrate the challenges of using PE estimates of behavioral parameters to analyze the effects of policies that affect the whole economy, but also the rewards of using them in combination with a model to infer what the full equilibrium effect would be. In the world described by our model, the PE effects of demographic changes substantially overestimate the GE effect. It is important to consider this key fact – that accounting for general equilibrium effects can substantially change the relationship between fertility and savings implied by partial equilibrium results – because many important macroeconomic policies depend on the anticipated results of imminent changes in aggregate fertility caused by factors such as the end of China’s fertility restrictions, or the end of Japan’s demographic “collapse.”

At the same time, our study highlights the sensitivity of the model-derived quantitative effects to the parameters that are used. Thus, an important endeavor for future studies on the effect of aggregate fertility change is to obtain reliable parameter estimates from careful micro-empirical estimates.

There are, of course, many caveats to keep in mind in interpreting our main result. Most importantly, rational expectations about the relatively distant future play an important role in our argument. In our model, parents react to the fact that the current boom in fertility will raise interest rates in the future when these children join the labor force. In contrast, if parents do not make the connection between current fertility changes and future price changes, the PE predictions would be the right ones. Finding reliable evidence that helps us determine the plausibility of this assumption remains a very important part of this research agenda.
References


Table 1: Means

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Table 2: The Effect of Family Planning on Fertility

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<td></td>
</tr>
</tbody>
</table>

Notes: All estimates control for city fixed effects. Standard errors, clustered at the level of birth year-sex-city are presented in parentheses. There are 131 clusters. The sample uses households that have their first child during 1967-77, and where the age of the household head is 50-65. Source: RUMiC (2008).
<table>
<thead>
<tr>
<th>Dep. Var Mean</th>
<th>Baseline</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>17162</td>
<td>17162</td>
<td>17162</td>
<td>17162</td>
<td>17162</td>
<td>17162</td>
<td>17162</td>
<td>17162</td>
<td>17162</td>
</tr>
<tr>
<td>1st Born 1972+</td>
<td>6,175</td>
<td>7,355</td>
<td>5,673</td>
<td>13,453</td>
<td>14,361</td>
<td>12,730</td>
<td>13,466</td>
<td>11,417</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1,689)</td>
<td>(4,065)</td>
<td>(2,366)</td>
<td>(470)</td>
<td>(1,042)</td>
<td>(512)</td>
<td>(1,226)</td>
<td>(968)</td>
<td></td>
</tr>
<tr>
<td>1st Born 1972+ x 1st is a son</td>
<td>-13,104</td>
<td>-12,613</td>
<td>-12,692</td>
<td>-12,979</td>
<td>-13,119</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2,687)</td>
<td>(2,638)</td>
<td>(2,663)</td>
<td>(2,718)</td>
<td>(2,710)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st is a Son</td>
<td>11,097</td>
<td>10,989</td>
<td>10,981</td>
<td>11,795</td>
<td>11,783</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1,741)</td>
<td>(1,701)</td>
<td>(1,715)</td>
<td>(1,868)</td>
<td>(1,870)</td>
<td></td>
<td></td>
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<tr>
<td>Controls</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HH Head Age</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>HH Head Age Squared</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>HH Head Years of Edu</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>HH Head Years of Edu Squares</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>HH Head Age &gt;55</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>Age of Youngest Child</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>Youngest Child Age &lt; 22</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>Mother is HH Head</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>475</td>
<td>475</td>
<td>475</td>
<td>475</td>
<td>475</td>
<td>475</td>
<td>475</td>
<td>475</td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.106</td>
<td>0.129</td>
<td>0.125</td>
<td>0.125</td>
<td>0.147</td>
<td>0.144</td>
<td>0.156</td>
<td>0.152</td>
<td></td>
</tr>
<tr>
<td>Joint: 1st Born 1972+ 1st Born 1972 x 1st is a Son</td>
<td>348.4</td>
<td>1748</td>
<td>37.94</td>
<td>487.2</td>
<td>-1703</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>0.893</td>
<td>0.521</td>
<td>0.988</td>
<td>0.871</td>
<td>0.543</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: All estimates control for city fixed effects. Standard errors, clustered at the level of birth year-sex-city are presented in parentheses. There are 131 clusters. The sample uses households that have their first child during 1967-77, and where the age of the household head is 50-65. Source: RUMiC (2008).
Table 4: The Effect of Fertility on Household Savings

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dep Var Means</td>
<td>17162</td>
<td>17162</td>
<td>17162</td>
<td>17162</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td># Kids</td>
<td>-14122</td>
<td>-14155</td>
<td>-18571</td>
<td>-18574</td>
<td>-0.110</td>
<td>-0.110</td>
<td>-0.158</td>
<td>-0.158</td>
</tr>
<tr>
<td></td>
<td>(5828)</td>
<td>(5780)</td>
<td>(1752)</td>
<td>(1741)</td>
<td>(0.063)</td>
<td>(0.062)</td>
<td>(0.028)</td>
<td>(0.028)</td>
</tr>
<tr>
<td># Kids x 1st is a Son</td>
<td>11052</td>
<td>11163</td>
<td>0.118</td>
<td>0.116</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(6846)</td>
<td>(6806)</td>
<td>(0.098)</td>
<td>(0.098)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st is a Son</td>
<td>803</td>
<td>-19910</td>
<td>-30358</td>
<td>0.007</td>
<td>-0.215</td>
<td>-0.026</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1723)</td>
<td>(12401)</td>
<td>(15853)</td>
<td>(0.025)</td>
<td>(0.184)</td>
<td>(0.261)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Controls</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>Regional Sex Ratio x 1st is a Son</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>475</td>
<td>475</td>
<td>475</td>
<td>475</td>
<td>475</td>
<td>475</td>
<td>475</td>
<td>475</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.008</td>
<td>0.008</td>
<td>0.001</td>
<td>0.003</td>
<td>0.018</td>
<td>0.018</td>
<td>0.006</td>
<td>0.007</td>
</tr>
<tr>
<td>F-stat (1st Stage)</td>
<td>53.51</td>
<td>55.86</td>
<td>20.02</td>
<td>19.94</td>
<td>53.51</td>
<td>55.86</td>
<td>20.02</td>
<td>19.94</td>
</tr>
</tbody>
</table>

Notes: All estimates control for baseline controls: city fixed effects, age of the household head and its squared term, education of the household head and its squared term. Standard errors, clustered at the level of birth year-sex-city are presented in parentheses. There are 131 clusters. The excluded instruments are: a dummy variable for whether the first child is born after 1972, and the interaction terms between whether the first child is male and dummy variables for if the first child is born after 1972. The sample uses households that have their first child during 1967-77, and where the age of the household head is 50-65. Source: RUMiC (2008).
Table 5: The Effect of Fertility on Income

<table>
<thead>
<tr>
<th></th>
<th>(1) Wages</th>
<th>(2) Other Labor</th>
<th>(3) Business</th>
<th>(4) Interest &amp; Rent</th>
<th>(5) Pension</th>
<th>(6) Welfare</th>
<th>(7) Dep Var Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dep Var Means</td>
<td>19708</td>
<td>120.1</td>
<td>2132</td>
<td>1800</td>
<td>25650</td>
<td>288.8</td>
<td></td>
</tr>
<tr>
<td># Kids</td>
<td>-11,235.635</td>
<td>-8,803.966</td>
<td>-65.054</td>
<td>-1,851.312</td>
<td>-411.131</td>
<td>-211.735</td>
<td>60.291</td>
</tr>
<tr>
<td></td>
<td>(1,973.216)</td>
<td>(1,637.266)</td>
<td>(123.057)</td>
<td>(726.407)</td>
<td>(394.466)</td>
<td>(718.815)</td>
<td>(169.558)</td>
</tr>
<tr>
<td># Kids x 1st is a Son</td>
<td>8,636.034</td>
<td>-2,890.272</td>
<td>-897.833</td>
<td>5,317.645</td>
<td>1,397.633</td>
<td>3,020.509</td>
<td>1,760.605</td>
</tr>
<tr>
<td></td>
<td>(9,044.960)</td>
<td>(8,178.309)</td>
<td>(450.710)</td>
<td>(4,210.454)</td>
<td>(1,850.359)</td>
<td>(4,090.164)</td>
<td>(1,083.093)</td>
</tr>
<tr>
<td>1st is a Son</td>
<td>-11,361.338</td>
<td>5,893.019</td>
<td>1,778.735</td>
<td>-7,844.857</td>
<td>-2,742.532</td>
<td>-3,919.576</td>
<td>-2,679.567</td>
</tr>
<tr>
<td>Observations</td>
<td>475</td>
<td>475</td>
<td>475</td>
<td>475</td>
<td>475</td>
<td>475</td>
<td>475</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.200</td>
<td>0.126</td>
<td>-0.081</td>
<td>0.160</td>
<td>0.198</td>
<td>0.346</td>
<td>-0.079</td>
</tr>
</tbody>
</table>

Notes: The excluded instruments are the dummy variable for the 1st child born in 1972+ and its interaction with whether the first child is male. All estimates control for baseline controls: city fixed effects, age of the household head and its squared term, education of the household head and its squared term. Standard errors, clustered at the level of birth year-sex-city are presented in parentheses. There are 131 clusters. The sample uses households that have their first child during 1967-77, and where the age of the household head is 50-65. Source: RUMiC (2008).
Table 6: Parameter Values

<table>
<thead>
<tr>
<th></th>
<th>$\lambda = 0$</th>
<th>$\lambda = \frac{1}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>10.18%</td>
<td>0%</td>
</tr>
<tr>
<td>$\tau$</td>
<td>8.77%</td>
<td>15.32%</td>
</tr>
</tbody>
</table>
Figure 1: Household Net Transfer Income (as a Fraction of Total Household Income) as Function of the Age of the Main Respondent and the Number of Children – CHARLS (2011) Nationally Representative Sample, Urban Households
Figure 2: The Fraction of Urban Parents Cohabitng with at Least One Child – UHIES (1990, 1998, 2005) from 19 Cities in 9 Provinces
Figure 3: Responses to the Survey Question “Whom do you think you can (most) rely on for old-age support?” as a Function of the Age of the Main Respondent – CHARLS (2011) Nationally Representative Sample, Urban Households
Figure 4: Distribution of Age and Savings in RUMiC Sample – Kernel Density with Gaussian Kernel Function

(a) Age of the First Child

(b) Age of the Household Head

(c) Age of the Youngest Child

(d) Savings

(e) Savings Rate
Figure 5: PE vs. GE for $\lambda = 0$
Figure 6: PE vs. GE for $\lambda = \frac{1}{3}$
A Data Appendix

The sample frame used in the RUMiC is the same as the one used in the National Bureau of Statistics (NBS) *Annual Urban Household Income and Expenditure Survey* (UHIES). Sample selection is based on several stratifications at the provincial, city, county, township, and neighborhood community levels. Households are randomly selected within each chosen neighborhood community. The RUMiC covers 19 cities in nine of the provinces.\(^{36}\) The sample aims to include 0.01% of households in the population. This sampling frame typically misses migrant laborers. For our study, this is an advantage since we assume that urban households we observe in 2008 also had urban status when they had their first child.

The survey was conducted in March and April, 2008. In addition to general information (including fertility) for household members, the questionnaire also included the demographic characteristics, education, and employment situation of other family members who are not residing with the household head and spouse, including parents, children, and siblings.\(^{37}\) This allows us to know the total fertility history and characteristics of adult children such as sex, age and marital status. In our study, total fertility is synonymous with the total number of living children. In our sample, the total number of living children is very similar to the total number of children ever born since infant mortality during the early 1970s was very low (Banister and Hill, 2004).

The information on household income and expenditure from the RUMiC in China are directly recorded from the UHIES survey (which is administered to the same households), which records income and expenditure variables using a diary. Specifically, households are required to record each item (disaggregated for hundreds of product categories) purchased and income received for each day for a full year (in our case, 2007). Enumerators visit sample households once or twice each month to review the records, assist the household with

\(^{36}\) The provinces included in the RUMiC urban survey are: Shanghai, Guangdong, Jiangsu, Zhejiang, Henan, Anhui, Hubei, Sichuan, and Chongqing. The detailed list of cities can be found at [http://rumici.anu.edu.au](http://rumici.anu.edu.au)

\(^{37}\) The questionnaires are available from [http://rumici.anu.edu.au](http://rumici.anu.edu.au)
questions, and take away the household records for data entry and the aggregation of the annual data at the local Statistical Bureau Office.

The UHIES data is the best available data on urban household economic variables. According to interviews with NBS statisticians and a detailed examination of income and expenditure distributions conducted by researchers in study of the income distribution and income taxation using the UHIES data, researchers concluded that the households that refuse to participate are typically the poorest and the richest households (Piketty and Qian, 2009). This makes it difficult to use the UHIES to study the extreme tails of the income distribution, but should not affect our study, which focuses on the mean household.

In our data, total household income is the sum of incomes from labor, business, property, pension and retirement allowances and other social welfare benefits. Total expenditure is the sum of consumption expenditure (e.g. food; clothing; housing; family equipment; service; health; transpirations and communication; education; cultural and entertainment; other commodity and services), operational expenditure, property expenditure, social security expenditure (e.g. individually paid pension fund, individually paid public housing fund, individually paid health care fund, individually paid unemployment fund, and other social security).  

Other recent studies of Chinese household savings or expenditures have mainly used the following surveys, which we discuss briefly to motivate the need to collect a new survey. The UHIES (1988– ), which we discuss above, surveys contain high quality income and expenditure data, but do not report total fertility. The China Health and Nutritional Surveys (CHNS) urban sample is small. The China Household Income Project (CHIP) does not report completed fertility and has a very small urban sample. The China Health and Retirement Longitudinal Survey (CHARLS, 2008, 2011) contains similar information to our survey and in addition, report transfers. We use these data for our descriptive statistics. Once we apply our sample restrictions, the CHARLS and RUMiC provide similar sample sizes for our study. Unfortunately, we are unable to use the CHARLS for the regression analysis because the ID

38 Food expenditure is the sum of expenditure on the following categories: grain, wheat, and rice coarse grains; pork, beef, and mutton; edible vegetable oil, fresh vegetables, dried vegetables, poultry, meat, eggs, fish; sugar, cigarettes, liquor, fruit, wine, beer, fresh melons and fruit cake; and milk.
variables required for linking the transfer data to other household data are not yet available for the full-sized surveys (they are only available for the 2008 pilot, which we use and discuss in Section 2).

B Details on Further Generalizations

B.1 Endogenous Fertility Choice

We now extend the model of Section 4.1 and allow parents to optimally decide how many children to raise. Children are an indivisible good, so that parents may choose $n_{i,t+1} \in \mathbb{N}$, where $\mathbb{N}$ is the set of non-negative integers. We also assume that parents have heterogeneous costs of raising children, in order to have a non-degenerate distribution of fertility choices, and we let the cost of raising children to be convex in the number of children itself. This assumption is necessary in order to have a unique optimal solution for each household. The parameter $\gamma > 1$ controls the degree of convexity. Last, fertility is constrained by a possibly binding constraint $\Lambda$. As an example, the relaxation of the One Child Policy can be modeled in this context as an increase in $\Lambda$. The household problem now reads as

$$
\max_{c_{i,t}^Y, c_{i,t+1}^O, n_{i,t+1} \in \mathbb{N}} \log (c_{i,t}^Y) + \beta \log (c_{i,t+1}^O)
$$

subject to

$$
c_{i,t}^Y + \frac{c_{i,t+1}^O}{1 + r_{t+1}} \leq A_{t} w_{t} \left( 1 - \tau - \theta_i n_{i,t+1}^\gamma \right) + \frac{A_{t+1} w_{t+1}}{1 + r_{t+1}} \tau n_{i,t+1}
$$

$$
n_{i,t+1} \leq \Lambda
$$

The optimal savings rate of the model is identical to the one of Section 4.1, and is given by

$$
s_{i,t} = \left[ \frac{\beta}{1 + \beta} \right] \left[ (1 - \tau - \theta_i n_{i,t+1}^\gamma) - \frac{\tau n_{i,t+1}^\gamma}{\beta (1 + r_{t+1})} \left( \frac{A_{t+1} w_{t+1}}{A_{t} w_{t}} \right) \right].
$$

The difference with the baseline model is that now the optimal number of children is endogenous. In order to describe household behavior is useful to first consider the latent number of
children, $\tilde{n}_{i,t+1}$, that would be optimally chosen if household could have any real number of children. This is given by

$$\tilde{n}_{i,t+1} = \left( \frac{A_{t+1}w_{t+1}}{A_tw_t} \right) \left( \frac{\tau}{\gamma(1+r_{t+1})} \right) \left( \frac{1}{\theta_i} - \tilde{\mu}_i \right)^{\frac{1}{\gamma-1}}$$

where $\tilde{\mu}_i \geq 0$ is the rescaled multiplier on the constraint $n_{i,t+1} \leq \Lambda$. It is immediate to notice that, as long as the constraint is not binding, $\tilde{n}_{i,t+1}$ is strictly decreasing in $\theta_i$. However, households cannot have a fraction of a child, so that true fertility, $n_{i,t+1}$, jumps discretely. In particular, it is easy to verify that for each value $n = \{1, 2, \ldots, \Lambda\}$ there exists $\theta_n, \theta_{n-1}$ such that if $\theta_i = \theta_n$ then $n_{i,t+1} = n$ and if $\theta_{n-1} \leq \theta_i < \theta_n$ then $n_{i,t+1} = n-1$.

In order to understand the implications of this model for the PE estimates on the relationship between savings and fertility, it is interesting to compare two households which are identical, but for the observed number of children. In particular, let’s assume that household 1 has $\theta_1 = \theta_n$ and household 2 has $\theta_2 = \theta_1 - \epsilon$, where $\epsilon$ is a very small number. Household 1 is going to have $n$ children, while household 2 is going to have $n-1$ children. We can then compare the savings rates of the two households: since $\epsilon$ is very small, it is immediate to see that $s_2 > s_1$: household 2 has less children and thus saves more. The model therefore is consistent with the PE evidence that shows, comparing households that are identical but for the number of children, that fertility and savings display a negative relationship.

Let’s now discuss the GE implications of the model for aggregate fertility changes. As an illustrative example, let’s consider the effect on savings of an aggregate reduction in fertility as caused by a tightening of the fertility constraint. Within the model, we thus consider the effect on fertility and savings of a decrease in $\Lambda$. The reduction in fertility is going to have the same GE effects on prices as in the baseline model of Section 4.1. Specifically, the reduction in fertility is going to reduce the interest rate and, as long as the economy is out of steady state, increase the growth rate of wage. However, the effect of the decrease in $\Lambda$ on households’ behavior is going to be different for different groups of households. In particular we need to distinguish between two different possibilities. The first type of households is represented by those that are constrained by the tightening of $\Lambda$. Those households are going
to decrease their fertility, and for them the analysis is identical to the case with exogenous fertility reduction: the extent to which their savings rate is going to increase depends on the relative strength of the consumption and transfer channels and on the responses of prices. There is, however, a second type of households, namely those that are not constrained even after the tightening of $\Lambda$. Those households are going to increase fertility on average. This is easy to see from the fact that, keeping $\tilde{\mu}_i$ fixed at zero (since those households are not constrained the multiplier is zero), the latent number of children is going to increase due to the fact that $\frac{w_{t+1}}{w_t}$ increases and $1 + r_{t+1}$ goes down. Hence, this second group of households is going to increase fertility and consequently reduce savings. As a consequence, the effect of the tightening of $\Lambda$ on aggregate savings rate is further dampened by the GE effects on this second group of individuals, beyond what it is in the case with exogenous fertility.

### B.2 Endogenous Investment in Human Capital

We extend the model of Section 4.1 and allow parents to optimally invest in their children’s human capital. We first consider the case in which in PE there is no quantity-quality tradeoff, so that neither the costs nor the benefits of investing in children human capital depend on the number of children itself. We model human capital as an increase in individual productivity. The wage income of an individual $i$ at time $t$ is thus given by $A_t w_t h_{i,t}$. Aggregate income is produced, as in the baseline case, with a Cobb-Douglas production function, where labor is now calculated in efficiency units, as standard in the human capital literature, so that $Y = K_t^\alpha (A_t h_t L_t)^{1-\alpha}$, where $h_t$ is the average human capital of the working population. Due to the assumption of competitive markets, the interest rate is $1 + r_t = \alpha k_t^{\alpha-1} h_t^{1-\alpha}$ and wage per efficiency unit is $w_t = (1 - \alpha) h_t^{-\alpha} k_t^\alpha$. Parents may invest in the human capital, $h_{i,t+1}$, of their children paying a convex cost $A_t w_t h_{i,t+1}^\gamma$, where $\gamma > 1$. Parents are willing to invest in the human capital of their children in order to increase received transfers: if children have more human capital they earn more and thus transfer more to parents. The problem of a household reads as follows:
\[
\begin{align*}
\max_{c_{i,t},c_{O,i,t+1},h_{i,t+1}} \quad & \log (c_{Y,i,t}) + \beta \log (c_{O,i,t+1}) \\
\text{s.t.} \quad & c_{Y,i,t} + \frac{c_{O,i,t+1}}{1 + r_{t+1}} \leq A_{t}w_{t}h_{i,t} \left( 1 - \tau - \theta \frac{h_{i,t+1}^{\gamma} n_{i}}{h_{i,t}} \right) + A_{t+1}w_{t+1}^{1} (\tau h_{i,t+1} n_{i})
\end{align*}
\]

Solving the first order conditions of the model we obtain an equation for optimal savings rate and one for optimal human capital investment

\begin{align*}
\begin{aligned}
s_{i,t} &= \left[ \frac{\beta}{1 + \beta} \right] \left[ \left( 1 - \tau - \theta \frac{h_{i,t+1}^{\gamma} n_{i}}{h_{i,t}} \right) - \frac{\tau h_{i,t+1} n_{i}}{\beta (1 + r_{t+1})} \left( A_{t+1}w_{t+1}^{1} A_{t}w_{t} h_{i,t} \right) \right], \\
h_{i,t+1} &= \left[ \frac{A_{t+1}w_{t+1}^{1} \tau}{\gamma A_{t}w_{t} \theta (1 + r_{t+1})} \right]^{\frac{1}{\gamma - 1}}.
\end{aligned}
\end{align*}

(14)

The second equation shows that, at the household level, optimal human capital does not depend on the number of children, but only on parameters that are identical across households, so that \( h_{i,t+1} = h_{t+1} \forall i \).

Next, we focus on steady states and substitute 15 into 14 to get

\begin{align*}
\begin{aligned}
s_{i} &= \left[ \frac{\beta}{1 + \beta} \right] \left[ \left( 1 - \tau - \frac{(1 + g) \tau n_{i}}{\gamma (1 + r)} \right) - \frac{(1 + g) \tau n_{i}}{\beta (1 + r)} \right], \\
h_{i,t+1} &= \left[ \frac{A_{t+1}w_{t+1}^{1} \tau}{\gamma A_{t}w_{t} \theta (1 + r_{t+1})} \right]^{\frac{1}{\gamma - 1}}.
\end{aligned}
\end{align*}

(15)

(16)

from which we see that, even in the presence of endogenous human capital investment, fertility and savings are negatively related at the household level, through both the expenditure and the transfer channels.

Let’s now solve for the GE. The law of motion of capital is given by

\[
k_{t+1} = (1 - \alpha) \frac{s_{t}}{(1 + g) n_{t+1}} h_{t}^{1-\alpha} k_{t}^{\alpha}
\]

so that in steady state

\[
k_{t}^{\alpha-1} h_{t}^{1-\alpha} = \frac{n(1 + g)}{s(1 - \alpha)}
\]
and hence, using the definition of the interest rate, we get that in steady state

\[ 1 + r = \frac{n (1 + g) \alpha}{s (1 - \alpha)}. \]

Substituting the equilibrium interest rate into 16 and summing over all households yield a formula for the aggregate savings rate

\[ s = \frac{\alpha \beta \gamma (1 - \tau)}{\alpha \gamma (1 + \beta) + \tau (1 - \alpha) (\beta + \gamma)} \]

which is independent from aggregate fertility. As such, despite the fact that at the household level fertility and savings are negatively related, aggregate fertility and aggregate savings are not related.

This result comes straight from the equation 15 for human capital investment. At the household level, human capital investment does not depend on the number of children, but is decreasing in the interest rate. At the aggregate level, however, human capital investment is decreasing in fertility: an increase in fertility increases the interest rate which makes the returns from investing in children human capital smaller. A quantity-quality tradeoff thus emerges in GE, due to the role of fertility on the interest rate. Due to the assumptions about the functions made in the model, the decrease in human capital investment exactly compensate the “expenditure channel” relationship between fertility and savings. Moreover, the “transfer channel” is muted in GE for the reasons discussed in the main text. The consequence is that in GE there is no relationship between fertility and savings.

**PE Quantity-Quality Tradeoff** Alternatively, we could consider the case in which a quantity-quality tradeoff is present also in PE. A PE quantity-quality tradeoff can be modeled as a cost of human capital investment that is increasing in the number of children, so that the cost of investing in children human capital is now given by \( \zeta (n_{t+1}) A_t w_t h_t^\gamma \), where \( \frac{\partial \zeta (n_{t+1})}{\partial n_{t+1}} > 0 \). This would imply that households with more children invest less in the human
capital of each one of them. The savings rate and optimal human capital are now given by
\[ s_{i,t} = \left[ \frac{\beta}{1 + \beta} \left(1 - \tau - \theta \frac{h_{i,t+1}^n}{h_{i,t}} \zeta(n_i) n_i \right) \right] - \frac{\tau h_{i,t+1} n_i}{\beta (1 + r_{t+1})} \left( \frac{A_{t+1} w_{t+1}}{A_t w_t h_{i,t}} \right), \]
\[ h_{i,t+1} = \left[ \frac{A_{t+1} w_{t+1} \tau}{\gamma \zeta(n_i) A_t w_t \theta (1 + r_{t+1})} \right]^{\frac{1}{\gamma - 1}}. \]

Substituting the optimal human capital into the savings rate, and focusing to a steady state in which the number of siblings of parents and children is identical, we obtain again 16, so that the presence of PE quantity-quality tradeoff does not change the results previously shown.\(^{39}\)

### B.3 Endogenous Transfers to Parents

We extend the model of Section 4.1 and let transfers from children to parents to be an endogenous outcome. In order to do so, we develop the model along the lines of Boldrin, De Nardi, and Jones (2005).\(^{40}\) Individuals value their own consumption and the wealth of their parents. Every individual thus solves

\[
\max_{c_{i,t}^Y, c_{i,t+1}^O, \tau_{i,t}} \log(c_{i,t}^Y) + \beta \log(c_{i,t+1}^O) + \delta \log(e_{i,t-1}^Y)
\]

s.t.

\[
c_{i,t}^Y + \frac{c_{i,t+1}^O}{1 + r_{t+1}} \leq A_t w_t (1 - \tau_{i,t} - \theta n_i) + \frac{A_{t+1} w_{t+1}}{1 + r_{t+1}} (\tau_{t+1} n_i)
\]

\(^{39}\)The assumption that number of siblings of parents and children is identical implies that \(h_{i,t} = h_{i,t+1}\), which is useful to simplify the algebra and have simple results. We can relax this assumption and show that the GE relationship between fertility and savings is muted up to a covariance term. These results are available upon requests.

\(^{40}\)The Boldrin, De Nardi, and Jones (2005) setting is slightly different than ours. Boldrin, De Nardi, and Jones (2005) uses a utility function of the form

\[ U = \log(c_i) + \beta \log(c_{i+1}) + \delta \log(c_i^O) \]

such that children value the consumption of their parents when parents are old, rather than parents well-being over the whole life. This assumption implies that parents have a strategic incentive not to save in the first period because savings crowd out transfers from children. We introduce the assumption that children care about the total wealth of the parents to abstract from parents strategic behavior in savings. Conceptually, we are assuming that children have the ability to commit to a level of transfer that does not depend on the parents’ behavior in the first period, but only on the parents’ income and macroeconomic condition.
\[ \epsilon_{i,t-1}^Y \leq A_{t-1}w_{t-1} + \frac{A_tw_t}{1+r_t}(\tau_{i,t} + \tilde{\tau}_t (\tilde{n}_i - 1)). \]

The previous notation applies. Also notice that when deciding how much money to transfer to parents, individuals take as given the number of their siblings, \( \tilde{n}_i \), and the transfer of their siblings, \( \tilde{\tau}_t \). We focus on a symmetric solution, so that in equilibrium \( \tau_i = \tilde{\tau} \).

Solving the first order conditions of the model, we obtain the usual equation for optimal savings rate and an additional equation that comes from solving for the optimal transfer rate

\[ s_{i,t} = \left[ \frac{\beta}{1+\beta} \right] \left[ (1 - \tau_{i,t} - \theta n_i) - \frac{\tau_{t+1}n_i}{\beta (1+r_{t+1})} \left( \frac{A_{t+1}w_{t+1}}{A_tw_t} \right) \right], \quad (17) \]

\[ c_{i,t}^Y = \frac{1}{\delta} c_{i,t-1}^Y (1 + r_t). \quad (18) \]

Using 17, 18, and the budget constraints, we solve for the optimal transfer rate as a function of the number of siblings and children

\[ \tau_t (n_i, \tilde{n}_i) = \left( \frac{1}{1+\beta} \right) \left[ \frac{(1-\theta n_i)}{1+\beta} + \frac{A_{t+1}w_{t+1}}{A_tw_t} \frac{\tau_{t+1}n_i}{(1+r_{t+1})(1+\beta)} \right] - \frac{1}{\delta} (1+r_t) \frac{A_{t-1}w_{t-1}}{A_tw_t} \frac{1}{1+\beta} + \frac{\tilde{n}_i}{\delta}. \]

The analysis of the optimal transfer rate is informative about the model implications for the PE and GE relationships between savings and fertility. The presence of endogenous transfer rate does not change the PE relationship between fertility and savings, which is still given by the usual equation 17.\(^{41}\) In GE however, the interest rate has now two effects on savings: (i) a wealth effect through the change in the value of transfers, which was present also in the model with exogenous transfer rate; (ii) a change in the transfer rate from each child. Both effects (i) and (ii) go in the same direction, so that GE forces are larger in the model with endogenous transfer rate. As an example, let’s consider the foreseeable effects of the relaxation of the One Child Policy. The increase in aggregate fertility puts an upward pressure on the interest rate. The increase in the interest rate decreases the total value of

\(^{41}\)The model implies that household level savings and fertility are negatively related, as long as we restrict the parameter set to obtain the natural assumption that households with more children receive more transfers.
transfers, so that parents save more with respect to the PE prediction. This mechanism is identical to the case with exogenous transfer rate. But, in addition, the increase in the interest rate implies that each child transfers less, due to the fact that parents value future transfers less, and this reduces the total amount of transfers and thus again increases savings. This additional channel means that the GE effect of an aggregate increase in fertility predicted by the model with endogenous transfer rate is smaller than the one predicted by the model with exogenous transfer rate.

To sum up, this analysis showed that if we believe that children transfer to parents as a result of altruistic behavior, then the GE relationship between fertility and savings is even weaker than if we assume the transfer rate to be exogenous.

C Alternative Calibrations

In the main calibration exercise we used point estimates from Table 4. However, as already discussed, two of the three coefficients of interest are not precisely estimated, and are in fact not significant, with p-values of respectively 0.23 and 0.24. For this reason, in this section we perform a robustness exercise to understand the implications of our model for different sets of parameters. We allow the three coefficients of interest, namely the coefficient on the number of kids \( \phi \) henceforth, the coefficient on the interaction between the number of children and the firstborn being a male \( \kappa \), and the coefficient on the firstborn being a male \( \xi \), to take one of three possible values: (i) the baseline value, which is simply the point estimates as shown in the Table 4; (ii) the baseline value minus one third of its standard deviation; and (iii) the baseline value plus one third of its standard deviation. The choice of one third is motivated by the fact that we want the transfer rate \( \tau \) implied by the model to be non-negative, and the maximum values of \( \phi, \kappa, \) and \( \xi \) that are consistent with \( \tau \) being non-negative are in fact \( \phi + \frac{1}{3}\sigma_\phi, \kappa + \frac{1}{3}\sigma_\kappa, \) and \( \xi + \frac{1}{3}\sigma_\xi \). We have three values for each of the three coefficients of interest, hence we have 27 possible combinations. For each of them we find the primitive parameters, \( \tau, \theta \) and \( \beta \), such that the model generates an average savings rate of 0.26 and matches the three coefficients of interest. We then use the calibrated model to perform the
counterfactual exercises of increasing fertility by one child both in general and in PE. We show that for almost all possible combinations of parameters the general and PE effects of fertility on savings are very different. We now describe the results in more details.

For brevity, we focus on our preferred estimates, the one with $\rho = 1$ and $\lambda = 0$. In Table 7 we report the calibrated transfer rate ($\tau$) for each triple of coefficients. Each matrix corresponds to one value for the coefficient on the firstborn being a male ($\xi$), each row to one value for the coefficient on the number of kids ($\phi$), and each column to one value for the coefficient on the interaction between the number of kids and the firstborn being a male ($\kappa$). In Table 8 we report the calibrated consumption per child ($\theta$). In Table 9 we report the percentage of the PE effect on savings that is still present in GE. Table 9 shows that for almost all combinations of coefficients the difference between PE and GE effects are sizable. The only exception is the case in which both $\xi$ and $\kappa$ take a high value. The reason is intuitive. When $\xi$ and $\kappa$ are high, the difference in savings rates between households with only one son and households with only one daughter is very small. This difference identifies the transfer rate, which is the driver of the GE effects. Indeed when $\xi$ and $\kappa$ are high the transfer rate is almost identical to zero, which implies that the relationship between fertility and savings is purely driven by the consumption channel. And, as shown in the paper, the consumption channel is identical in PE and GE.
Table 7: Transfer Rate

(a) Low $\xi$

<table>
<thead>
<tr>
<th></th>
<th>Low $\kappa$</th>
<th>Baseline $\kappa$</th>
<th>High $\kappa$</th>
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<tr>
<td>Low $\phi$</td>
<td>25.02%</td>
<td>15.70%</td>
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<tr>
<td>Baseline $\phi$</td>
<td>21.87%</td>
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<td>20.02%</td>
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(b) Baseline $\xi$

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</tr>
</thead>
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<tr>
<td>Low $\phi$</td>
<td>13.51%</td>
<td>8.81%</td>
<td>5.29%</td>
</tr>
<tr>
<td>Baseline $\phi$</td>
<td>12.76%</td>
<td>8.44%</td>
<td>5.11%</td>
</tr>
<tr>
<td>High $\phi$</td>
<td>12.12%</td>
<td>8.11%</td>
<td>4.94%</td>
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(c) High $\xi$

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<tr>
<td>Low $\phi$</td>
<td>6.47%</td>
<td>3.05%</td>
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<tr>
<td>Baseline $\phi$</td>
<td>6.20%</td>
<td>2.95%</td>
<td>0.23%</td>
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<tr>
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<td>5.96%</td>
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<td>0.24%</td>
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Table 8: Consumption per Child

(a) Low $\xi$

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<td>Low $\phi$</td>
<td>4.42%</td>
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<td>3.25%</td>
<td>3.79%</td>
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<td>1.83%</td>
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(b) Baseline $\xi$

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<td>10.47%</td>
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(c) High $\xi$

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<td>17.12%</td>
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<td>14.71%</td>
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Table 9: Ratio between the GE and PE Effect on Saving

(a) Low $\xi$

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<th>Baseline $\kappa$</th>
<th>High $\kappa$</th>
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<td>13.84%</td>
<td>17.12%</td>
<td>21.80%</td>
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<td>10.17%</td>
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<td>High $\phi$</td>
<td>5.86%</td>
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(b) Baseline $\xi$

<table>
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<td>32.68%</td>
<td>24.89%</td>
<td>52.76%</td>
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<td>29.81%</td>
<td>37.77%</td>
<td>49.56%</td>
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<td>High $\phi$</td>
<td>26.44%</td>
<td>33.99%</td>
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(c) High $\xi$

<table>
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<td>72.62%</td>
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<td>High $\phi$</td>
<td>54.34%</td>
<td>70.96%</td>
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