Lecture 9: Supplementary Exercises

1) Value of the Tax Shield. When I valued the tax shield in class I assumed that even if the firm defaulted, the firm would still receive future tax savings from the debt tax shield. I assumed that after a default, the firm would be restructured with the same amount of debt ($1000). This is an upper limit on the value of the tax shield. In this question, I want you to make a different assumption. Assume that if the firm defaults, the firm is liquidated. The firm does not issue new debt and thus there is no tax shield after (or in the year in which the firm defaults.

A) Details of the bond issue. The bond has a face value of $1000 and a promised rate of return of 10%. The bond defaults 20 percent of the time and pays the $1000 principal but no interest if it defaults. When the bond does not default, it pays the $1000 principal plus the $100 interest. Assume that if the bond does not default, the debt is rolled over.[1] What is the expected tax savings at the end of the first year. Assume that the corporate tax rate is 34 percent.

B) What is the expected tax savings the second year? If you are having trouble with the logic of this question, see question 2 on Homework 1.

C) If we assume that the firm is liquidated once it defaults on its debt, what is the value of the tax shield. You can use the following formula. You should understand why the first portion of the expression is correct. Getting from the sum to the final expression is just math.

$$V = \sum_{t=1}^{\infty} \frac{(1 - p)^t \text{ Cashflow}}{(1 + r)^t} = \frac{(1 - p) \text{ Cashflow}}{r + p} \qquad \textbf{(1)}$$

[1] This means the firm pays the bond holder $1100, but then sells a new one year bond to the bond holder for $1000. The maturity of the bond is only one year, but the debt is expected to be outstanding longer than one year.

Lecture 9: Supplementary Exercise Answers

1) Value of the Tax Shield.

A) Expected tax savings in year one. At the end of the first year, the firm will pay either \$0 of interest or \$100 of interest. Thus the firm's tax savings due to the interest payments are \$0, 20 percent of the time. If the bond does not default, which occurs 80 percent of the time, the tax savings is \$34 (100 * .34). The expected tax savings at the end of the first year is thus:

$$E[\text{ tax savings in year 1 }] = 0.2\ (.34)\ 0 + 0.8\ (.34)\ 100 = 27.2 \qquad \textbf{(2)}$$

B) To pay interest the second year and thus to receive the tax shield the second year, the firm must not default in the first year and not default in the second year. The probability of not defaulting two years in a row is 64 percent (80% * 80%). The probability of defaulting in the second year is 16 percent (80% * 20%). The firm can only default in the second year if it first does not default in the first year. Thus the expected tax savings the second year is:

$$\begin{aligned} E[\text{ tax savings in year 2 }] &= 0.8\ [\ 0.2\ (.34)\ 0 + 0.8\ (.34)\ 100\ ] \\ &= 0.8\ 0.8\ (.34)\ 100 = 21.76 \end{aligned} \qquad \textbf{(3)}$$

C) The firm receives the tax savings due to the interest tax shield only if it doesn't default. The probability that it makes it through t years with out defaulting is 0.8 raised to the power t. Thus, the expected tax savings in year t is equal to:

$$E[\text{ tax savings in year t }] = 0.8^{t}\ (.34)\ 100 \qquad \textbf{(4)}$$

The value of the tax shield is the discounted value of the expected tax savings. Thus the value of the tax shield given our assumptions is:

$$\text{NPV}[\text{ Tax shield }] = \sum_{t=1}^{\infty} \frac{0.8^{t-1}\ .34\ 0.08\ 1000}{(1 + .08)^{t}} = \frac{0.34\ 0.08\ 1000}{0.08 + 0.20} = 97$$

$$\text{NPV}[\text{ Tax shield }] = \frac{\tau\ r_{Debt}\ \text{Face Value}}{r_{Debt} + \Pr[\text{ Default }]} \qquad \textbf{(5)}$$

A couple of observations. First this value is smaller than the number we calculated in class (340). In class we assumed that value of the tax shield wasn't lost if the firm defaulted on its debt. If we assume that the value of the tax shield is lost when the firm defaults, this reduces the value of the tax shield by 72 percent. Our assumption matters a lot in this case.

The other observation is a constant probability of default is similar to a negative expected growth in the expected cashflows. Notice that the expected cashflows to the tax shield (the expected tax savings) are declining by 20 percent per year. Compare equation (1) to equation (4) in the review of financial concepts. Since the first years cashflow (C_1) is 0.8*0.34*100, if your replace g by -p the formulas are identical.