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Selecting the Best? Spillover and Shadows in Elimination Tournaments

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We consider how past, current, and future competition within an elimination tournament affect the probability that the stronger player wins. We present a two-stage model that yields the following main results: (1) a shadow effect wherein the stronger the expected future competitor, the lower the probability that the stronger player wins in the current stage; and (2) an effort spillover effect wherein previous effort reduces the probability that the stronger player wins in the current stage. We test our theory predictions using data from high-stakes tournaments. Empirical results suggest that shadow and spillover effects influence match outcomes and have already been priced into betting markets.

Keywords: elimination tournament; dynamic contest; contest design; effort choice; betting markets

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Competition for employment and education, innovation funding, and design opportunities can all be framed as multistage elimination tournaments in which players are knocked out over successive stages of the event. These contests are often designed to increase player effort; indeed, much of the theoretical and empirical literature focuses on contests as incentive mechanisms. Yet tournaments may also serve as selection mechanisms, identifying the “best” candidates as overall winners. In labor tournaments where employees’ latent talents are not directly observable, firms may organize contests to reveal their workers’ relative abilities.¹

In this paper, we study how the strategies of heterogeneous players in matched-pair elimination tournaments are shaped by past, current, and future competition. More specifically, we examine how these intertemporal effects influence a tournament’s ability to reveal the strongest player as the winner. Past exertion may make current effort more costly and may depress performance, and the shadow of tough future competition decreases a player’s expected future payoffs and may also lead to lower current effort. The differential impact of past and future competition across players in a given match changes the effectiveness of tournaments as a selection mechanism. We find that both past exertion

and tough future competition increase the probability that a weak candidate wins.

Our results have practical implications. Whether the contest’s objective is to encourage effort, select a strong winner, or both, we find evidence suggesting that firms and other contest designers may need to consider the role of past and future competition in structuring incentives.

In personnel tournaments, workers risk elimination as they advance through corporate management levels. In most contexts, retention of the highest-quality worker is most desirable. For example, General Electric (GE)’s former chief executive officer, Jack Welch, designed an explicit elimination tournament to select his successor (Konrad 2009). Competition between firms may also be knockout events. In 2010, GE announced a three-stage elimination tournament, the Ecomagination Challenge, to award \$200 million to the firm that developed the best smart grid technologies. More commonly, architectural firms may compete for large contracts, and investment banks may compete for new clients over several elimination stages. Political races also may involve elimination—candidates must win their party’s primary election to compete in the general election to hold office. Many sporting events are also structured as elimination tournaments.

In each of these examples, effort is clearly important: firms want to hire managers, designers, bankers, and innovators who will invest heavily in the activity at hand; voters want their representatives to work hard

¹ In contrast, Lazear (1986) discusses how performance pay may attract higher-quality workers to the firm when the firm cannot readily observe innate worker ability.

on their behalf; and spectators enjoy high-action games. However, selection may also be a prime objective of the contest organizer: a client may desire the most creative design firm, voters may value the most skilled politician, and a board may want the smartest executive to lead the company.

We explore elimination tournaments as selection mechanisms with a two-stage matched-pair model. One particular strength of our model is that its predictions are framed in terms of outcomes. As a result, they are testable: in contrast with effort that is notoriously difficult to measure in the field, tournament wins and losses are readily observable. We test our theoretical predictions using the outcomes of high-stakes matches, exploiting the random assignment of players in professional tennis tournament draws. Examining the effect of changes in the skill of the expected competitor in the next round, we find strong evidence of a shadow effect. In addition, spillover in tennis tournaments appears to have a particularly negative impact on the stronger player. We also examine tennis betting markets and find that bookmakers' prices reflect both spillover from past competition and the shadow of future opponents.

The literature on the type of tournament that we study—sometimes called “knockout” tournaments—begins with Rosen's (1986) model of a multistage contest where players have Tullock-style contest success functions (Tullock 1980). One significant difference between Rosen's work and our current paper is that we use the contest success function that appeared earlier in Lazear and Rosen (1981). More importantly, Rosen is not focused on shadow and spillover effects; instead, his main result explains the skewed compensation distributions found in many firms. Harbaugh and Klumpp (2005) study a special case of Rosen's model with a single prize and introduce a version of spillover. In contrast to our result that effort in the first stage has a relatively larger impact on the stronger player's probability of success, they model a contest in which low-skilled players are disadvantaged in the final round. This result is generated by a set of assumptions that differs from those in our model; in particular, Harbaugh and Klumpp assume that effort is costless and therefore completely exhausted in the final stage, and the total supply of effort is fixed and equal for all players. In their setup, the stronger player conserves effort in anticipation of stiff competition in a final-stage match against an equally skilled opponent, whereas the weaker player always exerts more effort than the stronger player in the first stage. The weaker player's first-stage exertion “spills over” into the next stage and further reduces that player's chance of winning the event.

Our effort spillover prediction also relates to previous work on fatigue in dynamic competition. Ryvkin (2011) presents a winner-take-all model where homogeneous

players face a binary effort decision and effort has no explicit cost. These features are in stark contrast to our model where players are heterogeneous and effort is a continuous and costly choice variable in a multiprize tournament. In his work, fatigue accumulates across stages and players have no opportunity to refresh their effort resources. Among other results, he finds that equilibrium effort is decreasing in fatigue, similar to our notion of negative spillover between tournament stages. Schmitt et al. (2004) study the opposite phenomenon, positive spillover, in rent-seeking contests. They find both theoretical and experimental evidence that positive spillover leads to more first-period expenditure. Our contribution complements and extends these theoretical and experimental results to the field. Moreover, we consider the shadow of expected future competition in addition to the impact of spillover from the past.

Related to our interest in the effect of a future competitor, Ryvkin (2009) considers the elasticities of a player's equilibrium effort with respect to the abilities of opponents across several tournament formats. In elimination tournaments with weakly heterogeneous players, he finds that the abilities of opponents in the more distant future have a lower impact on a player's equilibrium effort than does the ability of the current opponent. Whereas Ryvkin focuses on players' effort, we are particularly concerned with tournament outcomes.

Several papers have explored the use of tournaments as a selection mechanism. Searls (1963) compares the statistical properties of single- and double-elimination contests and predicts that single-elimination events—the type of tournament that we consider in this paper—are most likely to select the highest-ability player as the winner. Whereas our model allows players to make strategic effort decisions in response to past and future competition, Ryvkin and Ortmann (2008) and Ryvkin (2010) compare the selection efficiency of three tournament formats when contestants do not choose effort. In these models, as in the one that we present below, a player's success is probabilistic. In contrast, Groh et al. (2012) model an environment in which heterogeneous players choose their level of effort but participate in a perfectly discriminating contest. In an all-pay auction, they explicitly consider various contest designer's objectives, including selection. They find that common seeding rules that match weakest to strongest players in the semifinals maximize the probability that the strongest player wins overall. Clark and Riis (2001, 2007) also examine one-stage, perfectly discriminating contests and explore how various prize rules can improve selection. Modeling a different type of strategic choice, Hvide (2002), Cabral (2003), and Hvide and Kristiansen (2003) study outcomes in contests in which competitors choose their degree of risk taking.

Of course, a tournament's ability to select the best is only important when contest designers face a field of heterogeneous competitors. Empirically, Sunde (2009) tests the incentive effect of player heterogeneity in professional tennis tournaments. He finds that heterogeneity impacts the effort choice of the stronger player more than it changes the effort of the weaker player in a match; for an equal change in rank disparity, the increase in the number of games won by the stronger player is smaller than the decrease in the number of games lost by the weaker player. In addition to the concurrent heterogeneity studied in Sunde's work, we examine heterogeneity across multiple stages of an event, exploring the incentive impact of ability differences with past, current, and (expected) future opponents. The effects of player heterogeneity on effort in one-shot tournaments has been studied both theoretically (e.g., Baik 1994, Moldovanu and Sela 2001, Szymanski and Valletti 2005, Minor 2011) and empirically (e.g., Knoeber and Thurman 1994, Brown 2011).

The paper is organized as follows: Section 1 presents a two-stage model of an elimination tournament. We derive several propositions and outline the testable hypotheses. In §2, we describe our data and empirical strategy for testing these predictions. Section 3 describes the results from past tournaments and discusses the spillover and shadow effects in the context of betting markets. We conclude in §4 and discuss the implications of our findings for contest designers.

1. Theory

We study a theory of knockout tournaments in which matches within a given stage are staggered over time. Players in later matches learn the identity of their potential future opponent from outcomes of earlier matches. However, players in these earlier events can only form expectations about their future opponent. This formulation captures both sequential and simultaneous features of competition—just as players in simultaneous matches must form expectations about the outcomes of parallel games, so too must players in early matches of sequential tournaments form expectations about later matches.

This tournament format is often found in practice. For example, in firms, simultaneous promotions to division vice president may be rare. Instead, the identity of the new appointee is known to other workers still competing for a parallel executive spot—the hopeful workers now know their future opponent for further advancement. This structure is in contrast with other models of elimination tournaments where all matches in a given stage occur simultaneously rather than sequentially (e.g., see Stracke 2011) or all participants compete against each other simultaneously in pools, rather than as pairs (Fu and Lu 2012). We use an

additive noise model, as in Lazear and Rosen's (1981) foundational work on one-shot labor tournaments, to focus on the dynamics of a multistage elimination tournament.²

In the following subsection, we explore the role of past and future competition on tournament outcomes. We present a model that is simple enough to clearly inform our empirical tests yet rich enough to capture common features of high-stakes, multistage tournaments. Specifically, we model an elimination tournament with heterogeneously skilled players engaging in sustained competition—one could imagine professionals of varying abilities competing over months or years for a prized position within the firm.³ For expositional ease, we present the spillover and shadow effects separately in §§1.2 and 1.3. Then, in §1.4, we present an analysis of the effects operating simultaneously and also enrich the notion of cross-round spillover.

Our theory results describe the probability that the stronger player wins in different stages of the elimination event. These predictions speak directly to our broader research question of "selecting the best." That is, our comparative statics results provide predictions about when the strongest player is more likely to advance to future rounds of competition and, ultimately, win the tournament.

1.1. Model Setup

Consider a two-stage elimination tournament with four risk-neutral players, where the players who win in the first stage advance to the final stage. The overall tournament winner receives a prize of V_W , and the second-place competitor receives a prize V_L . Let $V_W > V_L > 0$, and define the prize spread $\Delta V = V_W - V_L$. For simplicity, we assume no discounting across stages. Let player i 's total cost be a quadratic function of effort x_i and cost type c_i .⁴ The convexity assumption on the cost function is common in the literature on tournaments and captures the notion that additional units of effort are increasingly costly for competitors.⁵

For simplicity, we denote a player's cost function as $\frac{1}{2}x^2$ and a player i 's total cost as $c_i \frac{1}{2}x_i^2$. We assume that

² Additive noise models have been used in much of the labor tournaments literature since Lazear and Rosen (1981); for examples, see Konrad (2009).

³ In a sports context, our model better reflects the dynamics of an endurance event (e.g., tennis) than competition requiring short bursts of effort (e.g., powerlifting).

⁴ One could define a mapping $E: \mathbb{R}_+^N \rightarrow \mathbb{R}_+^1$ that collapses levels of N effort-generating activities to the real line. The overall cost of effort is then strictly increasing in the resultant scalar x_i .

⁵ Our results hold for more general convex cost functions, $\gamma(x_i)$, provided that both players' effort choices are sufficiently sensitive to a change in marginal benefit. In particular, we require that $\gamma''(x_1^*)/\gamma''(x_2^*) < c_2/c_1$. Versions of the model with linear and other quasi-convex costs also produce similar results.

cost types, c_i , vary across all players and are commonly known among competitors.⁶ We will describe players with relatively low costs as being “stronger” than players with relatively high costs.

Recall that matches in the first stage are sequential. Assume that players 3 and 4 compete first. Then, player 1 faces player 2 knowing the outcome of the previous match. Players 3 and 4 will form an expectation of the strength of future competition, knowing only the identity of two potential opponents. Without loss of generality, we assume that player 3 won the match against player 4.

1.1.1. Final Stage of the Tournament. Assume that player 1 won the first-stage match. To find the equilibrium of the multistage game, we begin by analyzing the strategies of player 1 and the opponent, player 3, in the *final* stage. Define player 1’s expected payoff function as

$$\pi_{1, \text{final}} = P_1(x_1, x_3)\Delta V - \frac{1}{2}c_1x_1^2 + V_L, \quad (1)$$

where the probability of winning takes the following form:

$$P_1(x_1, x_3) = \begin{cases} 1 & \text{if } x_1 + \varepsilon_1 > x_3 + \varepsilon_3, \\ \frac{1}{2} & \text{if } x_1 + \varepsilon_1 = x_3 + \varepsilon_3, \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

where $x_i + \varepsilon_i$ is player i ’s level of output, which is a function of both effort x_i and a random noise term ε_i . In definition (2), the probability that player 1 wins is increasing in player 1’s own effort and decreasing in the effort of the opponent.

Define $\varepsilon = \varepsilon_3 - \varepsilon_1$, and let ε be distributed according to some distribution G such that probability (2) can be written as

$$P_1(x_1, x_3) = \Pr(x_1 - x_3 > \varepsilon) = G(x_1 - x_3). \quad (3)$$

Now, player 1’s payoff function (1) can be written as

$$\pi_{1, \text{final}} = G(x_1 - x_3)\Delta V - \frac{1}{2}c_1x_1^2 + V_L, \quad (4)$$

and player 1’s first-order condition is

$$\frac{\partial \pi_{1, \text{final}}}{\partial x_1} = G'(x_1 - x_3)\Delta V - c_1x_1 = 0. \quad (5)$$

⁶ For ease of exposition, we model heterogeneity through players’ cost types. However, several alternative models produce identical results: for example, we can also capture heterogeneity across valuations by defining a player’s valuation of the prize as V/c_i or allow the impact of an additional unit of effort on the probability of winning to vary across competitors. It can also be shown that capturing heterogeneity by varying cost function convexity leads to similar results.

Following Konrad (2009) and Ederer (2010), we assume that G is distributed uniformly with the following support:⁷

$$G \sim U[-\frac{1}{2}a, \frac{1}{2}a],$$

and therefore,

$$G' = \frac{1}{a}.$$

The assumption that G is uniformly distributed removes the strategic interdependence of players’ *current* period effort choices (Konrad 2009).⁸ This allows us to isolate the consequences of past effort choices and potential future competition on current-stage effort. In a firm context, this would assume that a worker’s optimal effort choice is independent of the identity of the current opponent; of course, in earlier stages, a worker’s optimal effort depends on expectations about future opponents’ identities. The results hold broadly if we relax this assumption of same-stage independence and allow players’ optimal effort choices to depend on both their current and future opponents.⁹

Rewriting the first-order condition (5) yields

$$\frac{\partial \pi_{1, \text{final}}}{\partial x_1} = \frac{\Delta V}{a} - c_1x_1 = 0,$$

which we can rearrange as the following expression:

$$x_i^* = \frac{\Delta V}{ac_i} \quad \text{for } i = 1, 3. \quad (6)$$

Assume for the remainder of the analysis that player 1 is the stronger player ($c_1 < c_3$). Then, expression (6) implies that player 1 exerts more effort in the final stage ($x_1^* > x_3^*$). Therefore, the stronger player is more likely to win in the final stage relative to the weaker opponent (i.e., the better player is more likely to be “selected” as the overall tournament winner).

In the final round, since both players are guaranteed at least the second prize V_L , only the difference between first and second prize matters to competitors. As expected, a larger prize spread leads to more effort from both players, although the stronger player increases effort more than the weaker player. Also, increasing the noise around effort (i.e., increasing a , the width of the support of G) reduces effort, particularly for the stronger player.

⁷ To ensure that probabilities are well defined, we require that

$$0 < \frac{\Delta V/(ac_1) - \Delta V/(ac_3) + a/2}{a} < 1.$$

This condition ensures that $G(\cdot) \in (0, 1)$.

⁸ Results from Ryvkin (2009) support this simplifying assumption. He shows that, when players’ relative abilities are uniformly distributed, a “balanced” seeding can eliminate the dependence of a player’s equilibrium effort on the opponent’s ability.

⁹ A version of the model with more general distributions that allow for same-stage interdependence, including the normal distribution, is available in Brown and Minor (2014).

1.1.2. First Stage of the Tournament. Define z_1 and z_2 as the efforts of players 1 and 2 in the first stage, respectively. Player 1's expected payoff function in the first stage is

$$\pi_{1, \text{first}} = P_1(z_1, z_2)\tilde{V}_1 - \frac{1}{2}c_1z_1^2, \quad (7)$$

where \tilde{V}_1 is player 1's continuation value (i.e., payoff in the final stage):

$$\tilde{V}_1(x_1, x_3) \equiv \pi_{1, \text{final}} = G(x_1 - x_3)\Delta V - \frac{1}{2}c_1x_1^2 + V_L.$$

Equation (7) yields the first-order condition

$$\frac{\partial \pi_{1, \text{first}}}{\partial z_1} = \frac{\tilde{V}_1}{a} - c_1z_1 = 0,$$

which we can rearrange, for either player, as the following expression:

$$z_i^* = \frac{\tilde{V}_i}{ac_i} \quad \text{for } i = 1, 2. \quad (8)$$

Fixing a player's continuation value, that player's effort z_i^* is decreasing in c_i . Since the continuation value itself is also decreasing in c_i , first-stage effort z_i^* is increasing in a player's ability (decreasing in c_i).¹⁰

Recall that, at the start of their match, players 1 and 2 already know the outcome of the other first-stage match between players 3 and 4. Of course, this means that players 3 and 4 did not know exactly the identity of their future opponent. Instead, we assume that they formed an expectation of their continuation value as follows:

$$E[\tilde{V}_i] = p_{1|i}\tilde{V}_i(x_i^*, x_1^*) + (1 - p_{1|i})\tilde{V}_i(x_i^*, x_2^*) \quad \text{for } i = 3, 4,$$

where $p_{1|i}$ is the equilibrium probability that player 1 wins knowing that player i will be the opponent in the final stage.¹¹ Note that player i cannot influence this probability $p_{1|i}$ because it is a function of the realized outcome of the completed match between players 3 and 4. This simplifies our analysis, since player i 's first-stage effort z_i does not change this probability $p_{1|i}$. Thus, for players 3 and 4, we can express their effort as

$$z_i^* = \frac{E[\tilde{V}_i]}{ac_i} \quad \text{for } i = 3, 4,$$

and the analysis described above for players 1 and 2 applies similarly.

¹⁰ It can be shown, using expression (9), that

$$\frac{\partial \tilde{V}_i}{\partial c_i} = -\frac{1}{2} \frac{\Delta V^2}{a^2 c_i^2} < 0.$$

¹¹ When player 1 is stronger than player 2, $\tilde{V}_i(x_i^*, x_1^*) < E[\tilde{V}_i] < \tilde{V}_i(x_i^*, x_2^*)$ for $i = 3, 4$.

1.2. Shadow of Future Competition

We can use the model to understand the impact of known or expected future competition on the likelihood that stronger players advance to future stages of the tournament; of course, this then influences the likelihood that a high-skilled player is selected as the overall winner.

Consider an increase in the skill of the future opponent. This change has the effect of decreasing the continuation value for both players 1 and 2 in the first stage. In the following analysis, we show that if player 1 has a lower cost of effort than player 2, then the former will decrease first-stage effort more than the latter.

We can express player i 's first-stage effort as

$$z_i^* = \frac{\tilde{V}_i}{ac_i} = \frac{1}{ac_i} \left(\frac{\Delta V/(ac_i) - \Delta V/(ac_3) + (1/2)a}{a} \Delta V - \frac{1}{2}c_i \left(\frac{\Delta V}{ac_i} \right)^2 + V_L \right). \quad (9)$$

To identify the effect of a change in the effort cost of the future opponent, we take the derivative

$$\frac{\partial z_i^*}{\partial c_3} = \frac{\Delta V^2}{a^3 c_i c_3^2} > 0.$$

Thus, an increase in the skill of the future opponent (i.e., a decrease in c_3) decreases a player's effort in the first stage. This is consistent with Ryvkin (2009), who finds that in tournaments with weakly heterogeneous players, effort depends negatively on future opponents' skill levels.

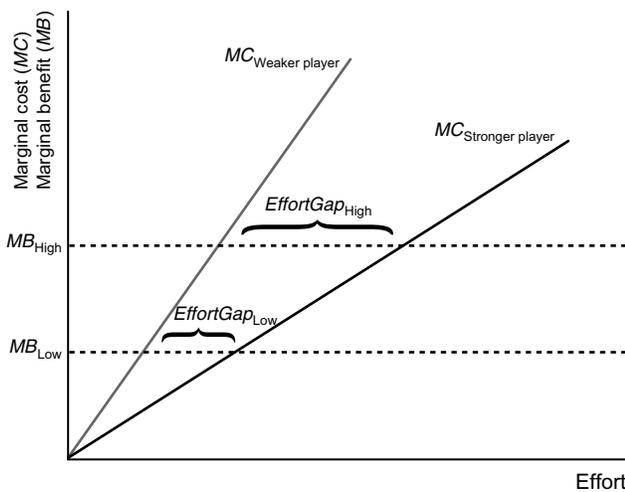
Since we are additionally concerned with tournament outcomes, we next ask, which first-stage player is more sensitive to the change in the future competition? Let $c_1 < c_2$. Then,

$$\frac{\partial z_1^*}{\partial c_3} = \frac{\Delta V^2}{a^3 c_1 c_3^2} > \frac{\Delta V^2}{a^3 c_2 c_3^2} = \frac{\partial z_2^*}{\partial c_3}.$$

This means that, for a given increase in the talent of the future competitor, player 1 decreases effort even more than player 2. This gives us the first proposition.

PROPOSITION 1. *As the skill of the future competitor in the final stage increases (declines), the stronger player becomes less (even more) likely to win in the first stage and thus less (even more) likely to be selected as the overall tournament winner.*

Figure 1 provides some intuition for the result. Marginal cost and benefit are presented on the vertical axis, and effort is shown on the horizontal axis. By definition, the marginal cost of the weaker player lies above the marginal cost of the stronger competitor. In the model, the marginal benefit of effort is always larger for the stronger player; however, for simplicity in the

Figure 1 Effect of the Shadow on Effort

figure, we make the conservative assumption that both players enjoy the same marginal benefit of effort. When the marginal benefit of effort is low, the difference between the stronger and weaker players' efforts is $\text{EffortGap}_{\text{Low}}$, and when the marginal benefit of effort is high, the difference is $\text{EffortGap}_{\text{High}}$. When the future competitor is more skilled, both of the current players experience a decrease in their marginal benefit of effort, a move from MB_{High} to MB_{Low} . Since $\text{EffortGap}_{\text{High}} > \text{EffortGap}_{\text{Low}}$, players facing a more skilled opponent provide more similar levels of effort, and this reduces the probability that the stronger player wins in the current stage. The reverse is true as the future competitor becomes less skilled; in this case, the gap between current players' efforts increases, thus improving the stronger player's chance of success.

A limit argument is also illustrative. Consider a current stage with two differently skilled players, where the winner advances to face some final typical opponent. Given superior skills, the stronger player is more likely to win in the current stage. Now, consider what happens when the next-round competitor is impossible to beat: both players in the current stage will exert almost no effort. This hurts the chances of the stronger player, since the likelihood of winning declines to 50%. In contrast, the weaker player in the current stage sees the probability of success improve to 50%. The prospect of an impossibly strong future opponent turns the current match into a coin flip and, as a result, reduces the chances that the stronger player wins. Similar intuition applies to the case where the future competitor changes from unbeatable to typical.

1.3. Effort Spillover

We can also examine effort spillover between stages of the tournament. Spillover can take either a positive or a negative form. Positive spillover might reflect learning-by-doing, skill building, or momentum within a firm.

For example, an innovation team whose proposal advances to a second stage of funding might benefit from its first-stage experiences, both technical and relational. With positive spillover, second-stage effort is less costly than first-stage effort. In contrast, negative spillover might reflect fatigue or reduced resources in later stages. For example, architects competing in design competitions might exhaust their creative resources in early stages and have only limited energy for second-stage proposals. In this case, second-stage effort is more costly than first-stage effort.¹²

Consider a scenario where effort expended by a player in the first stage influences the marginal cost of effort in the final stage.¹³ We can rewrite player 1's final-stage payoff as

$$\pi_{1,\text{final}} = G(x_1 - x_3)\Delta V - \frac{1}{2}kc_1x_1^2 + V_L,$$

where k reflects the change in total cost induced by first-stage effort.

To study a negative spillover effect, we let a player's marginal cost of effort in the final stage increase ($k > 1$), and final-stage effort is strictly decreasing in the degree of negative spillover. With positive spillover, a player's marginal cost of effort in the final stage decreases ($k < 1$), and final-stage effort is increasing in positive spillover.

Now, equilibrium effort is

$$x_1^* = \frac{\Delta V}{kac_1}.$$

Straightforward calculations show that negative (positive) spillover reduces (increases) a player's final-stage payoff. Consequently, first-stage effort decreases (increases) with negative (positive) spillover.

Thus, negative spillover implies a lower probability of success in the final stage, holding the opponent's effort and skill constant. Of course, the opposite is true for positive spillover. Since

$$\frac{\partial x_1^*}{\partial k} = -\frac{\Delta V}{k^2ac_1} < -\frac{\Delta V}{k^2ac_3} = \frac{\partial x_3^*}{\partial k},$$

when both players in a match suffer similar negative spillover, the stronger player is more adversely affected. As a result, the stronger player is relatively less likely to win. In the limit, $G(x_1^* - x_3^*) \rightarrow 0.5$ as the degree of negative spillover $k \rightarrow \infty$.

We summarize this finding in the second proposition.

PROPOSITION 2. *A common proportional increase in effective cost type decreases the probability that the stronger player wins.*

¹² Different notions of spillover have been explored in the literature in settings where players with exogenous, fixed resources make effort allocation decisions over multiple periods of play. For recent examples, see Harbaugh and Klumpp (2005) and Sela and Erez (2013).

¹³ If previous effort appears only as a fixed cost in the final stage, we would expect no change in final-stage effort.

Figure 2 Effect of Spillover on Effort

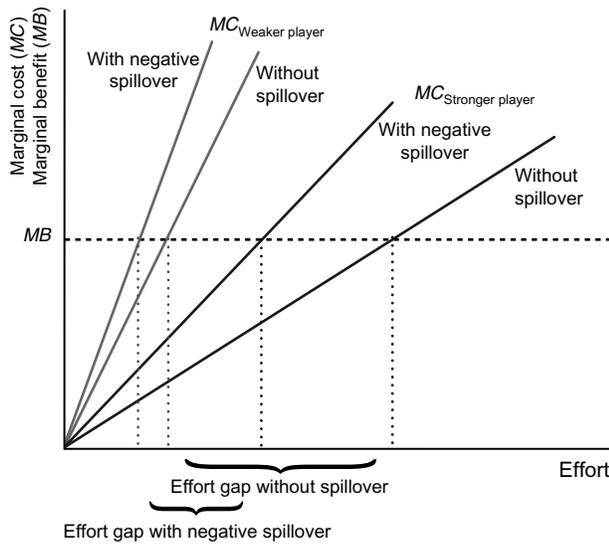


Figure 2 illustrates the spillover effect. For both players, negative spillover increases the marginal cost of effort and reduces the levels of effort exerted in the competition. However, a common proportional increase in marginal cost leads to a larger change in effort for the stronger player, relative to the weaker player. Since $EffortGap_{WithoutSpillover} > EffortGap_{WithNegativeSpillover}$, players experiencing negative spillover provide more similar levels of effort, and this reduces the probability that the stronger player wins in the current stage.

Again, a limit argument provides further intuition. Consider a current stage with two players who experience typical levels of negative spillover. Given superior skills, the stronger player is more likely to win in the current stage. Now, consider what happens when spillover increases dramatically. Facing very high costs, both players will exert similarly low levels of effort. This hurts the chances of the stronger player, whose likelihood of winning declines to 50%. In contrast, the weaker player in the current stage sees the probability of success improve to 50%. Thus, negative spillover evens the playing field.

Proposition (2) suggests that, with negative spillover, weaker players might support costlier competitive conditions—for example, a weaker player might advocate for more stringent common standards or more difficult tasks. Overall, however, the direction and impact of spillover depends on the context and, thus, is an empirical question.

Our result that negative spillover levels the playing field in both stages is in contrast to Harbaugh and Klumpp's (2005) finding that intertemporal trade-offs level the playing field for the first stage but do the opposite in the final stage. Their result is sensitive to the assumptions that effort is costless and that players' total efforts are equally constrained.

Spillover need not be modeled as a common proportional increase in marginal cost; in the next subsection, we allow the degree of spillover to be a function of first-stage effort and achieve similar results.

1.4. Combined Shadow and Spillover Effects

In the text above, we separately present the models of effort spillover and the shadow of future competition; now, we consider these effects simultaneously and allow spillover to be an increasing function of first-stage effort. Combining the effects does not change the general predictions of the previous analysis—the prospect of a stronger future competitor and the presence of negative spillover continue to even the playing field.

1.4.1. Spillover and Shadow: Final Stage. Under this formulation, our first-order condition for the final stage yields equilibrium effort choice

$$x_i^* = \frac{\Delta V}{k(z_i)ac_i},$$

where $k(\cdot)$ reflects the degree of spillover from the previous stage and is a strictly increasing function of first-stage effort z_i . As expected, greater first-stage effort results in lower effort in the final stage. Furthermore, this effect is amplified for the stronger type since $c_1 < c_2$. The final-stage spillover effect is

$$\frac{\partial x_i^*}{\partial k(z_i)} = \frac{-\Delta V}{k(z_i)^2 ac_i} < 0.$$

Since $\partial x_1^*/\partial k(z_1) < \partial x_2^*/\partial k(z_2) < 0$, a common level of spillover reduces the disparity between participants' efforts in the final stage. As a result, the stronger player is less likely to win in the final stage.

1.4.2. Spillover and Shadow: First Stage. Next, we consider effort decisions in the first stage. We write $k(z_i)$ as k_i to simplify the notation in this section and express player i 's payoff as

$$\begin{aligned} \pi_i = G_{\text{first}}(\cdot) & \left(\left(\frac{\Delta V/(k_i ac_i) - \Delta V/(k_j ac_j) + a/2}{a} \right) \Delta V \right. \\ & \left. - \frac{1}{2} k_i c_i \left(\frac{\Delta V}{k_i ac_i} \right)^2 + V_L \right) - \frac{1}{2} c_i z_i^2. \end{aligned}$$

The first-order condition for the first stage is

$$\begin{aligned} \frac{\partial \pi_i}{\partial z_i} = \frac{1}{a} & \left(\left(\frac{\Delta V/(k_i ac_i) - \Delta V/(k_j ac_j) + a/2}{a} \right) \Delta V \right. \\ & \left. - \frac{1}{2} k_i c_i \left(\frac{\Delta V}{k_i ac_i} \right)^2 + V_L \right) \\ & + G_{\text{first}}(\cdot) \left(-\frac{\Delta V^2}{2k_i^2 a^2 c_i} \right) \frac{\partial k_i}{\partial z_i} - c_i z_i = 0, \end{aligned}$$

which then gives us the following expression for first-stage equilibrium effort:

$$z_i^* = \left[\left(\frac{1}{a} \left(\frac{\Delta V}{k_i a c_i} - \frac{\Delta V}{k_j a c_j} + \frac{a}{2} \right) \right) \Delta V - \frac{1}{2} k_i c_i \left(\frac{\Delta V}{k_i a c_i} \right)^2 + V_L \right] \cdot \frac{1}{a c_i} + \frac{G_{\text{first}}(\cdot) (-\Delta V^2 / (2k_i^2 a^2 c_i)) (\partial k_i / \partial z_i)}{c_i} \quad (10)$$

shadow effect
spillover effect

With no spillover, the first term is the shadow effect that we described in §1.2. Again, the stronger player becomes less likely to win in the first stage as the skill of the future competitor increases.

The second term reflects spillover. Consider the effect of introducing spillover. Since $G_{\text{first}}(\cdot) \geq \frac{1}{2}$ and $c_1 < c_2$, the negative spillover effect is greater in magnitude for the stronger player. This increases the chances that the weaker player wins; thus, spillover has the effect of evening the playing field. That is, ceteris paribus, spillover increases the chance of an upset.

1.5. Model Predictions

The theory model outlined above provides the following main predictions.

1. *Shadow of future competitors:* The stronger the expected competitor in the next stage, the lower the probability that the stronger player wins in the current stage. Empirically, for a given pair of competitors, we expect that the stronger player is less likely to win when the winner of the current match will face a stronger future opponent.

2. *Effort spillover between stages:* Increased negative (positive) spillover decreases (increases) the probability that the stronger player wins in the final stage. Empirically, for a given pair of competitors experiencing negative spillover, we expect that similar levels of past exertion will make it less likely that the stronger player wins.

Although not the focus of the current paper, other predictions follow immediately from our analysis: (a) a steeper prize structure improves the stronger player’s probability of success in all stages; (b) the noisier the effort-to-output relationship, the lower the probability that the stronger player wins in either stage; and (c) fixing the competitors’ abilities and given a sufficiently large (small) second-place prize, the weaker player’s probability of winning is greater (smaller) in the final stage, relative to the first stage. Proofs for these additional results are available from the authors by request.

Note that the model’s main implications are framed in terms of outcomes, allowing us to readily test these

predictions by observing tournament winners. In the following sections, we describe our data and empirical analysis.

2. Data

Professional tennis offers an ideal environment in which to test the empirical implications of the theory.¹⁴ Tennis events are single-elimination tournaments—only winning players advance to successive stages until two players meet in the final stage to determine the overall winner. Prizes increase across stages with the largest prize going to the overall winner, and the distribution of prizes is known in advance for all tournaments. The financial stakes are substantial and vary across events; for example, the total purse for the 2009 U.S. Open singles competition was \$16 million, with a \$1.7 million prize for first place, whereas the total purse for the 2009 SAP Open was \$531,000, and the winner received \$90,925.

Our empirical analysis exploits the random nature of the initial tournament draw. By ATP rules, the top 20%–25% of players in an event (the “seeds”) are distributed across the draw: the top two seeds are placed on opposite ends of the draw, the next two seeds are randomly assigned to interior slots on the draw, the next four seeds are randomly assigned to other slots, etc. After the seeded players have been assigned, the remaining players are then randomly placed in matches prior to the start of the event. This variation provides the identification for our empirical approach, as we can observe the same skilled player compete against a variety of randomly assigned opponents. For example, in our data, we can observe the fourth-best player in the world play against competitors ranked 50th, 100th, and 250th in the first round of the same tournament over different years.¹⁵

The structure of tennis tournaments is particularly conducive to studying the shadow of future competition because both players (and the econometrician) know the competitors in the parallel match. In some cases, players know exactly who they will face in the next round; in other cases, they can make reasonable predictions about upcoming opponents. Measures of players’ abilities are also observable to competitors and researchers—past performance data, as well as world rankings statistics, are widely available.

Data from professional tennis have been used in other research. Walker and Wooders (2001) use video footage

¹⁴ Although tennis tournament organizers may have various objectives beyond selection, it is the *structure* of these tournaments that lends itself to our empirical tests. One would expect tournament competitors to respond to the structure and incentives, not the reason for that contest design.

¹⁵ In §3.3, we describe simulations that show that ATP seeding rules are not driving our empirical results.

and data from the finals of 10 Grand Slam events to identify mixed strategies. Malueg and Yates (2010) study best-of-three contests using four years of data from professional tennis matches with evenly skilled opponents. They find that the winner of the first set of a match tends to exert more effort in the second set than does the loser, and in the event of a third set, players exert equal effort. Forrest and McHale (2007) use professional tennis bookmaking data and find a modest long-shot bias. González-Díaz et al. (2012) use data from U.S. Open tournaments to assess individual players' abilities to adjust their performance depending on the importance of the competitive situation. They find that heterogeneity in this ability drives differences in players' long-term success. Using detailed data from the men's and women's professional tennis circuits, Gilsdorf and Sukhatme (2008a, b) find that larger marginal prizes increase the probability that the stronger player wins.

2.1. Professional Tennis Match Data

To test the predictions outlined in the theory, we examine the behavior of professional tennis players in 615 international tournaments on the ATP World Tour between January 2001 and June 2010. The data, available at <http://www.tennis-data.co.uk> (accessed October 18, 2010), include game-level scores and player ranks for men's singles matches. All of the tournaments are multiround, single-elimination events played over several days.

Tournament draws may include 28, 32, 48, 56, 96, or 128 players. Of the 615 events in the data, 432 tournaments consist of five rounds of play: rounds 1 and 2, quarterfinals, semifinals, and the final. Six rounds are played in 129 events. Fifty-four tournaments, including the Grand Slam events, consist of seven rounds of play. Most ATP events are best-of-three sets, whereas the Grand Slam events are best-of-five sets. Depending on the number of competitors, first-round byes may be awarded to the top-ranked players.¹⁶

World rankings are based on points that players accumulate over the previous 12 months. ATP points directly reflect the pyramid structure of tournaments; more points are awarded to players who advance in top tournaments. For example, a Grand Slam winner earns the maximum points awarded for a single event.¹⁷ ATP rankings are simply a rank order of all players by their accumulated points. In our analysis, we use the ATP rankings as a measure of players' skill levels.

Table 1 presents summary statistics from 25,758 men's professional tennis matches, reported separately for five-, six-, and seven-round events. The stronger player

Table 1 Summary Statistics

	Five-round tournaments	Six-round tournaments	Seven-round tournaments
No. of tournaments	432	129	54
No. of matches played			
Total	12,758	6,767	6,233
1st round	6,749	2,923	2,903
2nd round	3,435	2,043	1,716
3rd round	X	1,029	858
4th round	X	X	432
Quarterfinal	1,718	516	216
Semifinal	856	256	108
Means			
% of matches in which stronger player wins	64.2 (47.9)	62.9 (48.3)	68.9 (46.3)
Betting market prediction (%) that stronger player wins	62.7 (18.6)	62.4 (18.6)	67.3 (21.1)
Expected future opponent rank	48.35 (41.7)	25.88 (26.1)	29.21 (29.9)
Stronger player's previous games	15.85 (20.9)	14.29 (21.3)	25.43 (37.2)
Weaker player's previous games	16.52 (21.4)	20.75 (24.0)	29.96 (39.4)
Current rank ratio (worse/better rank)	5.79 (15.7)	6.71 (20.7)	9.09 (29.3)

Note. Values in parentheses are standard deviations. Matches from the final round of all tournaments are excluded from the counts, means, and standard deviations. The number of previous games for the stronger and weaker players is set to zero for the first round of all tournaments.

wins approximately 65% of the matches; on average, betting markets predict this outcome in approximately the same proportion. On average, matches are decided after approximately 17 games in five- and six-round tournaments and 28 games in seven-round tournaments, many of which are decided by best-of-five sets.

3. Results

In this section, we present empirical tests of the theory predictions. We first examine performance data from professional tennis matches, reporting empirical evidence of both spillover and shadow effects. Next, we ask whether shadow and spillover effects have been priced into betting markets. Although this additional analysis is not a direct test of the theory, it does provide further support for the importance of understanding these phenomena.

3.1. Spillover and Shadow Effects in Match Outcomes

Proposition 1 states that tougher future competition will decrease the stronger competitor's probability of success in the current stage. This prediction follows from the observation that, although stronger future competition will cause both players to decrease their effort in the current period, the current effort of the better-ranked player decreases more than the current effort of the worse-ranked opponent. Proposition 2

¹⁶ Byes automatically advance a player to the next round.

¹⁷ For details of the world rankings system, see the ATP World Tour Rulebook, available online at <http://www.atpworldtour.com> (accessed June 1, 2013).

considers the role of spillover in effort choice and predicts that negative spillover favors the weaker player. The direction of the spillover effect is often an empirical question; however, one might expect negative spillover in events that require intense effort exertion over a short period of time. In professional tennis, players may face a higher cost of effort if exertion in previous matches induced lasting fatigue.

The following specification allows us to study the effects of shadow and spillover simultaneously:

$$\begin{aligned}
 \text{StrongWins}_{mrs} = & \beta_0 + \beta_1 \text{Future}_{mrs} + \beta_2 \text{StrongPastGames}_{mrs} \\
 & + \beta_3 \text{WeakPastGames}_{mrs} + \beta_4 \text{Current}_{mrs} \\
 & + \alpha X_r + \gamma Z_s + \varepsilon_{mrs}, \quad (11)
 \end{aligned}$$

where StrongWins_{mrs} is a binary indicator of whether the better-ranked player in match m won in round r of tournament s ; Future_{mrs} represents the expected ability of the opponent in the next round, measured by the rank of the stronger of the possible opponents in the next round¹⁸; Current_{mrs} represents the degree of heterogeneity in players' skills in the current match, measured by the ratio of the ranks of the worse player and the better player; $\text{StrongPastGames}_{mrs}$ is the number of games played in all previous rounds of the tournament by the better-ranked player; $\text{WeakPastGames}_{mrs}$ is the number of games played in all previous rounds of the tournament by the worse-ranked player; X_r is a matrix of round fixed effects (e.g., first, second, quarterfinal); Z_s is a matrix of tournament-year fixed effects (e.g., 2008 U.S. Open) that capture average event-specific differences (e.g., temperature, purse, media attention); and ε_{mrs} is the error term.

We estimate all equations using a linear probability model with a robust variance estimator that is clustered at the tournament-year level to account for correlation in players' performances across matches in a given tournament in a given year. Results are very similar for a probit specification and are available from the authors by request.

We report results for regression (11) by tournament size, separating five-, six-, and seven-round events. This accounts for differences in tournament structures—for example, the quarterfinal competitor casts a shadow on the second round of a five-round tournament and the fourth round of a seven-round event, and accumulated spillover in a quarterfinal match of a five-round event may have a considerably different effect than in a seven-round event. We exclude final-round matches and set the number of previous games for the stronger and weaker players to zero for the first round of all tournaments.

¹⁸ Because of data limitations, the exact sequence of matches is not broadly available.

Table 2 Actual Match Outcomes

	DV = Stronger player wins in current match (0% or 100%)		
	Five-round tournaments	Six-round tournaments	Seven-round tournaments
<i>Expected future opponent rank</i>	0.0762*** (0.0113)	0.0844*** (0.0292)	0.1737*** (0.0224)
<i>Stronger player's previous games</i>	-0.3342*** (0.0800)	-0.3641*** (0.0752)	-0.3510*** (0.0615)
<i>Weaker player's previous games</i>	0.1998** (0.0774)	0.2926*** (0.0790)	0.1885*** (0.0491)
<i>Current rank ratio (worse/better rank)</i>	0.2747*** (0.0457)	0.1818*** (0.0650)	0.1323*** (0.0438)
Fixed effects			
Round	X	X	X
Tournament-year	X	X	X
R ²	0.05	0.05	0.06
No. of observations	12,758	6,767	6,233

Notes. Values in parentheses are robust standard errors, clustered by tournament-year (e.g., 2008 U.S. Open). Matches from the final round of all tournaments are excluded. The number of previous games for the stronger and weaker players is set to zero for the first round of all tournaments. DV, dependent variable. "X" denotes that fixed effects are included.

** $p < 0.05$; *** $p < 0.01$.

3.1.1. Results: Match Outcomes. Table 2 presents results for the main specification for five-, six-, and seven-round events. The coefficient on the shadow (Future_{mrs}) is positive and statistically significant in all three regressions ($p < 0.01$). This suggests that the stronger (i.e., better ranked) the future opponent, the lower the probability that the stronger player wins in the current round. For a one-standard-deviation decrease in future opponent's rank (i.e., increase in ability), we estimate that the probability that the stronger player wins in the current round decreases by approximately 3.2–5.7 percentage points, depending on the tournament size. Given that the probability that the stronger player wins is approximately 65%, on average, a one-standard-deviation increase in the shadow is associated with a 5%–9% decline in the probability of winning.

Coefficient estimates for the two spillover variables take on predicted signs and are statistically significant in all cases ($p < 0.01$ in five of six cases; $p < 0.05$ in one of six). More previous games for the stronger player decreases the probability of winning the current match, whereas more previous games for the weaker player increases the chance that the stronger player wins. A one-standard-deviation increase in the number of previous games for the stronger player is associated with a decline of approximately 7–13 percentage points in the probability of winning the current match; this represents a 10%–20% decline. A one-standard-deviation increase in the number of previous games for the weaker player is associated with a decline of

approximately 4–7 percentage points in the probability of winning the current match; this represents a 6%–11% decline in the probability that the weaker player wins.

The history of the stronger player appears to drive current success more than the history of the opponent—coefficient estimates on the number of games played by the stronger player are larger in magnitude than the coefficient estimates on the weaker player, although the magnitudes are significantly different only for the seven-round tournament ($p < 0.01$). This finding is not surprising, in light of the theory. Exertion from past rounds increases the marginal cost of effort for both players; however, a common proportional increase in marginal cost leads to a larger change in effort for the stronger player, pushing players to provide more similar levels of effort. This reduces the probability that the stronger player wins in the current stage.

As expected, the coefficient estimates on the skill disparity measure for the current match are all positive and statistically significant ($p < 0.01$). This suggests that increased player heterogeneity increases the probability that the stronger player wins. On average, a one-standard-deviation increase in the rank ratio—an increase in the disparity between players’ abilities—improves the probability that the stronger player wins by approximately 4 percentage points.

3.1.2. Alternative Specifications. *Tournament stakes:* Shadow and spillover results are robust to different tournament-level controls. Replacing the tournament fixed effects in our main specification with more detailed controls for the court type, surface type, and the natural log of the total tournament purse (in 2010 U.S. dollars) yields shadow and spillover effects similar to those presented in Table 2. Results are presented in Table 3.

Although not the focus of our current paper, this robustness exercise also allows us to assess whether the stronger player is more likely to win in higher-stakes events—a prediction that follows from the theory, if increases in the total purse do not materially change the shape of the distribution of prizes. In Table 3, the coefficient estimates on the total prize are positive and statistically significant, consistent with the prediction that larger prizes increase the probability that the stronger player wins ($p < 0.01$).

Long shadow: Motivated by the simple two-stage model in §1, the main empirical specification in Table 2 considers the impact of the expected opponent in the next round of the tournament. It is possible, however, that players respond to a “longer” shadow: in principle, players could look at the full tournament roster and adjust their effort in response to the overall presence of

Table 3 Alternative Specifications

	DV = Stronger player wins in current match (0% or 100%)					
	Tournament stakes			Long shadow		
	Five-round tournaments	Six-round tournaments	Seven-round tournaments	Five-round tournaments	Six-round tournaments	Seven-round tournaments
<i>Expected future opponent rank</i>	0.0586*** (0.0107)	0.0727*** (0.0265)	0.1703*** (0.0220)	0.0663*** (0.0110)	0.0780*** (0.0293)	0.1724*** (0.0222)
<i>Stronger player's previous games</i>	−0.3023*** (0.0742)	−0.3776*** (0.0637)	−0.2883*** (0.0529)	−0.2885*** (0.0787)	−0.3407*** (0.0750)	−0.3177*** (0.0612)
<i>Weaker player's previous games</i>	0.1551** (0.0747)	0.2474*** (0.0764)	0.1793*** (0.0488)	0.2113*** (0.0766)	0.3149*** (0.0779)	0.1796*** (0.0486)
<i>Current rank ratio (worse/better rank)</i>	0.2972*** (0.0438)	0.1857*** (0.0678)	0.1331*** (0.0433)	0.1720*** (0.0347)	0.1351** (0.0582)	0.0790** (0.0319)
<i>ln(Total purse) (in 2010 USD)</i>	4.5795*** (1.3409)	5.0128*** (1.2389)	11.3372*** (1.7053)			
<i>Rank of best player</i>				2.4543*** (0.2281)	3.9033*** (0.8130)	20.2632*** (1.7109)
Fixed effects						
Indoor/outdoor controls	X	X	X			
Surface-type controls	X	X	X			
Round	X	X	X	X	X	X
Tournament-year				X	X	X
R^2	0.02	0.02	0.04	0.07	0.06	0.07
No. of observations	12,549	6,767	6,233	12,758	6,767	6,233

Notes. Values in parentheses are robust standard errors, clustered by tournament-year (e.g., 2008 U.S. Open). Matches from the final round of all tournaments are excluded. The number of previous games for the stronger and weaker players is set to zero for the first round of all tournaments. *Rank of best player* generally equals the rank of the best player in the tournament; for matches involving the best-ranked player, this variable equals the rank of the second-best competitor in the event. DV, dependent variable. “X” denotes that fixed effects are included.

** $p < 0.05$; *** $p < 0.01$.

a very highly skilled player in the event. For example, these effort adjustments could take the form of changes in physically and mentally costly training activities for an upcoming tournament. To consider this possibility empirically, we expand the main specification to include the ATP rank of the most able opponent in the tournament.¹⁹

Results are reported in Table 3. In all cases, the coefficient estimates for the rank of the most able opponent in the tournament are positive and statistically significant ($p < 0.01$). That is, the presence of a strong player in the tournament—not necessarily even in the next immediate rounds—may lower the probability that the stronger player wins in the current match. The coefficient estimates associated with the more immediate shadow effect are similar to those in the main specification and remain statistically significant ($p < 0.01$). Both the short and long shadows cast by skilled competitors affect the probability that the stronger player wins in the current round.

3.2. Shadow and Spillover Effects in Betting Markets

In this section, we explore whether markets account for the shadow and spillover dynamics of multistage competition. Indeed, using data from professional betting markets, we find compelling evidence that subtle spillover and shadow effects have been incorporated into prices.

The efficiency of prediction and betting markets has been studied extensively in the literature; for examples, see the survey by Vaughan Williams (1999). Prediction markets are founded on the argument that by aggregating information, competitive markets should result in prices that reflect all available information (Fama 1970). Therefore, driven by aggregated information and expectations, prediction market prices may offer good forecasts of actual outcomes (Spann and Skeira 2003).

Similarly, betting odds reflect bookmakers' predictions of future outcomes. Betting odds may change as new information becomes available to the bookmaker and with changes in the volume of bets that may be driven by individual bettors' private information. As with formal prediction markets, we might expect betting odds to provide good forecasts. Spann and Skeira (2008) compare forecasts from prediction markets and betting odds using data for German premier soccer league matches. They find that prediction markets and betting odds provide equally accurate forecasts.

To examine whether betting markets incorporate information about shadow and spillover effects, we estimate a regression similar to Equation (11). Now,

¹⁹ For most players, this is the best-ranked player in the tournament; for matches involving the best-ranked player, the most able opponent is the second best-ranked competitor in the event.

instead of a binary indicator of the actual outcome, the dependent variable is the probability that the stronger player wins the match as implied by betting markets.

Our data include closing odds from professional bookmakers for prematch betting.²⁰ Woodland and Woodland (1999) note that bookmakers adjust odds based on the volume of bets, making the odds available as the betting market closes particularly rich in information. In our analysis, we use the median of the available odds data since the data from no single firm covered all matches.²¹ Overall, there was little variation between odds posted by different bookmakers for the same match, perhaps because participants in tennis betting markets tend to be specialists and there is little casual betting (Forrest and McHale 2007).

Table 1 reports the implied probabilities that the stronger player wins across rounds in five-, six-, and seven-round tournaments. On average, the stronger player is predicted to win; the betting market favors the stronger player approximately 63% of the time, with slightly more favorable predictions in high-stakes, seven-round tournaments. The accuracy of odds market predictions suggests that information beyond simple rankings are being priced into the market. Between 2001 and 2010, predictions from the market are correct for 69% of the 25,633 matches for which betting data are available. Given that the stronger player actually wins in 65% of the matches, one might not be surprised by this accuracy if the market always predicted that the better-ranked player wins. However, in 19% of the matches, the betting odds imply that the weaker player is expected to win. Interestingly, these market predictions are accurate nearly 63% of the time. That is, betting markets do almost as well predicting an upset as they do predicting a win by the stronger player. This is particularly notable since a naive assessment of the ATP rankings in these matches might suggest that the odds are still solidly against the weaker player; in predicted upsets, the mean rank of the weaker player is 98, roughly 1.7 times higher than the opponent's rank of 59.

3.2.1. Results: Betting Market Predictions. Table 4 reports results for regressions where the dependent variable is the probability that the stronger player wins as implied by the betting market. Overall, coefficient estimates suggest that the betting predictions incorporate information about players' past, current, and expected future competition.

²⁰ Data from 11 betting firms are included in our main data set, obtained from <http://www.tennis-data.co.uk>. Several betting firms also offer in-play betting, but we focus our analysis on prematch bets only.

²¹ We calculate the probability odds from the decimal odds in the original data. Probability odds are $1/(\text{decimal odds} - 1)$.

Table 4 Betting Market Data

	DV = Implied probability that the stronger player wins in current period (%)		
	Five-round tournaments	Six-round tournaments	Seven-round tournaments
<i>Expected future opponent rank</i>	0.0314*** (0.0048)	0.0422*** (0.0116)	0.1065*** (0.0122)
<i>Stronger player's previous games</i>	-0.1341*** (0.0256)	-0.2103*** (0.0308)	-0.2224*** (0.0280)
<i>Weaker player's previous games</i>	0.0520* (0.0276)	0.1196*** (0.0305)	0.1076*** (0.0244)
<i>Current rank ratio (worse/better rank)</i>	0.2311*** (0.0332)	0.1568*** (0.0522)	0.1258*** (0.0402)
Fixed effects			
Round	X	X	X
Tournament-year	X	X	X
R ²	0.10	0.10	0.12
No. of observations	12,634	6,711	6,148

Notes. Values in parentheses are robust standard errors, clustered by tournament-year (e.g., 2008 U.S. Open). Matches from the final round of all tournaments are excluded. The number of previous games for the stronger and weaker players is set to zero for the first round of all tournaments. DV, dependent variable. "X" denotes that fixed effects are included.

* $p < 0.10$; *** $p < 0.01$.

Coefficient estimates for the effect of a stronger future opponent are positive and statistically significant for the three regressions ($p < 0.01$). For a one-standard-deviation decrease in future opponent's rank (increase in ability), we estimate that the implied probability that the stronger player wins in the current round decreases by approximately 1.1–3.2 percentage points; this represents a roughly 3% decline in the implied probability of winning.

Since betting markets close at the start of the match, players' past exertion information is readily available to bookmakers. Indeed, coefficient estimates for the stronger and weaker players' number of previous games are statistically significant and take on the expected signs in all cases ($p < 0.01$ in five of six cases; $p < 0.1$ in one of six). More previous games played by the stronger player is associated with a decrease in the expectation that he succeeds, whereas more previous games played by the weaker player is associated with an increase in the expectation that the stronger player wins. The magnitudes of these effects also align with predictions from our model—stronger players are more sensitive to an additional unit of spillover compared with weaker players.

Greater heterogeneity in players' abilities may increase the market's expectation that the stronger player wins—the coefficient on rank ratio is positive and statistically significant in all cases ($p < 0.01$). Overall, we find strong evidence that prices in tennis betting markets reflect the shadow and spillover effects predicted by our model.

3.2.2. Unobserved Player Heterogeneity Across Rounds.

One advantage of the betting market data is that we can identify things that might otherwise be outside the econometrician's observation. In particular, we can identify when there is a *predicted* upset—this prediction is based on observations of the bookmaker and not simply the ranks of the players. For example, if a player has a minor injury or seems to be in the midst of a short winning streak, his world rank would not reflect this transient state. However, bookmakers could integrate this information into their predictions about match outcomes.

We can identify predicted upsets by comparing the implied probability of the betting odds to the rank-based outcome prediction (i.e., the prediction that the stronger player is more likely to win). If the betting odds predict that the worse-ranked player has a better than 50% chance of winning, then there is some unobserved (by us) positive shock for him (and/or negative shock for the stronger player). Deviations from the ranked-based predictions that persist over multiple rounds suggest a state-dependent component of play. We take a conservative approach to identify this state dependence.

There are 2,085 predicted upsets in the data, representing roughly 7% of all matches. In 67 cases, a single player was predicted to cause multiple upsets in the same event; 57 of these instances involved two upsets in the same tournament, and 10 cases involved three predicted upsets. This means that more than 96% of predicted upsets did not persist beyond a single round. Overall, we find little evidence that match outcomes are driven by unobserved state dependence.

3.3. The Role of ATP's Seeding Rules

One might be concerned that ATP seeding rules introduce a mechanical relationship in the data—although seeded and unseeded players are randomly matched in the first round, the identity of potential opponents in the future rounds can be constrained by the initial draw. To understand the roles of ATP's seeding protocol, we undertook several simulation exercises and conclude that mechanical relationships are not driving our empirical results.

First, we simulate an initial draw for a 32-player tournament using ATP seeding rules. We then simulate match outcomes probabilistically. Specifically, the stronger player wins if

$$61.7 + 0.379 \left(\frac{Rank_{weak}}{Rank_{strong}} \right) > x, \quad (12)$$

where $Rank_{strong}$ and $Rank_{weak}$ are the tournament ranks (i.e., values 1–32) of the stronger and weaker players in the match, respectively, and x is a random draw

from a uniform distribution with support 0–100.²² Otherwise, the weaker player wins. The parameters used in expression (12) are obtained from a regression of an indicator of whether the better-ranked player won on a constant and the ratio of the players' ranks, using the main ATP data set for the first round of five-round events.

With our simulated draws and results, we can estimate the following regression using a linear probability model:

$$\text{StrongWins}_m = \gamma_0 + \gamma_1 \text{Future}_m + \varepsilon,$$

where StrongWins_m is a binary indicator of whether the better-ranked player in match m won in a stated round of a tournament and Future_m represents the ability of the stronger opponent in the next round, as determined by the initial simulated draw. Note that it is not necessary to include tournament fixed effects because the tournaments are identical in all respects except draws and outcomes.

Concerned with a potential mechanical shadow effect caused by tournament seeding, we focus our attention on the magnitude and statistical significance of γ_1 . We adapt the simulation procedure to generate second and quarterfinal data, advancing players according to the simulated outcomes.

3.3.1. Simulation Results. With the simulation data, we can test whether there is a mechanical relationship between the expected ability of the opponent in the next round and the stronger player's success in the current round. Results in Table 5 show that when we include all first-round matches, there is a small, positive, and statistically significant relationship between the indicator that the stronger player wins and the skill of the expected future opponent. This suggests that we could be overestimating the magnitude of the shadow effect in the first round. However, this relationship does not appear in later rounds, relieving concerns of a pervasive mechanical effect.

One solution that we propose is to drop matches that include the top two seeds of the tournament, at least in the first round—recall that the top two seeds are the only players who are not randomly assigned to a position in the draw. The results from regressions using simulation data are reassuring. Without the top two seeds, we now do not observe any statistically significant relationship between the indicator that the stronger player wins and the skill of the expected future opponent in any round. That is, we do not observe any mechanical relationship that could be driving our empirical results.

²² All simulation results are qualitatively identical if we instead use a simulated world ranking, in which 8 players are randomly assigned unique values between 1 and 50 and 24 players are assigned values between 51 and 170.

Table 5 Simulation Results

Round	Shadow effect	All players	Excluding top two seeds
First	Coefficient estimate	0.117**	0.041
	Standard error	0.052	0.057
	No. of observations	16,000	14,000
Second	Coefficient estimate	0.002	0.039
	Standard error	0.084	0.090
	No. of observations	8,000	6,668
Third	Coefficient estimate	0.070	0.062
	Standard error	0.124	0.137
	No. of observations	4,000	3,080

Notes. Each simulation run includes 1,000 32-player tournaments. The ability measure was rank in tournament.

** $p < 0.05$.

3.3.2. ATP and Betting Market Results Excluding the Top Two Seeds. In light of the simulation results, we estimated the main specification using the ATP data set and excluding matches with the top two seeded players; results are presented in Table 6. Coefficient estimates in these specifications are very similar in magnitude to those in the main specification and are identical to the main results in terms of the pattern of statistical significance. That is, the shadow effect survives dropping the top seeded players from the analysis and the observed effect does not appear to be driven by the ATP's seeding protocol.

4. Conclusion

In this paper, we present a two-stage, matched-pair tournament model that provides two sharp results: (a) a shadow effect of future competition, wherein the tougher the expected competitor in the final stage, the lower the probability that the stronger player is selected as the winner in the first match; and (b) an effort spillover effect, wherein negative spillover has a stronger adverse effect on the higher-skilled player, relative to its impact on the weaker player's probability of success.

We test our two main theoretical hypotheses using data from professional tennis matches. We find evidence of a substantial shadow effect and also identify a negative spillover effect in tennis tournaments. In a second analysis, we use probability odds data from bookmakers to show that betting markets recognize and price in these spillover and shadow effects.

Our findings have implications in terms of the structure of elimination tournaments. Tournaments are often designed to identify high-ability candidates in environments where the contest organizer cannot readily observe innate talent. In a firm context, our results suggest ways by which a manager can improve the likelihood of promoting the strongest candidate.

Shrouding the skill of a strong future opponent increases players' continuation values, relative to the

Table 6 Regressions Excluding Top Two Seeds

	DV = Stronger player wins in current match (0% or 100%)			DV = Implied probability that the stronger player wins in current period (%)		
	Five-round tournaments	Six-round tournaments	Seven-round tournaments	Five-round tournaments	Six-round tournaments	Seven-round tournaments
<i>Expected future opponent rank</i>	0.0631*** (0.0125)	0.0673** (0.0297)	0.1659*** (0.0238)	0.0266*** (0.0054)	0.0331*** (0.0120)	0.1040*** (0.0122)
<i>Stronger player's previous games</i>	-0.3863*** (0.0934)	-0.3252*** (0.0829)	-0.4355*** (0.0728)	-0.1904*** (0.0352)	-0.2219*** (0.0357)	-0.2535*** (0.0332)
<i>Weaker player's previous games</i>	0.2054** (0.0947)	0.2678*** (0.0928)	0.1886*** (0.0538)	0.0809** (0.0386)	0.1101*** (0.0372)	0.1202*** (0.0270)
<i>Current rank ratio (worse/better rank)</i>	0.2425*** (0.0419)	0.1653*** (0.0630)	0.1281*** (0.0423)	0.2077*** (0.0303)	0.1432*** (0.0492)	0.1222*** (0.0392)
Fixed effects						
Round	X	X	X			
Tournament-year	X	X	X			
R ²	0.06	0.06	0.07	0.10	0.11	0.12
No. of observations	10,339	5,781	5,706	9,898	5,621	5,571

Notes. Values in parentheses are robust standard errors, clustered by tournament-year (e.g., 2008 U.S. Open). Matches from the final round of all tournaments are excluded. The number of previous games for the stronger and weaker players is set to zero for the first round of all tournaments. DV, dependent variable. "X" denotes that fixed effects are included.

** $p < 0.05$; *** $p < 0.01$.

case where the player faces a stronger rival with certainty. This will elicit more effort, particularly from the stronger player, and improve the probability that the stronger player will win in the current match. Of course, the opposite is true if the contest designer shrouds the identity of a weaker future opponent. Overall, a shrouding policy could elicit more effort (and thus improve the likelihood of selecting a strong winner) in a setting where the future opponent is more likely to be strong, rather than weak. In promotion contests within the firm, a manager who suspects that workers will face a particularly high-skilled competitor should be discouraged from posting explicit information about the skill and identity of this future threat. In practice, a credible shrouding policy could be implemented by always delaying the announcement of winners from parallel competitions.

Limiting negative spillover by allowing competitors opportunities to refresh their resources between stages may also increase the probability that the stronger type wins. For example, in an innovation contest, firms should be given adequate time between stages to raise additional funds and pursue more advanced technology improvements. Similarly, a firm may wish to institute a "work-life balance" program that promotes employee wellness, discourages career-related burnout, and improves the probability that the firm's labor tournament promotes the strongest workers.

In addition, firms may want to encourage positive spillover through learning. For example, managers could analyze and provide constructive feedback about workers' performances, allowing them to better accumulate skills over stages of the promotion tournament.

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