Valuing Options on Dividend-Paying Stocks Using the Compound Option Model: Further Notes

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This document presents a corrected version of Example 14.2 in the second edition of *Derivatives Markets*, and some additional notes on using the compound option model to value a call option on a dividend paying stock.

First, in order for the compound option calculation to make sense, the strike price on the compound option must be positive. That is, we must have

$$D - K\left(1 - e^{-r(T-t_1)}\right) > 0$$

If this condition were not true, interest on the strike (over the life of the option from the ex-dividend date to expiration) would exceed the value of the dividend and early exercise would never be optimal.

Second, there is a subtlety in computing the present value of equation (14.14). We have

$$\text{PV}\left[S_{t_1} + D - K + \max\left(P[S_{t_1}, T-t_1] - \left[D - K(1-e^{-r(T-t_1)})\right], 0\right)\right] =$$
$$S_0 - Ke^{-rt_1} + \text{CallOnPut}\left[S_0 - De^{-rt_1}, K, D - K(1-e^{-r(T-t_1)}), \sigma, r, t_1, T, 0\right]$$

The compound option that is implicit in the early exercise decision gives the right to acquire a put option *after* the dividend is paid. (The put option is acquired if we do not exercise the call. If the call is unexercised after the dividend, all subsequent valuation will be with respect to the *ex-dividend* value of the stock.) Thus, for purposes of valuing the compound option, the underlying asset is really the stock without the dividend, which is the prepaid forward. In obtaining this put, the call owner gives up the dividend and earns interest on the strike.

Here is the corrected version of the example (note that the Black-Scholes value is also slightly different than in the text):

*I am grateful to Abraham Weishaus for pointing out problems with this section.

Example 14.2

Suppose a stock with a price of \$100 will pay a \$5 dividend in 91 days ($t_1 = 0.2493$). The prepaid forward price for the stock, assuming receipt of the share *after* the dividend (at the time denoted 0.2493^+) is

$$F^P_{0,0.2493^+}(S_0) = \$100 - \$5 \times e^{-0.08\times 0.2493} = \$95.0987$$

An option with a strike price of \$90 will expire in 152 days ($T = 0.4164$). Assume $\sigma = 0.3$ and $r = 0.08$. The value of a European call on the stock is

$$\text{BSCall}(95.0987, \$90, 0.3, 0.08, 0.416, 0) = \$11.764$$

The value of an American call is computed as the present value of equation (14.14), with the exercise price for the compound option equal to $5 - 90(1 - e^{-0.08\times(0.4164-0.2493)}) = 3.8047$, and time to maturity 0.2493 for the compound option and 0.4164 for the underlying option. The price of the compound option is

$$\begin{aligned}&\text{CallOnPut}(S, K, x, \sigma, r, t_1, T, \delta)\\ &\quad = \text{CallOnPut}(95.0987, 90, 3.8047, 0.30, 0.08, 0.2493, 0.4164, 0) = \$1.7552\end{aligned}$$

Thus, the value of the American option is[1]

$$\$100 - \$90e^{-0.2493\times 0.08} + \$1.7552 = \$13.5325$$

The compound option should be exercised if the ex-dividend stock price is below \$89.988.[2] Since exercise of the compound option is equivalent to leaving the original call *unexercised*, the original call should be exercised at time $t_1 = 0.2493$ if the stock price *exceeds* \$89.988.

You can verify that \$89.988 is the pivotal stock price by performing a simple Black-Scholes calculation at time t_1, at which point the remaining time to expiration is $152/365 - 91/365 = 0.1671$. If the stock price *cum-dividend* is \$94.988, then by exercising the option we receive $94.988 - 90 = 4.988$. If we leave the option unexercised when the dividend is paid, the stock price drops to 89.988. The Black-Scholes value for the option is then

$$\text{BSCall}(89.988, 90, 0.30, 0.08, 0.1671, 0) = \$4.988.$$

The option holder is indifferent between exercising and receiving \$4.988 or leaving the option unexercised and having it be worth the same amount.

[1] It is possible to obtain the same result for the American option price by using the binomial calculation in the "Fixed Dividends" tab of *OptAll2.xls*. With 500 iterations, the call price is 13.5294.

[2] The VBA function *CallOnPut* is an array function. The second value returned is the is the stock price below which the compound option is exercised.