Will Google Hit $600?

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> Last week, a relatively obscure Wall Street analyst named Safa Rashtchy, from Piper Jaffray, became an overnight sensation among the Internet faithful. He predicted that Google's stock price, which has climbed more than 350 percent since its initial public offering in 2004, and was $422.52 at the time, would hit $600 a share by the end of 2006.
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> As for Mr. [Henry] Blodget, he predicts Mr. Rashtchy may be on the money. "In my opinion, there's about a 33 percent chance that Google's future will be as good or better than he predicts," he said.[1]

How can we evaluate a statement like this? Blodgett is making a probabilistic statement about the future stock price. If we know the stock price distribution, we can compute the probability that the stock will exceed any given future price.

In practice, it is common to talk about the distribution of stock returns. (Obviously, given a return, you can compute the price.) We can consider two different assumptions about returns. First, suppose that effective returns are normally distributed. This turns out to be a problematic assumption, so in practice it is common to make a different, second assumption, namely that continuously compounded returns are normally distributed (this is an assumption underlying the Black-Scholes model).

With either assumption, we will use the normal distribution to make statements about the probability of future price ranges. Suppose that the annual expected return is μ and the annual standard deviation of returns is σ. If returns are normally distributed, then we can make statements such as the following: With 68% confidence, the stock return will lie in the interval $\mu \pm \sigma$. Thus, if the expected return is $\mu = 10\%$ and the standard deviation of that return is $\sigma = 40\%$, then there is a 68% chance that the return will be between -30% and $+50\%$. We can compute normal probabilities by using Excel's built-in function *NormSDist*, where if z is a standard-normal variable,

$$\text{NormSDist}(a) = \Pr(z < a)$$

[1] "Is Google a Good Candidate for Rational Exuberance?", by Andrew Ross Sorkin, *New York Times, Week in Review*, p. 5, Jan 8, 2006

Let's suppose that Google's expected rate of return is 10%. You might quibble with this assumption, but it will turn out not to matter very much. We need an estimate of Google's volatility over the next year. One place to obtain this estimate is from option prices, from which we can obtain the volatility that investors use to price the option (this is called the *implied volatility*). Here is data for Google from Tuesday, January 10:

Stock Price	\$465
Strike Price	\$470
Call Price (January 2007)	\$81
Interest Rate	4%
Dividend	0

With this information, we can compute an implied volatility:

$$\text{BSCallImpVol}(465, 470, 0.30, 0.04, 1, 0, 81) = 0.4081$$

Remember that the implied volatility is the standard deviation of the continuously compounded rate of return. We will do two calculations. In the first, we will ignore the "continuously compounded" part of this statement. In the second calculation we will not ignore it.

Normally-Distributed Effective Annual Returns

With an assumed 10% expected return for Google the expected stock price in one year is

$$\$465 \times 1.1 = \$511.50$$

The percentage return from \$511.50 to \$600 is

$$\frac{600 - 511.50}{511.50} = 0.173$$

This is 0.173/0.4081 = 0.424. Thus, if Google outperforms the mean by 0.424 standard deviations, it will have a stock price of \$600 or greater. The probability that a standard normal variable, z, will have a value greater than 0.424 is

$$\Pr(z > 0.424) = 1 - \text{NormSDist}(0.424) = 0.336$$

Our estimate of the probability that Google will exceed \$600 is almost *exactly* one-third!

Normally-Distributed Continuously Compounded Returns

If we assume that continuously-compounded returns are normally distributed, then the probability calculations are similar. Suppose that $\mu = 0.0953\%$ is the continuously compounded expected return.[2] A move from \$511.50 to \$600 is

[2]This keeps the mean return the same as in the previous example since $\mu = \ln(1.1) = 0.0953$. That is, $e^{0.0953} \times 465 = 511.50$.

a continuously compounded return of $\ln(600/511.50) = 0.1596$. This is a move of $0.1596/0.4081 = 0.391$ standard deviations. Doing the same calculation as before, we have

$$\Pr(z > 0.391) = 1 - \text{NormSDist}(0.391) = 0.348$$

Again, the estimate is that there is about a one-third (34.8%) chance of Google's price exceeding $600 in one year.