Expected Returns on Equity and Debt

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Finance 465

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When we view the liabilities of a firm as options, we can compute the expected return on debt and equity. This handout illustrates how to do this using two examples. This exercise is useful because it provides an example illustrating how seniority affects expected returns. However, it also illustrates the use of delta and elasticity in an application where their use is not obvious at first glance.

In all the examples we will assume that the assets of the firm are lognormally distributed with volatility $\sigma = 40\%$, expected return $r_A = 11\%$, and the continuously compounded risk-free rate is 6%. The firm makes no payouts, hence the dividend yield $\delta = 0$. We assume there are no taxes.

1 Example 1: Equity and a Single Debt Issue

Suppose the firm has a single zero-coupon debt issue outstanding, with a promised principal repayment of $\bar{B} = \$120$ in 5 years. We first compute the values of debt and equity. Equity is a call option on the assets with a strike price of \$120, hence the value of equity at time 0 is

$$\begin{aligned} E_0 &= \text{BSCall}(A_0, \bar{B}, \sigma, r, T, \delta) \\ &= \text{BSCall}(100, 120, 0.4, 0.06, 5, 0) \\ &= \$38.425 \end{aligned} \tag{1}$$

The value of the Debt, B_0, is

$$B_0 = A_0 - E_0 = \$100 - \$38.425 = \$61.575 \tag{2}$$

This calculation takes the value of the assets and allocates it between the debt and equity.

We can also compute the yield to maturity on the debt, y. The debt costs \$61.575 and pays \$120 in 5 years, so the yield is

$$y = \frac{1}{5}\ln(120/61.575) = 13.345\% \tag{3}$$

We next compute the expected return on equity and debt by computing their *elasticities*, which we denote Ω_E and Ω_B. The expected return on equity, r_E, is then

$$r_E = r + \Omega_E(r_A - r) \tag{4}$$

That is, the risk premium on equity is proportional to the risk premium on the assets of the firm, with the elasticity providing the measure of proportionality.

Elasticity is defined as the percentage change in the value of the equity divided by the corresponding percentage change in the value of the assets. If assets change by \$1, the percentage change in the value of the assets is $1/A$. The percentage change in the value of the equity is Δ_E/E, where Δ_E is the *dollar* change in the value of the equity when the assets change by \$1. Elasticity is thus

$$\Omega_E = \frac{\Delta_E/E}{1/A} = \frac{A\Delta_E}{E} \tag{5}$$

Although the variable have different names, you may recognize this formula. Since equity is a call option, this is the standard elasticity calculation for a call option, $\Omega = A\Delta/C$, where Δ is the delta of the call and C its price. Using the spreadsheet function for elasticity, we have

$$\Omega_E = \text{BSCallElast}(100, 120, 0.4, 0.06, 5, 0) = 1.870$$

Using equation (4), the expected return on equity is therefore

$$r_E = .06 + 1.870 \times (0.11 - 0.06) = 15.351\% \tag{6}$$

We can compute the expected return on the debt in much the same way. First, however, we need to compute the elasticity of debt.

The elasticity of debt is the percentage change in the price of debt divided by the percentage change in the value of the assets. We can obtain a formula for this in the same way as with equity. Suppose the assets change in value by \$1. The percentage change in the value of assets is $1/A$. Since the debt is equal to the value of assets less the value of equity, the change in the value of the debt is $1 - \Delta_E$, where Δ_E is the delta of the equity. The percentage change in the value of debt is $(1 - \Delta_E)/B$. We have

$$\Omega_B = \frac{(1 - \Delta_E)/B}{1/A} = \frac{A(1 - \Delta_E)}{B} \tag{7}$$

In our example, debt elasticity is

$$\Omega_B = \frac{100(1 - \text{BSCallDelta}(100, 120, 0.4, 0.06, 5, 0))}{61.575} = 0.457 \tag{8}$$

The expected return on debt is therefore

$$r_B = r + \Omega_B \times (r_A - r) = .06 + 0.457 \times (0.11 - 0.06) = 8.285\% \tag{9}$$

The expected return on debt is substantially less than the yield, because the yield is the *greatest* return the debt holders could earn, and there is a chance they will earn less if assets are worth less than \$120 after 5 years (i.e., the firm is bankrupt).

Another way to compute the expected return on debt is to use the fact that the elasticity of a portfolio is a *weighted average* of the elasticities of the portfolio components. Thus, using the fact that the elasticity of assets is 1, we have

$$\begin{aligned}\Omega_B &= \frac{A}{A-E} \times \Omega_A - \frac{E}{A-E} \times \Omega_E \\ &= \frac{100}{100 - 38.425} \times 1 - \frac{38.425}{100 - 38.425} \times 1.870 \\ &= 0.457\end{aligned} \tag{10}$$

This the same elasticity as that we obtained in equation (8).

2 Example 2: Equity and Multiple Debt Issues

Now suppose that the firm in Section 1 still has \$120 of debt outstanding, but suppose that there are three classes of debt: senior, with face value $\bar{B}_S = \$40$; intermediate, with face value $\bar{B}_I = \$30$; and junior, with face value $\bar{B}_J = \$50$. As discussed in Chapter 16, we can compute the values of debt and yields by treating each class of debtholder as having bought an option and sold an option. We will work through the example of the intermediate class.

The intermediate class of bondholders is paid in full in year 5 provided that the assets in year 5 are worth at least $\bar{B}_S + \bar{B}_I = \$70$. If the assets in year 5 are worth less than $\bar{B}_S = \$40$, the intermediate bondholders are paid nothing. Thus, the intermediate bondholders own a call option on the assets with a strike price of $\bar{B}_S = \$40$ and have written a call option with a strike price of $\bar{B}_S + \bar{B}_I = \$70$. The value of the debt is

$$B_I = \text{BSCall}(100, 40, 0.40, 0.06, 5, 0) - \text{BSCall}(100, 70, 0.40, 0.06, 5, 0) = \$15.842 \tag{11}$$

Seniority	Principal	Value	Yield	Elasticity	Expected Return
Junior	\$50	\$17.917	20.525%	0.908	10.539%
Intermediate	\$30	\$15.842	12.771%	0.526	8.631%
Senior	\$40	\$27.815	7.266%	0.127	6.636%

Table 1: Values, yields, and expected returns of debt when there are three classes of debt outstanding. All debt is zero coupon, and matures in year 5.

The yield on the intermediate debt, y_I, is

$$y_I = \frac{1}{5}\ln(30/15.842) = 12.771\% \tag{12}$$

We can compute the elasticity on the intermediate debt in the same way as before. The dollar change in the value of the intermediate debt for a \$1 change in the value of assets is

$$\begin{aligned}\Delta_I = {} & \text{BSCallDelta}(100, 40, 0.40, 0.06, 5, 0)\\ & - \text{BSCallDelta}(100, 70, 0.40, 0.06, 5, 0) = \$0.083\end{aligned}$$

The elasticity is therefore

$$\Omega_I = \frac{\Delta_I/B_I}{1/A} = \frac{0.083 \times 100}{15.842} = 0.526 \tag{13}$$

The expected return on intermediate debt is therefore

$$r_I = r + \Omega_I(r_A - r) = .06 + 0.526 \times (0.11 - 0.06) = 8.631\% \tag{14}$$

Table 1 summarizes yields, elasticities, and expected returns for all three classes of debt.

Finally, note that the weighted average of yields on equity and all classes of debt must equal the expected return on the assets. We have

$$\begin{aligned}&(38.245 \times 15.351\% + 17.917 \times 10.539\% + 15.842 \times 8.631\\ &\qquad + 27.815 \times 6.636\%)/100 = 11\%\end{aligned} \tag{15}$$