Lecture 5: Supplementary exercises

We know from Finance I that only systematic risk is priced (rewarded with a higher expected return), because we assume that investors can diversify away idiosyncratic risk (i.e. they don't bear it). If the intuition of why we used different discount rates in the coffee café example has not sunk in, the following set of exercises should help. To complete this exercise you will need to download the Excel spreadsheet supp_5.xls from the Corporate Finance web page.

4) Discount rates and systematic risk (version 1). The information from lecture and the spreadsheet is reproduced below. The probability of each state is the probability of preferences times the probability of the state of the economy (see spreadsheet). This question is based on the value of the project when we do not hire the market research firm.

	Future Years					
Preferences	Berkeley	Berkeley	Berkeley	Cleveland	Cleveland	Cleveland
Prob of preferences	80.0%	80.0%	80.0%	20.0%	20.0%	20.0%
State of the economy	High	Medium	Low	High	Medium	Low
Prob of state of economy	25.0%	50.0%	25.0%	25.0%	50.0%	25.0%
Cash flow (year 1-10)	32.0	22.0	12.0	22.0	12.0	2.0
Prob of state	20.0%	40.0%	20.0%	5.0%	10.0%	5.0%

A) Updated probabilities. If we build the coffee café today, next year when we discover the cash flow we will update our estimate of the probability that Chicago looks like Berkeley. Assume all you will see next year is the cash flow (i.e. you can't see the state of the economy). What is the probability that Chicago looks like Berkeley after we see the first year of cash flows. You can enter these numbers in row 23 of the spreadsheet.

B) The value of the project once we have made the investment is the NPV of the project (20.3) plus the initial investment (100). Thus, the value of the project today is 120.3[1] Calculate the value of the project next year in each of the six states. This is the value of the cash flow which is received next year plus the present value of the expected cash flow over the remaining nine years of the project. You should discount these cash flows to year 1 (as you are calculating a present value as of year 1).You will need the probabilities, which you calculated in A, to calculate the expected cash flows. Put your answers in row 24 of the spreadsheet.

C) Calculate the realized return in each state. The return is a function of the 'cum-dividend' asset value in year one (row 24) and the asset value in year zero (cell B22). You should find that the return is higher when the economy and stock market are doing well. You should also find that the return is higher when the probability that

[1] To calculate the NPV of the project, you can use the formula for a finite annuity from the Review of Financial Concepts.

Chicago looks like Berkeley is high. It should make sense why each of these two statements are correct.

D) Calculate the correct project discount rate. To do this you must first estimate the asset β. Remember β is estimated as the covariance of the excess return on the asset (the coffee café) with the excess return on the market divided by the variance of the excess return on the market. The excess return on the coffee café is the return on the coffee café (row 25) minus the risk free rate (cell B2). The excess return on the market (row 19) is the return on the market (row 18) minus the risk free rate (cell B2). The formula for beta from Finance I is:[2]

$$\beta_A = \frac{\text{Cov}\left(r_{market} - r_{free} , r_{asset} - r_{free} \right)}{\text{Var}\left(r_{market} - r_{free} \right)}$$

$$= \frac{\sum_{t=1}^{6} p_t \left(r_{A,t}-r_{f,t} - E[r_A-r_f]\right)\left(r_{m,t}-r_{f,t} - E[r_m-r_f]\right)}{\sum_{t=1}^{6} p_t \left(r_{m,t}-r_{f,t} - E[r_m-r_f]\right)^2} \quad \textbf{(1)}$$

$$= \frac{\sum_{t=1}^{6} \left(p_t \left(r_{A,t}-r_{f,t}\right)\left(r_{m,t}-r_{f,t}\right)\right) - E[r_A-r_f]\,E[r_m-r_f]}{\sum_{t=1}^{6} \left(p_t \left(r_{m,t}-r_{f,t}\right)^2 \right) - E[r_{m,t}-r_{f,t}]^2}$$

If you prefer pictures, a graph of the excess return on the assets versus the excess return on the market can be found on the third tab of spreadsheet. Is the discount rate you calculated the same one I used in lecture?

E) Value the project. In lecture we took the expected cash flow and discounted them at the risk adjusted rate. This time take the expected value of the possible asset values (row 24). Remember, the six states are not equally likely. You can use the sumproduct( ) function in Excel to calculate expected values. Next discount the expected year one cash flow (or asset value) at the correct discount rate. The risk free rate is 2.0 percent and the market price of risk is 8.5 percent. You calculated the beta in D). How does your value compare to the answer we obtained in class?

F) Extra question for the quantitatively inclined. You calculated the returns and the β conditional on an initial valuation of 120.3 (see cell B21). You should have noticed that to get this number, we had to assume the asset β was equal to one and the correct discount rate was 10.5 percent, even though our objective was to estimate the β and the discount rate. To convince yourself that this approach yields a uniquely correct

[2] You can't use the excel variance and co-variance functions to answer this question. They assume each observation is equally likely. That isn't true in this example, given the assumed probabilities (row 14 of the spreadsheet). You will therefore need to use the sumproduct function in excel. As an example, sumproduct(A1:A3,C1:C3) equals A1 * C1 + A2 * C2 + A3 * C3. Thus if the probabilities are in row A, this calculates the expected value of C.

answer, use a different estimate of the project's value (i.e. replace the 120.3 in cell B22). You may want to save your worksheet first. The correct estimate of the project's value (cell B22) is the one which produces a final estimate (cell B33) which is equal to your initial estimate (cell B22). An easy way to do this is to use Excel's Goal Seek or Solver. Tell Excel to adjust your initial guess (cell B22) until the difference between your initial guess (B22) and the final value (cell B33) is zero. This difference is contained in cell B34.

5) Discount rates and systematic risk (version 2). In this question, we will calculate the project beta, discount rate, and project value for the case where we hire the market research firm. To answer this question, make a copy of the Excel spreadsheet tab which contains your answer to the previous question.[3]

A) Updated probabilities. If we hire a Kellogg marketing professor, they will tell us next year with complete accuracy what the probability that Chicago looks like Berkeley. Thus in each of the 6 possible states we will have a probability that Chicago looks like Berkeley. You can enter your probabilities in row 23 of the spreadsheet. This should require no calculation. As a check, what is the expected probability that our marketing professor will tell us Chicago looks like Berkeley.[4]

B) Value of the project next year. Calculate the value of the project next year in each of the six states. Remember, the value of the project is different than what we calculated in the previous question. When we are told the preferences of Chicago customers (Berkeley or Cleveland) we will make a decision to invest 100 (a cash outflow) in exchange for the future cash flows from the coffee café (a cash inflow) or we will decide not to invest. The value of the project next year is the NPV of the project in each state. Put your answers in row 24 of the spreadsheet.

C) Calculate the value of the project today. The following steps should already have been calculated in your spreadsheet (you entered the formulas in the previous question). To value the project today you will need to discount the expected value next year discounted at the correct risk adjusted rate (cell B32). The discount rate depends upon the project beta (cell B31). Remember, this analysis depends upon an initial guess of the project value (cell B22). Initially you can leave this equal to your previous answer (120.3). Then use solver to calculate the correct value once all the formulas are entered.

Once you have a numerical answer, go back and look at the project beta you estimated. Does it make sense why this must be the beta. If not, graph the excess return on the project (row 26), against the return on the excess return on the market (row 19). What is the slope of the line which best fits the data? This is the project beta.

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[3] If you are not sure how to do this, right click on the tab. Then click on Move or Copy. Click the Create a copy box and click on OK.

[4] To calculate this use the command sumproduct(C15:H15,C23:H23).