Lecture 2: Answers to supplementary exercises.

4) Calculating Asset Values and Asset Betas (Case 1 – Assets).

A) The market price of risk is the expected excess return on the market over the riskfree. This excess return ranges from a high (in states 17-20) of 38.2% to a low (in states 1-4) of -21.2. The average of 8.5 is the market price of risk. This is also the expected return on the market (13.1%) minus the riskfree rate (4.6%). Notice, that the standard deviation of the excess return is 21%, exactly the number from Lecture 2.

B) The asset returns range from a low of -56.6% (severe recession and customers don't like our product) to a high of 82.8% (terrific boom and customers love our product). The asset is risky – and this risk includes both systematic and idiosyncratic risk. You could calculate the mean asset return and its standard deviation, but since they are based on our guess of the asset value, at the moment these would be meaningless.

C) The covariance and thus the asset β (which are estimates until we get the correct asset value) are 0.049 and 1.111 when we guess the asset has a value of 90.

D) The discount rate is the riskfree rate (4.6%) plus the asset β times the market price of risk (8.5%). When we use an asset value of 90, and thus an asset β of 1.111, the asset discount rate is 14.0%.

E) Now that we have the full loop (from guess at asset value, to asset β, to asset discount rate, to asset value), we can find the correct value of the assets. The asset value is 100. Based on this, the asset β is 1.0. The return on the asset is expected to rise one for one with the market. It may rise more or less – due to idiosyncratic risk -- but on average it rises one for one. This implies the discount rate is 13.1% (the same as the expected return on the market. Finally discounting the expected cashflow (113.1) at the asset discount rate of 13.1% confirms your guess is correct – the asset is worth $100.

5) Calculating Debt and Equity Values and Betas (Case 1 – Debt & Equity).

A) Value the debt. To value the debt, there are two basic steps. Calculate the expected cashflows and calculate the correct discount rate.

1) Expected cashflows. The cashflows to the debt holders are the minimum of the promised payment (in this case $20) and the assets of the firm. Enter the actual debt cashflows in row 23. Then the expected cashflow in E23. This is the numerator in our debt valuation.

2) Discount rate.

a) The discount rate depends upon the covariance of debt returns and the market returns. Calculate the debt returns in row 25. This is the cashflow to debt divided by the purchase price of debt minus one. The cashflow to debt you calculated in row 23. The purchase price of debt is its value and for the time being use your guess (C24). Returns are low when the debt defaults and the cashflow is less the promised. Returns are high when the debt does not default (asset cashflow is greater than promised payment on debt) and is equal to the promised return. Notice we did not make an assumption about the promised return. It will be determined by the market. Calculate the expected return and put this in E25

b) β. Now that we have the returns, calculate the covariance of the debt returns with the market return. Enter this in E27. This number is positive if the returns on the debt are above average (E25), when the market is above its average (E9). The β of the debt is the covariance you just calculated (E27) divided by the variance of the market (F9 squared). Enter this in E28.

c) Discount rate. The discount rate for debt is the riskfree rate (C2) times the debt β (E28) times the market price of risk (E10). When you are done, this will also equal the expected return on the debt (E25). Enter your answer in E29.

3) Valuing the debt. Now to value the debt, discount the expected cashflow (E23) at the required return on the debt (E29). Enter your answer in E30. This may not match your guess (C24). If it does not match, you need to change your guess until it does. If you entered the numbers correctly, the button Price the Debt will do the work for you. You will find that the value of the debt is 19.12. This implies that the debt β is zero and the expected return on the debt is 4.6% (the riskfree rate).

B) Promised return. The debt β is zero. This doesn't necessarily imply the debt is riskfree, only that it is free of systematic risk. In this case, the promised return is also 4.6% ((20/19.12)-1), which is equal to the riskfree (default free) rate. The bond has no risk. Notice that the promised payment is less than the minimum value of the assets (43.4). The bond never defaults. Notice that the cashflow to the bondholders is always 20. The bond has no risk.

C) Value of the equity. To value the equity, we go through the same procedure as we did for the assets and the debt. It should be more obvious, however, what the equity is worth. Since the assets are worth 100 and the debt has a value of 19.12, then the equity should be worth 80.88. Now to check this.

1) Expected cashflows. First, we calculate the cashflows (row 33). This is the asset cashflow minus the debt cashflow. Notice that it is always positive. This also implies the debt is riskfree (given we are assuming absolute priority). The expected cashflow (E33) is 93.10.

2) Discount rate. First calculate the returns (row 35). Given theses, we calculate the covariance of the equity returns with the market return. Since the assets have systematic risk, the equity will have systematic risk. Given a guess of 80.88 for the equity, the covariance is 0.055. The β (covariance/variance of the market return) is 1.236. Notice this is higher than 1, the asset β. You knew that had to be true. The equity discount rate is therefore 15.1%.

3) Value of the equity. Discount the expected cashflow (E33) to equity at he equity discount rate (E39) and you get the value of the equity (E40) which is 80.88. Our guess was correct. This is one of the implications of an asset pricing model that prices only systematic risk. This kind of calculation will always work – not just for CAPM.

D) Why is the equity riskier. When the firm was all equity, the equity ended up with all of the risk from the assets. When the firm is levered with riskfree debt, the equity still ends up with all of the risk from the assets. When the value of the assets falls from an expected cashflow of 113.10 to a low of 43.40 (a fall of 69.70), the value of

the equity falls from an expected cashflow of 93.10 to a low of 23.40 (a fall of 69.70). The reason the equity is riskier, is we are measuring risk per dollar of investment. We are measuring risk in returns. The worst case for the assets is a fall of 56.6% (H15). The worst case for the equity (in the levered firm) is a fall of 71.1%. All of the risk of the assets has now been concentrated on a smaller equity base. This makes the equity riskier. Bad states have much more negative returns (H15 versus H35) and good states have much more positive returns (AA15 versus AA35).

6) Leverage and βs (Case 2). Now the promised payment on debt is 60.

A) Since we haven't changed the assets, the asset β and the value of assets are the same. We can go directly to valuing the debt.

1) Debt value. Once the promised payment rose above 43.40, the debt is now risky. We don't know yet the type of risk. Sometimes the bond pays the promised payment, sometimes it pays less. The expected cashflow must therefore be less than the promised. In this case it is 59.08. Solving for the value of the debt, it is now worth 56.01. The value of the debt has obviously risen, since the promised payment rose.

2) Equity Value. The equity value has fallen. Since the value of the firm has not changed, this must be true. It has fallen to 43.99.

B) The promised rate on the debt is 7.1% (60.00/56.01 - 1). Since the debt now has a positive probability of default, the promised rate must rise above the riskfree rate. This is partially because you don't always get the promised return. Sometimes you get less. Thus you must promise more than the riskfree rate, to get at least the riskfree rate on average. In addition, if the debt β is now positive (we will check below), then the promised return must rise even more to get the expected return up high enough to cover the additional systematic risk.

C) The expected return in default is -9.2%. This is the average of the returns when the firm defaults on its debt. In this case, it defaults in two states (state 1 and state 5). The average of these two returns is -9.2%.

D) The expected return on debt is the probability of default (10%) times the return in default plus the probability of no default (90%) times the promised return. The expected return is 5%.

$$r_D = (1-p)\, r_{promised} + p\, r_{default} = 0.90\ (0.071) + 0.10\ (-0.092) = 0.055 \quad \textbf{(1)}$$

Notice this is equal to the debt discount rate (E25) that we calculated based on the debt β.

E) The debt and equity β both rise when we lever up the firm (issue debt with a promised payment of 60 opposed to 20). The debt β rises from zero (for riskfree debt) to 0.105. The default risk is partially systematic and partially idiosyncratic.[1]

[1] To know whether there is any idiosyncratic risk, we need to calculate the variance of the epsilon (idiosyncratic risk). Total variance is equal to systematic risk plus idiosyncratic risk.

The equity β also has risen. It rose from 1 (when there was no debt) to 2.139. The average β is 1.0 - which is exactly the asset β.

$$\begin{aligned}\beta_A &= \beta_D \frac{D}{D+E} + \beta_E \frac{E}{D+E} \\ &= 0.105 \frac{56.01}{56.01+43.99} + 2.139 \frac{43.99}{56.01+43.99} \\ &= 1.00\end{aligned} \qquad \textbf{(2)}$$

7) Leverage and βs (Case 3). Now the promised payment on debt is 80.

A) Since we haven't changed the assets, the asset β and the value of assets are the same. The value of the debt has risen to 70.71 and the value of the equity has fallen to 29.29.

B) The promised rate on the debt has risen even further. It is now 13.1% (80.00/70.71 - 1). The increase in the promised rate is partially due to a higher probability of default (now 25%) and partially due to a higher required rate of return (7.2%) since the β has increased (0.309).

C) The expected return in default has fallen to -10.5%. This is the average of the returns when the firm defaults on its debt. The firm now defaults in five states or 25% of the time.

D) The debt and equity β both continue to rise as we increase the leverage further. The debt β rises to 0.309 and the equity β rises to 2.667. The average β is still 1.0 since the assets of the firm have not changed. - which is exactly the asset β.

$$\begin{aligned}V[\, r_D \,] &= V[\, r_{riskfree} + \beta_D (r_{market} - r_{riskfree}) + \varepsilon \,] \\ \sigma_D^2 &= 0 + \beta^2 \sigma_{market}^2 + \sigma_{idiosyncratic}^2 \\ 0.065^2 &= 0.105^2 (0.210)^2 + \sigma_{idiosyncratic}^2 \\ \sigma_{idiosyncratic}^2 &= 0.061^2\end{aligned} \qquad \textbf{(2)}$$

Thus a large fraction of the debt risk is idiosyncratic. The idiosyncratic risk is 6.1%.