Lecture 2: Supplementary exercises.

3) Bond Pricing. In the previous lecture (see Lecture 1 handout), you calculated the correct (arbitrage free) price of the third bond based on the prices of the first two bonds. This example is similar. I will give you the rates of return on five zero coupon government bonds. The bonds make one payment (face value) at maturity (1-5 years). The zero coupon rates are:

Maturity	1	2	3	4	5
Zero Rate	4.31%	5.67%	6.38%	6.80%	7.31%

A) What is the price of each of the five zero coupon bonds. The bonds have a $1000 face value.

B) What are the five one year forward rates? The forward rates is the rate you can lock in today for a one year loan that starts in year t and matures in year t+1. The forward rate that starts today (t=0) is the same as the one year rate: 4.31%.

C) Pricing 2 year coupon bonds. Bonds that pay interest give you money prior to maturity. When the term structure is not flat, the discount rate for cashflows one year from today is not the same as cashflows which occur two years from today.

1) What is the price of a two year coupon bond with a coupon rate of 5.67%? Assume that only interest is paid before maturity. All principal is repaid at maturity. If the bond doesn't sell for par ($1000 face value), why not?

2) What must the coupon rate be for the bond to sell at par? Hint: It is easier to answer this question if you use the goal seek function in Excel.

D) Pricing the other bonds. For the 3, 4, and 5 year bonds calculate the coupon rate which the market will required for the bonds to sell at par. These are related to but not the same as the zero rates you were given above.

Lecture 2: Answers to supplementary exercises.

3) The way bonds are priced is to discount their expected cashflows at the expected rate of return. Alternatively, you can discount the promised payments at the promised rate of return. Since we are assuming that default risk is zero, these are the same in this example.

Maturity	1	2	3	4	5
Zero Rate	4.31%	5.67%	6.38%	6.80%	7.31%
Bond Price	958.68	895.56	830.65	768.63	702.75

A) Bond prices. The price of the bond is $1000 discounted back at the maturity matched discount rate for the correct number of years. Thus for the five year bond, discount $1000 back at 7.31% for five years.

$$P_5 = \frac{1000}{(1 + r_{5\ \text{year}})^5} = 702.75$$

B) Forward rates. The one year forward rate is the rate you can lock in today for a future one year loan. You can calculate the one-year forward rate by building the following replicating portfolios (see Lecture 2 notes). As long as both portfolios have identical payoffs in all possible states of the world, they must have identical prices.

Portfolio 1: Suppose that you want to loan $1 to the U.S. government for two years. Portfolio 1 would consist of a long position in $1 worth of two-year zero coupon bonds. At the end of two years, this portfolio would be worth $1(1.0567)(1.0567) = $1.1166.

Portfolio 2: Portfolio 2 would consist of a long position in a one-year zero coupon bond and a commitment to invest the proceeds from this one year investment at the one-year forward rate. The forward rate must be agreed on today for the rate to be fixed. The value of this portfolio after two years would by $1(1.0431)(1+$r_{1,1}$) where $r_{1,1}$ is the one year forward rate. Since both portfolios require you to loan the U.S. government $1 for two years, they must have the same value at the end of two years. Therefore:

$$\$1(1.0431)(1+r_{1,1}) = 1.1166$$
$$\Rightarrow r_{1,1} = 0.0705$$

How do you enter into a commitment to invest at the current forward rate one year from now? One way is to build a portfolio by investing one dollar in the two-year zero coupon bond and borrowing $1 for one year (by shorting the one-year bond). Your cashflows are as follows:

Cashflow	Today	Year 1	Year 2
Invest $1 @ 5.67% for 2 years	-1.0000	0.0000	1.1166
Borrow $1 @ 4.31 for one year	1.0000	-1.0431	0.0000
Net cashflows	0.0000	-1.0431	1.1166

Thus you will invest 1.0431 in one year and get back 1.1166 in two years. Your rate of return is therefore 7.05% – the forward rate. The rest are calculated in a similar manner. We saw this in Lecture 2.

Maturity	1	2	3	4	5
Zero Rate	4.31%	5.67%	6.38%	6.80%	7.31%
Forward Rates	4.31%	7.05%	7.81%	8.07%	9.37%

C) Price of a two year bond.

1) To value the bond we discount the cashflows at the correct discount rate. For one year cashflows, the correct discount rate is 4.31%. For two year cashflows the correct discount rate is 5.67%. A bond with a 5.67% coupon will sell for 1000.07 which is slightly more than par. The reason the bond sells for more than par is because the discount rates for the one year cashflow is less than 5.67%. It is only 4.31%.

$$P_{2\ ,\ r_c=5.67\%} = \frac{1000*.0567}{(\ 1\ +\ 0.0431\ )^1} + \frac{1000*(1+.0567)}{(\ 1\ +\ 0.0567\ )^2} = 1000.70$$

2) The bond sells for more than par. Thus to sell at par, we need to lower the coupon rate slightly. A 2 year bond with a 5.63 percent coupon rate will sell for par.

$$P_{2\ ,\ r_c=5.63\%} = \frac{1000*.0563}{(\ 1\ +\ 0.0431\ )^1} + \frac{1000*(1+.0563)}{(\ 1\ +\ 0.0567\ )^2} = 1000.00$$

D) The coupon rates on the other bonds are as follows:

Maturity	1	2	3	4	5
Zero Rate	4.31%	5.67%	6.38%	6.80%	7.31%
Coupon Rate (to sell @ face)	4.31%	5.63%	6.31%	6.70%	7.15%

Notice that since the zero rate term structure is always upward sloping (it increases between each pair of years), the coupon rate on the bonds are always less than the zero of the same maturity.