Lecture 2: Supplementary Exercises.

1) Risk and return on government bonds (version 1). This question should give you practice on calculate asset values and rates of return. It will also illustrate how we determine whether an asset is risky and what type of risk this is. The current rate on a one year government bond is 6 percent. Next year, the one year rate on one year government bonds will be either 4 percent or 8 percent with equal probability. This future on year rate (4 or 8 percent) is the rate an investor will earn if they buy a one year government bond next year. Assume that all government bonds are zero coupon bonds. The price of the one year government bond is 94.34 today. The price of the two year government bond is 89.03.

	Today	Next Year	
		State 1	State 2
One year rate (%)	6.0	4.0	8.0
Price of 1 year bond	94.34		
Price of 2 year bond	89.03		

A) What is the return on the one year government bond in State 1? What is the return on the one year government bond in state 2?

B) What is the one year return on the two year government bond in State 1? What is the return on the two year government bond in state 2? To calculate the one year return on the two year government bond, you first need to calculate the price at which you can sell the two year government bond for next year.

C) What is the expected return on the two year government bond over the next year?

D) Is the two year government bond risky? If so, what kind of risk?

2) Risk and return on government bonds (version 2). The current rate on a one year government bond is 6 percent. Next year, the one year rate on one year government bonds will be either 4 percent or 8 percent with equal probability. This one year rate (4 or 8 percent) is the rate an investor will earn if they buy a one year government bond next year. This bond will mature in two years. Assume that all government bonds are zero coupon bonds. The price of the one year government bond is 94.34 today. The price of the two year government bond is 88.61. Notice the only change in the problem is the price of the two year government bond.

	Today	Next Year	
		State 1	State 2
One year rate (%)	6.0	4.0	8.0
Price of 1 year bond	94.34		
Price of 2 year bond	88.61		

A) What is the one year return on the two year government bond in State 1? What is the one year return on the two year government bond in state 2? To calculate the one year return on the two year government bond, you first need to calculate the price at which you can sell the two year government bond for next year.

B) What is the expected return on the two year government bond over the next year?

C) Is the two year government bond risky? If so, what kind of risk?

D) Do you expect the stock market return to be above average in state 1 or state 2? You will need to use your answer to C).

Lecture 2: Supplementary Exercises Answers.

1) Risk and return of government bonds (version 1).

	Today	Next Year	
		State 1	State 2
One year rate (%)	6.0	4.0	8.0
Price of 1 year bond	94.34	100	100
Price of 2 year bond	89.03	96.15	92.59

A) The one year government bond will mature next year. Since the bond is a zero coupon bond it will pay $100 next year in each state. Thus the return on the one year government bond is 6.0 percent.

$$r_{1 \text{ year govt bond}} = \frac{100}{94.34} - 1 = 0.060$$

B) Next year the two year government bond will be a one year government bond with a promised (and expected) payment of $100 due in one year (which will be year 2). In state one, this bond will sell for 96.15. The bond will sell for 92.59 in state 2.

$$P_{\text{Bond in state 1}} = \frac{100}{1 + .04} = 96.15$$

$$P_{\text{Bond in state 2}} = \frac{100}{1 + .08} = 92.59$$

Since the bond can be purchased for 89.03 today and sold for 96.15 in state 1, the one year return on the two year government bond is 8 percent in state 1. The return on the two year government bond is 4 percent in state 2.

C) Since the two year government bond will have a return of 8 percent (state 1) or 4 percent (state 2) with equal probability, the expected return on the two year government bond is 6 percent.

D) The two year government bond is risky. Sometimes it has a return of 4 percent; sometimes it has a return of 8 percent. However, the risk is not systematic. Notice that the expected return on the (risky) two year government bond is the same as the return on the (riskfree) one year government bond. If the expected return on an asset is equal to the risk free rate, the β is zero. The asset has no systematic risk. Since the bond is risky, the risk must all be idiosyncratic.

2) Risk and return of government bonds (version 2).

	Today	Next Year	
		State 1	State 2
One year rate (%)	6.0	4.0	8.0
Price of 1 year bond	94.34	100	100
Price of 2 year bond	88.61		

A) Next year the two year government bond will be a one year government bond with a promised (and expected) payment of $100 due in one year. In state one, this bond will sell for 96.15. The bond will sell for 92.59 in state 2.

$$P_{\text{Bond in state 1}} = \frac{100}{1 + .04} = 96.15$$

$$P_{\text{Bond in state 2}} = \frac{100}{1 + .08} = 92.59$$

Since the bond can be purchased for 88.61 today and sold for 96.15 in state 1, the return on the two year government bond in state 1 is 8.5 percent. In state 2, the return on the two year government bond in state 2 is 4.5 percent.

B) Since the two year government bond will have a return of 4.5 percent or 8.5 percent with equal probability, the expected return on the two year government bond is 6.5 percent.

C) The two year government bond is risky. Sometimes it has a return of 4.5 percent; sometimes it has a return of 8.5 percent. Since the expected return on the (risky) two year government bond is greater than the return on the (riskfree) one year government bond, the risk is systematic. If the expected return on an asset is greater than the risk free rate, the β is positive. The asset therefore has systematic risk.

D) Since the β on two year government bonds is positive, the return on two year government bond positively covaries with the stock market. Thus when the stock market return is higher than average, the return on two year government bonds is higher than average. The expected return on two year government bonds is 6.5 percent. In state 1, the return on two year government bonds is above average (8.5 percent). Thus, the return on the stock market is expected to be higher than average in state 1 and correspondingly lower than average in state 2.