Lecture 10: Supplementary Exercise

3) Cost of Underpricing and Nash Equilibrium. In Lecture 10, we derived the wealth of old shareholders a couple of ways. In the lecture, I argued the only equilibrium is where the firm (the old shareholders) issued and invested only in State 2. You have studied equilibrium in your micro economics class and maybe in a game theory class. One form of equilibrium is a Nash Equilibrium. The equilibrium we derived is a Nash equilibrium. This exercise will let you convince yourself of this. There are two players: the firm (old shareholders) and the market (new shareholders). Each player has two possible strategies. The firm can either always issue equity and invest or issue equity and invest only in state 2. The market can either believe that the firm always issues equity and invests or believe that the firm issues equity and invests only in state 2. You will see that the beliefs don't have to be correct, but will be in equilibrium.

A) What is the payoff to old shareholders (the firm). Since there are four possible combinations, given each player has two strategies, you need to calculate four numbers. You may use the table at the end of the questions to record your answers. Payoffs should be the expected cash flows conditional on the strategy choose by the firm and by the market. Use the numbers from Lecture 10. Table I is reproduced below.

Table I	State 1	State 2	E[Value]
Assets in place	150	50	100
Investment opportunity (NPV)	20	10	15
Value of Firm: Pre-issue (P)	170	60	115
Equity Investment (E)	100	100	100
Value of Firm: Post-issue (P+E)	270	160	215

B) What is the payoff to the market. Since there are four possible combinations, given each player has two strategies, you need to calculate four numbers.

C) What is the Nash equilibrium? To answer this question fill in the following table. Remember, in a Nash equilibrium no player (the firm nor the market) wants to unilaterally deviate. There is a unique Nash equilibrium in this game.

Firm's Issuing Strategy	Market's Beliefs	
	Firm Always Issues	Firm Issues in State 2 Only
Firm Always Issues		
Firm Issues in State 2 Only		

3) To derive the Nash equilibrium, we first need to figure out the payoff for each player's strategies conditional on the strategy chosen by the other player.

A) Firm's (old shareholder's) payoff. The old shareholders have two strategic choices. They can always issue equity (state 1 and state 2) or they can issue equity in only state 2. The market also has two strategies. They can believe that the firm always issues equity or they can believe that the firm only issues equity in state 2. Thus we want to write down the payoff for the old shareholders (the firm) for each of their strategies conditional on the market's belief. The payoff to the firm is the value of the assets in place, plus the NPV of the project (conditional on them issuing equity and investing) plus the mispricing in the equity (this can be positive, negative, or zero). First the payoffs assuming the firm always issues equity. If the market expects the firm to always issue equity, they will demand 46.5% of the firm in exchange for \$100.[1] Thus the payoff to the firm is:

$$\text{Payoff}[\ \text{Firm}|\ F{=}1\&2\ \ M{=}1\&2\ ] = 100 + 15 + [\ 100 - 0.465\ (215)\ ] = 115.0 \qquad \textbf{(1)}$$

I have used the notation F=1&2 to mean the firm issues equity in state 1 and 2 and the notation M=1&2 to mean the market believes the firm issues equity in state 1 and 2.

Now to calculate the payoff if firm always issues equity, but the market believes they issue equity only in state 2. In this case, the market will demand 62.5% of the firm, since they assume the firm is worth 160 (as they believe the probability of state 2 is 100% if the firm is issuing equity). Since the firm always issues, the total equity is worth 215 (issue in both states), so the equity which is given away is worth 134 (0.625*215). The payoff to the firm (old shareholders) is thus:

$$\begin{aligned}\text{Payoff}[\ \text{Firm}|\ F{=}1\&2\ \ M{=}2\ ] &= 0.5\ (\ 150 + 20 + [100 - 0.625(270)]\ ) \\ &\quad + 0.5\ (\ 50 + 10 + [100 - 0.625(160)]\ ) \\ &= 100 + 15 + [\ 100 - 0.625\ (215)\ ] \\ &= 80.6\end{aligned} \qquad \textbf{(2)}$$

The next two payoffs are based on the firm issuing only in state 2. Again we do this for both if the market believes the firm always issues (state 1 and 2) and if the market believes the firm issues only in state 2. In these cases, the firm receives the NPV of the project only if it invests. Thus the expected NPV of the project is only 5 (0.5 * 10). Also remember that there is a one-half probability that the market will buy equity at all, so we multiply the mispricing of equity by one-half. If the market believes the firm always issues and invests, but the firm only issues and invests in state 2, the payoff to the firm is:

$$\text{Payoff}[\ \text{Firm}|\ F{=}2\ \ M{=}1\&2\ ] = 100 + 0.5(10) + 0.5[\ 100 - 0.465\ (160)\ ] = 117.8 \qquad \textbf{(3)}$$

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[1] This percentage is always the money the market pays for the equity (100) divided by the post-money value of the firm, conditional on the market's expectation. The firm is worth 270 or 160 post-money. Since the market believes, in this case, that each state is equally likely (because they believe the firm always issues equity), the expected value of the firm is 215.

Now if the firm issues and invests only in state 2, and this is what the market believes, the payoff to the firm is:

$$\text{Payoff}[\ \text{Firm}|\ F=2\ M=2\ ] = 100 + 0.5(10) + 0.5[\ 100 - 0.625\ (160)\ ] = 105.0 \quad \textbf{(4)}$$

B) Market's (new shareholder's) payoff. This time around, the same strategies apply, but we are looking at the payoffs to the market. This time, the payoff to the market is the value of assets they purchase minus the amount they pay. We start with the market assuming the firm always issues. What happens if the market believes that the firm always issues? Then the market believes the firm is worth 215 (the expectation of the firm's value), so it is willing to give up \$100 in exchange for 46.5% of the firm's equity. If the firm does always issue, then the market's payoff is:[2]

$$\text{Payoff}[\ \text{Market}|\ F=1\&2\ M=1\&2\ ] = -100 + 0.465\ (215) = 0 \quad \textbf{(5)}$$

Now consider the case where the market still believes the firm always issues, but the firm issues only in state 2. The market will still pay \$100 for 46.5% of the firm (which it believes to be worth 215). The firm will issue only half the time and only when the firm is worth 160. Thus the payoff to the market is:

$$\text{Payoff}[\ \text{Market}|\ F=2\ M=1\&2\ ] = 0.5[-100 + 0.465\ (160)] = -12.8 \quad \textbf{(6)}$$

Now we consider the cases where the market believes that the firm issues in state 2 only. Since the market believes that the firm is worth only 160 (conditional on the firm issuing equity), it is willing to pay \$100 for 63.5% of the firm. If the firm always issue, the expected value of the firm is still 215. The total payoff to the market is therefore:

$$\text{Payoff}[\ \text{Market}|\ F=1\&2\ M=2\ ] = -100 + 0.625\ (215) = 34.4 \quad \textbf{(7)}$$

The final case is when the market believes that the firm will only issue in state 2, and the firm only issues in state 2. In this case, the firm is worth 160, conditional on the firm issuing equity. Thus the payoff to the market is:

$$\text{Payoff}[\ \text{Market}|\ F=2\ M=2\ ] = 0.5[-100 + 0.625\ (160)] = 0.0 \quad \textbf{(8)}$$

We now have the four payoffs to the market and the firm, conditional on all possible combinations of strategies.

C) Nash Equilibrium. The Nash equilibrium for a game is the combination of strategies from which neither player has an incentive to unilaterally deviate. The table below has the payoff to each player (firm and market) given their strategic choice and the

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[2] You should notice that the payoff to the market has already been calculated in A). This is the last term in equations (1) through (4), except the sign must now be reversed. Thus if the mispricing is a positive for the firm, it is a negative of the same number for the market.

strategic choice of the other player. I have reported the payoff to the firm in the top of each box and the payoff to the market in the bottom of each box. As you can see, the bottom right cell is the unique Nash equilibrium. The top right cell has a higher total payoff, but is not a Nash equilibrium. Independent of what the firm does (always issue or only in state 2), the market is always better off believing that the firm issues only in state 2 (34.4 > 0 and 0 > -12.8). The same is true for the firm. Independent of what the market believes, the firm is always better off issuing only in state 2 (117.8 > 115 and 105 > 80.6). Although the total payoff is higher if the firm always issues and the market believes it, this isn't the outcome we will see.

Firm's Issuing Strategy	Market's Beliefs	
	Firm Always Issues	Firm Issues in State 2 Only
Firm Always Issues	115.0	80.6
	0.0	34.4
Firm Issues in State 2 Only	117.8	105.0
	-12.8	0.0