Lecture 10: Supplementary Exercise

2) Columbia Amusements markets party favors in British Columbia. Columbia entered this industry last year and partially funded their investment with a debt issue. The debt issue is a zero coupon bond which is due next year. The bond has a $100 face value. The payoff to the firm's asset's next year will be 40, 80, 120, or 160 with equal probability. Columbia Amusement's has no other investment opportunities. The cashflow to assets is uncorrelated with the market. The risk free rate is 10 percent.

	State 1	State 2	State 3	State 4	Expected
Asset	40	80	120	160	100
Debt					
Equity					

A) Calculate the value of Columbia Amusement's assets, debt and equity.

B) Columbia has just discovered the possibility of expanding their capacity. For an investment of $85 today, they can double their capacity. Thus if the cashflow next year is 120 with out the investment, it will be 240 with the investment. In an MM world, would you invest in the project?

C) In the real world, the MM assumptions don't always hold. Assume that you own all of Columbia's equity. The debt is owned by financial guru, Professor Thompson. If you had to contribute the additional equity yourself, would you invest in doubling Columbia's capacity? Remember, your objective is to maximize your own wealth. You may find the following table useful.

	State 1	State 2	State 3	State 4	Expected
Asset					
Debt					
Equity					

D) Instead of financing the project with equity, you could finance the project with debt. Professor Thompson is a very sophisticated investor, however. His loan to you included a covenant that restricted you from issuing any new debt with an equal or higher priority. Thus the new debt must be junior to Professor Thompson's loan. What must be the promised payment on the new debt for it to be correctly priced? The new debt will sell for $85.

	State 1	State 2	State 3	State 4	Expected
Asset					
Senior Debt					
Junior Debt					
Equity					

E) If the debt carries the promised rate you calculated in D), should you issue the debt to finance the project?

Lecture 10: Answers to Supplementary Exercise

2) Columbia Amusements. Project evaluation when capital structure is relevant.

A) Value of assets, debt and equity. To value each of these assets/securities, discount the expected cashflow at the appropriate discount rate. Since the cashflow to assets -- and therefore debt and equity -- are uncorrelated with the market, the correct discount rate is the riskfree rate -- 10 percent.

	State 1	State 2	State 3	State 4	Expected
Asset	40	80	120	160	100
Debt	40	80	100	100	80
Equity	0	0	20	60	20

$$V_{Assets} = \frac{100}{(1 + .1)} = 90.91$$

The promised payment on the debt is $100. The debt defaults and pays less than promised in states 1 and 2. The value of debt is:

$$V_{Debt} = \frac{80}{(1 + .1)} = 72.73$$

The value of equity is just the difference (91.91 - 72.73). The equity is worth 18.18. You could also value the equity by discounting the cashflows to equity.

$$V_{Equity} = \frac{20}{(1 + .1)} = 18.18$$

B) NPV of project in an M&M world. Value the project by discounting the cashflows. We can ignore how the project is funded in this case.

$$NPV_{Project} = -85 + \frac{\frac{1}{4}160 + \frac{1}{4}120 + \frac{1}{4}80 + \frac{1}{4}40 +}{(1 + .1)} = 5.91$$

You should invest in this project in an M&M world.

C) Project evaluation with equity financing.

Cashflow with the With Project	State 1	State 2	State 3	State 4	Expected
Asset	80	160	240	320	200
Debt	80	100	100	100	95
Equity	0	60	140	220	105
Change in Cashflow					
Asset	40	80	120	160	100
Debt	40	20	0	0	15
Equity	0	60	120	160	85

To evaluate a project when the assumptions of M&M fail, we value the project assuming capital structure is irrelevant, and then add the NPV of financing. This gives us the change in the value of old equity holders wealth. Based on the above table, we can calculate the changes in the value of your equity if you invest in the project. By investing, you must contribute an additional $85 today and will receive an extra $85 next year (105 instead of 20). The change in your equity value is therefore:

$$\text{Change in equity value} = -85 + \frac{85}{(1 + .1)} = -7.73$$

You shouldn't invest in the project if you have to contribute the equity, even though the project has a positive NPV on a stand alone basis.

Our answer is different than our answer in B) because capital structure is relevant -- i.e. the NPV of financing is not zero. The investment project lowers the risk of the debt -- it no longer defaults in state 2. Since the expected cashflow to debt is higher, the value of the debt is higher. The expected cashflow to debt rises by $15 (see table) and the value of the debt goes up by $13.64. This is a transfer of wealth from equity holders to debt holders. Professor Thompson would like you to make this investment. When you sum the NPV of the project and the NPV of financing, the investment lowers the wealth of equity holders by 7.73 -- same as our answer above.

$$\begin{aligned} \text{NPV}[\text{ Project }] &= \text{NPV}[\text{ Project } | \text{ CSI }] + \text{NPV}[\text{ Financing }] \\ &= 5.91 - 13.64 = -7.73 \end{aligned}$$

D) Junior debt issue.

Cashflow with the With Project	State 1	State 2	State 3	State 4	Expected
Asset	80	160	240	320	200
Senior Debt	80	100	100	100	95
Junior Debt	0	60	140	174	93.5
Equity	0	0	0	46	11.5

We want to set the promised rate on the debt so the debt will sell for $85. This is the capital infusion we need to finance the project. The expected cashflow to the junior debtholders must therefore be 93.5 (85 * 1.1). A promised payment of 174, will generate an expected cashflow of 93.5. You need to solve for this. Using a spreadsheet is the easiest way to do it.

Notice the junior debt is high risk debt. It defaults in three of the four states. It is also high yield debt. It has a promised rate of return of 105 percent. This is high -- even by junk bond standards.

E) Project evaluation with junior debt financing.

Change in Cashflow	State 1	State 2	State 3	State 4	Expected
Asset	40	80	120	160	100
Senior Debt	40	20	0	0	15
Junior Debt	0	60	140	174	93.5
Equity	0	0	-20	-14	-8.5

Our answer is surprisingly similar to our previous answer. Financing the project, even when it is financed with the junior debt holders money, is a bad idea for equity. It lowers the value of your equity by $7.73.

$$\text{Change in equity value} = \frac{-8.5}{(1 + .1)} = -7.73$$

The investment raises the value of the firm's assets by 5.91. It raises the wealth of the old debt holders (Professor Thompson) by 13.64. The change in the wealth of the new (junior) debt holders is zero. This has to be true. Since the debt is correctly priced, the new debt holders contribute $85 in exchange for a debt security worth $85. If the debt security wasn't worth $85, they wouldn't purchase the debt for $85. The change in old equity holders

wealth is therefore -7.73. These are the same numbers we calculated when the investment was financed by equity.

$$\begin{aligned} \text{NPV}[\ \text{Project}\ ] &= \text{NPV}[\ \text{Project} \mid \text{CSI}\ ] + \text{NPV}[\ \text{Financing}\ ] \\ &= 5.91 - [\ 13.64 + 0\ ] = -7.73 \end{aligned}$$