Homework Assignment 3

1) Calculating option payoffs: Portfolio 1. Drawing the payoffs for complicated securities is an excellent way to understand how the securities work and how they are constructed. With practice you will be able to look at a payoff diagram and write down the building blocks (a risk-free bond, the asset, and calls and puts on the asset) which were used to construct the portfolio. In this question, I will describe the portfolio and you will draw the payoff diagram.

   A) Gross payoff. Graph the gross payoff at maturity as a function of the value of the underlying asset for the following portfolio: Purchase one call option with a strike price of 45 and sell one call option with a strike price of 50. Assume the options are European and they have the same maturity.

   B) Net payoff. Graph the net payoff at maturity as a function of the value of the underlying asset for the above portfolio. The price of the call option with a strike price of 45 is 7.75. The price of the call option with a strike price of 50 is 3.50.

   C) Beliefs. Given the current stock price is 45, what beliefs would make this portfolio a good investment? Think about how the investor's beliefs about the mean return of the stock or the variance of the stock's return must differ from the market's beliefs. If an investor's beliefs are identical to the market's then the financial transaction must be a zero NPV investment.

2) Calculating option payoffs: Portfolio 2. Now consider a portfolio that is long one call option with a strike price of 80 (the option premium is 8.4), long one call option with a strike price of 100 (the option premium is 1.9), and short two call options with a strike price of 90 (the option premium is 4.2).[1]

   A) Graph the gross payoff at maturity as a function of the value of the underlying asset for the above portfolio.

   B) Graph the net payoff at maturity as a function of the value of the underlying asset for the above portfolio.

   C) What beliefs would make this portfolio a good investment? Think about how the investor's beliefs about the mean return of the stock or the variance of the stock's return must differ from the market's beliefs.

[1] These prices are based on options with one year to maturity, an annual volatility of 20 percent, a 5 percent risk-free rate, a dividend yield of 0, and a current stock price of 80. I used Black-Scholes to value the options.

3) Option Values. We will not learn how to price options in this class.[2] However, I would like you to be familiar with what factors which affect option values and why they affect option values. That is the purpose of this question. We will practice computing option prices using the Black-Scholes formula. We will use an Excel function which gives you the Black-Scholes option price when you enter the six inputs.[3] Report your answers to two decimal places (e.g. 1.00).

A) Stock price effects. We know from lecture that changes in the current stock price changes the value of an option. I want you to price options with a strike price of 100. Assume the options have a one year maturity, the risk-free rate is 8 percent, the dividend yield is 4 percent, and the annual volatility is 30 percent. Report your answer:

1) How much does the price of the option change when you raise the stock price from 80 to 81?

2) How much does the price of the option change when you raise the stock price from 100 to 101?

3) How much does the price of the option change when you raise the stock price from 120 to 121?

4) Why does the amount by which the option price changes, per dollar change in the stock price, change as the stock price rises from 80 to 100 to 120? In other words explain the pattern in your answers to 3-A-1 to 3-A-3.

B) Dividend yield effects. Start with the above option when the stock price is 120 and raise the dividend yield from 4% to 22%. Remember the options have a one year maturity, a strike price of 100, the risk-free rate is 8 percent, and the annual volatility is 30 percent.

1) What happens to the Black-Scholes value of the option?

2) Why does the value of the option change?

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[2] I highly recommend you take the derivatives course (Finance 465) to learn how to price options. Familiarity with option pricing, like knowledge of discounted cash flow, has evolved from a skill only needed by technicians to a skill needed by anyone involved in finance.

[3] The excel function: BSCall(stock price, exercise price, volatility, risk-free rate, expiration, dividend yield) calculates the value of a call option using Black-Scholes. To use the Black-Scholes function, click on the function icon in Excel (the down arrow next to the summation sign) then click on More functions. Under "Or select a category" click on user defined functions, then search for BSCall. Alternatively, you could type BSCall into a cell of the spreadsheet.

The six inputs for Black-Scholes are the current asset value or stock price, the strike price, the maturity of the option, the risk-free rate, the dividend yield and the volatility. It doesn't matter what the units the last four inputs are expressed in (e.g. years or months) as long as you are consistent. An excel spreadsheet which contains the Black-Scholes function can be obtained from the homework page on the web. These are copy written by Professor McDonald who has kindly let us use them for this class.

C) Volatility effects. Assume that the stock price is 100, the option has a one year maturity, a strike price of 100, the risk-free rate is 8 percent, and the annual dividend yield is 4 percent.

1) What happens to the Black-Scholes value of the option when you raise the annual volatility from 30% to 50%?

2) Why does the value of the option change?

