Homework Assignment 1

1) Riskless Arbitrage:

Find a riskless arbitrage among the following three government bonds. The first two are zero coupon bonds, meaning they pay no interest. They make a bullet payment at maturity. The third bond is a two-year bond that has a five percent coupon.

	Bond price	Bond cash flows$_1$	Bond cash flows$_2$
Bond 1	95	100	0
Bond 2	85	0	100
Bond 3	95	5	105
	20	0	0

A) Start by finding the one year risk-free rate implied by the price of bond 1.

B) Next, find the two year risk-free rate implied by the price of bond 2.

C) What is the price of bond 3 implied by your answers to A and B?

D) A risk-free arbitrage creates positive cash flows today and either zero or positive total cash flows at each future date. Start with either buying or selling (you decide which is the better strategy!) 20 Bond 3's. How many bonds 1 and 2 will you have to buy or sell to offset the cash flows from your bond 3 position in years 1 and 2? What are your arbitrage profits today?

	Cash flows$_0$	Cash flows$_1$	Cash flows$_2$
20 Bond 3's			
Total			

E) What do you expect to happen to the prices of the three bonds given what you have learned? What lesson does this teach us about market efficiency?

F) Assume bonds 1 and 2 were correctly priced. What is the one year forward rate one year from now?

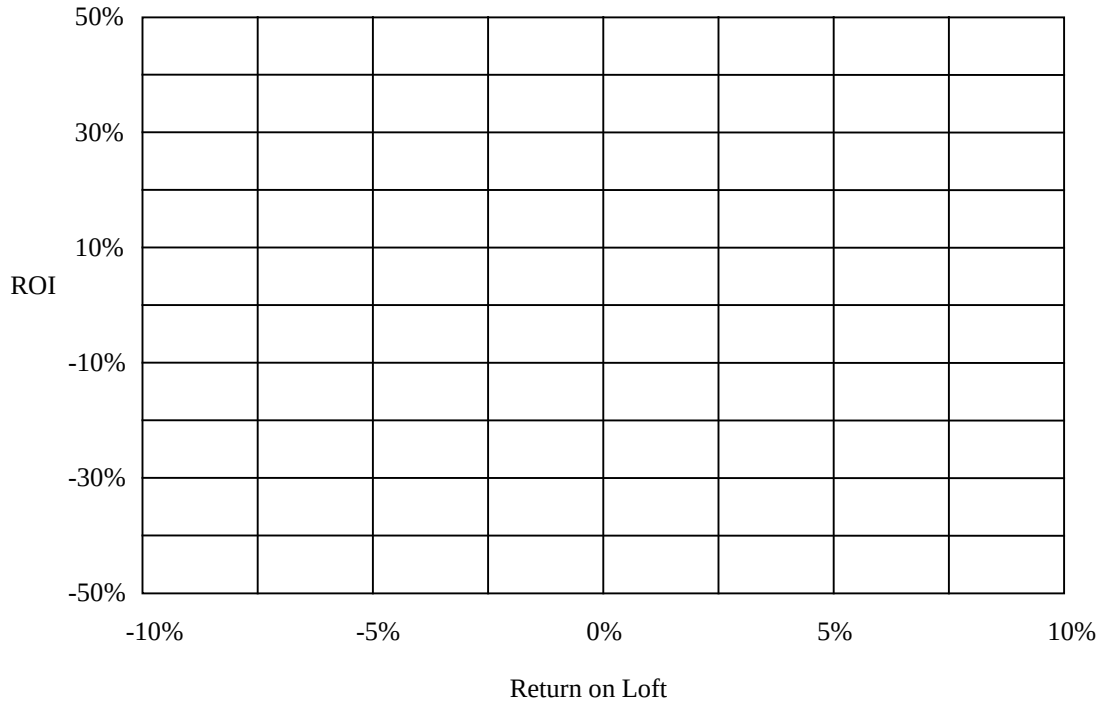
2) Leverage and risk. This question examines leverage. It is in the context of a personal investment in a home. However, the basic idea applies generally to investments made with borrowed money.

With your diploma from the Kellogg School of Management in hand you have decided to purchase a loft in Wicker Park. The loft costs $200,000.

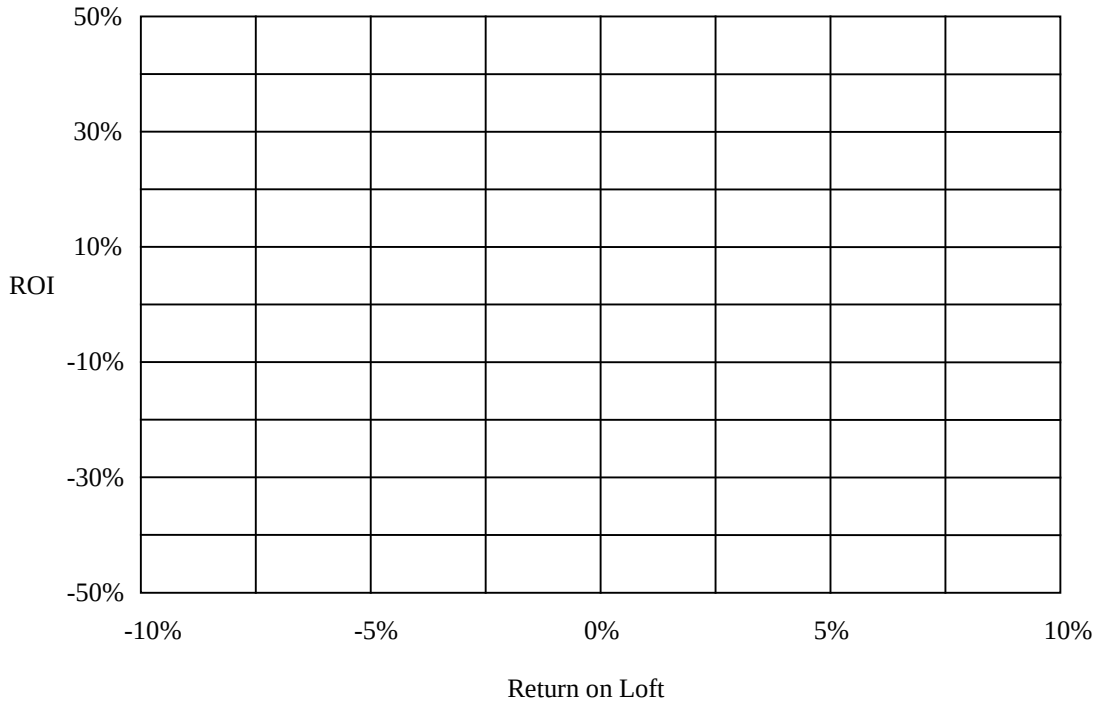
A) I have calculated the percentage return on the loft as a function of possible loft prices next year.

Loft Price (next year)	$180K	$190K	$200K	$210K	$220K	$230K
Return	-10%	-5%	0%	5%	10%	15%

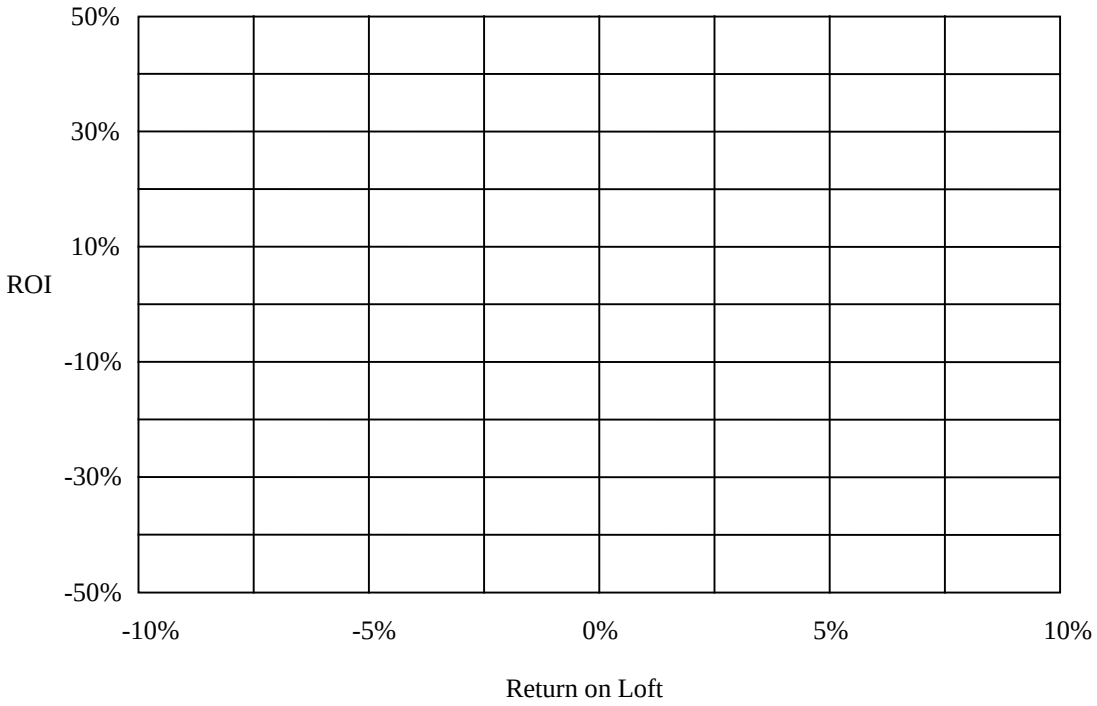
Assume you purchase the loft with cash. This means you don't have to borrow any money to purchase your new home. Graph the one year realized net return on your $200,000 investment as a function of the percentage return on the loft. Recall that the realized net return is calculated by dividing the payoff from the investment by the initial investment and subtracting one.



B) Now assume that you only have $100,000 to invest in your new home. Your current shortage of funds has not deterred you from purchasing the $200,000 loft. You borrow the remaining $100,000 from a bank, promising to repay the loan plus 10% interest in one year. Graph the return on your $100,000 investment as a function of the percentage return on the loft. Remember to account for the bank's stake in the investment in both the payoff and in the initial investment.



C) Now assume that you have only $50,000 to invest in your new home. You still want to purchase the same loft for $200,000, so you borrow $150,000 from the bank. You promise to repay the loan plus 10% interest in one year. Graph the return on your $50,000 investment as a function of the percentage return on the town house.



D) You have three graphs of the return on your investment as a function of the return on the underlying asset (you answers to A, B, and C). You should notice that the graphs have different slopes. How are the slopes related to the fraction of the loft's value which you, as opposed to the bank, contributed? What have you learned about leveraged investing in risky assets?

3) Junk Bond Valuation. This question looks at the expected default premium which is implicit in the pricing of junk bonds. It is based on a *Wall Street Journal* article from January 31, 1991. Your job will be to calculate the expected probability of default each year for the Fort Howard junk bonds. These bonds are due in 1997 and have a coupon rate of 12.375%. The coupon payments are made annually, the next one will be made in a year. The "appropriate discount rate" for junk bonds is 12.4%. This rate is based on a one year risk-free rate of 6.9% and a debt beta of 0.65.

A) What would the bond sell for if the bonds never defaulted and you knew at the time of purchase that they would not default? The bonds have a $1,000 face value. Assume the yield curve is flat, i.e., the risk-free rate for all maturities is 6.9%.

B) Assume that the probability of default on the junk bonds is p per year. This probability is the same each year. What is the probability that the bond makes its promised payment at the beginning of 1992? Your answer should be a function of p. What is the probability that the bond makes its promised payment at the beginning of 1993? Assume that once the bonds default, no further interest or principal payments are made.

C) The bonds have been selling at a significant discount to face value. On January 1, these bonds were selling for $750 per $1,000 face value. Assume that probability of default for Fort Howard's junk bonds is the same in every year and that the probability of default is equal to p. Based on the market price of the bonds, what is the implied probability of default (p)? Assume that if the bonds default, no more interest or principal payments are made. Hint: Use a spreadsheet.

D) On January 4th, Fort Howard announced an equity infusion of $250 million and its plans to repurchase some of its debt. The price of its junk bonds jumped to $870 per $1,000 face value. What is the implied probability of default now? You should still assume that if the bonds default, no more interest or principal payments are made. Assume that the first interest payment is still one year away.

4) Stone Container is a major producer of cardboard boxes. Stone Container has $10M in outstanding equity. In addition, it has $2M in outstanding debt. The debt is a ten-year mortgage and is rated AAA. This is low risk debt. $2M is both the book and market value of the debt. In addition to its cardboard box production and sales facilities, Stone Container also has a portfolio of 3 month government t-bills. These are currently worth $3M. The market price of risk, $E[\ r_m - r_f\ ]$ is 8.4 percent. Assume that the yield curve is flat.

A) Stone Container's debt has a beta of 0.20. The equity beta was estimated using the following equation:

$$r_{\text{Stone Container's Equity}} - r_{\text{Risk Free}} = 0.0 + 1.4\left(r_{\text{Stock Market}} - r_{\text{Risk Free}}\right) + \varepsilon$$

Calculate the beta which measures the risk of Stone Container's assets.

B) Stone Container is considering expanding its capacity by 15 percent. It will do this by building a new production facility. It will also expand its sales force by 15% to market the additional cardboard boxes. The total investment required for this project is $2M. The firm will liquidate part of its T-bill portfolio to pay for the investment. Since Stone Container will lose the 3 percent yield on the bonds, should 3 percent be the discount rate it uses for evaluating its capacity expansion? Explain.

C) An alternative method for deriving a discount rate is to use the Capital Asset Pricing Model. What discount rate for the capacity expansion investment does CAPM suggest?

D) A year after Stone Container builds the cardboard box production facility you could estimate their equity beta. Using only equity return data from the year following the construction of the cardboard production facility, do you expect your estimate to be above, below, or approximately equal to 1.4? Explain.

5) Swing and Pool is a small publicly traded firm. You will find a partial pro-forma income statement and balance sheet for the next five years in a spreadsheet on the corporate finance web page. The numbers in the table are year-end figures reported in thousands of dollars. The firm has assets in place which had an initial acquisition price of $50,000. These assets are being depreciated for tax purposes using straight line depreciation over fifteen years. Any new capital expenditure will also be depreciated over 15 years, beginning the following year. Swing and Pool faces a 34% tax rate on its income.

A) Calculate the cash flow to Swing and Pool's assets for the years 2014 to 2018. You should use the spreadsheet from the course web page. Hand in a copy of the balance sheet, income statement, and cash flow statement with your answers. These are tabs Q4_Balance, Q4_Income, and Q4_Cashflow in the spreadsheet.

B) Firms are sometimes valued as a multiple of the firm's cash flow. If you could sell the firm in the year 2018 for 12 times the year 2018 cash flow, what would the sales price be?

C) What is the price of one share of Swing and Pools stock at the end of 2013? You will need to make a few assumptions. Assume the discount rate for assets (r_A) is 15%. The next cash flow will arrive at the end of 2014. There are currently 1.8 million shares outstanding. To value cash flows beyond the year 2018, use the exit multiple.[1] Assume that the value of cash flows in the year 2018 and beyond discounted back to the year 2018, is 12 times the year 2018 cash flow.

D) If cash flows of an asset grow at a given rate forever, then you can write the value of the asset as a function of next year's cash flow (C_1), the long term expected growth rate (g), and the discount rate (r). (Hint: Use the formula for a growing perpetuity from the Review of Financial Concepts in your course packet.)

If Swing and Pools' cash flows will grow at a constant rate forever, what growth rate is consistent with the exit cash flow multiple we assumed in part (C).

E) Calculate the price to cash flow ratio (P_0/C_1) for the years 2013 to 2017. Why does this cash flow multiple fall. If it helps, you might want to calculate the long term growth rate implied by the multiple.

[1] The thought experiment is that you operate the project for the next five years (2014-2018) – and receive the cash flow from operations. At the end of the fifth year, suppose that you will sell the business for its then current value. This is sometimes called the "exit value" (you are exiting the business). It is also often called a "terminal value" (the value at the end or terminal date of the valuation).

