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Snowball: A dynamic oligopoly model with indirect network effects[☆]

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Abstract

Allowing for innovation dynamics in the software market, this paper studies the conditions under which standardization in the hardware market arises and persists over time. In the model, software firms repeatedly invest in quality upgrades, compete in the product market, and make exit as well as entry decisions. The results show that, in general, *excess inertia* does not occur. A platform becomes the standard in a market only if it is better than the competing platforms. Furthermore, low overall rates of innovation always lead to variety; conversely, the higher the speed of innovation, the more likely standardization is.

JEL Classification: D62; L13; L15

Keywords: Network effects; Oligopoly; Dynamics; Market structure

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1. Introduction

In May 1998, the U.S. Department of Justice, claiming a number of violations of Sections 1 and 2 of the Sherman Act, filed suit against Microsoft Corporation.[1] The powerful market forces favoring both competition and monopoly in the Microsoft

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[☆] An earlier version of this paper was circulated under the title ‘Snowball—The Evolution of Dynamic Oligopolies with Network Externalities’.

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[1] *United States v. Microsoft*, Civil Action No. 98-1232.

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antitrust case have attracted a great deal of attention and have raised disagreements among economists (e.g., Evans et al., 2000; Davis and Murphy, 2000; Bresnahan, 2001a, b). Some think that the industry is already perfectly competitive and that, despite network effects, Windows would quickly be replaced were a superior alternative to come along (see Evans et al., 2000). Others believe that sunk costs on the part of users and developers, together with the network effects associated with Windows, are so strong that Windows could not be displaced at all (see Economides, 2000). While the court conceded that, 'There is insufficient evidence to find that, absent Microsoft's actions, Navigator and Java already would have ignited genuine competition in the market for Intel-compatible PC *operating systems*,'[2] the following question remains: absent Microsoft's interference, would a different market structure prevail? Or, more generally, absent regulatory or any other strategic interference, what would be the long-run structure of a market with indirect network effects?

Oligopolies with indirect network effects are characterized by strong interrelations between firms' and consumers' actions: the more users a platform has, the more software will be available for it; the more available software a platform has, the more users it will attract. These interrelations have also been studied empirically and have been found to hold in various industries (e.g., Gandal et al., 1999, 2000). Since the value of a platform depends largely on the quality of the software available on it, hardware manufacturers generally benefit from encouraging the production of high-quality complements. This suggests that hardware manufacturers might act strategically to intervene in the complementary software market in order to strengthen their platform's competitive position. Microsoft's and Netscape's 'browser war,' as well as Intel's strategy of stimulating innovation in complementary markets, are good examples of this type of intervention. In the 'browser war' case, competition from Netscape created incentives for Microsoft to innovate and to invest not only in the quality of the browser but also in the quality and variety of compatible applications (see Bresnahan, 2001a, b). Note that, in this example, the browser is the hardware, while the applications that run on the browser are the software. As for Intel, as part of its strategy to stimulate innovation, Intel encourages outside complementors to innovate by picking one firm (called a 'rabbit') to be a leading proponent of the new technology or standard. Intel then invests in marketing and public relations support to focus industry attention on the rabbit as well as on the market Intel is trying to promote (see Gawer and Cusumano, 2002).

In order to assess the effect of Microsoft's and Intel's actions, or, more generally, the effect of any strategic or regulatory action on a market's long-term structure, one might be interested in understanding the *natural* structure of the market in the absence of interference. The core questions in this context are: What kind of economic environment allows for the coexistence of different platforms ('variety') and when will we see standardization? What are the conditions for 'tipping,' when a small advantage for one platform 'snowballs' into overall dominance by this platform? And, finally, under what conditions is it possible to escape from 'excess

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[2] See the court's findings of fact in *United States v. Microsoft*, November 5, 1999.

inertia' that can lead to the dominance of an inferior platform?[3] Note that in this context, 'inferior' platform means that all software on that platform has lower quality than software on the alternative platform.

The questions above have been extensively studied in the literature (e.g., Farrell and Saloner 1985a, b; Katz and Shapiro, 1985), mostly in the context of a static analysis. However, industries characterized by network effects are among the most dynamic industries around. In the literature, this dynamic element has been assumed away due to the unavailability of suitable modeling techniques, despite the fact that most network industries, in particular software markets, demonstrate high levels of innovation. Because innovation is one (if not the major) driver of competition, this calls for a renewed inspection that takes innovation dynamics explicitly into account. This paper analyzes these core questions in a dynamic framework, concentrating on software innovation as well as on entry and exit in the software market.

I propose a dynamic model with indirect network effects that extends the framework presented in Ericson and Pakes (1995) to allow for dynamics in demand as well as in supply. In particular, this model allows software firms to choose the platform for which they want to produce, to invest in quality upgrades on an ongoing basis, and to compete repeatedly in the product market. In addition to making investment and pricing decisions, incumbent firms decide whether to remain in the industry; potential entrants decide whether to enter, and, if so, onto which platform. Consumers first choose hardware and then choose among the differentiated software each platform offers, where hardware lives longer than software. In contrast to direct network effects, where consumers' utility directly depends on the size of the network, in this model of indirect network effects consumers' utility is derived from software quality. I solve numerically for the Markov perfect Nash equilibrium (MPE) in order to characterize industry dynamics and identify circumstances under which standardization arises and persists over time.

Firms invest in an ongoing basis to improve the quality of their product. When a software firm upgrades the quality of its product, it may attract consumers from competitors producing for the same or for a different hardware. For example, applications compatible with the Windows operating system are challenged by applications compatible with Linux. Furthermore, an innovative firm may also attract consumers who have so far chosen the outside good: the attractiveness of the PC market is affected by the development of thin client terminals; sales of Sony's playstation and Microsoft's Xbox games and consoles are affected by advances in the computer games market. Going back to the motivating example, one can think of BlackBerry as competing with browsers in providing connectivity to the World Wide Web. Consequently, I measure software qualities relative to the quality of the outside good. In particular, I assume that an advance in the technology of substitute goods depreciates the quality of all available software in the analyzed market.

Investment, entry, and exit allow firms to partially control the evolution of the market. The incentives that underlie the investment decisions are affected by the

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[3] More precisely, excess inertia refers to the case where a new technology is not adopted, even though it would be in the interest of most consumers to adopt it, and in the long run an inferior technology remains as the standard in the market.

distribution of consumers and firms across the platforms, while the probability that each platform dominates the market depends on the market structure to which investments have led. Since the outcome of the investment and entry processes is stochastic, over time, firms and consumers will find themselves under different market structures, facing different incentives. This allows for an explicit analysis of the interaction between market structure, software firm choices, and consumer choices.

Defining standardization as the case where at least 95% of the consumers own the same hardware, the results show that when consumers care about quality and not variety, *excess inertia* does not occur. An initial advantage can lead a platform to dominate the market only if the compatible software sustains its superiority over the software available on the competing platform.In addition, overall low rates of innovation always lead to variety. Thus, the higher the speed of innovation, the more likely we are to see standardization.

Standardization, however, is not necessarily bad for consumers. The high speed of innovation leading to standardization provides fast, quality growth within the industry as well as in related industries that produce substitute goods. Moreover, whenever we see standardization, there are incentives for more than one software firm to produce for the winning platform. The literature on network effects has traditionally highlighted at least three important issues: expectations, coordination, and compatibility, and has generally focused on normative economics. This paper, in contrast, focuses on the positive economics of when and why the market would exhibit the 'snowball effect,' arriving at standardization as the long-run equilibrium.

This paper contributes to the literature in several ways. First, it studies innovation dynamics in the software industry, whereas the existing literature typically analyzes hardware firms' decisions. In the previous literature, the variety of software is either ignored or almost instantaneously determined by the platform's market share. Consequently, firms in these models cannot affect their quality and there are no quality differences between software firms. For example, Farrell and Saloner (1985b) study the sequential decisions of firms regarding whether or not to adopt a new technology. They show that, in a model with incomplete information, there is always excess inertia and the inferior standard is chosen. Farrell and Saloner (1985a, 1986) explore how consumers choose between two incompatible technologies, finding that the outcome can be standardization or variety when there is a case of suboptimal standardization. Katz and Shapiro (1986) analyze technology adoption in the case where the pattern of adoption depends on whether technologies are sponsored. They show that, in the absence of sponsors, there is a tendency to standardize on the technology that is initially superior even though it is the 'wrong' technology. Concentrating on the hardware market, Kandori and Rob (1998) and Auriol and Benaim (2000) propose dynamic models to study the adoption decision of consumers. Auriol and Benaim find that, when consumers' utility functions are convex, there will be standardization; when the utility functions are concave, the solution is predictable but not necessary optimal. Church and Gandal (1992) provide one of the few models that analyze the software market, albeit in a static framework. Allowing software firms to choose the platform for which they want to supply software, the authors show that suboptimal standardization as well as suboptimal variety is possible. Note, however, that in their model, software firms cannot choose prices or quality levels.

Second, this paper allows for market structure dynamics in the software market. Allowing for entry and exit in the software industry enables analysis of standardization as well as of the expected lifespan of a standard. Third, the model allows, in a simplified manner, for competition from producers of substitute goods outside the industry to affect competition within the market of interest. Finally, this paper formally models both demand and supply in the market, and it is the first to formally consider the dynamic interrelations between firms and consumers. These features of the model help in achieving the main goal of identifying and evaluating the main drivers of standardization when markets are driven by software innovation and consumers care about quality.

2. Model

Following Bresnahan and Greenstein (1999), I define a platform to be a hardware technology around which buyers and sellers coordinate their efforts.[4] I adapt the framework presented in Ericson and Pakes (1995) and the algorithm for computing it presented in Pakes and McGuire (1994) to allow for dynamics in the demand side of the market. The model is cast in discrete time and has an infinite horizon to avoid end effects. Consumers derive utility from the software they purchase and care about the set of available software offered by a platform, both in terms of quality and variety. Compatible hardware is needed to operate the software, where consumers have preferences over the available hardware. I assume that consumers are forward-looking: they evaluate the benefits of currently available software on each platform, as well as expected potential quality upgrades, and choose hardware and software accordingly. Software producers develop knowledge that is specific to a platform and, therefore, they find it infinitely costly to switch platforms. Consequently, software firms choose their exit, entry, and investment strategies based on expectations about their own performance, the future performance of their competitors, and the future performance of their platform.

I assume that there are two incompatible platforms and that each platform can accommodate up to N software firms. The timing of the game is as follows: first, consumers choose hardware and incumbent software firms decide whether to exit, while a potential entrant decides whether to enter. A firm that chooses to stay in the market has to choose how much to invest in quality upgrades. Next, incumbents compete on prices and consumers buy one unit of software or the outside good. Finally, investment realization determines the outcome of the firms' investments and whether a technological innovation in substitute markets has devalued the quality levels of all software producers on both platforms.

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[4] For ease of exposition, this definition slightly departs from Bresnahan and Greenstein (1999), who define a platform as a 'bundle of standard components around which buyers and sellers coordinate efforts.' While their definition implicitly includes software and peripherals, the definition used here excludes both.

First, some definitions. The analysis for both platforms is analogous, I therefore discuss only platform A:

- Let $Q = \{0, 1, 2, \dots, K\}$ be a finite set of quality values for each firm, and let $a_j \in Q$ characterize firm j's quality level of software compatible with platform A. A quality level of zero indicates that the firm is not active and thus there is room for entry. The vector $a \equiv (a_1, \dots, a_N)$ represents the quality level of all firms on platform A. Vector $a' \equiv (a'_1, \dots, a'_N)$ represents the quality in the next period. Vectors b and b' are defined analogously.
- σ is the percentage of consumers who own a unit of hardware A.[5] σ is a discrete variable with a discretization step κ.
- The state $S \equiv (\sigma, a, b)$ represents the structure of the industry.

2.1. Consumers' choice

Every period, one-half of the consumers on each platform purchase one unit of hardware,[6] which only facilitates the consumption of software and therefore provides no stand-alone benefit. The value a consumer gets from the hardware he adopts depends solely on the quality and price of the software he uses, where software provides services for a single period. The one-period utility consumer i gets from the consumption of hardware A and software j with quality level a_j and price p_j^{A} is, $U_{ij}^{\mathrm{A}}(a) - p_j^{\mathrm{A}} = a_j - p_j^{\mathrm{A}} + \varepsilon_{ij}$, where ε_{ij} represents taste differences among consumers.[7]

Software choice. Each consumer purchases one unit of software or the outside good, which gives a utility of ε_0. Assuming that consumers' preferences, ε, are independently and identically distributed according to a standard double exponential distribution, the demand for product j is

$$D_j(a, p) = M \frac{\exp(a_j - p_j)}{1 + \sum_{k \neq 0} \exp(a_k - p_k)}, \tag{1}$$

where $M > 0$ is the total size of the market, and $p = (p_1, \dots, p_N)$.

Hardware choice: Given that software j maximizes consumer i's utility from software in the current period, and software k maximizes his utility from software in the next period, the expected benefit consumer i gets from purchasing hardware A is

$$W_i^{\mathrm{A}}(\sigma, a, b) = \mathrm{E}\{[U_{ij}^{\mathrm{A}}(a) - p_j]|\sigma, a, b\} + \beta \mathrm{E}(\mathrm{E}\{[U_{ik}^{\mathrm{A}}(a') - p'_k]|\sigma, a, b\})) + \varepsilon_i^{\mathrm{A}}, \tag{2}$$

where $\mathrm{E}U_{ij}^{\mathrm{A}}(a)$ and $\mathrm{E}(\mathrm{E}[U_{ik}^{\mathrm{A}}(a')])$ are the benefits the consumer expects to get from purchasing software $j \in \{1, \dots, N\}$ in the current period and software $k \in \{1, \dots, N\}$

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[5] Some authors refer to these shares as the 'installed base' (e.g., Greenstein, 1993; Farrell and Saloner, 1986).

[6] This assumption simplifies the market-share law of motion and avoids the need for additional state variables.

[7] Note that the model can easily accommodate any other utility function, e.g., a function with direct network effects where utility also directly depends on the size of the network.

in the next period, respectively. $\varepsilon_i^{\mathrm{A}}$ represents consumer *i*'s additional random benefit for platform A (e.g., in the operating system market, some consumers prefer the Windows platform, while others favor Linux). When consumers make their hardware decisions, they already know the prices and qualities of software available in the current period but not their idiosyncratic tastes. Thus, consumers compute the expected maximum utility from software compatible with platform A, $\mathrm{E}\{[U_{ij}^{\mathrm{A}}(a) - p_j]|S\}$. Looking at the next period, consumers take the current state (σ, a, b), and form expectations of future available qualities and prices. These are then used to compute the expected maximal utility.[8] Note that Eq. (2) reflects the fact that utility is mainly derived from software. However, once a consumer chooses a platform, he can only buy software compatible with this platform.

Consumer *i* will purchase hardware A if and only if it gives him a higher utility than purchasing hardware technology B. That is, if and only if $W_i^{\mathrm{A}}(\sigma, a, b) - P^{\mathrm{A}} > W_i^{\mathrm{B}}(\sigma, a, b) - P^{\mathrm{B}}$, where P^{A} and P^{B} are the prices of hardware A and B, respectively. Assuming, again, that consumers' preferences, ε_i^k, are independently and identically distributed according to a standard double exponential distribution, the market share of platform A is then[9]

$$\Psi(\sigma, a, b; P^{\mathrm{A}}, P^{\mathrm{B}}) = \frac{\exp(W^{\mathrm{A}} - P^{\mathrm{A}})}{\exp(W^{\mathrm{A}} - P^{\mathrm{A}}) + \exp(W^{\mathrm{B}} - P^{\mathrm{B}})}. \tag{3}$$

Given that in each period only half of the consumers buy a new hardware, platform A's market share in the following period is given by

$$\sigma'(\sigma, a, b; P^{\mathrm{A}}, P^{\mathrm{B}}) = \sigma/2 + \Psi(\sigma, a, b; P^{\mathrm{A}}, P^{\mathrm{B}})/2. \tag{4}$$

Note that since platform A's market share is discretized, market shares computed by Eq. (4) might sometimes not be a multiple of the discretization step κ. In these cases, I take the weighted average of the two potential market shares within which σ'falls. This induces a transition probability for σ', denoted by $\Lambda(\sigma, a, b; P^{\mathrm{A}}, P^{\mathrm{B}})$.

2.2. Software firms

The software market is a differentiated good oligopoly where each firm produces only one type of software compatible with one of the platforms. The vector *a* (equivalently, *b*) describes the software market specific to platform A (B). Thus, the entire software market is completely described by the tuple (a, b). Software firms have three strategies: exit, entry, and investment. Firms' profits are determined in the price competition stage.

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[8] Using properties of the logit distribution, $\mathrm{E}(\mathrm{E}\{[U_{ij}^{\mathrm{A}}(a') - p_j]|S\}) = \sum_{\substack{\text{possible}\\ \text{next}\\ \text{states}}} \ln(1 + \sum_{k, a'_k \neq 0} \exp(a'_k - p'_k))$.

[9] Note that when making hardware decisions, consumers are not allowed to choose the outside good. This assumption simplifies the computation as it avoids the need for an additional state variable.

2.2.1. Incumbent firms

In each period, incumbent firms have to choose whether to stay in the market or exit. An exiting firm gets a scrap value of ϕ and vanishes. A software firm will therefore exit if and only if the expected discounted value of its future net cash flow is less than ϕ. Let $\chi_j(S) \in \{0, 1\}$ indicate firm j's exit policy, where $\chi_j(S) = 1$ indicates that incumbent firm j will exit the market.

Incumbents that remain in the market invest in order to upgrade the quality level of their product. Each firm's quality level in the next period is determined by a Markov process which depends on the firm's quality level today, its level of investment today, and the level of competition from substitute industries. The more a firm invests, the higher is the probability of a quality upgrade. In particular, if firm j invests x_j, I assume the probability of a quality upgrade to be $x_j/(1 + x_j)$. At the same time, technological advances in substitute markets may erode the advantage enjoyed by all software firms (on both platforms) within the discussed industry. For example, looking at the video games market, an advance in the computer games market would negatively affect Microsoft's Xbox as well as Sony's Playstation. More generally, such a depreciation can be a result of any exogenous process affecting the relative quality evaluations of all software firms in the market of interest. I assume that innovation in substitute markets depreciates the quality of all software on both platforms by an equal amount. I take the probability of depreciation to be δ and denote its realization by $v \in \{0, 1\}$. Hence if $P(a_j'|a_j, x_j, \chi_j, v)$ denotes the probability that firm j will be at quality level a_j' in the next period, given that its quality level in the current period is a_j, then for $a_j \in \{2, \ldots, K-1\}$

$$P(a_j'|a_j, x_j, \chi_j, v=0) = \begin{cases} (1-\chi_j)\dfrac{x_j}{1+x_j} & \text{if } a_j' = a_j + 1, \\ (1-\chi_j)\dfrac{1}{1+x_j} & \text{if } a_j' = a_j, \\ \chi_j & \text{if } a_j' = 0, \end{cases}$$

$$P(a_j'|a_j, x_j, \chi_j, v=1) = \begin{cases} (1-\chi_j)\dfrac{x_j}{1+x_j} & \text{if } a_j' = a_j, \\ (1-\chi_j)\dfrac{1}{1+x_j} & \text{if } a_j' = a_j - 1, \\ \chi_j & \text{if } a_j' = 0. \end{cases} \tag{5}$$

Since firms in the highest (lowest) quality level cannot increase quality further up (down), I set $P(a_j' = a_j|a_j, x_j, \chi_j, v = 0) = 1$ for $a_j = K$, and $P(a_j' = a_j|a_j, x_j, \chi_j, v = 1) = 1$ for $a_j = 1$. For all other cases the probabilities are as in Eq. (5).

2.2.2. Entry

In each period only one potential entrant may enter the industry by paying a one-time sunk entry fee, ϕ^{e}. The entry fee is random and is assumed to be distributed uniformly between $\phi_{\mathrm{L}}^{\mathrm{e}}$ and $\phi_{\mathrm{H}}^{\mathrm{e}}$. The entrant then chooses whether to enter on platform A, enter on platform B, or stay out of the industry. I assume that entrants are short-lived and do not consider the possibility of delaying entry. Upon committing to entry, the entrant undergoes a setup period. The entrant then becomes an incumbent in the following period. In the case of entry on platform A (analogously for B), the entrant enters at quality level a^{e} if there is no outside shock and at $a^{\mathrm{e}}-1$, otherwise. That is, outside competition affects entrants immediately in the period of entry. Let $\lambda^{\mathrm{A}}(\sigma,a,b)\in[0,1]$ and $\lambda^{\mathrm{B}}(\sigma,a,b)\in[0,1]$ indicate the probability of entry on platforms A and B, respectively. Given that $\lambda^{\mathrm{A}}(\sigma,a,b)>0$, the probability that an entrant enters on platform A with quality level a'_j is

$$P(a'_j|\lambda^{\mathrm{A}},v)=\begin{cases}\lambda^{\mathrm{A}} & \text{if } a'_j=a^{\mathrm{e}}-v,\\ 1-\lambda^{\mathrm{A}} & \text{if } a'_j=0.\end{cases} \tag{6}$$

2.2.3. Transition probabilities

If the industry is in state $S=(\sigma,a,b)$ today, then the probability that the industry will be in state $S'=(\sigma',a',b')$ in the next period is determined jointly by the consumers' hardware choices, the incumbents' exit and investment decisions, and the entrant's entry decision, as well as by the level of competition from substitute industries. Assuming that the firms' transitions are independent of each other and of the transitions of substitute industries, the transition probability $T(S'|S,x^{\mathrm{A}}(S),x^{\mathrm{B}}(S),\chi^{\mathrm{A}}(S),\chi^{\mathrm{B}}(S),\lambda^{\mathrm{A}}(S),\lambda^{\mathrm{B}}(S),\delta)$ can be written as

$$\left[\begin{array}{l}\prod\limits_{\{j|a_j>0\}}P(a'_j|a_j,x^{\mathrm{A}}_j(S),\chi^{\mathrm{A}}_j(S),v=0)\cdot\prod\limits_{\{j|b_j>0\}}P(b'_j|b_j,x^{\mathrm{B}}_j(S),\chi^{\mathrm{B}}_j(S),v=0)\cdot P(a'_j|\lambda^{\mathrm{A}},v=0)\\ \quad\cdot P(b'_j|\lambda^{\mathrm{B}},v=0)\cdot(1-\delta)+\prod\limits_{\{j|a_j>0\}}P(a'_j|a_j,x^{\mathrm{A}}_j(S),\chi^{\mathrm{A}}_j(S),v=1)\\ \quad\cdot\prod\limits_{\{j|b_j>0\}}P(b'_j|b_j,x^{\mathrm{B}}_j(S),\chi^{\mathrm{B}}_j(S),v=1)\cdot P(a'_j|\lambda^{\mathrm{A}},v=1)\cdot P(b'_j|\lambda^{\mathrm{B}},v=1)\cdot\delta\end{array}\right]\cdot\Lambda(\sigma'|S),$$

where $x^{\mathrm{A}}_j(S)\,(x^{\mathrm{B}}_j(S))$ gives firm j's investment decision given that firm j produces software compatible with platform A (B). $P(\cdot)$ is given by Eqs. (5) and (6), and $\Lambda(\sigma'|S)$ is the transition probability of σ'. Firms' investment, exit and entry decisions – $x^{\mathrm{A}}_j(S)$, $x^{\mathrm{B}}_j(S)$, $\chi^{\mathrm{A}}_j(S)$, $\chi^{\mathrm{B}}_j(S)$, $\lambda^{\mathrm{A}}(S)$ and $\lambda^{\mathrm{B}}(S)$ – are derived from the optimizing behavior of incumbents and entrants, and are discussed in more detail below.

2.2.4. Price competition

While investment decisions are dynamic, the pricing game is a static game with no future effects or dynamics. Software firms (on each platform) set prices oligopolisticly and software demand is then determined according to Eq. (1).[10] Assuming that marginal cost is the constant c, for any vector of prices, the profit-

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[10] Note that although the pricing game is static and prices do not directly depend on the structure of the competing platform, profits are also a function of the firm's own platform market share, which in turn depends on the quality level and structure of the competing platform.

maximization problem of firm j on platform A is

$$\max_{p_j \geqslant 0} \sigma * D_j(a,p) * (p_j - c), \tag{7}$$

where σ is the percentage of consumers who own platform A, and $D_j\ (a,p)$ is firm j's demand. Firm j's first-order condition (FOC), the derivative of (7) with respect to p_j, is then

$$0 = 1 - \frac{1 + \sum_{k \neq 0, k \neq j} \exp(a_k - p_k)}{1 + \sum_{k \neq 0} \exp(a_k - p_k)} (p_j - c).$$

It can be shown that there exists a unique Nash equilibrium $p^*(a)$ of the pricing game (Caplin and Nalebuff, 1991). This Nash equilibrium can be computed by numerically solving the system of FOCs. The per-period profit of firm j in the Nash equilibrium of the pricing game is then

$$\pi_j(\sigma, a, p) = \sigma * D_j(a, p^*(a)) * (p_j^*(a) - c). \tag{8}$$

2.2.5. The incumbent's problem

If the industry is in state $S = (\sigma, a, b)$, then an incumbent firm solves an intertemporal maximization problem in order to reach its exit and investment decisions. Let $V_j^{\mathrm{A}}(S)$ be the expected future payoff of software firm j on platform A. Firm j then solves the following Bellman equation:

$$\begin{aligned} V_j^{\mathrm{A}}(S) = \max\Bigg\{ \phi, \sup_{x_j \geqslant 0} \Bigg[\pi_j(S) - x_j + \beta \sum_{\sigma', a', b'} V_j^{\mathrm{A}}(S') \cdot T(S' | S, x_j, \chi_j^{\mathrm{A}} = 0, \\ x_{-j}^{\mathrm{A}}(S), x^{\mathrm{B}}(S), \chi_{-j}^{\mathrm{A}}(S), \chi^{\mathrm{B}}(S), \lambda^{\mathrm{A}}(S), \lambda^{\mathrm{B}}(S), \delta) \Bigg] \Bigg\}, \end{aligned} \tag{9}$$

where $x_{-j} = (x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_N)$ and $\chi_{-j} = (\chi_1, \ldots, \chi_{j-1}, \chi_{j+1}, \ldots, \chi_N)$. The max operator compares the exit value (ϕ) to the firm's continuation value, where the firm shuts down if ϕ is larger. If $\chi_j^{\mathrm{A}}(\sigma, a, b)$ is the exit indicator function, then in equilibrium, firm j sets:

$$\chi_j^{\mathrm{A}}(\sigma, a, b) = 1 \quad \text{iff } V_j^{\mathrm{A}}(\sigma, a, b) \leqslant \phi, \text{otherwise } \chi_j^{\mathrm{A}}(\sigma, a, b) = 0. \tag{10}$$

A continuing firm earns current profits in the pricing game, $\pi_j(\sigma, a, p)$, plus the expected discounted value of future returns.

2.2.6. The entrant's problem

An entrant faces a similar optimization problem. The value to a potential entrant from entering platform A with quality level a_j is

$$V_j^{\mathrm{e,A}}(S) = \beta \sum_{\substack{\sigma', a', b', \\ a_j' > 0}} V_j^{\mathrm{A}}(S') \cdot T(S' | S, x^{\mathrm{A}}(S), x^{\mathrm{B}}(S), \chi^{\mathrm{A}}(S), \chi^{\mathrm{B}}(S),$$

$$\lambda^{\mathrm{A}}(S)>0, \lambda^{\mathrm{B}}(S)=0, \delta). \tag{11}$$

As noted before, an entrant must incur (sunk) setup costs of $\phi^{\mathrm{e}} \sim \mathrm{U}[\phi^{\mathrm{e}}_{\mathrm{L}}, \phi^{\mathrm{e}}_{\mathrm{H}}]$ upon entry and spends a period building its plant. An entrant would choose to enter market A if and only if $V^{\mathrm{e,A}} > \phi^{\mathrm{e}}$ and $V^{\mathrm{e,A}} > V^{\mathrm{e,B}}$. Let $I^{\mathrm{A}}_{\mathrm{e}}$ be an indicator function where $I^{\mathrm{A}}_{\mathrm{e}} = 1$ iff $V^{\mathrm{e,A}} > V^{\mathrm{e,B}}, I^{\mathrm{A}}_{\mathrm{e}} = 0$ otherwise. The probability of entry on platform A is therefore:

$$\lambda^{\mathrm{A}}(\sigma, a, b) = \begin{cases} I^{\mathrm{A}}_{\mathrm{e}} & \text{if } V^{\mathrm{e,A}} > \phi^{\mathrm{e}}_{\mathrm{H}}, \\ \dfrac{V^{\mathrm{e,A}} - \phi^{\mathrm{e}}_{\mathrm{L}}}{\phi^{\mathrm{e}}_{\mathrm{H}} - \phi^{\mathrm{e}}_{\mathrm{L}}} \cdot I^{\mathrm{A}}_{\mathrm{e}} & \text{if } V^{\mathrm{e,A}} > \phi^{\mathrm{e}}_{\mathrm{L}}, \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

$(V^{\mathrm{e,A}} - \phi^{\mathrm{e}}_{\mathrm{L}})/(\phi^{\mathrm{e}}_{\mathrm{H}} - \phi^{\mathrm{e}}_{\mathrm{L}})$ is the probability that the entrant's expected value is greater than the entry fee. $((V^{\mathrm{e,A}} - \phi^{\mathrm{e}}_{\mathrm{L}})/(\phi^{\mathrm{e}}_{\mathrm{H}} - \phi^{\mathrm{e}}_{\mathrm{L}})) \cdot I^{\mathrm{A}}_{\mathrm{e}}$ is then the probability that $V^{\mathrm{e,A}}$ is greater than the entry fee as well as than the expected value from entry onto market B. No entry occurs when $\phi^{\mathrm{e}} > V^{\mathrm{e,A}}$ and $\phi^{\mathrm{e}} > V^{\mathrm{e,B}}$.

2.3. *Equilibrium in the industry*

A subgame perfect equilibrium for the above game consists of a collection of strategies and value functions that constitute a Nash equilibrium for every history of the game. I consider only Markov strategies – i.e., the class of strategies that depend only on the 'payoff relevant' states. This means that the strategies are defined for every state of the game regardless of how this state has been reached. Formally, a Markov perfect equilibrium for the game is defined by the

- Investment and exit strategies $x^{\mathrm{h}}_{j}(\sigma, a, b), \chi^{\mathrm{h}}_{j}(\sigma, a, b)$ for $j = 1, \ldots, N$; $\mathrm{h} = \mathrm{A}, \mathrm{B}$ and every possible state (σ, a, b).
- Entry strategies $\lambda^{\mathrm{A}}(\sigma, a, b), \lambda^{\mathrm{B}}(\sigma, a, b)$ for every possible state (σ, a, b).
- Value functions $V^{\mathrm{h}}_{j}(\sigma, a, b), V^{\mathrm{h,e}}_{j}(\sigma, a, b)$ for $i = 1, \ldots, N$; $\mathrm{h} = \mathrm{A}, \mathrm{B}$ and every possible state (σ, a, b).

Such that:

(i) The strategies are optimal given the value functions $V^{\mathrm{h}}_{j}(\sigma, a, b), V^{\mathrm{h,e}}_{j}(\sigma, a, b)$.
(ii) For every state (σ, a, b), the value functions describe the present value of profits realized when all firms play the equilibrium investment, exit, and entry strategies.

Proposition 1. *There exists an anonymous symmetric MPE in pure strategies.*

In light of Proposition 4 in Doraszelski and Satterthwaite (2007), the key to the proof is to show that the value function has at most one extreme point in a large enough interval of investment levels, $(0, \bar{x})$. Computation (available upon request) easily shows that this condition holds in this model. An outline of the proof can be found in Appendix A.

While uniqueness cannot be guaranteed in general, my computations always lead to the same value and policy functions irrespective of the starting point and the particulars of the algorithm.

2.4. Computing the equilibrium

To compute the MPE, I extend the algorithm described in Pakes and McGuire (1994) to accommodate dynamics in demand as well as in supply. The computational algorithm is iterative and uses the value function approach. The algorithm works as follows: first, the algorithm initiates incumbents' and entrants' value functions $V^0(S)$ and investment levels $x^0(S)$ for all states. Value functions $V^0(S)$ are initiated with an arbitrary number, and investment levels $x^0(S)$ are set to zero. The algorithm then works iteratively. To move from iteration k to iteration $k+1$, the algorithm first takes the value functions $V^k(.)$, policy functions $x^k(.)$, exit strategies $\chi^k(.)$, and entry strategies $\lambda^k_{\mathrm{A}}(.)$ and $\lambda^k_{\mathrm{B}}(.)$ as inputs and using Eq. (3) calculates consumers' hardware choices, $\psi^{k+1}(.)$. $V^k(.)$, $x^k(.)$, $\chi^k(.), \lambda^k_{\mathrm{A}}(.), \lambda^k_{\mathrm{B}}(.)$, and $\psi^{k+1}(.)$ are then plugged into Eqs. (10) and (12) to calculate exit, χ^{k+1}, and entry, λ^{k+1}, decisions, respectively. Finally, $V^k(.)$, $x^k(.)$, $\chi^{k+1}(.), \lambda^{k+1}_{\mathrm{A}}(.)$, $\lambda^{k+1}_{\mathrm{B}}(.)$, and $\psi^{k+1}(.)$ are taken as inputs to the Bellman equation (9) to update the incumbents' value and policy functions. This calculation is done separately for each firm: in each iteration the algorithm takes $V^k_j(.)$, $x^k_{-j}(.)$, $\chi^{k+1}(.)$, $\lambda^{k+1}_{\mathrm{A}}(.)$, $\lambda^{k+1}_{\mathrm{B}}(.)$ and $\psi^{k+1}(.)$ from memory and feeds them into Eq. (9) to solve for firm j's investment strategy, $x^{k+1}_j(.)$. It then takes the calculated $x^{k+1}_j(.)$ and computes firm j's value function, $V^{k+1}_j(.)$. The same calculations are then performed to compute $\{V^{k+1}_j(.), x^{k+1}_j(.)\}$ for all j. Using a stopping criteria of $\imath = 10^{-4}$, the algorithm iterates over the value functions and the investment strategies and stops when $\{V^k(.), V^{k+1}(.)\}$; and $\{x^k(.), x^{k+1}(.)\}$ are very close, pointwise, between iterations.

The equilibrium value functions, exit, entry, and investment strategies and consumers' hardware choices (i.e., $V^*(.), x^*(.)$, $\chi^*(.)$, $\lambda^*_{\mathrm{A}}(.), \lambda^*_{\mathrm{B}}(.)$ and $\psi^*(.)$) are used to construct the transition matrix that characterizes industry dynamics. Relying on stochastic process theory as well as simulations, the transition matrix is then used to find the limiting distribution. Finally, the states in the limiting distribution are used to characterize the long-run market structure: while σ indicates whether the long-run market structure is variety or standardization, the vector (a, b) specifies the market structure on each platform. Section 3 presents a detailed analysis of the attributes of the long-run market structure for different δ values.

2.4.1. Parameterization

Since investment realization is a relatively slow process, I take a period to be one year and set the discount factor to be $\beta = 0.92$. The highest quality level any software firm can achieve, K, is endogenously determined in the model. Starting with a very high arbitrary K, the model finds that a monopolist with quality level of 6 will not find additional investment aimed at achieving a higher quality level to be sufficiently rewarded by consumers and, therefore, chooses to not invest at all. Accordingly, I set K to 6. This upper bound represents the maximum difference that an incumbent can obtain relative to a potential entrant or to any other player, including those in related industries that produce substitute goods. In other words, any player in the industry, including potential entrants, can always acquire enough knowledge from publicly available sources so that it will fall no further behind than this maximum number of quality steps.

All other values are set as follows: sell-off value, $\phi = 0.1$; sunk entry cost, $\phi^{\mathrm{e}} \sim U[0.75, 1.25]$; entrant's quality level, $a^{\mathrm{e}} = b^{\mathrm{e}} = 2$; market size, $M = 10$. For simplicity, the hardware prices are set to zero, $P^{\mathrm{A}} = P^{\mathrm{B}} = 0$. In order to keep the model computational tractable, I restrict the maximum number of software firms on each hardware platform, N, to 2. Finally, platform A's market share, σ, runs from 0% to 100% in increments of 5%.[11]

3. Results

I define standardization to be the case where, in the long run, one of the platforms enjoys a market share of at least 95% – states (σ, a, b) where $\sigma > 0.95$ or $\sigma < 0.05$ – and there is no software available for the competing platform. Variety is defined as the case where, in the long run, both platforms are active in the market, each with a market share of 0.95 or less. I restrict the analysis to industry-level results and focus on the structure of the industry in the long run.[12] More specifically, I analyze the conditions under which the industry exhibits the snowball effect and only one standard prevails in the long run, as well as the expected average life span of a standard. In order to find the long-run market structure, i.e., the ergodic distribution, I use the equilibrium policy functions to simulate the market for 1,000,000 periods. I then take the average over 100 simulations of 1,000,000 periods each.[13]

In general, the results show that the ergodic distribution puts probability mass either on states that represent standardization or on states representing variety. Overall, two main factors drive the dynamics in the model: competition within the industry and competition from substitute industries. The effect of these two factors, however, differs tremendously. While the level of competition from substitute industries determines the ergodic distribution, inter-platform competition, intra-platform competition, and the

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[11] In order to characterize the whole space of parameter values, I calculated the equilibrium for all combinations of quality levels, initial market shares, and levels of outside competition. Unfortunately, the programs were unsuccessful in finding the equilibrium for some levels of outside competition, δ. I therefore do not report the results for $\delta = 0.2$, 0.3 and 0.8, where the program did not converge.

[12] The short-run aspects of the model, in particular the effect of δ on firms' investment strategies, are analyzed in Markovich and Moenius (2006a, b). Markovich and Moenius (2006b) find that a firm's optimal investment depends predominantly on its quality level relative to its competitors on the same hardware, and on the quality level of all software firms on the same hardware relative to other hardware platforms. Markovich and Moenius (2006a) find that indirect network effects tie together the performance of firms on the same platform.

[13] I first use stochastic process theory in order to find the communicating classes, and in all cases I find only one communicating class. Because of computer memory limitations, I then use simulations rather than theory to find the ergodic distribution. In order to make sure the ergodic distribution does not depend on a specific starting state, I run the simulations with different starting conditions.

industry's initial state affect the speed of transition to the limiting distribution. However, they do not affect the ultimate industry structure. Before continuing, I first introduce some simplifications that facilitate the presentation of the results.

The envelope sum of qualities[14]: Since there is no clear *ex ante* ranking for the 49 possible quality combinations on each platform, characterizing these results is somewhat problematic. I therefore introduce the following ranking to improve transparency: each combination of qualities on a platform is assigned a single number, indicating the sum of qualities (e.g., 3-3 and 4-2 would both be assigned the sum 6).[15] Then, for each sum of qualities, initial platform share, and level of outside competition, I pick the quality combination that, for the corresponding platform, delivers the highest expected platform market-share in the following period. I use only these 'strong' combinations for presenting the results. This reduces the number of states per platform to 13, ranging from zero (no active firms) to 12 (both firms enjoy a quality level of six). I define these combinations as the *envelope sum of qualities* (*ESQ*). Other quality combinations with lower expected market shares can be thought of as observationally equivalent to a lower sum of qualities. These weaker cases deliver results 'in-between' the strong cases. For example, for the sum of qualities of 6 the envelope sum of qualities is (3,3), and for 5 it is (3,2). This implies that for the corresponding platform, next period's market share for the quality levels of 4-2, 5-1 and 6-0 lie in-between 3-2 and 3-3. Furthermore, as the results show, these same combinations also maximize total investment on the corresponding platform. That is, for the sum of qualities of 6, a market structure of (3,3) results in higher total investment than a market structure of (4,2), (5,1) or (6,0). The intuition behind this latter result is that consumers base their hardware decisions on future expected quality levels, which in turn are determined by the firms' level of investment. The following Propositions summarize the last two observations.

Proposition 2. *Total investment on a platform is maximized at the envelope sum of qualities.*

Proposition 3. *For a given sum of qualities, Z, next period's market share for non-envelope sum of qualities' combinations are 'in-between' next period's market share for the envelope sum of qualities for Z and Z-1.*

$$\sigma'\left(q_1, q_2 \,\middle|\, \begin{array}{c} (q_1, q_2) \in ESQ, \\ q_1 + q_2 = Z - 1 \end{array}\right) < \sigma'\left(q_1, q_2 \,\middle|\, \begin{array}{c} (q_1, q_2) \notin ESQ, \\ q_1 + q_2 = Z \end{array}\right)$$
$$< \sigma'\left(q_1, q_2 \,\middle|\, \begin{array}{c} (q_1, q_2) \in ESQ, \\ q_1 + q_2 = Z \end{array}\right).$$

A numerical proof for both propositions is available upon request from the author.

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[14] I thank Johannes Moenius for giving me this idea.

[15] In order to avoid the use of fractions for qualities, I am using the sum rather than the average.

In general, the following pattern was found: for low levels of δ, monopolies attract more consumers than duopolies up until the sum of qualities is 3, after which duopolies promised higher market shares. Furthermore, when possible, symmetric duopolies are always more attractive to the consumers than asymmetric ones. For higher levels of δ, the cut-off point for monopolies moves to a quality level of 4 instead of 3. The intuition behind this result is as follows: consumers value quality and therefore, given a certain sum of qualities, the value from a monopoly is higher than the value from a duopoly. However, once the sum of qualities reaches a certain level, consumers expect the competitive environment to promote innovation. As the level of δ increases, innovation becomes harder, thus a higher sum of qualities is required for consumers to prefer a duopoly over a monopoly. Finally, a symmetric structure is always more competitive than an asymmetric one, making the environment more attractive for consumers. Table 1 shows the *envelope sum of qualities* in boldface for $\delta > 0.4$.[16]

Standardization and its determinants: Software firms' profits depend on their quality level relative to the quality level of their competitors on the same platform, as well as to those on the competing platform. The vector of relative qualities, in turn, determines the platforms' market shares. Since hardware is long-lived (two periods) and software is short-lived (one period), one might expect standardization to be determined by the mix of the following four factors: competition from substitute industries, inter-platform competition, intra-platform competition, and the initial market share of each platform. The results, however, give a clear-cut – and potentially surprising – answer. Whether the long-run industry structure is standardization or variety is determined only by the level of competition from substitute industries. Competition within the industry only affects the speed at which the market achieves this state.

3.1. *The effect of outside competition*

In general, the sole driver of standardization in the model is how fast innovations in the industry would be devalued by innovation in (potential) substitute industries. Fig. 1 shows how quickly standardization occurs depending on the outside competition parameter, δ. The figure presents the cumulative probability of standardization over time on either platform A or B, starting with state (0.5; 4,4; 4,4)[17] for $\delta = 0.1$, 0.4, 0.5, 0.7, and 0.9.

Fig. 1 shows that if $\delta \leqslant 0.1$, the long-run industry structure is variety, while for $\delta \geqslant 0.4$, standardization characterizes the industry structure in the long run. That is, there exists $0.1 < \delta^* < 0.4$[18] such that, for $\delta > \delta^*$, the industry exhibits the snowball effect and, in the long run, only one standard prevails in the market; if $\delta < \delta^*$ the long-run equilibrium is variety.

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[16] The pattern for other values of δ is similar and can be obtained upon request from the author.

[17] The state (0.5; 4,4; 4,4) represents an industry where consumers are equally distributed across the two platforms. Each platform has two firms producing compatible software, all with a quality level of four.

[18] Since the program could not find the equilibrium for δ values in between 0.1 and 0.4, I cannot specify the exact value at which the long-run industry structure switches from variety to standardization.

Table 1
Envelope sum of qualities for $\delta > 0.4$

Sum of qualities	Market structure
0	**(0,0)**
1	**(1,0); (0,1)**
2	(1,1)
	(2,0); (0,2)
3	(2,1); (1,2)
	(3,0); (0,3)
4	(2,2)
	(3,1); (1,3)
	(4,0); (0,4)
5	**(3,2); (2,3)**
	(4,1); (1,4)
	(5,0); (0,5)
6	**(3,3)**
	(4,2); (2,4)
	(5,1); (1,5)
	(6,0); (0,6)
7	**(4,3); (3,4)**
	(5,2); (2,5)
	(6,1); (1,6)
8	**(4,4)**
	(5,3); (3,5)
	(6,2); (2,6)
9	**(5,4); (4,5)**
	(6,3); (3,6)
10	**(5,5)**
	(6,4); (4,6)
11	**(6,5); (5,6)**
12	(6,6)

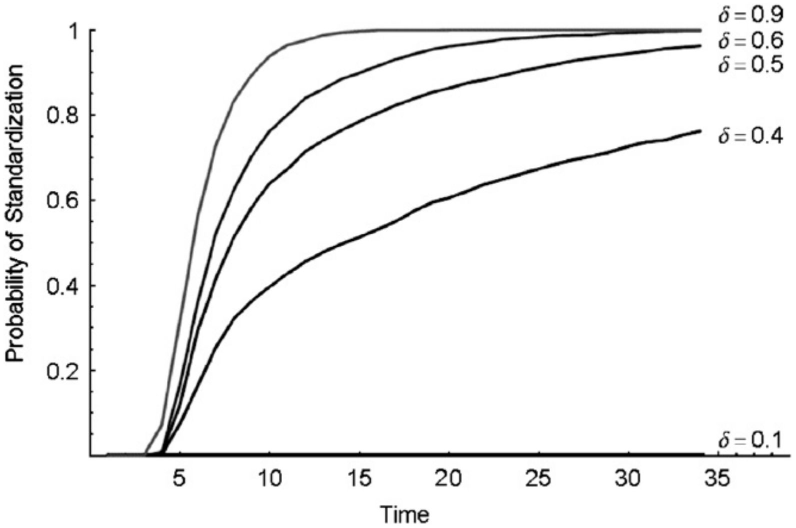

Fig. 1. Cumulative standardization-probability over time.

Furthermore, Fig. 1 shows that for $\delta \geqslant 0.4$, as the level of outside competition increases, the standardization process speeds up. For example, when $\delta = 0.4$, the standardization process takes more than 35 periods in about 25% of all cases, while if $\delta = 0.9$, the process of standardization is completed within 20 periods and typically takes less than 10 periods. That is, fiercer competition from substitute industries increases the probability of standardization and speeds up the standardization process.

The economic intuition behind this result is as follows: stronger outside competition increases the riskiness of investment in software, since each achievement may be devalued instantaneously, as in the case when $\delta = 1$. This intensifies the fight for survival, and firms reinvest very high shares of their profits into quality upgrades. In addition, while the exogenous shock is an industry-specific shock hitting all firms in the industry equally hard, each firm's probability of quality upgrade depends also on its own level of investment. This investment, however, may fail. A stumbling firm might get too far behind and, thus, find it unprofitable to catch up with the competition. This, in turn, puts the compatible platform in a disadvantageous position. All this leads to a snowball that rolls faster as the level of outside competition increases.

3.1.1. Standardization and market structure

Up until now, the discussion has focused on industry structure in terms of the number of active platforms. However, it has said nothing about the within-market structure one would expect to see – in particular, about the number of active firms on each platform and their quality levels. In principle, multiple solutions are possible. There could be either a software monopoly or duopoly on one platform (in the case of standardization), and there could be duopolies or monopolies on either platform (in the case of variety). The results show that, in general, whenever a platform establishes itself there will be a duopoly.

I start with the variety case. When $\delta = 0$, innovations are never depreciated and, therefore, once the industry gets to state (0.5; 6,6; 6,6), it stays there forever. Once we add competition from substitute industries to the game (i.e., $\delta > 0$), the industry becomes dynamic and the quality levels of the modal state depend on the value of δ. In this case, unlike in the static literature, the long-run structure is not necessarily symmetric. In fact, the industry might move from states where the consumers are equally divided between both platforms to states where most of the consumers own the same platform. Nevertheless, the most likely split of consumers remains symmetric, in which case the market may still be asymmetric in terms of the firms' quality levels. Fig. 2 gives the limiting distribution when $\delta = 0.1$. The left panel gives the limiting distribution of platform A's market share, and the right panel gives the limiting distribution of qualities given that the consumers are equally divided between both platforms.

According to the left panel in Fig. 2, when $\delta = 0.1$, the two competing platforms split the consumers equally with 66% probability. Given an even split, the modal state (right panel) is (6,6;6,6). Since the probability that substitute markets would depreciate the quality of products in this market is pretty small, the limiting

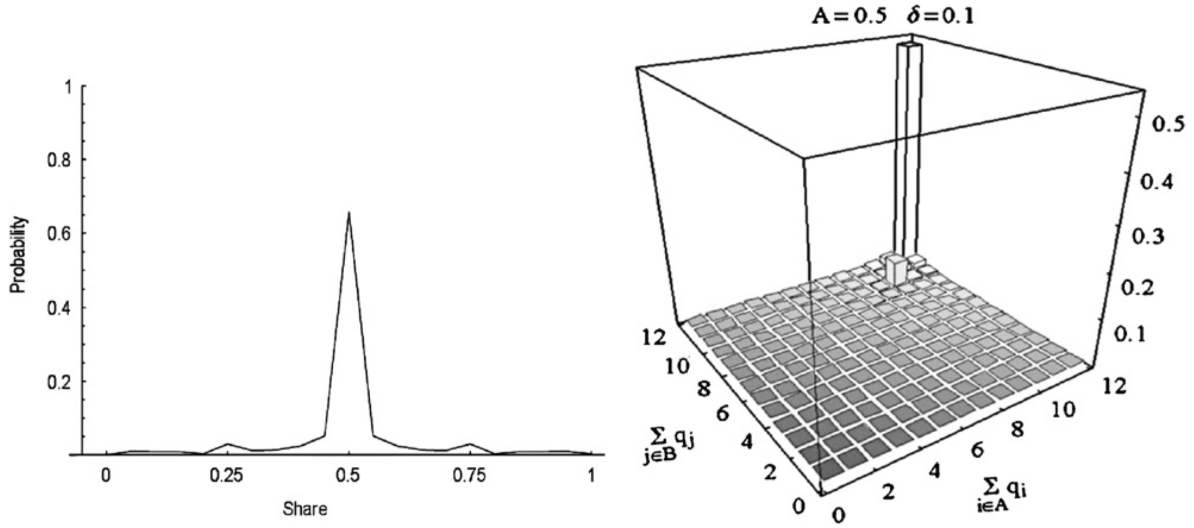

Fig. 2. Limiting distribution for $\delta = 0.1$.

distribution puts probability mass only on states where all firms in the market offer a product with a quality level of 4 or higher.

The standardization case is also mostly characterized by duopolies, where the quality levels of the modal state decrease as δ increases. Fig. 3 shows this observation for $\delta = 0.4$ and 0.9. Using the envelope sum of qualities, the top panel of Fig. 3 shows the across-platform limiting distribution. The bottom panel then gives the corresponding within-platform limiting distribution.

In both cases, the limiting distribution is bimodal. When $\delta = 0.4$, the modal states are (1;6,6;0,0) and (0;0,0;6,6), each with probability of 0.17. The probability of being in states (1;5,5;0,0) or (0;0,0;5,5) and (1;4,4;0,0) or (0;0,0;4,4) is 0.15 and 0.03, respectively. As competition from substitute industries increases, the probability of lower quality products increases as well, such that when $\delta = 0.9$, the modal states are (1;1,1;0,0) and (0;0,0;1,1), each with probability of 0.18. In this case the industry is very dynamic with a lot of exit and entry. Firms' life spans are fairly short; however, an exiting firm would most likely be quickly replaced by an entrant. Nevertheless, the probability of a monopoly in this case is relatively high, in particular the probability of states (1;1,0;0,0) and (0;0,0;1,0) is 0.12, each.

It is necessary to take these results with a grain of salt: generally, consumers have a taste for variety (in addition to quality), so whether we will see monopolies or duopolies will depend on how strong this taste for variety is. Moreover, for computational reasons, I do not allow for more than two firms on each platform. Thus, we do not know whether more variety is always better. Finally, one would expect firms' profitability to decrease in the number of firms competing in the market. Consequently, it is not clear that the symmetric market structure would also hold in oligopolies with more than 2 firms.

However, there remains a deeper and more subtle point to be emphasized. In the model, outside competition, unfavorable circumstances, and competitors' advances can destroy the value of a firm's investment. Thus, investment in the

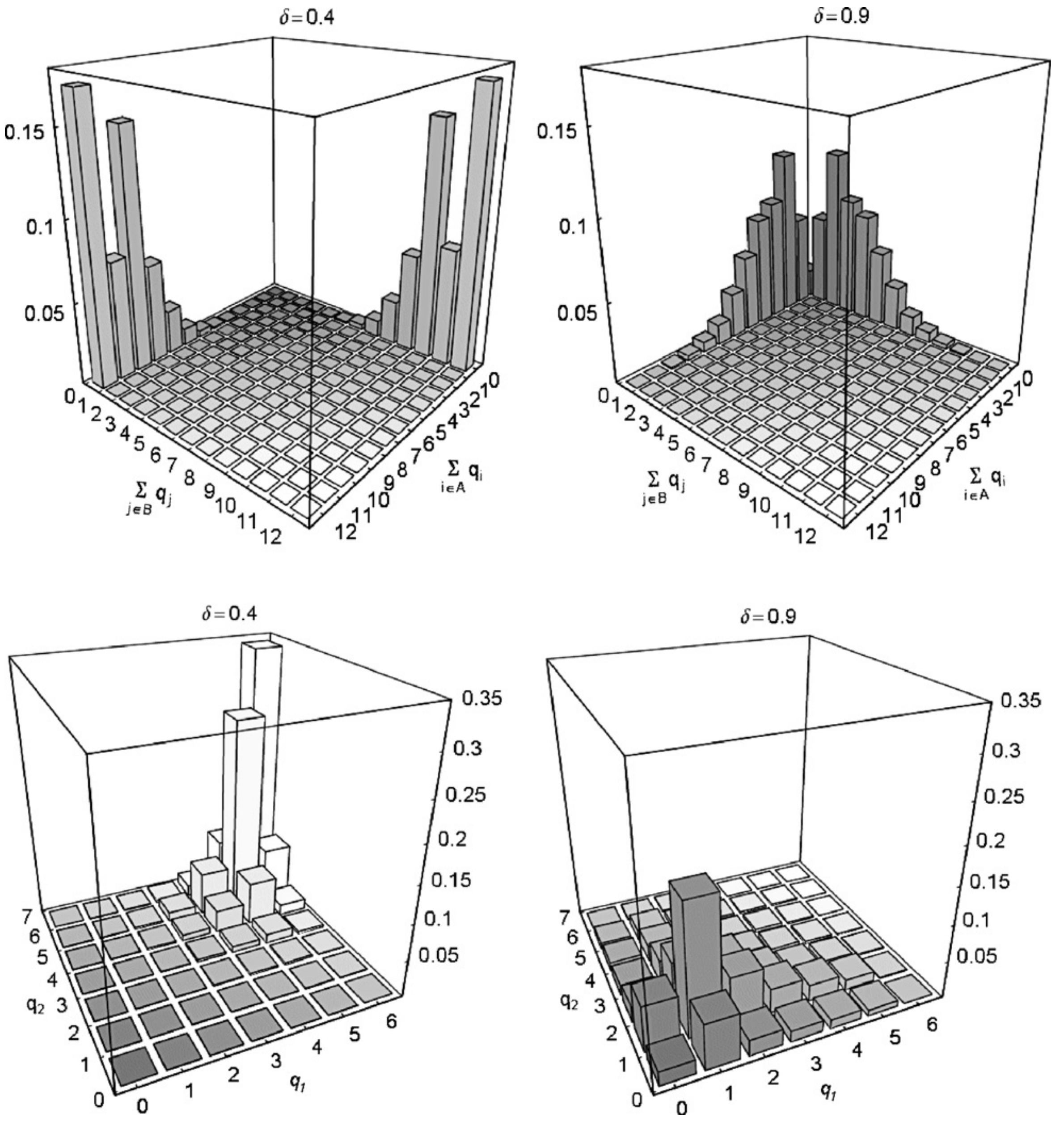

Fig. 3. Limiting distribution.

model is inherently risky. Competition within the market – as well as from substitute industries – may reduce the lead an incumbent can gain, thereby opening up a window for an entrant. Entry reduces the incumbent's profits, but it strengthens the platform overall, as more consumers will be attracted to buying the hardware of this platform. This, in turn, increases the platform's probability of survival, and both the incumbent and the entrant benefit from reduced risk. That is, entry here is a double-edged sword: while new entry can be thought of as life insurance for the incumbent, the incumbent has to pay a premium for it – a share of the market.[19]

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[19] This idea is further explored in Markovich and Moenius (2006a).

3.1.2. Standards replacement

The static literature in this area finds that, once a standard is established, we would tend to observe inertia, allowing the standard to live forever. The dynamic setting, however, allows for replacement, where an established standard would be superseded by a different and (perhaps) better standard. One might therefore be interested in analyzing the expected average life span of a standard. Focusing only on the standardization case, I approach this analysis in two steps. First, I begin by looking at the probability of entry. I then study the standards' expected average life span.

I start with studying how likely it is to observe entry onto a second platform given that only one exists at the outset. I analyze this question under the stark assumption that this second platform is readily available and that the only setup cost is the sunk cost that the entrant in the software market has to pay.

Given that one of the platforms enjoys a market share of at least 95% and that there are no active firms on the second platform, the results show that the probability of entry on the second platform is positive for $\delta = 0.4$, 0.5, 0.6 and 0.7. In this case, if $\delta = 0.5$, 0.6 or 0.7, an entrant would enter on the competing platform only when the qualities offered on the existing platform are (1,1); for $\delta = 0.4$, the probability of entry is positive for quality levels of (1,1) and (2,1).[20] That is, when the probability of an outside shock is relatively high, a competing platform would emerge only if the entrant can offer a product which is better than the existing ones. However, when it is relatively easier to upgrade its product (lower δ), an entrant might enter with a quality that is similar to the highest quality that currently exists. This suggests that firms should never rest in peace, even when being active on the standard platform. Low innovation invites competition onto the competing platform, which may then change the statues quo in the market.

This is a very important result for which we can also find support in reality. For example, Nintendo's dominance in the 8-bit market was suppressed by the 16-bit technology; the market for LP records disappeared once compact disks were introduced. Furthermore, although in both cases there were some periods at which both standards were active in the market, eventually the market converged to only one of the standards. Also, the speed of convergence depended on the level of competition from substitute industries.

Thus, standards can rise and fall; however, the final outcome in terms of standardization or variety remains unchanged. This notion that standardization and variety are both 'natural' outcomes is in strong contrast to the 'tipping' and 'inertia' results highlighted in the static literature, with important implications. While the static literature suggests a role for policy to prevent unwanted outcomes determined by initial conditions, this model suggests that it would be extremely costly for policy

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[20] When $\delta = 0.9$ and 1, if one platform owns at least 95% of the market, the probability of entry on the competing platform is zero for all possible states. The probability of entry becomes positive only when the established platform's market share is at most 85% for $\delta = 0.9$ and 70% for $\delta = 1$. In these cases, consumers anticipate the software firms' entry and, based on these expectations, they purchase the competing platform before actual entry takes place.

Table 2
Expected lifetime of a standard

δ	0.4	0.5	0.6	0.7	0.9	1
Expected life span	85,000	300,000	38,000	650	30	7

makers to prevent a natural outcome from taking place. If outside competition were fierce, preventing standardization would be extremely difficult, since as long as firms keep improving their quality, standardization happens even if the incumbent platform possesses only a slight quality advantage over the entrants. However, if outside competition were low, variety would prevail. Higher initial quality advantages of the incumbent platform would reduce exit and entry, while lower initial quality advantages would spur this activity. Furthermore, worries about standardization might be unwarranted. If we slightly stretch the model, we can continue to argue that once industries mature, and innovation rates slow down, more variety would emerge, including new platforms.

Having established that standards can be replaced, I next turn to study the average expected life span of a standard. Based on 100 simulations of 1,000,000 periods each, I calculate the average life span of a standard; i.e., the number of consecutive periods during which one of the platforms had more than 95% market share and there is no software available for the competing platform. As the table below shows, the expected average life span of a standard depends on the level of competition from substitute industries as well. Table 2 gives the expected number of periods an established standard would live as a function of the level of competition from substitute industries, δ.

The results show that a standard's expected lifetime peaks at $\delta = 0.44$ with and expected lifetime of almost 400,000 periods. That is, as δ increases beyond 0.44, the convergence process speeds up; however, an established standard would live for a shorter period. The intuition behind this result is as follows. When δ increases, two contrasting effects emerge: on one hand, the probability of getting to states with quality levels of (1,1) increases, thereby increasing the probability of achieving states where entry onto a competing standard is possible. On the other hand, as δ increases, the average lifetime of entrants decreases, decreasing the expected value from entry and therefore the probability of entry. The product of these two contrasting effects seems to be minimized at $\delta = 0.44$, causing the life span of a standard to peak at this point. Based on these results, we should, therefore, expect markets to go through cycles of standards, interrupted by short convergence periods in which more than one standard is active in the market.

3.2. *The effect of initial competitive structure*

I turn, now, to analyzing the effect of the initial competitive structure of the industry on the convergence process. I first study the effect of initial differences across platforms; I then turn to the effect of initial differences across firms within the same platform. Finally, I analyze the effect of the initial distribution of consumers across the platforms on the convergence process. As noted before, the results show that the initial state affects the speed of convergence but not the long-run structure of the market.

3.2.1. The effect of inter-platform competition

The static literature on standardization predicts 'tipping.' That is, an initial advantage of one technology will eventually lead to standardization on that platform. In a dynamic setting, leads may be contested and lost market shares may potentially be won back in later periods. In order to analyze the effect of inter-platform competition on standardization, I study the importance of quality asymmetries across platforms. Fig. 4 shows the probability of standardization after a given number of periods, depending on the initial structure of the industry. In order to facilitate the representation of a 3-dimensions graph, I take the initial industry structures to be the *envelope sum of qualities* on each platform, where, for each platform, only those combinations of qualities that guarantee the highest market share for this platform are considered. Platform A's initial market share is assumed to be 0.5. The following set of graphs give the cumulative probability of standardization for $\delta = 0.4$ and 0.7.

In each panel, the left graph gives the probability that the market standardized – one of the platforms achieved a market share of at least 95%, and there is no software available for the competing platform – after 5 periods or less. The middle graph adds up two additional cases: (1) the probability of standardization after 6–10 periods and (2) the probability of standardization after 11–20 periods. Finally, the graph on the right adds the cases where the convergence process takes 21–40 periods or takes 41–100 periods.

In general, if $\delta = 0.4$, in few cases does standardization take longer than 100 periods. In particular, when quality differences across platforms are large, standardization is the most frequent outcome within the first five periods. It is almost guaranteed within 10 periods. Longer standardization processes (indicated by the 'canyon') only happen when quality differences are small across platforms and are large relative to the outside good.

Higher levels of outside competition do not change the results by much, but certainly speed up the process. In the case of $\delta = 0.7$, we see almost no cases where variety prevails for more than 40 periods. The standardization process happens faster and the state space that supports longer standardization processes gets narrower, indicating that only very close competition across platforms prevents early standardization.

3.2.2. The effect of intra-platform competition

I analyze the effect of intra-platform competition in an analogous way. I fix the initial quality levels on platform A to be the maximum for both firms (i.e., six each), fix platform A's initial market share to 0.5, and plot the probability of standardization depending on the quality level of both software firms on platform B.[21] Fig. 5 shows the cumulative probability of standardization after a given number of periods for the case of $\delta = 0.4$.[22]

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[21] The results stay qualitatively the same for other quality levels on platform A.

[22] The case of $\delta = 0.7$ can be found in Fig. 7 in Appendix B.

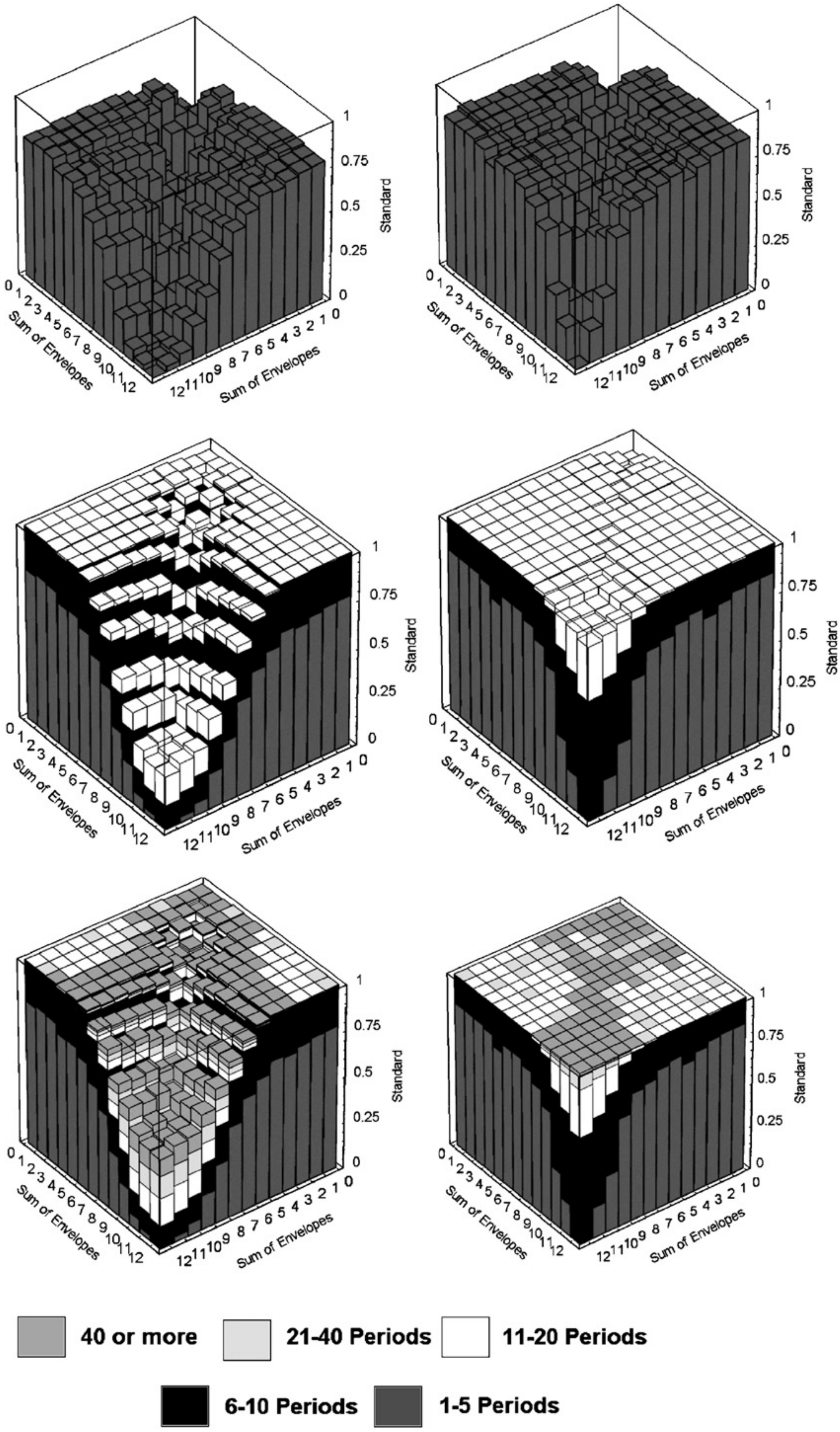

Fig. 4. Cumulative probability of standardization for $\delta = 0.4$ and 0.7.

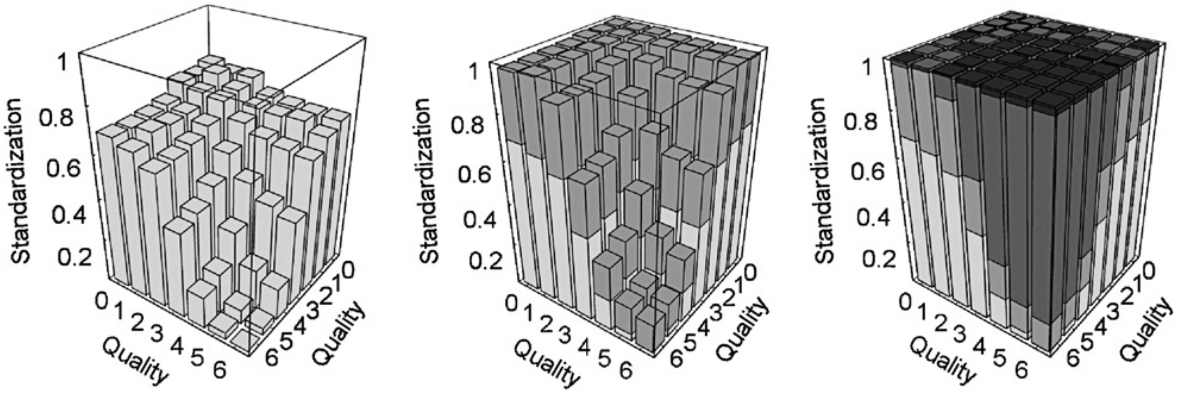

Fig. 5. Cumulative probability of standardization for $\delta = 0.4$.

As before, the figure presents the cumulative probability of standardization after 5, 6–10, and more than 10 periods. The figure shows that standardization takes place quite rapidly. Furthermore, initial similarities in qualities are not enough to ensure survival in the market for long – high quality relative to the outside good is required, as well. As suggested before, this process is even more pronounced when the level of outside competition is higher. In this case, only high-quality close competitors can survive for prolonged periods of time.

3.2.3. *The effect of initial market share*

Finally, I consider the effect of the platforms' initial market shares. In order to study this effect, I again plot the cumulative probability of standardization after 5, 6–10, and more than 10 periods. Fig. 6 plots the cumulative probability of standardization as a function of platform A's initial market share and the level of outside competition, δ. In the initial state, all software firms' quality levels are fixed at 4.[23]

Fig. 6 shows that, when the level of outside competition is high, a large initial market share for either platform speeds up the standardization process. However, when δ is low, even when starting with one standard, the long-run market structure will always be variety. That is, as noted before, the initial distribution of consumers across the platforms affects the short-run dynamics but does not determine the long-run market structure.

4. The drivers of standardization in a dynamic context

The discussion above allows the identification of the main trade-offs in hardware–software settings when the dynamics in the market are driven by the

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[23] I arbitrarily set the initial quality levels at 4. While higher (lower) initial qualities would slow down (speed up) the standardization process, the results stay qualitatively the same. Obviously, initial quality levels favoring one of the platforms will speed up the standardization process, and change the symmetry of the results. Still the effect of δ is unchanged.

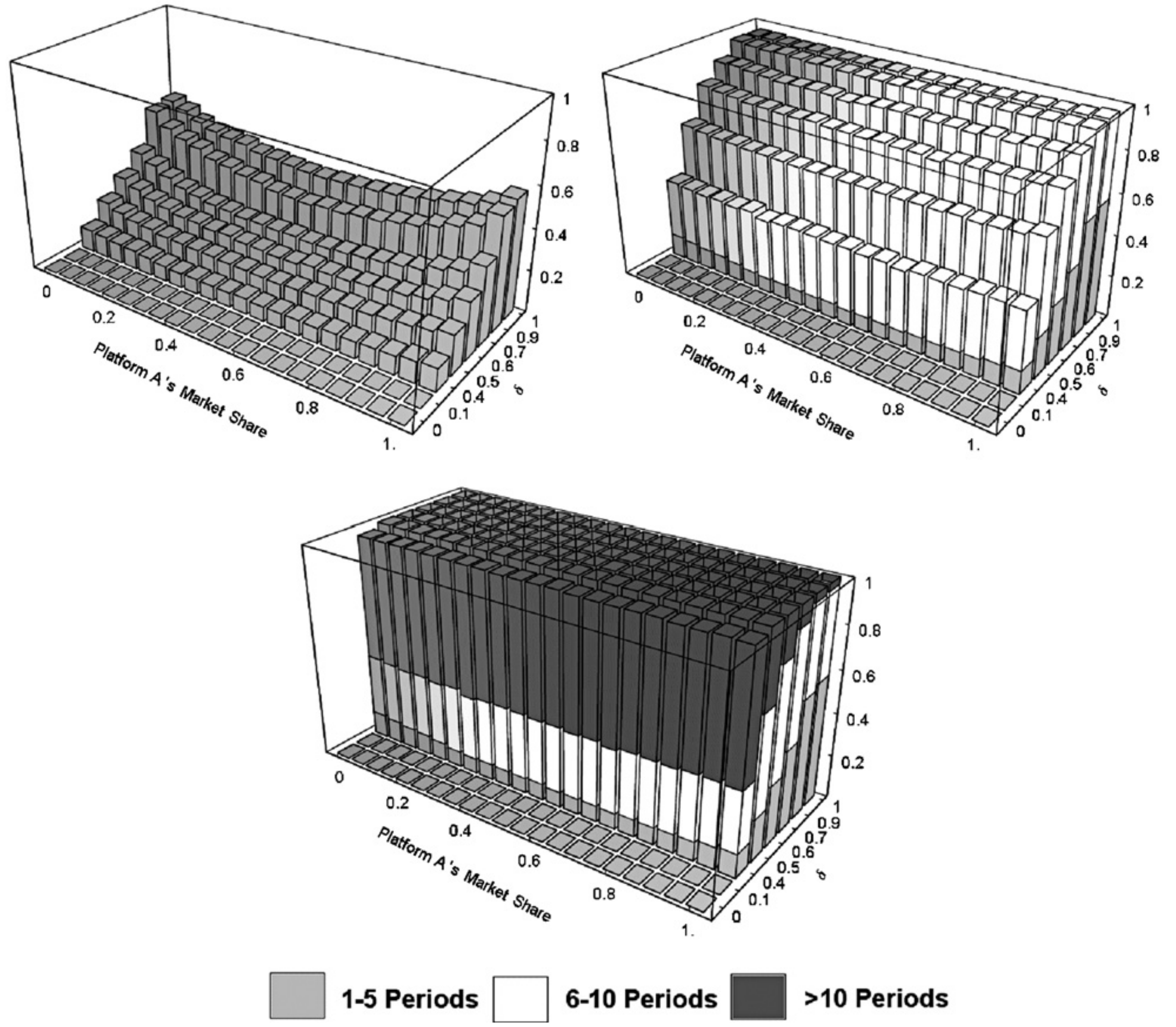

Fig. 6. Cumulative probability of standardization.

quality and availability of software and by competition from potential substitute markets. Under the model's assumptions, the number of platforms that the market can support crucially depends on the degree of competition from substitute markets: stronger competition from substitute goods leads to a smaller number of platforms. On the other hand, if there is little competition from substitute industries, standardization is unlikely, since individual profit considerations entice risk-neutral firms to enter the industry. Furthermore, if the standard platform is monopolized, entry into that platform is very lucrative, since the existing firm already attracts customers to the platform – some of which will then switch to the new software entrant.

The model's results tend to support an overall balanced variety across markets. Standardization will occur only if there are sufficiently up-to-date substitute goods produced in other markets. Coexistence of platforms arise if there is no strong competition from substitute goods. In addition, incentives to innovate further disappear once the industry matures. In other words, if an industry is far ahead of outside competition, it will cease to innovate. Taking the model literally, this implies the stark result that lack of standardization should be of higher concern than standardization, since it is an indicator of potentially too-slow competition from substitute industries and likely too-low competition, overall. Of course, this conclusion should be reached with care. Nevertheless, it has some important implications for policy: standardization should not be prevented wherever it occurs as the result of a competitive market process. Lack of standardization should not be treated with enforced standardization, either, but should rather draw attention to related industries which have the potential to supply substitute products.

At this point, it seems worthwhile to compare again these results with those of the static literature. Church and Gandal (1992) and Farrell and Saloner (1985a) find that the main drivers of standardization are the level of consumer preferences over the hardware technologies relative to how much consumers value variety in software and relative to their benefit from the size of the network, respectively. Note, however, that while the static models explicitly specify the effect of network effects in the consumer's utility function, in this paper, network effects are a result rather than an assumption. Consumers do not value a large network or a large variety of software, *per se*, but rather they value quality. These preferences together with the firms' desire to maximize profits create indirect network effects such that, the larger the market share of a specific platform, the more software firms would want to produce software for this platform and, therefore, the more intense competition would be. Competition increases the quality level and decreases prices, which in turn increase consumers' utility.

Furthermore, the static models predict that the markets will exhibit *excess inertia*. However, as noted before, in the present paper the market would never get 'stuck' on an inferior platform. This does not necessarily mean that the market would immediately switch to the better platform. On the contrary, as in the case of the 'browser war,' the most likely scenario is that the incumbent standard would fight back by improving its own quality. Then, depending on the level of competition from substitute markets, the market would either split between both platforms or would standardize on one of them. Nevertheless, in the long run, the inferior platform would not stay as the market's status quo. Going back to the 'browser war,' we can see that Microsoft has fully understood the rules above. And, indeed, during the time in question, the firm invested hundreds of millions of dollars developing technologies like ActiveX and VB-script to help software firms with developing high-quality applications for Internet Explorer. Note that using these tools also prevented the applications from running on Netscape's Navigator.

An important point underlies the differences above. In the previous literature, the variety of software was either ignored or almost instantaneously determined by the platform's market share. Consequently, firms could not affect their quality and there were no quality differences between software firms. In practice, however, software firms tend to invest large sums in R&D and in improving their products. For example, in 2003, IBM invested over $5 billion in research, development, and engineering, out of which $2.3 billion was software-related.[24] Microsoft invested

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[24] IBM, Annual Report, 2004, p.104.

during this same year \$4.7 billion dollars in R&D, and Oracle invested \$1.2 billion in R&D (corresponding to more than 16% of gross revenues and more than 50% of net income).[25,26] In this model, therefore, the prediction is just the opposite of that in the static literature: the quality levels of software firms determine the platforms' market share, and gradual quality upgrades in the software market can correct initially unfavorable market shares. However, if one firm gets behind in a highly competitive environment, it almost definitely must exit. While in the early days of DOS, UNIX, and Windows, one might be able to ascribe a substantial share in utility to be delivered by a certain platform relative to competing platforms, especially its ease of use, this seems less convincing when we think of today's computers or cell-phones. I, therefore, think it is important to be aware of how much both hardware and software contribute to utility before one can judge which one of these two approaches is more appropriate.

5. Conclusions

This paper analyzes the drivers of standardization when platforms' market shares are driven by innovation in the software market. I find that competition from substitute industries is the sole determinant of standardization: fierce outside competition leads to standardization, while the lack of outside competition leads to variety with multiple platforms. Inter- and intra-platform competition, as well as the initial market shares of platforms, affect only the probability of 'snowballing' to each platform and the duration of the standardization process. Standardization will be replaced by entry if market conditions are favorable for variety, but initial variety will not prevent standardization if market conditions favor standardization. Finally, I find neither excess inertia nor tipping – asymmetric development comes exclusively from unfavorable investment outcomes, which prove to be fatal more quickly in the face of high competition, but which can be reversed and salvaged in less-competitive environments.

These results suggest that when market forces are in play, consumers will have the choice of variety supplied from within or, although not directly analyzed, outside the industry. Furthermore, consumers will be able to choose from higher-quality goods overall when there is fierce competition, which, however, will unavoidably involve standardization. In these cases, policies that try to prevent standardization may be wrong and may actually weaken the industry, since each firm will not have sufficient funds to meet competition from substitute producers.

Finally, it is important to acknowledge the limitations of this study. While the model itself is general and rich, the results presented here are based on specific parameters. Furthermore, the paper shows that when consumers care only about quality, the market will not exhibit excess inertia. Whether this result would still hold in the case where consumers care about both quality and variety remains as a

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[25] Microsoft, Annual Report, Form K-10, 2004, p. 23.

[26] Oracle, Condensed Consolidated Statements of Operations, 2004, pp. 1–2.

question. While a full analysis is beyond the scope of this paper, one would expect the results in this case to depend on the importance of quality relative to variety.

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Appendix A

Proposition 4 Doraszelski and Satterthwaite (2007), hereafter DS (2005), requires the following to hold for a symmetric and anonymous equilibrium to exist:

(a) The state space is finite.
(b) Profits are bounded.
(c) Investments are bounded.
(d) Firms discount future values.
(e) The distributions of scrap values and setup costs have continuous and positive densities and bounded support.
(f) Firms' local income function is a continuous function of their strategies and value function.
(g) The transition function is UIC admissible.
(h) $\bar{x}$ and $\bar{x}^{\mathrm{e}}$ are finite and larger than $\beta(\bar{V}^* - \underline{V}^*)$.
(i) The local income functions are symmetric and exchangeable.

Assumptions (a)–(d), (h) and (i) are straightforward and do not require a proof. According to DS (2005), if the profit function is additively separable from investment, the continuity in assumption (f) merely requires continuity of the transition function. Consequently, the only assumption that has to be shown that holds in this model is assumption (g). Computation easily shows that the expected value of firms' future cash flow satisfies the separability condition (25) and the derivative condition (26) in DS (2005). That is, the transition probability in this paper satisfies condition 1 in DS (2005) and is, therefore, UIC admissible. Finally note, while condition 4 assumes that both the scrap value and the setup costs are random variables – assumption (e) – in this model in practice it is sufficient to assume that the distribution of the scrap value is degenerate.

Appendix B

Fig. 7 presents the cumulative probability of standardization after 5, 6–10, and more than 10 periods (from left to right), as a function of the initial qualities on

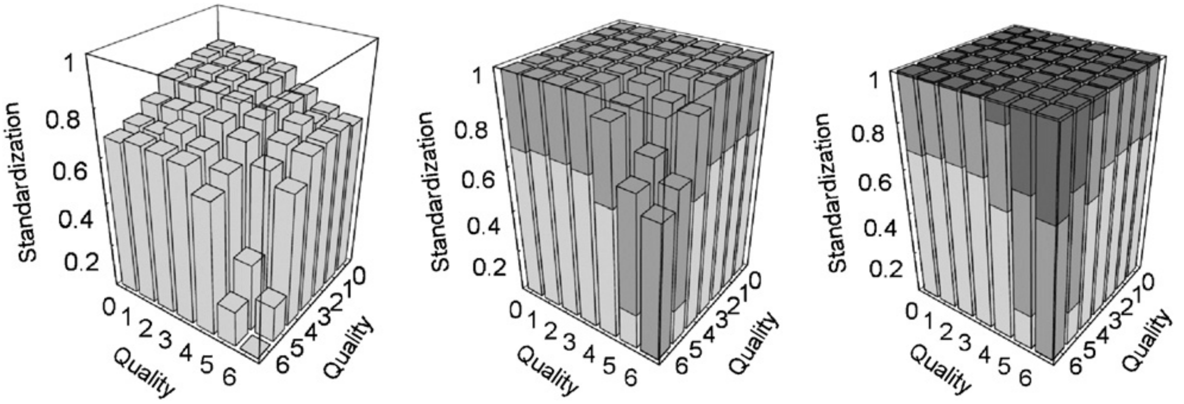

Fig. 7. Cumulative probability of standardization for $\delta = 0.7$.

platform B. Initial quality levels on platform A are fixed at six for both firms, and platform A's initial market share is fixed at 0.5.

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