Advertising Dynamics and Competitive Advantage*

Ulrich Doraszelski
Department of Economics, Harvard University, and CEPR†

Sarit Markovich
Recanati Graduate School of Business Administration, Tel Aviv University‡

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Abstract

Can advertising lead to a sustainable competitive advantage? To answer this question, we propose a dynamic model of advertising competition where firms repeatedly advertise, compete in the product market, and make entry as well as exit decisions. Within this dynamic framework, we study two different models of advertising: In the first model, advertising influences the goodwill consumers extend towards a firm ("goodwill advertising"), whereas in the second model it influences the share of consumers who are aware of the firm ("awareness advertising"). We show that asymmetries may arise and persist under goodwill as well as awareness advertising. The basis for a strategic advantage, however, differs greatly in the two models of advertising. We show that tighter regulation or an outright ban of advertising may have anticompetitive effects and discuss how firms use advertising to deter and accommodate entry and induce exit in a dynamic setting.

1 Introduction

During 2003 close to 250 billion dollar was spent on advertising in the U.S., well above 2% of GDP. Practitioners know very well the value of advertising to achieving their long-term market share and profitability goals. A 1999 survey of senior executives, conducted by the American Advertising Federation, reveals that 82.9% somewhat or strongly agree that good advertising can provide their company with an edge over the competition in the marketplace.

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†Cambridge, MA 02138, U.S.A., doraszelski@harvard.edu.

‡Ramat Aviv, Tel Aviv 69978, Israel, saritm@post.tau.ac.il.

Furthermore, 86.8% somewhat or strongly agree that advertising is a long-term investment that contributes to the financial growth and stability of their company. In 1991 the chairman and CEO of Procter & Gamble Co., one of the world's largest advertisers, Edwin L. Artzt summarized his view of advertising: "Think of advertising and promotion as exercise and recreation. Advertising is exercise. It's something you need and it provides long-term benefits, but it's awfully easy to either cut or postpone because there's no immediate penalty for not exercising. ... When you want your brand to be fit, it's got to exercise regularly."[1]

Practitioners, it seems, presume that advertising is capable of giving them a sustainable competitive advantage over their rivals. The existing dynamic models of advertising competition, however, suggest quite the opposite. In these models there is a globally stable symmetric steady state (see e.g. Friedman 1983, Fershtman 1984, Chintagunta 1993, Cellini & Lambertini 2003).[2] Consequently, any differences among firms are bound to vanish over time, and there is no room for a sustainable competitive advantage, not even if firms enter the market one by one and thus differ in their strategic positions at the outset of the game (Fershtman, Mahajan & Muller 1990).

The goal of this paper is to reconcile theory and observation by showing that advertising can indeed have a lasting effect on the structure of an industry. That is, in contrast to the existing literature, we provide conditions under which persistent differences arise for strategic reasons as an equilibrium phenomenon among *ex ante* identical firms.

We propose a dynamic model of advertising competition that adapts the Markov-perfect-equilibrium framework presented in Ericson & Pakes (1995) to track the evolution of an industry. More specifically, we allow firms to advertise on an ongoing basis and to compete repeatedly in the product market, where consumers choose between the differentiated products on offer. In addition to making advertising and pricing decisions, incumbent firms decide whether to remain in the industry and potential entrants decide whether to enter. Within this dynamic framework, we first specify how advertising affects consumers and build up a model of product market competition. We then solve numerically for the symmetric Markov perfect equilibrium (MPE) in order to characterize industry dynamics and identify circumstances under which asymmetric industry structures arise and persist over time.

To study the implications of advertising for industry dynamics, we first need to understand the way in which advertising affects consumers. The literature (see Bagwell (2005) for a survey) has traditionally emphasized the persuasive nature of advertising: Its purpose

[1] J. Lawrence, "P&G's Artzt on ads: Crucial investment," *Advertising Age*, October 28, 1991.

[2] To be precise, the steady state is symmetric provided that the economic fundamentals are the same across firms. Of course, a competitive advantage can arise and persist in a game in which firms differ in their economic fundamentals (e.g., cost structures). However, rather than explaining asymmetries, this begs the question how such differences in "initial conditions" come about in the first place.

is to alter consumers' tastes for established brand names or company reputations (Dixit & Norman 1978). Stigler & Becker (1977) and Becker & Murphy (1993) argue somewhat differently that advertising is part of consumers' preferences in the same way as goods and that there are complementarities between advertising and goods. Hence, a more-advertised good is *ceteris paribus* preferred over a less-advertised good. Common to the persuasive and the complementary view is that advertising affects the utility derived from consuming a particular product. We capture this idea in a model of *goodwill advertising*.[3] Following Nerlove & Arrow (1962), we take goodwill to be a stock related to the flow of current and past advertisements. This stock depreciates over time, reflecting the loss in effectiveness of past advertising campaigns. A firm therefore advertises in order to keep up its stock of goodwill as well as to add to it.

Another strand of the literature views advertising as information: Advertising aids the operation of markets by helping to identify buyers and sellers or by making the terms of sale known (Stigler 1961). This is especially important in markets for differentiated products because, if products are differentiated, a consumer may be unaware of the very existence of a particular product unless she sees it advertised. We capture this informative role of advertising in a model of *awareness advertising*, where advertising influences the share of consumers who know about the firm and its product (Grossman & Shapiro 1984, Fershtman & Muller 1993, Boyer & Moreaux 1999).[4]

The informative role of advertising has previously been analyzed using static models. However, static models cannot tell us anything about whether or not a competitive advantage persists. Moreover, they suggest that a competitive advantage does not even arise in the first place. Butters (1977), Stegeman (1991), and Robert & Stahl (1993) study price advertising in markets for homogenous goods. Although the equilibrium in these models is characterized by price dispersion, all firms make zero profits and none has a competitive advantage. Turning to differentiated products and awareness advertising, no asymmetries arise in the equilibrium of Grossman & Shapiro's (1984) model. On the other hand, Fershtman & Muller (1993) and Boyer & Moreaux (1999) show in the context of a static game of awareness choice followed by price competition that firms may opt for less than full aware-

[3]That is, our goodwill model can be interpreted according to either view unless the goal is to make welfare statements. As Dixit & Norman (1978) note, making welfare statements is tricky at best if advertising is assumed to alter consumers' preferences. This is not an issue if the complementary rather than the persuasive view of advertising is adopted because then consumers have well-defined preferences over advertising.

[4]We restrict attention to directly informative advertising. In contrast, building on earlier work by Nelson (1974), Kihlstrom & Riordan (1984) and Milgrom & Roberts (1986), among others, analyze advertising for experience goods rather than search goods. In this context, advertising is indirectly informative because consumers draw inferences about a product merely from the fact that the firm spends on advertising. Advertising may also signal information about firms themselves. For example, a firm may use advertising to signal its low cost and the associated low price (Bagwell & Ramey 1994a, Bagwell & Ramey 1994b). Finally, advertising may help to coordinate the expectations of consumers about the purchasing decisions of others in markets with consumption externalities (Pastine & Pastine 2002).

ness even if advertising were costless. Yet, they are silent as to whether the equilibrium outcome is going to be symmetric or asymmetric.

In contrast to its informative role, the persuasive role of advertising has already been analyzed using dynamic models. Our model of goodwill advertising improves upon earlier work in a number of ways.[5] First, the existing dynamic games either abstract from product market competition altogether by assuming a constant markup (Fershtman 1984, Fershtman et al. 1990, Chintagunta 1993) or use a reduced-form demand specification that depends on stocks of goodwill in a somewhat arbitrary (linear-quadratic) fashion (Friedman 1983, Cellini & Lambertini 2003). In contrast, we start with consumer behavior and derive the model of product market competition. This enables us to specify in more detail why and how goodwill advertising affects consumer choice. Second, we compute the MPE rather than the open-loop equilibrium (e.g., Friedman 1983, Fershtman 1984). As is well known, open-loop equilibria may be based on threats and promises that are not credible and hence in general fail to be subgame perfect. Third, the recent literature on models of industry evolution points out the important role that idiosyncratic shocks play in explaining the great variation in the fate of similar firms over time (Jovanovic 1982, Hopenhayn 1992). We incorporate this insight by making the law of motion of a firm's stock of goodwill stochastic, whereas existing dynamic games are deterministic. Fourth, we incorporate entry and exit as key drivers of industry evolution (see e.g. Dunne, Roberts & Samuelson (1988) for empirical evidence).

The benefits from improving upon earlier work are most obvious in comparison to linear-quadratic games (Friedman 1983, Cellini & Lambertini 2003). Since the law of motion in such a game is given by a system of linear difference (or differential) equation, the dynamics are generically either explosive and thus inconsistent with equilibrium or there is a globally stable symmetric steady state. Hence, the very nature of a linear-quadratic game goes against the notion of a sustainable competitive advantage. Taken together, our departures from earlier work lead to a model of advertising competition that exhibits much richer dynamics. On the other hand, they force us to leave analytically tractable modelling frameworks such as linear-quadratic games behind.

Instead we rely on numerical methods to compute the MPE for each of our two models of advertising and analyze the evolution of the industry. We show that, under goodwill as well as awareness advertising, asymmetries arise and persist provided that one firm has a strategic advantage over the other. The tangible form of this advantage is that one firm can deter the other from advertising. The basis for a strategic advantage, however, differs

[5]The literature has traditionally taken a number of shortcuts to modelling dynamic advertising competition, including hazard rate models, passive rival models, and reaction function models. Only recently have dynamic games been developed (see Chapter 11 of Dockner, Jorgensen, Van Long & Sorger (2000) for a survey).

markedly in the two models of advertising.

Under goodwill advertising, the size of the market and the cost of advertising are key determinants of industry structure and dynamics. In particular, goodwill advertising leads to an extremely asymmetric industry structure with a large and a small firm if the market is small or if advertising is expensive. Because the marginal benefit of advertising is small relative to its cost, a small firm has only a weak incentive to advertise when competing against a large firm and, in fact, may choose not to advertise at all. If the market is large or if advertising is cheap, on the other hand, even a small firm has a fairly strong incentive to advertise. In this case we obtain a symmetric industry structure with two large firms.

In contrast to the cost/benefit considerations that give rise to a strategic advantage under goodwill advertising, whether or not asymmetries arise and persist under awareness advertising depends on the intensity of product market competition. If competition is soft, the industry evolves towards a symmetric structure, but it evolves towards an asymmetric structure if competition is fierce. Industry dynamics in this latter case resemble a rather brutal preemption race. During this race, both firms advertise heavily as long as they are neck-and-neck. Once one of the firms manages to pull even slightly ahead, however, its rival "gives up," thereby propelling the firm into a position of dominance. The ensuing asymmetric industry structure persists because it is in the self-interest of the smaller firm to stay behind. In fact, the nature of product market competition is such that once the smaller firm tries to grow, the larger firm responds aggressively by triggering a "price war," thereby pushing prices and hence profits down. This gives the smaller firm an overwhelming incentive to remain inconspicuous.

Our results yield novel insights into the link between advertising restrictions and industry concentration. Whereas the market power theory of advertising (Kaldor 1950, Bain 1956, Comanor & Wilson 1974) holds that restricting persuasive advertising aids competition, we show that tighter regulation or an outright ban of goodwill advertising may have anticompetitive effects. The key insight here is that regulating or banning advertising makes it harder and thus costlier for firms to reach consumers. Given that asymmetries are rooted in cost/benefit considerations in the case of goodwill advertising, this may pave the way for one firm to dominate the industry. Our results are consistent with the empirical finding that concentration has increased after regulation was implemented in industries like cigarettes (see e.g. Eckard 1991) and beer (Sass & Saurman 1995), where advertising is arguably persuasive rather than informative in nature.

Our dynamic framework also lends itself to studying the role of advertising as a barrier to entry. An incumbent deters entry by over-advertising and, in general, accommodates entry by under-advertising. While this contrasts with a part of the existing literature (Schmalensee 1983, Fudenberg & Tirole 1984), the intuition is the same: The incumbent aspires to make post-entry competition in the product market fiercer when it comes to deterring the entrant but to soften it when it comes to entry accommodation. In our model of awareness advertising, in turn, over-advertising makes product market competition fiercer, whereas under-advertising softens it. Our dynamic framework points to yet another consideration. If post-entry industry dynamics take the form of a preemption race, then the incumbent accommodates the entrant by over-advertising in a bid to gain a head start in the race and in this way improve its chances of eventually becoming the dominant player in the industry. This purely dynamic consideration, in fact, is strong enough to override the incumbent's desire to soften product market competition by under-advertising. Taking industry dynamics into account is thus crucial to analyzing barriers to entry.

In sum, this paper bridges the gap between the "micro foundations" of advertising competition and the existing dynamic games. By starting with consumer behavior and building up a model of product market competition, we are able to study different models of advertising while holding consumers' preferences constant. We do not restrict ourselves to the persuasive aspects of advertising, but are the first to study its informative aspects in a dynamic setting. This allows us to shed new light on sources of asymmetries in dynamic models of advertising competition. Understanding the mechanisms behind persistent asymmetries has important implications for regulatory policy and, in turn, aids our understanding of the role of advertising as a barrier to entry.

The remainder of this paper is organized as follows: We set up the basic model without entry and exit in Section 2. We present our results in Section 3. In Section 4 we discuss the link between advertising restrictions and industry concentration. In Section 5 we study firms' entry and exit decisions and their impact on the structure of an industry. Section 6 presents a number of robustness checks and an alternative interpretation of the model. Section 7 summarizes and concludes.

2 Model

The model is cast in discrete time and has an infinite horizon to avoid end effects. There are two firms with potentially different levels of goodwill or awareness. Each firm is in turn able to influence its goodwill (awareness) level through advertising.

Setup and timing. We assume that the goodwill consumers extend towards a firm is at one of L levels and set $-\infty < v_0 < v_1 < \ldots < v_{L-1} < \infty$. Similarly, the share of consumers who are aware of the firm is at one of L levels $0 \leq s_0 < s_1 < \ldots < s_{L-1} \leq 1$. In each period, a firm decides how much to advertise in order to add to its goodwill (awareness). At the same time consumers forget, and the firm is bound to lose some of its goodwill (awareness). In other words, goodwill (awareness) decays. The outcomes of the advertising and forgetting processes are assumed to be stochastic. Thus, even if a firm advertises, it is not guaranteed that its goodwill (awareness) increases. Moreover, the firm's goodwill (awareness) might decrease due to forgetting despite advertising.

After making their advertising decisions but before the outcomes of the advertising and forgetting processes are realized, firms compete in the product market. Profits from product market competition in each period are determined by firms' levels of goodwill (awareness) (v_i, v_j) $((s_i, s_j))$. To simplify notation we take (i, j) to mean that firm 1's level of goodwill (awareness) is v_i (s_i) and firm 2's level of goodwill (awareness) is v_j (s_j), and denote the profit functions of firm 1 and firm 2 by $\pi_1(i, j)$ and $\pi_2(i, j)$, respectively. We first provide details on the product market game under goodwill advertising and then under awareness advertising. Finally, we turn to the dynamic framework. We present the derivations for firm 1, the derivations for firm 2 are analogous.

Goodwill advertising. Suppose that firms' levels of goodwill are (v_i, v_j). Taking their goodwill levels as given, firms compete in the product market by setting prices (p_1, p_2). Our demand specification is similar to the discrete-choice models that are often used in the empirical industrial organization literature (see e.g. Berry 1994, Berry, Levinsohn & Pakes 1995). Consumers are heterogeneous in their tastes. Each consumer purchases at most one good. The utility consumer m derives from purchasing from firm 1 is

$$v_i - p_1 + \epsilon_{m1},$$

where ϵ_{m1} represents taste differences among consumers. Note that the utility difference between consuming and not consuming good 1, $(v_i - p_1 + \epsilon_{m1}) - 0$, is increasing in v_i. This specification thus implies that advertising and goods are complementary in the sense of Becker & Murphy (1993).

Besides the two goods offered by the two firms, there is an outside good, good 0, which has utility ϵ_{m0}. In this way we allow advertising to have a market-size effect in addition to a market-share effect (see e.g. Roberts & Samuelson (1988), Slade (1995), and Kadiyali (1996) for empirical evidence).

Assuming that the idiosyncratic shocks ϵ_{m0}, ϵ_{m1}, and ϵ_{m2} are independently and identically type 1 extreme value distributed, the probability that a randomly chosen consumer purchases from firm 1 is

$$D_1(p_1, p_2; i, j) = \frac{\exp(v_i - p_1)}{1 + \exp(v_i - p_1) + \exp(v_j - p_2)}.$$

The profit-maximization problem of firm 1 is thus given by

$$\max_{p_1 \geq 0} M D_1(p_1, p_2; i, j) p_1$$

where $M > 0$ is the size of the market (the measure of consumers) and, in the interest of parsimony, we abstract from marginal and fixed costs of production.[6] The first-order condition (FOC) is

$$0 = 1 - \frac{1 + \exp(v_j - p_2)}{1 + \exp(v_i - p_1) + \exp(v_j - p_2)} p_1.$$

It can be shown that there exists a unique Nash equilibrium $(p_1^*(i,j), p_2^*(i,j))$ of the product market game (Caplin & Nalebuff 1991). The Nash equilibrium can be computed easily by numerically solving the system of FOCs. The per-period profit of firm 1 in the Nash equilibrium of the product market game under goodwill advertising is then given by $M\pi_1(i,j)$, where

$$\pi_1(i,j) \equiv D_1(p_1^*(i,j), p_2^*(i,j); i, j) p_1^*(i,j) \tag{1}$$

is the profit per consumer.

Awareness advertising. While the above model emphasizes the persuasive aspects of advertising, advertising is informative in our next model. Our product market game under awareness advertising is similar to Fershtman & Muller (1993) and Boyer & Moreaux (1999).

Suppose that firms' levels of awareness are given by (s_i, s_j). Since we now assume that advertising influences awareness rather than goodwill, the utility that consumer m derives from good 1 becomes

$$v - p_1 + \epsilon_{m1}.$$

We refer to v as the perceived quality of firms' products in order to clearly distinguish it from their levels of goodwill in the model of goodwill advertising. Note that the consumer perceives the products of both firms to be of the same quality, reflecting the informative nature of advertising. There is again an outside good, which has utility ϵ_{m0}.

All consumers are aware of the outside good. In addition, a share s_i of consumers is aware of firm 1 and a share s_j of consumers is aware of firm 2. Depending on their choice set, consumers can therefore be divided into four mutually exclusive and exhaustive segments: a group that is aware of neither good 1 nor good 2, a group that is aware of good 1 but not good 2, a group that is aware of good 2 but not good 1, and a group that is aware of both goods.

[6] Adding a constant to the marginal cost of production and the level of goodwill does not change the per-period profit. Setting marginal cost to zero is thus without loss of generality. A firm's advertising strategy is furthermore independent of its fixed cost of production.

Assuming that consumers are exposed to advertising at random, the event of being aware of firm 1 is independent of the event of being aware of firm 2. Hence, the probability that a randomly chosen consumer belongs to the four segments is $(1-s_i)(1-s_j)$, $s_i(1-s_j)$, $(1-s_i)s_j$, and $s_i s_j$, respectively. It follows that the probability that a randomly chosen consumer purchases from firm 1 is

$$D_1(p_1, p_2; i, j) = s_i(1-s_j)\frac{\exp(v-p_1)}{1+\exp(v-p_1)} + s_i s_j \frac{\exp(v-p_1)}{1+\exp(v-p_1)+\exp(v-p_2)}. \qquad (2)$$

Equation (2) shows that firm 1's demand is composed of a captive segment of consumers who do not know of firm 2 and a competitive segment of consumers who know of firm 2. The size of these segments is proportional to $1-s_j$ and s_j, respectively. Moreover, as the perceived quality v goes up, the inside goods become more attractive relative to the outside good to both segments of consumers, and competition thus intensifies.

The FOC arising from firm 1's profit-maximization problem is

$$\begin{aligned} 0 &= (1-s_j)\frac{\exp(v-p_1)}{1+\exp(v-p_1)}\left(1-\frac{1}{1+\exp(v-p_1)}p_1\right) \\ &\quad + s_j\frac{\exp(v-p_1)}{1+\exp(v-p_1)+\exp(v-p_2)}\left(1-\frac{1+\exp(v-p_2)}{1+\exp(v-p_1)+\exp(v-p_2)}p_1\right). \end{aligned}$$

In general, there may not be a Nash equilibrium in pure strategies.[7] However, by verifying that none of the two firms has a profitable unilateral deviation, it is easy to ensure that a numerical solution to the system of FOCs constitutes a Nash equilibrium. The per-period profits are then constructed in the same way as in the model of goodwill advertising (equation (1)).

State-to-state transitions. We now turn to the dynamic framework. Recall that we take (i, j) to mean that firm 1's goodwill (awareness) is v_i (s_i) and firm 2's goodwill (awareness) is v_j (s_j). Hence, the industry is completely described by the tuple $(i, j) \in \{0, \ldots, L-1\}^2$. We call (i, j) the state of the industry. Given that the industry is in state (i, j) today, it will be in state (i', j') tomorrow. Our next task is to specify the probability distribution that governs the state-to-state transitions.

Consider firm 1. Its transition between goodwill (awareness) levels depends on how much it advertises and on how easily consumers forget. We think of the advertising and forgetting processes as follows: In each period, firm 1 invests kx_1 in an advertising campaign,

[7] Note that a firm faces a choice between setting a low price in order to be competitive and exploiting its captive segment by setting a high price. This may give rise to a discontinuity in the firm's best reply function and ultimately lead to nonexistence of a Nash equilibrium in pure strategies. Our computations suggest that this actually happens for high values of v.

where $x_1 \geq 0$ is the amount of advertising and $k > 0$ measures the cost of advertising. For example, x_1 may represent the number of 30-second TV commercials or pages of advertisements in magazines and k the cost of such a commercial or page. More generally, the amount of advertising is expressed as the number of exposures and its cost as the cost-per-thousand exposures. The more a firm advertises, the higher is the probability that its campaign succeeds in creating goodwill (awareness). In particular, we take the probability of success to be $\frac{x_1}{1+x_1}$. Note that since the second derivative of this probability is negative the "production" of goodwill (awareness) is subject to diminishing returns. At the same time, consumers may forget and the firm may thus lose some of its goodwill (awareness). Forgetting can occur when the effect of past advertising on consumers wears out and is not reinforced by current advertising, when the current advertising campaign is ill-conceived and repels instead of attracts consumers, or when the firm suffers a public relations mishap. We take the probability of forgetting to be δ.

Hence, if $\Pr(i'|i, x_1)$ denotes the probability that firm 1 will be in state i' tomorrow given that it is in state i today, then we have

$$\Pr(i'|i, x_1) = \begin{cases} \frac{(1-\delta)x_1}{1+x_1} & \text{if} \quad i' = i+1, \\ \frac{1-\delta+\delta x_1}{1+x_1} & \text{if} \quad i' = i, \\ \frac{\delta}{1+x_1} & \text{if} \quad i' = i-1 \end{cases}$$

if $i \in \{1, \ldots, L-2\}$. Clearly, firm 1 cannot move further down (up) from the lowest (highest) state. We therefore set

$$\Pr(i'|i, x_1) = \begin{cases} \frac{x_1}{1+x_1} & \text{if} \quad i' = i+1, \\ \frac{1}{1+x_1} & \text{if} \quad i' = i \end{cases}$$

if $i = 0$, and

$$\Pr(i'|i, x_1) = \begin{cases} \frac{1-\delta+x_1}{1+x_1} & \text{if} \quad i' = i, \\ \frac{\delta}{1+x_1} & \text{if} \quad i' = i-1 \end{cases}$$

if $i = L-1$. Note that since we interpret the lowest state as minimal goodwill or zero awareness, it is natural to assume the absence of forgetting in the transition function for $i = 0$.

Bellman equation. Let $V_1(i, j)$ denote the expected net present value to firm 1 of being in the industry given that firm 1's goodwill (awareness) is v_i (s_i) and firm 2's goodwill (awareness) is v_j (s_j). In what follows, we first characterize the value function $V_1(i, j)$ under the presumption that the firm behaves optimally. In a second step, we derive the policy function $x_1(i, j)$. Throughout we take firm 2's advertising strategy $x_2(i, j)$ as given.

The Bellman equation is

$$V_1(i,j) = \max_{x_1 \geq 0} M\pi_1(i,j) - kx_1 + \beta \sum_{i'=0}^{L-1} W_1(i') \Pr(i'|i,x_1), \tag{3}$$

where $0 < \beta < 1$ is the discount factor and

$$W_1(i') = \sum_{j'=0}^{L-1} V_1(i',j') \Pr(j'|j, x_2(i,j)).$$

The Bellman equation adds the firm's current cash flow $M\pi_1(i,j) - kx_1$ and its discounted expected future cash flow. Note that $\sum_{i'=0}^{L-1} W_1(i') \Pr(i'|i,x_1)$ is the expectation over all possible future states (i',j') calculated under the presumption that firm 1 chooses to advertise x_1 and firm 2 chooses to advertise $x_2(i,j)$ in the current state (i,j).

Two remarks are in order. First, since multiplying market size M and advertising cost k by the same constant rescales the value function but preserves the policy function, either M or k could be normalized to unity without loss of generality. Instead, we treat the ratio $\left(\frac{M}{k}\right)$ as the parameter of interest to remind the reader that it is the relative values of market size and advertising cost rather than their absolute values that matter for industry structure and dynamics. Second, while spending kx_1 on an advertising campaign secures the firm a probability of $\frac{(1-\delta)x_1}{1+x_1}$ of adding to its stock of goodwill (awareness), one intuitively expects the required expenditures to vary with market size. In this case, k is implicitly a function of M, and the question is how the ratio $\left(\frac{M}{k}\right)$ changes with M. Empirical evidence (see e.g. p. 37 of Greer 1998) suggests that reaching a given number of consumers is cheaper in larger markets than in smaller ones. The ratio of market size to advertising cost thus continues to be increasing in market size.

Advertising strategy. The FOC for an interior solution is

$$-k + \beta \sum_{i'=0}^{L-1} W_1(i') \frac{\partial \Pr(i'|i,x_1)}{\partial x_1} = 0.$$

Consider $i \in \{1, \dots, L-2\}$. Solving the FOC for x_1 yields

$$-1 + \sqrt{\frac{\beta}{k} \left((1-\delta)(W_1(i+1) - W_1(i)) + \delta(W_1(i) - W_1(i-1))\right)}.$$

The second-order condition (SOC) reduces to

$$-\left((1-\delta)(W_1(i+1) - W_1(i)) + \delta(W_1(i) - W_1(i-1))\right) < 0.$$

Hence, the SOC is satisfied whenever a solution to the FOC exists. Moreover, the objective function equals $M\pi_1(i,j) + \beta\{(1-\delta)W_1(i) + \delta W_1(i-1)\}$ at $x_1 = 0$ and approaches $-\infty$ as x_1 approaches ∞. This implies that the objective function is decreasing when a solution to the FOC fails to exist. (To see this, suppose to the contrary that the objective function is increasing at some point. Since the objective function approaches $-\infty$, it must then have a local maximum and a solution to the FOC would exist.) Thus,

$$x_1(i,j) = \max\left\{0, -1 + \sqrt{\frac{\beta}{k}\left((1-\delta)(W_1(i+1) - W_1(i)) + \delta(W_1(i) - W_1(i-1))\right)}\right\} \quad (4)$$

if this is well-defined and $x_1(i,j) = 0$ otherwise. If $i = 0$ or $i = L-1$, the advertising strategy of firm 1 can be derived using similar arguments.

Equilibrium. Both models of advertising give rise to symmetric profit functions, i.e., $\pi_1(i,j) = \pi_2(j,i)$. We therefore define $\pi(i,j) \equiv \pi_1(i,j)$, note that $\pi_2(i,j) = \pi(j,i)$, and restrict attention to symmetric Markov perfect equilibria (MPE). Hence, if $V(i,j) \equiv V_1(i,j)$ denotes firm 1's value function, then firm 2's value function is given by $V_2(i,j) = V(j,i)$. Similarly, if $x(i,j) \equiv x_1(i,j)$ denotes firm 1's policy function, then firm 2's policy function is given by $x_2(i,j) = x(j,i)$. Existence of a symmetric MPE in pure strategies follows from the arguments in Doraszelski & Satterthwaite (2003) provided that we impose an upper bound on advertising. To compute the MPE we use a Gauss-Seidel variant of the Pakes & McGuire (1994) algorithm.[8] While in general it cannot be guaranteed that the MPE is unique, our computations always led to the same value and policy functions irrespective of the starting values and the particulars of the algorithm.[9]

Parameterization. Since advertising is fairly fast paced, we think of a period as a quarter and accordingly set the discount factor to $\beta = \frac{1}{1.02}$.

While there is little doubt that the passage of time renders advertising less effective, the available estimates of the decay in the impact of advertising on sales differ widely (Clarke 1976). Empirical studies also provide little guidance on the decay of goodwill. Roberts & Samuelson (1988), using yearly data, estimate the rate of retention in the stock of goodwill to be 0.831 and 0.892 for low- and high-tar cigarettes, respectively. This corresponds to decay rates of 0.045 and 0.028 per quarter, respectively, and suggests a small but positive value of δ. In contrast, Jedidi, Mela & Gupta (1999) take the decay rate to be around

[8] Programs are available from the authors upon request.

[9] Our default starting values are $V(i,j) = 0$ and $x(i,j) = 0$. Alternative starting values such as $V(i,j) = \frac{\pi(i,j)}{1-\beta}$ and $x(i,j) = 0$ never led to multiple equilibria. Neither did a simple continuation approach that uses as starting values the equilibrium from a "nearby" parameterization. We have also worked with a Gauss-Jacobi variant of the Pakes & McGuire (1994) algorithm and experimented with different dampening factors.

0.4 per quarter. We choose a probability of forgetting of $\delta = 0.3$ as our baseline, but note that this is not directly comparable because both empirical studies assume that the decay is proportional to the stock of goodwill whereas we take the probability of forgetting to be constant in most of what follows.

Starting with goodwill advertising, we assume that the goodwill consumers extend towards a firm is at one of the $L = 21$ levels given by $v_0 = 0$, $v_1 = 0.5$ up to $v_{20} = 10$. That is, if the industry is in state (i, j), then firm 1's level of goodwill is $0.5i$ and firm 2's level of goodwill is $0.5j$. Note that we pick the lower bound on goodwill small enough and the upper bound large enough such that a firm's market share ranges from close to 0% to close to 90%. The size of the market and the cost of advertising turn out to be critical for industry structure and dynamics. The cost per consumer varies widely across geographic markets and product categories (see e.g. Hilke & Nelson (1989) for coffee and Greer (1998) for beer) as well as across media and across vehicle within the same medium (see e.g. Chapter 12 of Shimp 2000). Consequently, we explore a very wide range of values of $\left(\frac{M}{k}\right)$ between 0.1 and 100.

Turning to awareness advertising, we set $L = 21$ with $s_0 = 0$, $s_1 = 0.05$ up to $s_{20} = 1$. That is, awareness runs from 0% to 100% in steps of 5%. We fix the ratio of market size to advertising cost at $\left(\frac{M}{k}\right) = 10$ and focus on the role of the perceived quality of firms' products. In particular, we explore a range of values of v between -1 and 9.

The above parameters yield reasonable cross-price elasticities under both models of advertising. In the equilibrium of the product market game under goodwill advertising, the cross-price elasticity of firm 1's demand with respect to firm 2's price ranges between 0.03 in state $(20, 0)$ and 6.78 in state $(0, 20)$. Under awareness advertising with $v = 4$, for example, the equilibrium cross-price elasticity is 0.03 in state $(20, 1)$ and reaches 0.95 in state $(13, 20)$; with $v = 8$, it is 0.04 in state $(20, 1)$ and reaches 1.23 in state $(12, 20)$. In this sense the results reported below do not hinge on unrealistic parameterizations of the product market game. In fact, as we argue in the next section, the chosen parameters imply advertising-to-sales ratios that are roughly consistent with empirical evidence.

In the next section we look at a wide range of values for the critical parameters, i.e., $\left(\frac{M}{k}\right)$ in case of goodwill advertising and v in case of awareness advertising. In order to establish the robustness of our conclusions we also vary the remaining parameters such as the probability of forgetting. In Section 6 we present a number of further robustness checks.

3 Results

In this section, we present the results for the two models of advertising: goodwill advertising and awareness advertising. Throughout, our approach is to use the equilibrium policy functions to construct the probability distribution over tomorrow's state (i', j') given today's state (i, j), i.e., the transition matrix that characterizes industry dynamics. This allows us to use tools from stochastic process theory to analyze the Markov process of industry dynamics rather than rely on simulation. We discuss the short-run (transitory) dynamics of this Markov process first and then turn to its long-run (steady-state) dynamics. Finally we comment on the performance of the industry.

3.1 Goodwill Advertising

Industry dynamics. Under goodwill advertising, the evolution of the industry depends on the size of the market and the cost of advertising. To illustrate we contrast a case where the size of the market is small relative to the cost of advertising ($\left(\frac{M}{k}\right) = 2$) with a case where this ratio is large ($\left(\frac{M}{k}\right) = 10$).

If the market is small or if advertising is expensive ($\left(\frac{M}{k}\right) = 2$), then how much a firm advertises depends crucially on its competitor's goodwill as the policy function in the top left panel of Figure 1 shows. In particular, a firm has a strategic motive to advertise as it can deter its rival from advertising by growing large. To see this, note that $x(0, j) = 0$ iff $j \geq 5$; $x(1, j) = 0$ iff $j \geq 7$; $x(2, j) = 0$ iff $j \geq 10$; and $x(3, j) = 0$ iff $j \geq 17$. That is, a large firm has a strategic advantage over a small rival because the smaller firm "gives up" if it is sufficiently far behind. This suggests that the industry will evolve towards an extremely asymmetric structure with a large and a small firm. However, such a strategic advantage cannot be gained over a medium or large rival because $i \geq 4$ implies $x(i, j) > 0$ for all j,[10] thereby suggesting a symmetric industry structure with two large firms.

In the short run, an extremely asymmetric structure and a symmetric structure are indeed both possible as the top left panels of Figures 2, 3, and 4 show. The top left panel of Figure 2 depicts the transient distribution of states (i, j) after $T = 15$ periods, starting from state $(0, 0)$.[11] This tells us how likely each possible industry structure is after $T = 15$ periods when both firms had minimal goodwill at the outset of the game. The top left panels of Figures 3 and 4 depict the same after $T = 25$ and $T = 50$ periods, respectively. The transient distribution is bimodal: after $T = 15$ periods, its modes are states $(0, 9)$ and $(9, 0)$ and each have a probability of 0.02; after $T = 25$ periods, its modes are states $(0, 14)$ and $(14, 0)$ and each have a probability of 0.02; and after $T = 50$ periods, its modes are states $(0, 20)$ and $(20, 0)$ and each have a probability of 0.08. That is, the most likely industry structure becomes more asymmetric over time. On the other hand, a lot of probability mass remains concentrated around state $(8, 8)$ after $T = 15$ periods, around state $(12, 12)$

[10] With the exception that, if $16 \leq j \leq 20$, then $x(20, j) = 0$ to be precise.

[11] Let P be the $L^2 \times L^2$ transition matrix of the Markov process of industry dynamics. The transient distribution after T periods is given by $a^{(T)} = a^{(0)} P^T$, where $a^{(0)}$ is the $1 \times L^2$ initial distribution.

after $T = 25$ periods, and around state $(19, 19)$ after $T = 50$ periods. Hence, a symmetric industry structure is also quite likely.

Given that multiple industry structures are possible in the short run, it is natural to ask which ones survive in the long run. We therefore compute the limiting distribution (ergodic distribution) which gives the fraction of time that the Markov process of industry dynamics spends in each state.[12] The result is depicted in the top left panel of Figure 5. States $(0, 17)$ and $(17, 0)$, states $(0, 18)$ and $(18, 0)$, states $(0, 19)$ and $(19, 0)$, as well as states $(0, 20)$ and $(20, 0)$ each have a probability of 0.01, 0.05, 0.17, and 0.25, respectively. Consequently, in the long run, the industry evolves towards an extremely asymmetric structure with a large and a small firm.

The transition to the limit is slow: A symmetric industry structure is still quite likely after 10^6 periods. However, a symmetric industry structure is not sustainable in the long run because, sooner or later, one of the firms has a string of bad luck. If this bad luck lasts long enough to annihilate most of the firm's goodwill, the industry reaches the region of the state space where the small firm ceases to advertise, thereby locking the small firm into a marginal position. Since such a long string of bad luck is unlikely to occur, it may take a long time before a symmetric industry structure is broken up.

The transition to the limit speeds up considerably if we decrease the size of the market or increase the cost of advertising. For example, if $\left(\frac{M}{k}\right) = 1$, then the probability mass quickly concentrates around the modes of the limiting distribution, states $(0, 20)$ and $(20, 0)$. In fact, the transient distribution after $T = 50$ periods puts probability mass only on states (i, j) and (j, i) with $i = 0$ and $14 \leq j \leq 20$. Repeating this exercise for other values of $\left(\frac{M}{k}\right)$ shows that a quick transition to the limit is the rule. We have chosen a parameterization that causes the transition to be slow in order to illustrate that knowing the short-run dynamics of an industry may be at least as important as knowing its long-run dynamics. While a static model may be suitable to analyzing the steady state, our dynamic model of advertising competition lends itself to analyzing the transition path.

If the market is large or if advertising is cheap ($\left(\frac{M}{k}\right) = 10$), then how much a firm advertises is fairly insensitive to its rival's goodwill (top right panel of Figure 1). A firm strives to build up maximal goodwill on its own more or less irrespective of its rival's goodwill. In particular, a firm cannot deter its rival from advertising by growing large. This suggests a symmetric industry structure with two large firms.

A look at the transient distribution after $T = 15, 25, 50$ periods (top right panels of

[12]The Markov process of industry dynamics turns out to be irreducible. That is, all its states belong to a single closed communicating class and the $1 \times L^2$ limiting distribution $a^{(\infty)}$ solves the system of linear equations $a^{(\infty)} = a^{(\infty)} P$, where P is the $L^2 \times L^2$ transition matrix. While the limiting distribution assigns positive probability to all states, we abstract from states that have probability of 0.005 or less in what follows in order to simplify the discussion.

Figures 2, 3, and 4) confirms that the industry evolves towards a symmetric structure. After $T = 15$ periods, the modal state is $(9, 9)$ and has a probability of 0.03; after $T = 25$ periods, the modal state is $(14, 14)$ and has a probability of 0.02; and after $T = 50$ periods, the modal state is $(20, 20)$ and has a probability of 0.29. The transition to the limit is quick as a comparison of the transient distribution after $T = 50$ periods in top right panel of Figure 4 and the limiting distribution in the top right panel of Figure 5 shows. In the long run, the most likely industry structure is state $(20, 20)$ with probability of 0.36. More generally, the limiting distribution puts probability mass on states (i, j) with $i \geq 18$ and $j \geq 18$ as well as on states $(17, 20)$ and $(20, 17)$. In contrast to the extremely asymmetric industry structure that arises in case of a small market/expensive advertising, in case of a large market/cheap advertising, both firms have high goodwill almost all the time. Due to the presence of idiosyncratic shocks it is nevertheless improbable that firms have the same goodwill at all times. As firms' stocks of goodwill change period after period, so do their optimal advertising expenditures. This gives rise to fluctuations in the temporal pattern of advertising. Any differences in firms' competitive positions, however, are temporary because, even if the leading firm has already accumulated maximal goodwill, the lagging firm continues to add to its own goodwill until the industry returns to a symmetric structure with two large firms.

Industry performance. To evaluate the long-run performance of the industry, we use the limiting distribution to compute the expected value of profits per consumer from product market competition. In addition to $E\left(\pi(i,j)\right)$ we also compute the expected profits of the larger firm, $E\left(\pi^L(i,j)\right)$, and the smaller firm, $E\left(\pi^S(i,j)\right)$.[13] Table 1 shows the expected profits per consumer along with the expected advertising-to-sales ratio $E\left(\frac{kx(i,j)}{M\pi(i,j)}\right)$.[14]

$\left(\frac{M}{k}\right)$	$E\left(\pi(i,j)\right)$	$E\left(\pi^L(i,j)\right)$	$E\left(\pi^S(i,j)\right)$	$E\left(\frac{kx(i,j)}{M\pi(i,j)}\right)$
2	3.29	6.49	0.04	0.03
10	1.01	1.13	0.90	0.10

Table 1: Expected profits per consumer and advertising-to-sales ratio. Goodwill advertising.

Recall that in case of a small market ($\left(\frac{M}{k}\right) = 2$), a high-goodwill firm competes against a low-goodwill firm, whereas two high-goodwill firms compete head on in case of a large market ($\left(\frac{M}{k}\right) = 10$). Consequently, product market competition is fiercer in a large market, and expected profits decrease. Moreover, the much higher expected advertising-to-sales ratio is reminiscent of Sutton's (1991) "competitive escalation of outlays by firms" (p. 11)

[13] More formally, we define $\pi^L(i,j) = \pi\left(\max(i,j), \min(i,j)\right)$ and $\pi^S(i,j) = \pi\left(\min(i,j), \max(i,j)\right)$.

[14] Recall that we abstract from marginal costs of production. Sales and profits therefore coincide.

in industries with endogenous sunk costs. In a small market, the profit gap between the large firm and the small firm is huge because the large firm enjoys a dominant position with an expected market share of 87% while its rival is marginalized with an expected market share of 4%. This, in turn, reflects the fact that a small market cannot support two high-goodwill firms because head-on competition is too fierce to allow both firms to be profitable.

How much do firms spend on advertising as a fraction of sales? The advertising-to-sales ratios of some exemplary U.S. industries in 2001 are 15.1% for distilled and blended liquors, 9.7% for soaps, detergents, and toilet preparations, 7.5% for malt beverages, 4.7% for apparel, 1.8% for cigarettes, and 1.1% for newspaper publishing and printing.[15] The advertising-to-sales ratios in Table 1 are seen to be in line with the empirical evidence. This suggest that the chosen parameters are fairly representative of a wide range of industries.

We have also computed advertising elasticities. Given that this period's advertising affects next period's goodwill, we compute the elasticity of next period's expected demand with respect to this period's advertising. Since this elasticity differs from state to state, we once again use the limiting distribution to take the expectation. In case of a small market/expensive advertising ($\left(\frac{M}{k}\right) = 2$) this yields an advertising elasticity of 0.0006 with 0.0013 for the large firm and zero for the small firm that ceases to advertises. The advertising elasticity is 0.0106 in case of a large market/cheap advertising ($\left(\frac{M}{k}\right) = 10$). Compared to the empirical literature these elasticities are small but not unheard of. For example, in their survey of the literature Leone & Schultz (1980) give a range from 0.003 to 0.482 and Sethuraman & Tellis (1991) report an average of 0.1 over 262 studies but also that "several advertising elasticities .. are close to zero (of the order of 0.001)" (p. 168).

Table 2 shows the contemporaneous and intertemporal correlations of levels of goodwill. The contemporaneous correlation $\rho\left(v_{i,t}, v_{j,t}\right)$ between firms' levels of goodwill at time t measures the strength of the link between firms in equilibrium. The intertemporal correlation $\rho\left(v_{i,t}, v_{i,t-h}\right)$ between a firm's goodwill at time t and its goodwill at time $t-h$, where $h \geq 1$, is a measure of the degree of persistence in a firm's level of goodwill.

In case of a small market/expensive advertising ($\left(\frac{M}{k}\right) = 2$), the contemporaneous correlation is negative and large. That is, firms' fortunes are negatively correlated. The intertemporal correlations are declining barely in the lag h, indicating that past goodwill is a strong predictor for current goodwill. Taken together, the contemporaneous and intertemporal correlations confirm that one firm gains and maintains an advantage over the other. This differential in positions persists over time due to the strategic nature of the competitive interactions.[16] In contrast, the contemporaneous correlation is small but nega-

[15] *Advertising Age*, www.adage.com, QwickFIND ID AAN96C. See also Table 4.4 in Sutton (1991) for advertising-to-sales ratios for various industries and countries.

[16] Note that a dominant position cannot be maintained indefinitely. Ultimately a role reversal occurs

$\left(\frac{M}{k}\right)$	$\rho\left(v_{i,t}, v_{j,t}\right)$	$\rho\left(v_{i,t}, v_{i,t-1}\right)$	$\rho\left(v_{i,t}, v_{i,t-5}\right)$	$\rho\left(v_{i,t}, v_{i,t-25}\right)$
2	-0.9963	0.9989	0.9972	0.9963
10	-0.0160	0.6581	0.1818	0.0011

Table 2: Contemporaneous and intertemporal correlations. Goodwill advertising.

tive and the intertemporal correlations are declining rapidly in case of a large market/cheap advertising ($\left(\frac{M}{k}\right) = 10$). This shows that neither firm is able to gain a lasting advantage over its rival. On the contrary, industry leadership quite frequently changes hands under goodwill advertising in a large market.

Discussion. Under goodwill advertising, the evolution of the industry depends on the size of the market and the cost of advertising. If the market is small or if advertising is expensive, a large firm has a strategic advantage over a small rival, but it cannot deter a medium-sized or large competitor from advertising. In the long run, this leads to an extremely asymmetric industry structure with a large and a small firm (and an expected combined market share of 0.91 in case of $\left(\frac{M}{k}\right) = 2$). As the size of the market increases or as the cost of advertising decreases, the industry moves towards a symmetric structure with two large firms (and an expected combined market share of 1.00 in case of $\left(\frac{M}{k}\right) = 10$). Differences in firms' competitive positions are now temporary because neither firm is able to gain a strategic advantage over the other.

To gain some intuition, consider the marginal cost and benefit of advertising. In the two scenarios discussed above, hold the marginal cost (as determined by the cost of advertising k and the probability of forgetting δ) fixed. Roughly speaking, the marginal benefit of advertising is determined by the increase in profits from product market competition that results from an increase in goodwill.[17] Because per-period profits are proportional to the size of the market, the marginal benefit of advertising increases with market size. In other words, in a small market, the marginal benefit of advertising is small. Moreover, as a look at the left panel of Figure 6 confirms, the marginal benefit decreases in the rival's goodwill. That is, an increase in its stock of goodwill is worth less to the firm when its rival is strong than when its rival is weak. Intuitively, a strong rival makes it harder for the firm to extract much of their increased willingness to pay from consumers. As a result a low-goodwill firm has a weak incentive to advertise when competing against a high-goodwill firm, and, in fact, may choose not to advertise at all. This is the source of the strategic advantage that a

because both modes of the limiting distribution are contained in a single recurrent set. However, the expected time it takes the industry to move from one mode to the other is 1.84×10^{14} periods.

[17]The nature of our argument here is more suggestive than formal because it is in fact the value function that determines the marginal benefit of advertising (see equation (4)). At the same time, however, the value function reflects not only the policy function but also the profits from product market competition.

large firm enjoys over a small rival. On the other hand, the marginal benefit of advertising increases in the firm's goodwill (see again the left panel of Figure 6). That is, an increase in its stock of goodwill is worth more to a strong firm than to a weak firm because a strong firm is in a better position to partake in the increased willingness to pay of consumers.[18] This explains why a medium-sized or large firm is not as easily deterred from advertising as a small firm. Finally, as market size goes up, so does the marginal benefit of advertising. In a large market, therefore, the marginal benefit more than outweighs the cost irrespective of the rival's goodwill. Hence, neither firm is able to gain a strategic advantage over the other.

Increasing market size is analogous to decreasing advertising cost. Holding market size and thus the marginal benefit of advertising fixed, a high-goodwill firm is able to deter a low-goodwill firm from advertising if advertising is expensive but not if advertising is cheap. These cost/benefit considerations are also affected by the probability of forgetting δ. Clearly, a higher δ makes it costlier for a firm to add to its stock of goodwill. Holding market size and advertising cost fixed, increasing (decreasing) δ should therefore bias the industry towards an extremely asymmetric (symmetric) structure. Our computations confirm that this is indeed the case: With $\left(\frac{M}{k}\right) = 10$, increasing δ from 0.3 to 0.7 leads in the long run to an industry structure with a large and a small firm instead of two large firms; with $\left(\frac{M}{k}\right) = 2$, decreasing δ from 0.3 to 0.1 leads to two large firms instead of a large and a small firm.

While we have so far focused on a few specific examples to discuss the role of cost/benefit considerations in shaping the structure of an industry, the intuition is robust. Figure 7 illustrates how the market is split in the long run between the two firms and the outside good for a very wide range of values of $\left(\frac{M}{k}\right)$ between 0.1 and 100; it depicts the expected market share of the larger firm, $E\left(D^L(i,j)\right)$, the smaller firm, $E\left(D^S(i,j)\right)$, and the outside good, $1 - E\left(D^L(i,j)\right) - E\left(D^S(i,j)\right)$. As can be seen, there are three distinct regimes: First, if $\left(\frac{M}{k}\right) \geq 2.48$, then the market is split more or less evenly between the two firms. Our example of a large market/cheap advertising is part of this regime. Second, if $0.35 \leq \left(\frac{M}{k}\right) < 2.48$, then the market is split unevenly between the two firms. This is akin to our example of a small market/expensive advertising. It is worth noting that, while the regime is seen to switch abruptly in Figure 7, the value and policy functions are changing gradually in $\left(\frac{M}{k}\right)$. These very small changes, however, are compounded in the limiting distribution.[19] Third,

[18]Whether this property is satisfied depends on functional forms. It also holds if utility is taken to be linear in the logarithm of goodwill. More generally, there is a wide variety of models of product market competition in which the increase in the profit caused by an increase in the firm's state is decreasing (increasing) in its rival's (the firm's) state. As Athey & Schmutzler (2001) point out, this includes Bertrand and Cournot competition and models with vertical and horizontal product differentiation.

[19]If the expected market shares are computed using the transient distribution after $T \in \{15, 25, 50\}$ periods instead of the limiting distribution, then the regime is seen to switch much less abruptly.

if $\left(\frac{M}{k}\right) < 0.35$, then the market is again split evenly between the two firms. While we have not provided an example for this case, it is easily understood in terms of cost/benefit considerations. To see this note that the extreme case of $\left(\frac{M}{k}\right) = 0$ implies $x(i,j) = 0$ for all i, j in equilibrium. More generally, if the market is too small or if advertising is too expensive, then neither firm has a sufficient incentive to advertise. Consequently, the industry never takes off, i.e., both firms are stuck at the lowest possible goodwill level and thus split the market evenly. In sum, the size of the market and the cost of advertising are key determinants of the structure of an industry.

The strategic advantage that a large firm enjoys over a small (but not over a medium-sized or large) rival in our model of goodwill advertising can be traced back to two properties of the profit function: First, the increase in the profit from product market competition caused by an increase in the firm's goodwill is decreasing in its rival's goodwill, i.e., $\pi(i+1,j+1) - \pi(i,j+1) < \pi(i+1,j) - \pi(i,j)$. Second, the increase in the profit is increasing in the firm's goodwill, i.e., $\pi(i+2,j) - \pi(i+1,j) > \pi(i+1,j) - \pi(i,j)$. Athey & Schmutzler (2001) identify these properties as key conditions for the leading firm to invest more than the lagging firm ("weak increasing dominance") in special settings where firms are myopic or where they must commit to the entire time path of investments at the outset of the game. In order to ensure that the MPE of their game also entails weak increasing dominance in more general settings, Athey & Schmutzler (2001) are forced to make additional assumptions about the equilibrium strategies. Since these assumptions concern the equilibrium strategies rather than the model's primitives, their usefulness is limited because knowing whether or not they are satisfied requires computing the equilibrium in the first place. It turns out that (the finite-difference analogs of) these assumptions are violated in our model of goodwill advertising. Moreover, our computations show that the leader invests more than the follower in some states and less in others.[20]

Note that the key conditions for weak increasing dominance set forth by Athey & Schmutzler (2001) restrict the curvature of the profit function. Whether or not they are satisfied is therefore independent of market size (and, of course, advertising cost). Yet, depending on market size, we obtain quite different industry structures. In particular, our computations indicate a symmetric industry structure with two large firms under goodwill advertising in a large market. The reason is that cost/benefit considerations prevail over the curvature of the profit function: In a large market the marginal benefit of advertising is large and even the lagging firm has a fairly strong incentive to advertise, thus ultimately leading to a symmetric industry structure.

[20]There are numerous differences between their setup and ours. In particular, Athey & Schmutzler (2001) assume that investment projects are completed instantaneously as soon as the decision to invest has been made and that the state-to-state transitions are deterministic.

3.2 Awareness Advertising

Industry dynamics. In the model of awareness advertising, the perceived quality of firms' products is fixed, and firms advertise in order to add to their awareness. It turns out that under awareness advertising the size of the market and the cost of advertising play a lesser role in determining the evolution of the industry than under goodwill advertising. We therefore take as given that the size of the market is large relative to the cost of advertising ($(\frac{M}{k}) = 10$) and focus on the role of perceived quality by contrasting a case of low ($v = 4$) with a case of high ($v = 8$) perceived quality.

In the case of low perceived quality ($v = 4$), the policy function implies that, more or less irrespective of its rival's awareness, a firm advertises up to the point of full awareness (bottom left panel of Figure 1). While the firm reduces its advertising as it approaches the point of full awareness, it still advertises enough to fend off forgetting, thereby ensuring that it stays at (or at least near) the point of full awareness.

This advertising strategy results in a symmetric industry structure with two large firms. The bottom left panels of Figures 2, 3, and 4 show the transient distribution of states (i, j) after $T = 15, 25, 50$ periods. After $T = 15$ periods, the modal state is $(8, 8)$ with probability of 0.03; after $T = 25$ periods, the modal state is $(12, 12)$ with probability of 0.02; and after $T = 50$ periods, the modal state is $(19, 19)$ with probability of 0.06. The limiting distribution is unimodal as well and puts probability mass on states (i, j) with $i + j \geq 34$ in addition to $i \geq 16$ and $j \geq 16$ (bottom left panel of Figure 5). That is, most of the time, both firms have at least 80% awareness. The most likely industry structure, with probability of 0.12, is state $(19, 19)$ where both firms are enjoying an awareness level of 95%.

As we move from low ($v = 4$) to high perceived quality ($v = 8$), the shape of the policy function changes dramatically. As the bottom right panel of Figure 1 shows, how much a firm advertises depends crucially on its rival's awareness. In particular, a firm now has a strategic motive to advertise in order to deter its rival: $x(12, j) = 0$ iff $j \geq 16$; $x(13, j) = 0$ iff $j \geq 16$; $x(14, j) = 0$ iff $j \geq 17$; $x(15, j) = 0$ iff $j \geq 17$; $x(16, j) = 0$ iff $j \geq 18$; and $x(17, j) = 0$ iff $j \geq 19$. On the other hand, a firm always advertises until it has reached an awareness level of 60% (i.e., if $i \leq 11$, then $x(i, j) > 0$ for all j). Taken together, these two features of the policy function imply that a large firm has a strategic advantage over a medium-sized rival because the smaller firm gives up if it is sufficiently far behind.

The possibility of gaining a strategic advantage leads to industry dynamics that resemble a preemption race. In this race, both firms start off advertising heavily. Moreover, as long as their awareness levels are similar, they continue to advertise heavily. For example, both firms spend 6.59 on advertising in state $(0, 0)$ and 8.88 in state $(15, 15)$. This is astonishingly large given that the average level of advertising is 4.18. However, once one firm gains a slight edge over its competitor, there is a marked change in advertising activity. For example, if firm 1 moves even slightly ahead in the race (the industry moves from state $(15, 15)$ to state $(16, 15)$), then firm 2 scales back its advertising to 4.23 while firm 1 ratchets up its advertising to 12.17. This tends to further enhance the asymmetry between firms. Once the industry has reached state $(17, 15)$, firm 2 gives up, whereas firm 1 continues to advertise heavily. Eventually firm 1 secures itself a position of dominance.

In the case of high perceived quality, the industry moves towards an asymmetric structure as time passes. While the transient distribution of states after $T = 15$ periods is still unimodal (bottom right panel of Figure 2), the transient distribution of states after $T = 25$ and $T = 50$ periods is clearly bimodal (bottom right panels of Figures 3 and 4). After $T = 15$ periods, the modal state is $(9, 9)$ and has a probability of 0.03; after $T = 25$ periods, the modal states are $(11, 17)$ and $(17, 11)$ and each have a probability of 0.02; and after $T = 50$ periods, the modal states are $(11, 20)$ and $(20, 11)$ and each have a probability of 0.11. States $(11, 20)$ and $(20, 11)$ are also the most likely long-run industry structures as the limiting distribution in the bottom right panel of Figure 5 shows. Each of the modal states has a probability of 0.11. More generally, the limiting distribution puts probability mass on states (i, j) and (j, i) with $i = 18$ and $9 \leq j \leq 11$ or $19 \leq i \leq 20$ and $8 \leq j \leq 12$. The transition to the limit is quick because the asymmetric industry structure is the result of a preemption race. This race (and therefore the identity of the dominant firm) is, in effect, decided as soon as one firm gains a slight edge over the other. Hence, an instance of bad luck suffices to trigger an asymmetric industry structure.

Industry performance. Table 3 presents the expected value of profits per consumer from product market competition. As we increase the perceived quality from $v = 4$ to $v = 8$, two things happen. First, holding firms' levels of awareness fixed, the intensity of competition as measured by the cross-price elasticity goes up because more consumers now prefer one of the inside goods over the outside good. Second, while low perceived quality results in two large firms competing head on, high perceived quality results in a large firm with an expected market share of 58% competing against a medium-sized firm with an expected market share of 38%. That is, the industry shifts towards a less competitive structure. Overall, the second effect dominates the first, and expected profits rise sharply as Table 3 shows. However, while the large firm's expected profits almost triple, the small firm's expected profits do not even double. This is a direct consequence of the asymmetric market structure that arises with high perceived quality. The advertising-to-sales ratios in Table 3 are again in line with the empirical evidence, whereas the advertising elasticities are low with 0.0058 and 0.0086 for low ($v = 4$) and high ($v = 8$) perceived quality, respectively.

Table 4 shows the contemporaneous and intertemporal correlations of levels of aware-

v	$E(\pi(i,j))$	$E(\pi^L(i,j))$	$E(\pi^S(i,j))$	$E\left(\frac{kx(i,j)}{M\pi(i,j)}\right)$
4	0.95	0.99	0.90	0.06
8	2.17	2.87	1.47	0.04

Table 3: Expected profits per consumer and advertising-to-sales ratio. Awareness advertising.

ness. In case of low perceived quality ($v = 4$), the contemporaneous correlation $\rho(s_{i,t}, s_{j,t})$ between firms' levels of awareness at time t is small but negative, reflecting the fact that a firm's advertising is fairly insensitive to its competitor's awareness. The intertemporal correlations $\rho(s_{i,t}, s_{i,t-h})$ between a firm's awareness at time t and its awareness at time $t-h$ are declining rapidly in the lag h. Similar to goodwill advertising with a high ratio of market size to advertising cost, this indicates that neither firm is able to gain a lasting advantage over its competitor; rather firms repeatedly switch positions over time. In case

v	$\rho(s_{i,t}, s_{j,t})$	$\rho(s_{i,t}, s_{i,t-1})$	$\rho(s_{i,t}, s_{i,t-5})$	$\rho(s_{i,t}, s_{i,t-25})$
4	-0.0731	0.8556	0.4984	0.0503
8	-0.9663	0.9910	0.9749	0.9658

Table 4: Contemporaneous and intertemporal correlations. Awareness advertising.

of high perceived quality ($v = 8$), the contemporaneous correlation is negative and large because a firm's advertising depends critically on its competitor's awareness. Moreover, the intertemporal correlations are declining slowly, suggesting that past awareness is a strong predictor for current awareness. Taken together, this shows that one firm gains and maintains an advantage over the other. Similar to goodwill advertising with a low ratio of market size to advertising cost, this differential in positions persists over time due to the strategic nature of the competitive interactions.[21]

Discussion. Under awareness advertising, the evolution of the industry depends on the perceived quality of firms' products. If the perceived quality is low, awareness advertising results in a symmetric industry structure with two large firms (and an expected combined market share of 0.92 in case of $v = 4$), and differences in firms' competitive positions are temporary. If the perceived quality is high, a large firm has a strategic advantage over a medium-sized rival but it is unable to prevent a small competitor from advertising. The possibility of gaining a strategic advantage gives rise to a preemption race, which is effectively decided as soon as one firm gains a slight edge over the other. Compared

[21] While a role reversal occurs ultimately, the expected time it takes the industry to move from one mode of the limiting distribution to the other is 1.69×10^6 periods.

to goodwill advertising with a low ratio of market size to advertising cost, asymmetries are less pronounced; indeed, awareness advertising with high perceived quality leads to an asymmetric industry structure with a large and a medium-sized firm (and an expected combined market share of 0.96 in case of $v = 8$). Figure 8 demonstrates that this pattern is robust. As can be seen, the industry switches from a regime in which the two firms split the market unevenly in the long run to a regime in which they split it more or less evenly around $v = 5.86$. Indeed, if $-0.30 \leq v < 5.86$, then the mode of the limiting distribution is state $(19, 19)$, but firms' expected market shares decrease steadily as more and more consumers prefer the outside good over one of the inside goods. This means that firms may be small in terms of their expected market shares although they are large in terms of their stocks of awareness. If $-0.45 \leq v < -0.30$, then the modal states are $(0, 19)$ and $(19, 0)$. Finally, if the perceived quality of firms' product is too low, then neither firm has a sufficient incentive to advertise and the industry never takes off (the extreme case of $v = -\infty$ implies $x(i, j) = 0$ for all i, j in equilibrium).

To see why this is happening, contrast the profit function in the example with low perceived quality ($v = 4$, middle panel of Figure 6) with the profit function in the example with high perceived quality ($v = 8$, right panel of Figure 6). With high perceived quality, a firm's per-period profit from product market competition peaks in its own awareness provided that the awareness level of its rival is at least 75%. More precisely, the firm's profit increases up to an awareness level of 55% and decrease afterwards. Hence, it is often better for the small firm to be considerably smaller than the large firm rather than to be slightly smaller. To illustrate, suppose that the large firm is at the point of full awareness. If the small firm has an awareness level of 95%, then its per-period profit is 1.02 (per consumer). If the small firm, however, had an awareness level of 55%, it would earn a profit of 1.49. Consequently, when competing against a high-awareness firm, it is in the best interest of a medium-awareness firm to stay that way. This explains why a large firm has a strategic advantage over a medium-sized rival. Yet, the large firm cannot deter a small firm from advertising because the small firm can always increase its profit from product market competition by gaining some (but not full) awareness. With low perceived quality, by contrast, a firm's profit from product market competition increase in its own awareness regardless of the awareness level of its rival. Hence, matching one's competitor never hurts, and neither firm is able to gain a strategic advantage over the other.

The question therefore is: What causes the profit function to peak in case of high perceived quality but not in case of low perceived quality? Recall from equation (2) that firm 1's demand is composed of a captive and a competitive segment. Firm 1 therefore charges a price that lies between its monopolistic and its duopolistic price. As firm 2 adds to its stock of awareness, firm 1 puts less emphasis on its captive segment and more on the competitive segment and consequently lowers its price. This, in turn, puts firm 2's price under pressure.[22] Figure 9 illustrates prices in the Nash equilibrium of this product market game. Clearly, head-on competition between two high-awareness firms leads to a drop in prices. While this price drop is modest in case of low perceived quality (left panel), it is dramatic in case of high perceived quality (right panel). The reason is that, holding firms' levels of awareness fixed, the intensity of competition goes up with the perceived quality of firms' products because more consumers now prefer one of the inside goods over the outside good. Thus profits fall along with prices in case of high perceived quality, and a medium-sized firm is better off staying put rather than trying to grow when facing a large firm.

Put differently, under awareness advertising with high perceived quality, there is a benefit to assuming the posture of a "puppy dog" while allowing one's competitor to be a "top dog." As long as the puppy dog stays behind and does not threaten the top dog's dominance of the market, the top dog is willing to extend a "price umbrella" over the puppy dog. More formally, if $i > j$, then $p^*(i, j) > p^*(j, i)$, i.e., the large firm charges a higher price than the small firm. In fact, using the limiting distribution, the expected price of the large (small) firm is 4.96 (3.90). However, once the puppy dog tries to grow, the top dog responds aggressively by triggering a "price war," thereby pushing prices and profits down. This gives the puppy dog an overwhelming incentive to remain inconspicuous.

Under awareness advertising cost/benefit considerations continue to play a role in the sense that if the cost becomes too high or the benefit too low, then the low-awareness firm may choose not to advertise at all. With $v = 4$ ($v = 8$), we obtain an extremely asymmetric industry structure with a large and a small firm if we increase δ from 0.3 to 0.8 (0.9). Moreover, as we decrease the benefit of advertising relative to its cost by decreasing $\left(\frac{M}{k}\right)$ from 10 to 2, this sets in earlier, and we obtain an extremely asymmetric industry structure if we increase δ to 0.5 (0.7). Finally, as we approach the extreme case of $\left(\frac{M}{k}\right) = 0$, the industry never takes off, just as under goodwill advertising.

Yet, cost/benefit considerations play a lesser role here than under goodwill advertising. Because more awareness leads to less profits, the medium-sized firm is better off staying put rather than trying to grow even if advertising were costless. In fact, as was shown by Fershtman & Muller (1993) and Boyer & Moreaux (1999) in the context of a static game of awareness choice (at a cost of zero) followed by price competition, both firms may opt for less than full awareness in the subgame perfect Nash equilibrium. In contrast to goodwill advertising, where cost/benefit considerations give rise to a strategic advantage, the strategic advantage derives from the nature of product market competition. The central

[22] As pointed out by Boyer & Moreaux (1999), profits may peak even if goods are complements and not substitutes. This suggests that our results are robust to a wide range of demand specifications.

idea of our model of awareness advertising is that "more is less." This is a rationale for persistent asymmetries that has mostly been ignored in the literature on dynamic games.[23]

4 Advertising Restrictions and Industry Concentration

Whether advertising decreases or increases competition has long been a matter of dispute. Kaldor (1950), Bain (1956), and Comanor & Wilson (1974), among others, argue that advertising is anticompetitive as it allows the leading firms in an industry to increase product differentiation. This lowers the elasticity of demand and creates barriers to entry, thus giving a further advantage to the leading firms. In sum, advertising conveys market power and promotes industry concentration. On the other hand, Stigler (1961), Telser (1964), and Nelson (1970, 1974) focus on the informative rather than the persuasive aspects of advertising and argue that advertising is procompetitive as it disseminates information about the price and other product attributes more widely among consumers.

Based on this dichotomy, empirical studies routinely conclude that if restrictions on advertising led to an increase in concentration, then advertising must have been informative (e.g., Eckard 1991, Sass & Saurman 1995). This conclusion is unwarranted. In particular, we show that tighter regulation or an outright ban of advertising may have anticompetitive effects even if advertising is persuasive in nature.

Tighter regulation or an outright ban, in essence, reduce the efficacy of advertising, thereby making it costlier for firms to reach consumers. Our results for goodwill advertising in Section 3.1 therefore imply that tighter regulation may reduce a symmetric industry structure with two large firms that compete head on to an extremely asymmetric one. That is, given that asymmetries stem from cost/benefit considerations, regulating or banning advertising may enable one firm to dominate the industry. These anticompetitive effects stand in marked contrast to the market power theory of advertising.

The anticompetitive effects of advertising restrictions are consistent with the evidence. Eckard (1991), for example, shows that the 1970 ban on television advertising increased concentration in the U.S. cigarette industry. According to his results, small-share brands exhibited relatively better share growth than large-share brands before the ban on television advertising, an advantage that disappeared after the ban. In addition, he finds that the brand-level (firm-level) Herfindahl index decreases (decreases) over time before the ban and

[23] Besanko & Doraszelski (2004) show that asymmetries may arise and persist for exactly this reason in a dynamic model of capacity accumulation. There the production technology generates a competitive environment in which more capacity may lead to less profits. In our model of awareness advertising, in contrast, "more is less" because of consumer behavior. Moreover, their aim is to characterize the relationship between preemption races and investment reversibility, whereas we shed new light on possible sources of strategic advantage in models of advertising.

is constant (increases) afterwards, i.e., the cigarette industry grew more competitive before the ban and less competitive afterwards.[24]

Sass & Saurman (1995) present similar findings for the malt beverage industry. During the 1980s various states banned the advertising of beer in print media and/or on billboards and other outdoor signs, thus giving rise to cross-sectional variation in the cost of advertising. Sass & Saurman (1995) show that the state-level Herfindahl index increased in response to a ban. Moreover, advertising restrictions raised the state-level market share of the largest national brewer and reduced the market shares of most smaller ones.[25]

While both of the studies cited above conclude that their findings are "inconsistent with the market power theory of advertising" (Eckard 1991, p. 132) and "consistent only with the notion that advertising stimulates competition by providing valuable information to consumers" (Sass & Saurman 1995, p. 80), our results for goodwill advertising clearly show that the displayed patterns are not necessarily the result of informative advertising. In fact, it seems unlikely that advertising in the cigarette and beer industries serves the purpose of informing consumers about the price and other product attributes (see e.g. pp. 292 of Bauer & Greyser 1968). Our model of goodwill advertising provides a way to reconcile theory and evidence under the more plausible assumption that advertising is persuasive and alters consumers' tastes for established brand names.

5 Advertising and Barriers to Entry

In this section, we add entry and exit to our dynamic model of advertising competition. We first argue that, over a wide range of parameterizations, the long-run industry structure with entry and exit is the same as without entry and exit. Hence, the mechanisms behind persistent asymmetries remain operational in the presence of entry and exit. We then turn to the role of advertising as a barrier to entry and ask how an incumbent uses advertising to deter entry or accommodate entry. Finally we discuss how advertising is used to induce exit.

To study the effect of entry and exit on the advertising strategy as well as on the industry structure we extend the basic model of Section 2. We continue to use (i, j) to describe a duopolistic industry and, in addition, take (i) to mean that the monopolist's level of goodwill (awareness) is v_i (s_i). Our formulation of firms' entry and exit decisions

[24] Farr, Tremblay & Tremblay (2001) estimate a structural demand-and-supply model and conclude that both the Broadcast Advertising Ban and its predecessor, the Fairness Doctrine Act, limited competition in the U.S. cigarette industry.

[25] Lynk (1981) further supports the cost-concentration link. He argues that the explosive growth of television in the U.S. during the 1950s dramatically lowered the cost of advertising. Using data on sales of consumer nondurables in localized markets, he shows that this led to an increase in the sales of the smaller sellers at the expense of the larger ones, i.e., to a decrease in concentration.

is fully dynamic: In each period a potential entrant decides whether to actually enter the industry by paying a setup cost of ϕ^e. We indicate whether entry occurs by the function $\lambda(i) \in \{0,1\}$. If it does ($\lambda(i) = 1$), the entrant becomes an incumbent in the next period. Specifically, the entrant appears in state $j^e = 8$ with probability $(1-\delta)$ and in state $j^e - 1$ with probability δ. The entrant may subsequently decide to exit. Similarly, in each period, an incumbent decides whether to exit, and we use the indicator function $\chi(i,j) \in \{0,1\}$ to describe the exit policy. Upon exit ($\chi(i,j) = 1$) the incumbent receives a scrap value $\phi \leq \phi^e$ and perishes, thereby making room for additional entry.

Long-run industry structure. Entry and exit are key drivers of the structure of an industry. Tables 5 and 6 summarize their impact in case of awareness advertising with low ($v = 4$) and high perceived quality ($v = 8$), respectively.[26] The tables give the most likely long-run industry structure for different combinations of the setup cost ϕ^e and the scrap value ϕ. A cell lists the mode(s) of the limiting distribution: If $v = 4$, the industry likely consists of two large firms (state $(19,19)$) or of one large firm (state (20)). Table 5 designates these two possibilities as LL and L, respectively. If $v = 8$, the industry likely consists of one large and one medium-sized firm (states $(11,20)$ and $(20,11)$) or of one large firm (state (20)), labelled ML and L, respectively, in Table 6. In some cases, there is more than one closed communicating class. For example, if entry is very costly and exit is almost worthless (i.e., ϕ^e is large and ϕ is small), then the industry remains a duopoly (monopoly) provided that it starts as a duopoly (monopoly). Thus, one closed communicating class corresponds to a duopolistic industry structure and another to a monopolistic industry structure. Tables 5 and 6 list the mode(s) of the limiting distribution for each of them.

Over a wide range of setup costs and the scrap values the long-run industry structure with entry and exit is the same as without entry and exit, thus demonstrating that the results in Section 3 continue to hold. For larger setup costs and/or scrap values, the industry moves away from a duopoly towards a monopoly. This is due to the fact that as ϕ^e increases entry is discouraged, and therefore the industry becomes a monopoly, while as ϕ increases exit is encouraged, and again the industry becomes a monopoly.

Entry deterrence and accommodation. Entry deterrence and accommodation both occur if entry is possible but not inevitable, i.e., if entry takes place in some states but not in others. They are therefore best studied by comparing the incumbent's advertising strategy in two scenarios: in the first scenario, the setup cost ϕ^e is moderate so that entry is possible but not inevitable, whereas ϕ^e is large enough to render entry impossible in the second one.

[26] In the interest of brevity, we focus on awareness advertising in what follows. The results for goodwill advertising are similar. Details are available from the authors upon request.

$\phi\backslash\phi^e$	0	$\cdots$	250	275	300	325	350	375	400	425	450	475	$\cdots$	∞
0	LL	$\cdots$	LL	LL	LL	LL	LL	LL	LL	LL	LL; L	LL; L	$\cdots$	LL; L
$\vdots$		$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$
250			LL	LL	LL	LL	LL	LL	LL	LL	LL; L	LL; L	$\cdots$	LL; L
275				LL	LL	LL	LL	LL	LL	LL	LL	L	$\cdots$	L
300					LL	LL	LL	LL	L	L	LL	L	$\cdots$	L
325						LL	LL	L	L	L	L	L	$\cdots$	L
350							LL	L	L	L	L	L	$\cdots$	L
375								L	L	L	L	L	$\cdots$	L
400									L	L	L	L	$\cdots$	L
425										L	L	L	$\cdots$	L
450											L	L	$\cdots$	L
475												L	$\cdots$	L

Table 5: Most likely long-run industry structure. LL is shorthand for state $(19, 19)$, L for state (20). Awareness advertising with $v = 4$.

$\phi\backslash\phi^e$	0	$\cdots$	500	550	600	650	700	$\cdots$	1150	1200	$\cdots$	∞
0	ML	$\cdots$	ML	ML	ML	ML	ML	$\cdots$	ML	ML; L	$\cdots$	ML; L
$\vdots$		$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$		$\vdots$
500			ML	ML	ML	ML	ML	$\cdots$	ML	ML; L	$\cdots$	ML; L
550				ML	ML	ML	L	$\cdots$	L	L	$\cdots$	L
600					ML	ML	L	$\cdots$	L	L	$\cdots$	L
650						ML	L	$\cdots$	L	L	$\cdots$	L
700							L	$\cdots$	L	L	$\cdots$	L
$\vdots$								$\ddots$	$\vdots$	$\vdots$		$\vdots$
1150									L	L	$\cdots$	L
1200										L	$\cdots$	L

Table 6: Most likely long-run industry structure. ML is shorthand for states $(11, 20)$ and $(20, 11)$, L for state (20). Awareness advertising with $v = 8$.

The top panels of Figure 10 provide an example of this comparison for awareness advertising with low perceived quality ($v = 4$). A solid line pertains to $\phi^e = 375$. As can be seen, entry takes place in states (i) with $i \leq 9$ but not in states (i) with $i > 9$. In contrast, if $\phi^e = \infty$, then entry is impossible (dashed line). In states sufficiently far to the right of state (9), where entry does not take place, the incumbent's advertising strategy in the scenario with entry is the same as in the scenario without entry. In states sufficiently far to the left of state (9), where entry takes place, the incumbent advertises less in the scenario with entry than in the scenario without entry. That is, the incumbent *accommodates* entry by *under*-advertising. The reason is that the entrant will set a high price as long as its captive segment is large relative to its competitive segment. The relative size of the two segments, in turn, is determined by the stock of awareness of the incumbent. Consequently, a reduction in advertising by the incumbent softens the ensuing competition in the product market (see the left panel of Figure 9). Finally, advertising spikes quite dramatically in state (9) because the incumbent anticipates that by increasing its awareness a little it can keep the entrant out. The reason is that the entrant's profit decreases in the incumbent's awareness (see the middle panel of Figure 6). The incumbent thus *deters* entry by *over*-advertising.

The above results are in line with those of Boyer & Moreaux (1999) but in contrast to those of Schmalensee (1983) and Fudenberg & Tirole (1984). Despite these differences, however, the key is always that the incumbent aspires to soften product market competition when it comes to accommodating the entrant but to make it fiercer when it comes to entry deterrence.[27] In this sense the above results as well as the existing literature can be understood from a purely static perspective. Our dynamic framework points to yet another consideration. In fact, as we show next, taking industry dynamics into account is crucial to analyzing the role of advertising as a barrier to entry.

Recall that with low perceived quality entry accommodation takes the form of under-advertising. This is not necessarily the case with high perceived quality. The bottom panels of Figure 10 show the entry and advertising strategies for awareness advertising with $v = 8$. A solid line pertains to $\phi^e = 675$, where entry is possible but not inevitable, and a dashed line to $\phi^e = \infty$, where entry is impossible. The advertising strategy now has two pronounced peaks within the region of over-advertising, one in state (8) and another in state (12). The right peak coincides with the entry indicator switching from one to zero and is therefore due to entry deterrence, just as in the case of $v = 4$. In contrast, the left peak occurs in states where entry takes place and therefore must be consistent with entry accommodation. That is, the incumbent *accommodates* entry by *over*-advertising. This is quite striking because, in our setup, the incumbent makes the ensuing competition fiercer by advertising.

To see what is going on, note that under awareness advertising with $v = 8$ the incumbent over-advertises in situations where, once entry occurs, the two firms have equal or at least similar levels of awareness and are thus drawn into a preemption race. This is precisely the case here because the entrant appears either in state $j^e = 8$ or in state $j^e - 1$. Over-advertising is therefore motivated by the incumbent's desire to gain a head start in the post-entry preemption race. Put differently, over-advertising does not stem from the incumbent's desire to soften product market competition. Rather it arises out of a purely dynamic consideration. The incumbent foresees the post-entry preemption race and over-advertises in an attempt to better its chances of eventually becoming the dominant player in the

[27] In contrast to Boyer & Moreaux (1999) and our model, Fudenberg & Tirole (1984) assume that the captive market is immune to the entrant's advertisements, so that the entrant can only fight the incumbent over the remainder of the market. Consequently, a reduction in advertising by the incumbent makes the ensuing competition harsher in Fudenberg & Tirole's (1984) model, but softer in Boyer & Moreaux's (1999) and our model.

industry by gaining an early lead in the race.

Exit inducement. In a dynamic oligopoly with collusion and price wars Fershtman & Pakes (2000) show that a small firm is more prone to exiting the industry during a price war than under collusion. Hence, at states near the border of the collusive region, a large firm has an incentive to deviate from collusion and trigger a price war in order to induce exit. In our dynamic model of advertising competition, exit is induced through heavy advertising rather than low prices.

Figure 11 shows the exit policy of firm 2, $\chi_2(i,j) = \chi(j,i)$, and the advertising policy of firm 1, $x_1(i,j) = x(i,j)$, for awareness advertising with $v = 8$ and $\phi = 650$.[28] As can be seen, in addition to advertising heavily during the preemption race, there are other spikes in the advertising policy of firm 1. These spikes occur in situations where the value to firm 2 of remaining in the industry is close to the scrap value, and therefore even a small increase in the awareness gap suffices to push firm 2 out of the industry. In general, if one firm draws ahead of the other in a duopolistic industry, then the larger firm finds it profitable to increase the probability of becoming a monopolist through heavy advertising, thereby inducing the smaller firm to exit.

6 Robustness Checks and Related Applications

In this section, we first discuss a number of robustness checks concerning alternative specifications for the probability of forgetting, the effectiveness of advertising, the advertising technology, the goodwill levels, demand growth, and the nature of advertising. Then we argue that our model of goodwill advertising may be reinterpreted as a fully dynamic model of quality competition and compare our results to the extant literature.

6.1 Robustness Checks

Probability of forgetting. In the interest of simplicity we have specified a constant probability of forgetting in Section 2. In contrast, empirical studies usually assume that the decay is proportional to the stock of goodwill (awareness). The available estimates of the rate of decay are therefore not directly comparable to δ. An alternative specification of the forgetting process is to replace the constant probability of forgetting δ by the increasing probability of forgetting $\delta(i) = 1 - (1 - \mu)^i$. The latter is closer to proportional decay: For example, if μ is the probability that a "block" of goodwill decays, then $\delta(i)$ is the probability that at least one of i blocks decays. Because this alternative specification makes it more expensive for the larger firm than for the smaller firm to maintain/expand its stock

[28] Recall that, by symmetry, $\chi(j,i)$ indicates whether firm 2 exits if the industry is in state (i,j).

of goodwill (awareness), it tends to reduce any differences between firms. Yet, provided that μ is not too large, persistent asymmetries continue to emerge under the alternative specification.

Effectiveness of advertising. We have taken the probability that an advertising campaign is successful to be $\frac{x_1}{1+x_1}$ rather than $\frac{\alpha(i)x_1}{1+\alpha(i)x_1}$, where $\alpha(i)$ is a state-specific measure of the effectiveness of advertising. The theoretical literature (e.g., Butters 1977, Grossman & Shapiro 1984, Stegeman 1991, Robert & Stahl 1993) suggests that $\alpha(i)$ should be decreasing in i because it becomes increasingly difficult to reach ever higher fractions of the population of consumers. While this is easily taken into account, for example by setting $\alpha(i) = 1 - \frac{i}{(L-1)}$, our conclusions are robust. In particular, awareness advertising with $v = 8$ still gives rise to an asymmetric industry structure (the modes of the limiting distribution are states $(11, 18)$ and $(18, 11)$), just as goodwill advertising with $\left(\frac{M}{k}\right) = 2$ still gives rise to an extremely asymmetric industry structure (the modes are states $(0, 17)$ and $(17, 0)$).

Advertising technology. We have assumed that a successful advertising campaign creates one unit of goodwill (awareness). In reality, however, a blockbuster campaign may have a much greater impact on the stock of goodwill (awareness). To capture this idea we change the advertising technology to allow a successful campaign to increase the stock of goodwill (awareness) by either one or two units with probability γ and $(1-\gamma)$, respectively. Hence, the probability that firm 1 will be in state i' tomorrow given that it is in state i today becomes

$$\Pr(i'|i, x_1) = \begin{cases} \frac{(1-\gamma)(1-\delta)x_1}{1+x_1} & \text{if} \quad i' = i+2, \\ \frac{\gamma(1-\delta)x_1}{1+x_1} & \text{if} \quad i' = i+1, \\ \frac{1-\delta+\delta x_1}{1+x_1} & \text{if} \quad i' = i, \\ \frac{\delta}{1+x_1} & \text{if} \quad i' = i-1 \end{cases}$$

if $i \in \{1, \dots, L-3\}$. As before the transition probability and advertising strategy must be modified appropriately at the boundaries of the state space. We set $\gamma = \frac{2}{3}$.

Tables 7 and 8 compare the original "one-step" and the alternative "two-step" specification. As can be seen, industry structure and dynamics remain the same under awareness advertising and under goodwill advertising with $\left(\frac{M}{k}\right) = 10$ except, of course, that firms move to higher states in less time. In contrast, under goodwill advertising with $\left(\frac{M}{k}\right) = 2$ the industry evolves towards a symmetric structure. Given that industry dynamics under goodwill advertising are driven by cost/benefit considerations this is, however, not surprising as the two-step specification implicitly decreases the cost of acquiring a unit of goodwill. If we compensate by increasing the cost of advertising from $k = 1$ to $k = 1.5$, then the industry structure is again extremely asymmetric: The modes of the transient distribution are $(0, 12)$ and $(12, 0)$ after $T = 15$ periods, $(0, 19)$ and $(19, 0)$ after $T = 25$ periods, $(0, 20)$ and $(20, 0)$ after $T = 50$ periods, and the modes of the limiting distribution are states $(0, 20)$ and $(20, 0)$.

	$\left(\frac{M}{k}\right) = 2$		$\left(\frac{M}{k}\right) = 10$	
T	one-step	two-step	one-step	two-step
15	$(0, 9)$, $(9, 0)$	$(9, 9)$	$(9, 9)$	$(11, 12)$, $(12, 11)$
25	$(0, 14)$, $(14, 0)$	$(14, 15)$, $(15, 14)$	$(14, 14)$	$(20, 20)$
50	$(0, 20)$, $(20, 0)$	$(20, 20)$	$(20, 20)$	$(20, 20)$
∞	$(0, 20)$, $(20, 0)$	$(20, 20)$	$(20, 20)$	$(20, 20)$

Table 7: Most likely industry structure after T periods. One-step and and two-step specification. Goodwill advertising.

	$v = 4$		$v = 8$	
T	one-step	two-step	one-step	two-step
15	$(8, 8)$	$(10, 10)$	$(9, 9)$	$(12, 12)$
25	$(12, 12)$	$(15, 16)$, $(16, 15)$	$(11, 17)$, $(17, 11)$	$(11, 20)$, $(20, 11)$
50	$(19, 19)$	$(19, 19)$	$(11, 20)$, $(11, 20)$	$(11, 20)$, $(20, 11)$
∞	$(19, 19)$	$(19, 19)$	$(11, 20)$, $(11, 20)$	$(11, 20)$, $(20, 11)$

Table 8: Most likely industry structure after T periods. One-step and two-step specification. Awareness advertising.

Goodwill levels. Given the chosen parameters, firms regularly attain maximal goodwill. To ensure that the results above are not driven by end effects, we have tried enlarging the state space along the lines suggested by Pakes & McGuire (1994). Setting $L = 41$ with $v_0 = 0$, $v_1 = 0.5$ up to $v_{20} = 10$ and $v_{21} = \ldots v_{40} = 10$ may be interpreted as a particularly strong form of diminishing returns in the sense that no matter how much a firm advertises, it cannot increase its goodwill with consumers beyond a certain level. Table 9 compares the original linear and the alternative piecewise-linear specification. As can be seen, the upper bound of the state space is no longer reached. While a firm may push its state a little beyond maximal goodwill in order to insure itself against future losses from forgetting, this leaves our conclusions regarding industry structure and dynamics unchanged: An extremely asymmetric industry structure arises if the market is small or if advertising is expensive, and a symmetric industry structure arises if the market is large or if advertising is cheap.

Demand growth. Demand growth is easily modeled using demand states $d \in \{1, \ldots, D\}$ that evolve according to an exogenously given Markov process. We focus on the model of

	$\left(\frac{M}{k}\right)=2$		$\left(\frac{M}{k}\right)=10$	
T	linear	piecewise-linear	linear	piecewise-linear
15	$(0,9)$, $(9,0)$	$(0,9)$, $(9,0)$	$(9,9)$	$(9,9)$
25	$(0,14)$, $(14,0)$	$(0,14)$, $(14,0)$	$(14,14)$	$(14,14)$
50	$(0,20)$, $(20,0)$	$(0,21)$, $(21,0)$	$(20,20)$	$(22,22)$
∞	$(0,20)$, $(20,0)$	$(0,23)$, $(23,0)$	$(20,20)$	$(23,23)$

Table 9: Most likely industry structure after T periods. Linear and piecewise-linear specification. Goodwill advertising.

goodwill advertising where the size of the market is a key determinant of industry structure and dynamics and index the size of the market M_d by the state of demand d. If the industry is in demand state $d \in \{1, \ldots, D-1\}$ today, tomorrow it will stay in demand state d with probability γ and it will move to demand state $d+1$ with probability $(1-\gamma)$. We further assume that demand state D is absorbing. We set $D = 3$ and $\gamma = 0.99$ and let the ratio of market size to advertising cost grow from $\left(\frac{M_1}{k}\right) = 2$ over $\left(\frac{M_2}{k}\right) = 6$ to $\left(\frac{M_3}{k}\right) = 10$.

Since firms anticipate the final state of demand, their advertising strategies in earlier demand states are similar to their advertising strategies in the final demand state. Irrespective of the state of demand a firm can no longer deter its rival from advertising by growing large even though this is possible if the ratio of market size to advertising cost is held fixed at $\left(\frac{M}{k}\right) = 2$ in an industry without demand growth. That is, the policy function of firm 1 always resembles the one in the top right panel of Figure 1, never the one in the top left panel. Moreover, since all demand states but the final are transitory, the limiting distribution is identical to the one that results from an industry without demand growth.

While it does not have a lasting impact on the structure of the industry, the prospect of growth affects firms' advertising strategies in earlier demand states and therefore the transient distribution. Table 10 illustrates the transitional dynamics starting from the extremely asymmetric industry structure that emerges in an industry without demand growth. As the table shows, the small firm does not stay small for long. Once the small firm catches up with the large firm, the industry remains symmetric forever.

Nature of advertising. The nature of advertising varies greatly between industries, with advertising for beer and cigarettes being predominantly persuasive and advertising for optical goods and major appliances being predominantly informative (see e.g. pp. 292 of Bauer & Greyser 1968). Yet it also seems that in many (if not most) industries advertising has both persuasive and informative aspects to it. We explore this idea in the simplest possible setting by encoding both its goodwill and its awareness in the state of a firm. That is, if the industry is in state (i, j), then firms' levels of goodwill are (v_i, v_j) and their levels

T	demand	firms
5	1	$(1, 20)$, $(20, 1)$
10	1	$(2, 20)$, $(20, 2)$
15	1	$(4, 20)$, $(20, 4)$
25	1	$(7, 20)$, $(20, 7)$
50	2	$(20, 20)$
100	2	$(20, 20)$
150	3	$(20, 20)$
∞	3	$(20, 20)$

Table 10: Most likely industry structure after T periods. Demand growth. Goodwill advertising.

of awareness are (s_i, s_j). A successful advertising campaign increases both goodwill and awareness by one level. Goodwill runs from a lower bound of $\underline{v}$ to an upper bound of $\overline{v}$ and awareness runs from 0% to 100% in steps of 5%.

The left panel of Table 11 illustrates the evolution of the industry if $\underline{v} = -1$ and $\overline{v} = 9$. In case of a small market/expensive advertising ($\left(\frac{M}{k}\right) = 2$), a large firm has a strategic advantage over a small rival. In case of a large market/cheap advertising ($\left(\frac{M}{k}\right) = 10$), in contrast, a firm cannot deter its rival from advertising by growing large. Hence, like in the goodwill model, the industry evolves towards an extremely asymmetric structure in case of a small market and towards a symmetric structure in case of a large market.

	$\underline{v} = -1$, $\overline{v} = 9$		$\underline{v} = 6$, $\overline{v} = 9$		$\underline{v} = 6$, $\overline{v} = 9$
T	$\left(\frac{M}{k}\right) = 2$	$\left(\frac{M}{k}\right) = 10$	$\left(\frac{M}{k}\right) = 2$	$\left(\frac{M}{k}\right) = 10$	$\left(\frac{M}{k}\right) = 0.95$
15	$(7, 7)$	$(9, 9)$	$(8, 8)$	$(10, 10)$	$(6, 7)$, $(7, 6)$
25	$(0, 14)$, $(14, 0)$	$(14, 14)$	$(9, 14)$, $(14, 9)$	$(11, 17)$, $(17, 11)$	$(7, 12)$, $(12, 7)$
50	$(0, 20)$, $(20, 0)$	$(20, 20)$	$(10, 20)$, $(20, 10)$	$(10, 20)$, $(20, 10)$	$(10, 19)$, $(19, 10)$
∞	$(0, 20)$, $(20, 0)$	$(20, 20)$	$(10, 20)$, $(20, 10)$	$(10, 20)$, $(20, 10)$	$(0, 20)$, $(20, 0)$

Table 11: Most likely industry structure after T periods. Combined advertising.

Somewhat surprisingly, unlike in the awareness model, a large firm does not have a strategic advantage over a medium-sized rival. In fact, a closer inspection of the profit function $\pi(i, j)$ shows that a firm's per-period profit does not peak in its own state when competing against a large firm. Recall that in the awareness model the larger firm responds aggressively once the smaller firm tries to grow by triggering a price war. In the combined model this is made up for because the smaller firm benefits from the higher willingness to pay of consumers that results from its higher goodwill. Hence, given large enough additions to goodwill, the firm's profit continues to increase in its own state. Put loosely, goodwill offsets awareness if $\underline{v} = -1$ and $\overline{v} = 9$.

If the additions to goodwill are smaller, say because $\underline{v} = 6$ and $\overline{v} = 9$, then goodwill does not offset awareness. In fact, the reverse is true. Now a firm's profit peaks in its own state when competing against a large firm, thereby giving a large firm a strategic advantage over a medium-sized rival. As a consequence, as the middle panel of Table 11 illustrates, the industry evolves towards an asymmetric structure irrespective of the size of the market and the cost of advertising.

Depending on the parameterization it is possible that both sources of strategic advantage are present and reinforce each other. Consider $\underline{v} = 6$, $\overline{v} = 9$, and $\left(\frac{M}{k}\right) = 0.95$. Now a large firm can deter a small rival from advertising and it can almost (but not quite) deter a medium-sized rival from advertising. Consequently, as the right panel of Table 11 illustrates, the industry first evolves towards an asymmetric structure with a large and medium-sized firm and then towards an extremely asymmetric structure with a large and a small firm. That is, the combined model behaves much like the awareness model in the short-run and much like the goodwill model in the long-run.

Our combined model holds fixed the mix of persuasion and information in advertising. This sidesteps a number of interesting questions. In practice, firms have at least some control over the nature of advertising, and the optimal mix may vary, for example, with the product life cycle. Before it can build up goodwill for its product, first a firm has to build up awareness (see Mahajan, Muller & Kerin (1984) for the case of a monopolist). More interestingly, however, the marginal benefit of goodwill is presumably increasing in the stock of awareness and *vice versa*, and this complementarity between goodwill and awareness may give rise to very rich dynamics. Furthermore, the cost of a creating a unit of goodwill may differ from the cost of creating a unit of awareness, say if the former relies on TV commercials whereas the latter relies on a relatively more expensive direct sales force. This raises the important question of media planning in dynamic settings. We leave these questions to future research.

6.2 Quality Competition

By reinterpreting advertising as investment in R&D and firms' levels of goodwill (v_i, v_j) as the *actual* rather than the *perceived* quality of firms' products our model of goodwill advertising is seen to resemble a model of price competition with vertically differentiated products. Shaked & Sutton (1982) show that the subgame perfect Nash equilibrium of a static game of quality choice followed by price competition gives rise to quality differentials although quality is costless. In fact, one firm may offer the highest possible quality, the other the lowest (Anderson, de Palma & Thisse 1992, Section 8.3.2). The reason is that Shaked & Sutton (1982) assume that consumers differ in their marginal valuation of quality, but are identical in every other respect. A Bertrand-style argument thus implies that profits are zero whenever qualities are the same. The assumption that, at the same price, all consumers can agree on a ranking of the products is often criticized as being rather stringent (Shaked & Sutton 1987). Once it is relaxed by incorporating idiosyncratic shocks into the utility specification, both firms opt for maximal quality if quality is costless as in Shaked & Sutton (1982). On the other hand, if quality is costly, the degree of heterogeneity comes into play. If consumers are sufficiently heterogeneous, both firms offer the same quality, whereas the firms offer different qualities if consumers are sufficiently homogeneous. Finally, for intermediate degrees of heterogeneity there is one subgame perfect Nash equilibrium of each type (Anderson et al. 1992, Section 7.5.3).

In our setup, the idiosyncratic shocks ϵ_{m0}, ϵ_{m1}, and ϵ_{m2} capture taste differences among consumers. In order to explore how the degree of heterogeneity affects the structure and dynamics of the industry, we now respecify the utility consumer m derives from purchasing from, say, firm 1 as $v_i - p_1 + \sigma\epsilon_{m1}$, where $\sigma > 0$ is proportional to the standard deviation of the taste differences among consumers. Rescaling yields $\frac{1}{\sigma}(v_i - p_1) + \epsilon_{m1}$ and reveals that increasing σ has two effects. First, it makes the inside goods less attractive relative to the outside good. This is best seen in the extreme case of $\sigma = \infty$ in which the market is split equally between the two firms and the outside good irrespective of the quality of firms' products. This takes away the incentive to invest. Indeed, the extreme case entails $x(i, j) = 0$ for all i, j in equilibrium, and the industry is stuck in a symmetric structure with both firms at the lowest possible quality level. Second, increasing σ makes being the low-quality firm more attractive relative to being the high-quality firm. The ratio of the low-quality to the high-quality firm's profit from product market competition in states $(0, 20)$ and $(20, 0)$, for example, is 0.0010 if $\sigma = 0.5$, 0.0051 if $\sigma = 1$, and 0.0137 if $\sigma = 1.5$, and continues rising towards one. This gives the low-quality firm a stronger incentive to invest. Consequently, holding market size and advertising cost fixed, increasing (decreasing) σ should bias the industry towards a symmetric (extremely asymmetric) structure. In fact, with $\left(\frac{M}{k}\right) = 2$, increasing σ from 1 to 1.5 leads to two high-quality firms and, with $\left(\frac{M}{k}\right) = 10$, decreasing σ to 0.25 leads in the long-run to an industry structure with a high-quality and a low-quality firm.

This suggests that the basic conclusions regarding product selection that have been derived in static games continue to hold in dynamic settings in which product innovation proceeds step-by-step in an incremental fashion rather than at one fell swoop. A word of caution is nevertheless in order: While an industry may ultimately reach the same point in quality space in a dynamic model as in a static model, our model of goodwill advertising indicates that it may take the industry a long time to do so. In this case, knowing the transition path is arguably more important than knowing the steady state. Yet, a static model cannot speak to the former.[29]

Our model of goodwill advertising closely resembles the dynamic model of quality competition of Pakes & McGuire (1994).[30] Our results thus yield new insights into their model. First, depending on the parameterization, Pakes & McGuire's (1994) quality ladder model may lead to a symmetric or an asymmetric industry structure. Pakes & McGuire (1994) do not notice that asymmetries are not ubiquitous in their model because they confine themselves to a single parameterization. In addition, the possibility of gaining a strategic advantage does not lead to a preemption race in Pakes & McGuire's (1994) quality ladder model. In fact, firms barely put up a fight for market dominance, and one of the firms is stuck forever with a good of the lowest possible quality. Most important, however, we identify cost/benefit considerations as the driving force behind the persistent firm-size differences that arise in Pakes & McGuire's (1994) quality ladder model.

7 Conclusions

The idea that advertising shapes the structure of an industry dates back at least to Kaldor (1950). As Kaldor (1950) puts it, "the introduction of advertising causes a competitive field to become 'gravitationally unstable' " (p. 13) and therefore has a "concentration-effect" on the industry. Underlying this "concentration-effect" are scale economies in advertising and the assumption that larger firms are better able to finance expensive advertising campaigns than smaller firms. The scale economies that are at the heart of Kaldor's (1950) argument, however, are not borne out by the data (Bagwell 2005, pp. 33–36). In this paper, we are able to explain substantial and persistent differences among *ex ante* identical firms without relying on scale economies or financing constraints.

This paper proposes a dynamic model of advertising competition. We start by specifying in detail why and how advertising affects consumer choice. Given consumer behavior, we build up a model of product market competition and then study the persuasive as well as the informative aspects of advertising in a dynamic setting. We highlight the model of advertising, the size of the market, the cost of advertising, and the intensity of product

[29] In addition, since different strategies yield different payoffs, it is not evident how *ex ante* identical firms coordinate on an asymmetric equilibrium in the existing static models. Hence, the outcome of these games may depend on factors outside the model (e.g., social norms or pre-play communication). In contrast, we show how an asymmetric outcome can result from a symmetric equilibrium.

[30] The main differences are that Pakes & McGuire (1994) assume an industry-wide depreciation shock that aversely affects the quality of all firms in the industry whereas we assume idiosyncratic forgetting and that they map a firm's state into its level of quality using the nonlinear function $v_i = 3i - 7$ if $i \leq 6$ and $v_i = 12 + \ln(2 - \exp(-(3i - 19)))$ if $i > 6$ whereas we use the linear function $v_i = \frac{i}{2}$ to map a firm's state into its level of goodwill. To ensure that none of these differences is critical, we have replicated a duopoly version of Pakes & McGuire's (1994) quality ladder model. Details are available from the authors upon request.

market competition as key determinants of industry structure and dynamics and show that, under goodwill as well as awareness advertising, asymmetries arise and persist provided that one firm has a strategic advantage over the other. The basis for a strategic advantage, however, differs considerably in the two models of advertising. Under goodwill advertising cost/benefit considerations give rise to a strategic advantage. Under awareness advertising, in contrast, the larger firm is able to keep a smaller rival in check by threatening a price war. The strategic advantage is therefore grounded in the nature of product market competition. Industry dynamics in this latter case take the form of a preemption race.

Our results have important implications for regulatory policy. We show that tighter regulation or an outright ban of advertising may have anticompetitive effects. Because regulating or banning advertising makes it harder and thus costlier for firms to reach consumers, advertising restrictions may enable one firm to dominate the industry provided that asymmetries are due to cost/benefit considerations. Our results thus caution against the market power theory of advertising. They also help to explain why empirical studies find that concentration has increased after regulation was implemented in industries like cigarettes and beer, where advertising is likely to be persuasive in nature.

We apply our dynamic model of advertising competition to study firms' entry and exit decisions and their impact on the structure of an industry. Exit is induced and entry is deterred by over-advertising. Moreover, an incumbent generally accommodates entry by under-advertising. In a dynamic setting like ours, however, the incumbent's desire to soften product market competition by under-advertising may be overridden if post-entry industry dynamics take the form of a preemption race. In this case, the incumbent accommodates entry by over-advertising in an attempt to gain a head start in the race and improve its chances of eventually becoming the dominant player in the industry. This finding underscores the importance of accounting for industry dynamics.

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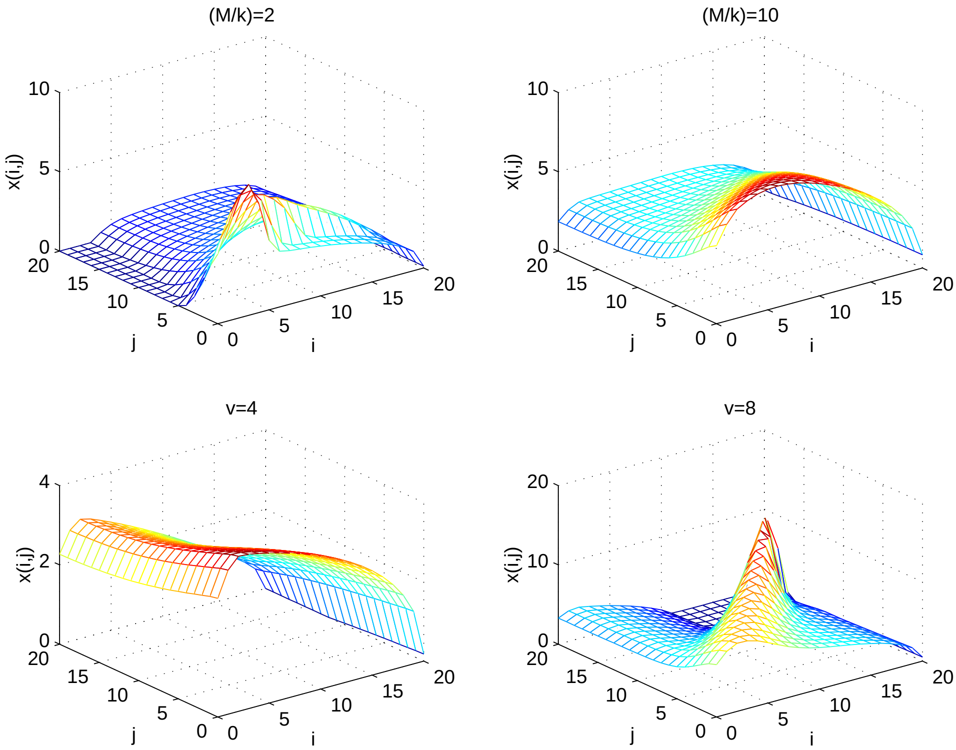

Figure 1: Policy function $x(i,j)$. Goodwill advertising with $\left(\frac{M}{k}\right) = 2$ (top left panel) and $\left(\frac{M}{k}\right) = 10$ (top right panel); awareness advertising with $v = 4$ (bottom left panel) and $v = 8$ (bottom right panel).

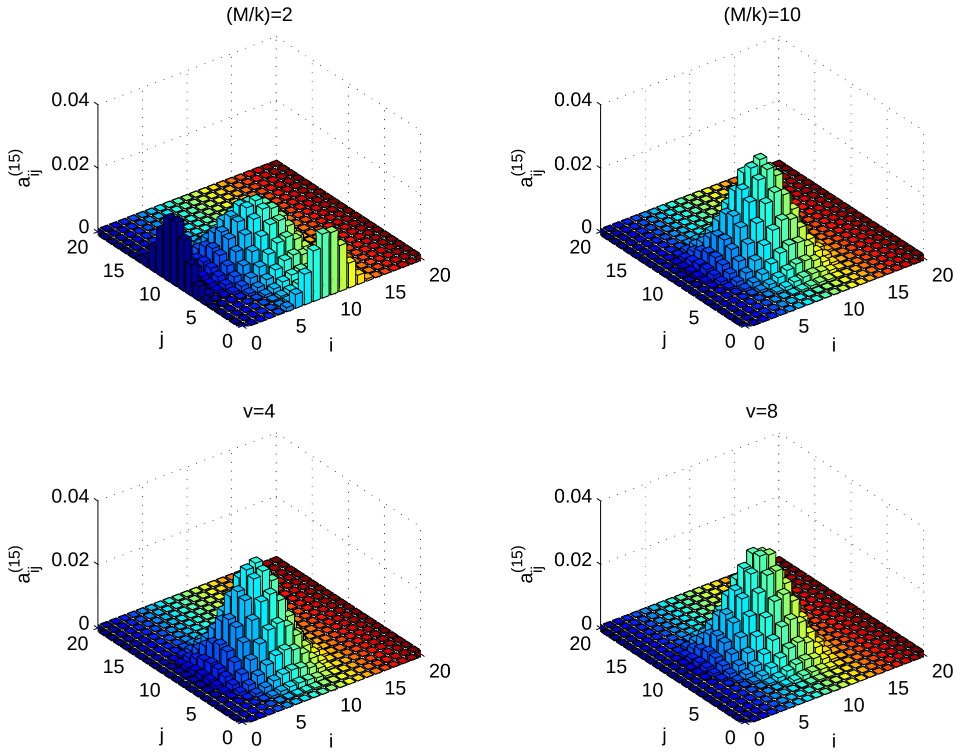

Figure 2: Transient distribution after $T = 15$ periods with $i_0 = j_0 = 0$. Goodwill advertising with $\left(\frac{M}{k}\right) = 2$ (top left panel) and $\left(\frac{M}{k}\right) = 10$ (top right panel); awareness advertising with $v = 4$ (bottom left panel) and $v = 8$ (bottom right panel).

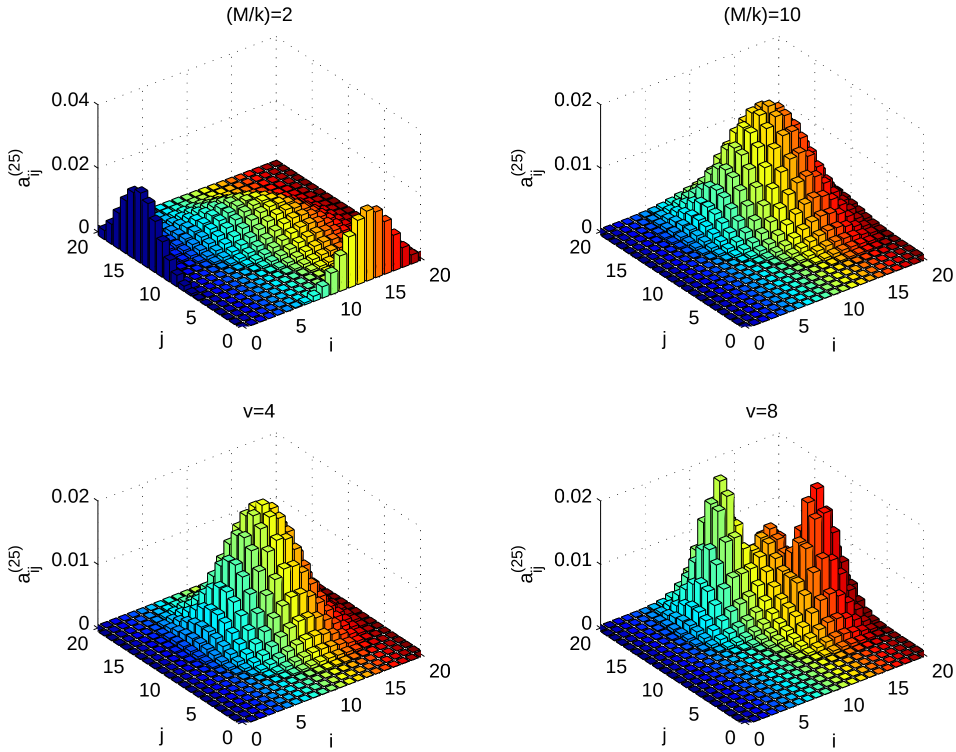

Figure 3: Transient distribution after $T = 25$ periods with $i_0 = j_0 = 0$. Goodwill advertising with $\left(\frac{M}{k}\right) = 2$ (top left panel) and $\left(\frac{M}{k}\right) = 10$ (top right panel); awareness advertising with $v = 4$ (bottom left panel) and $v = 8$ (bottom right panel).

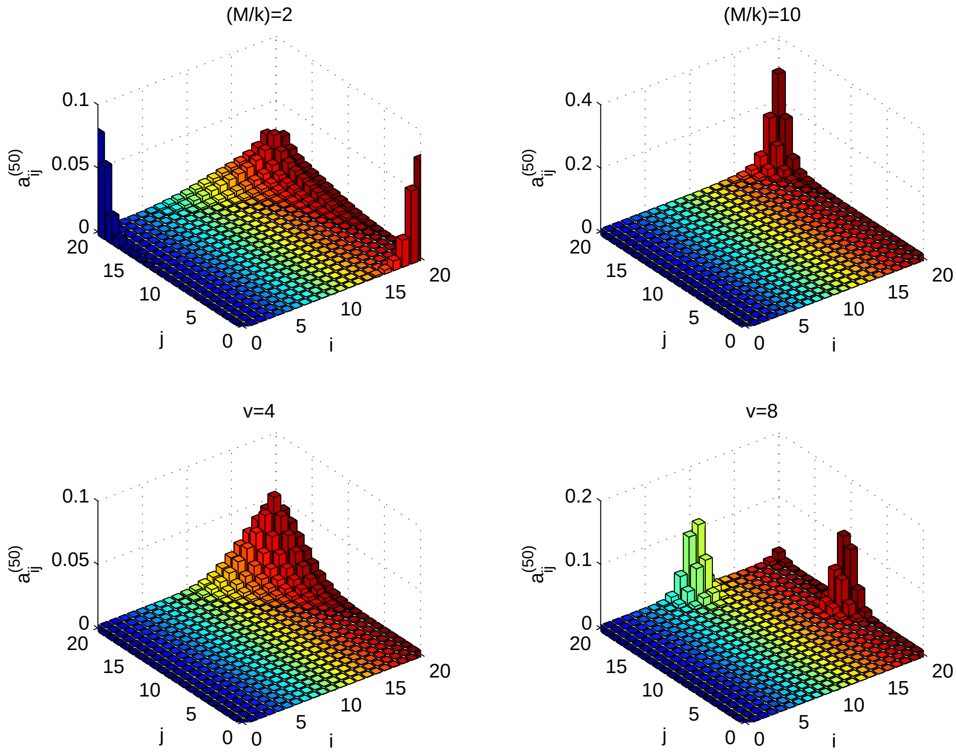

Figure 4: Transient distribution after $T = 50$ periods with $i_0 = j_0 = 0$. Goodwill advertising with $\left(\frac{M}{k}\right) = 2$ (top left panel) and $\left(\frac{M}{k}\right) = 10$ (top right panel); awareness advertising with $v = 4$ (bottom left panel) and $v = 8$ (bottom right panel).

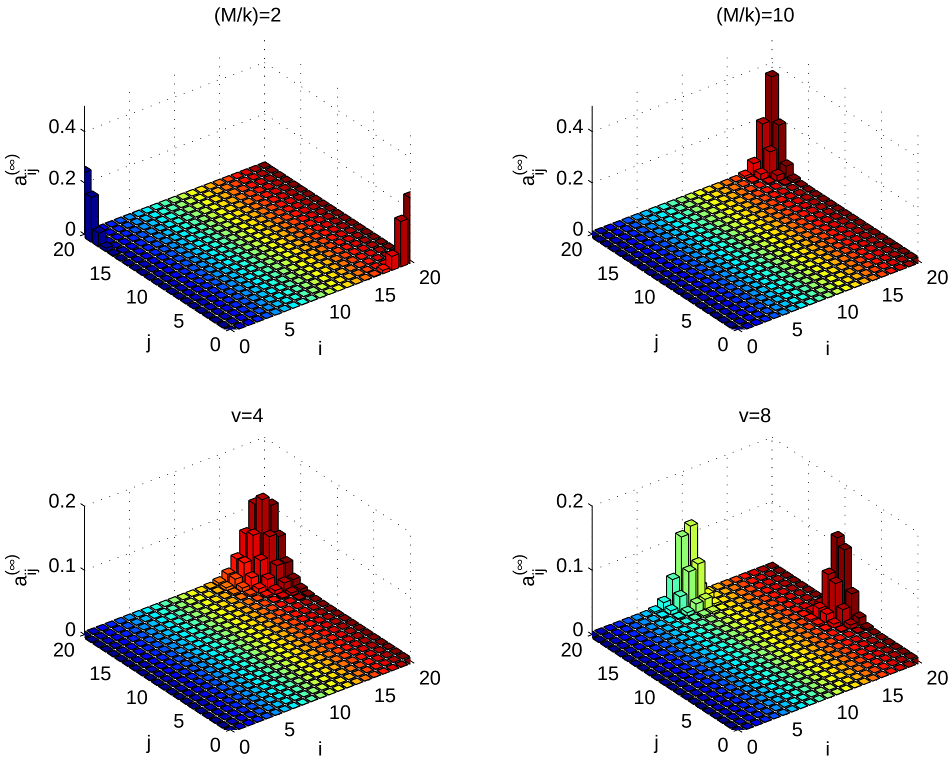

Figure 5: Limiting distribution. Goodwill advertising with $\left(\frac{M}{k}\right) = 2$ (top left panel) and $\left(\frac{M}{k}\right) = 10$ (top right panel); awareness advertising with $v = 4$ (bottom left panel) and $v = 8$ (bottom right panel).

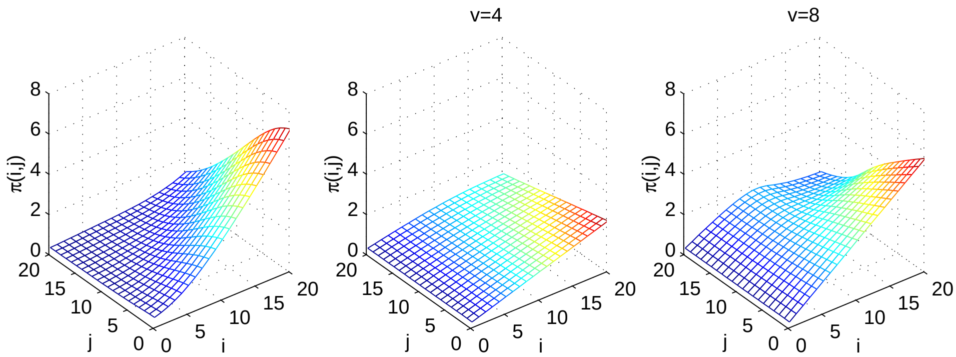

Figure 6: Profit function $\pi(i,j)$. Goodwill advertising (left panel), awareness advertising with $v = 4$ (middle panel), and awareness advertising with $v = 8$ (right panel).

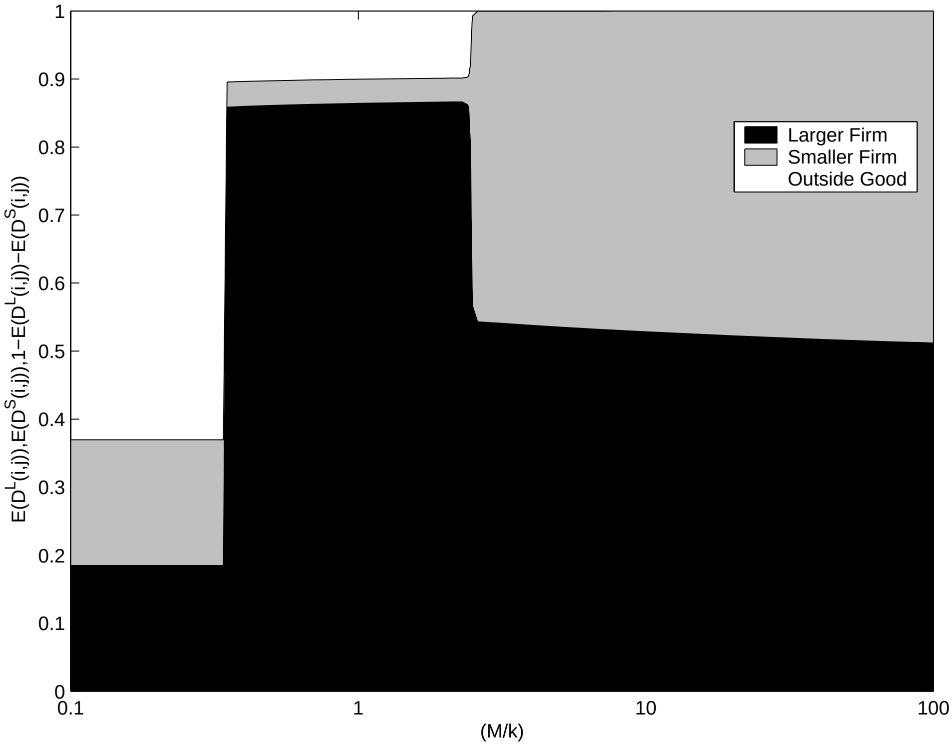

Figure 7: Expected market share of the larger firm $E\left(D^L(i,j)\right)$ (black), expected market share of the smaller firm $E\left(D^S(i,j)\right)$ (grey), and expected market share of the outside good $1 - E\left(D^L(i,j)\right) - E\left(D^S(i,j)\right)$ (white). Goodwill advertising.

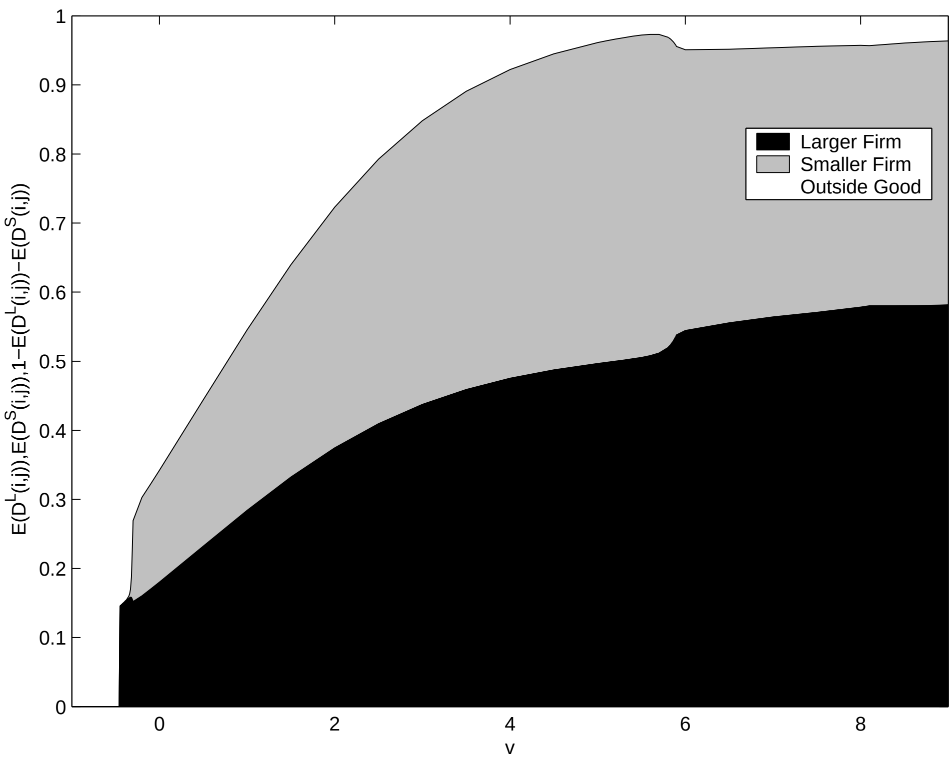

Figure 8: Expected market share of the larger firm $E\left(D^L(i,j)\right)$ (black), expected market share of the smaller firm $E\left(D^S(i,j)\right)$ (grey), and expected market share of the outside good $1 - E\left(D^L(i,j)\right) - E\left(D^S(i,j)\right)$ (white). Awareness advertising.

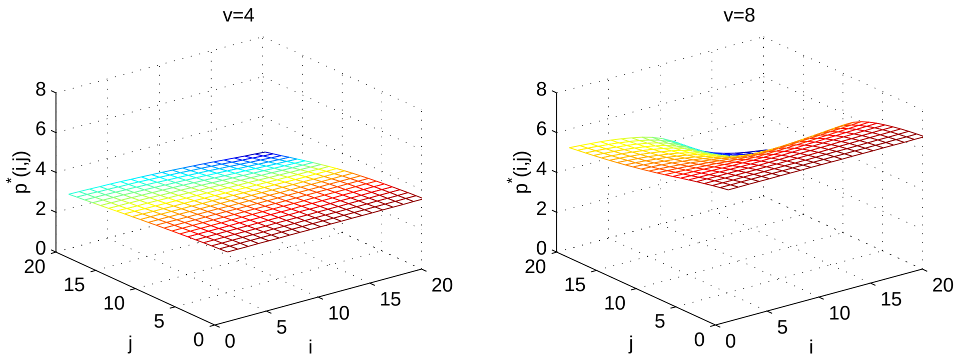

Figure 9: Equilibrium price $p^*(i, j)$. Awareness advertising with $v = 4$ (left panel) and $v = 8$ (right panel).

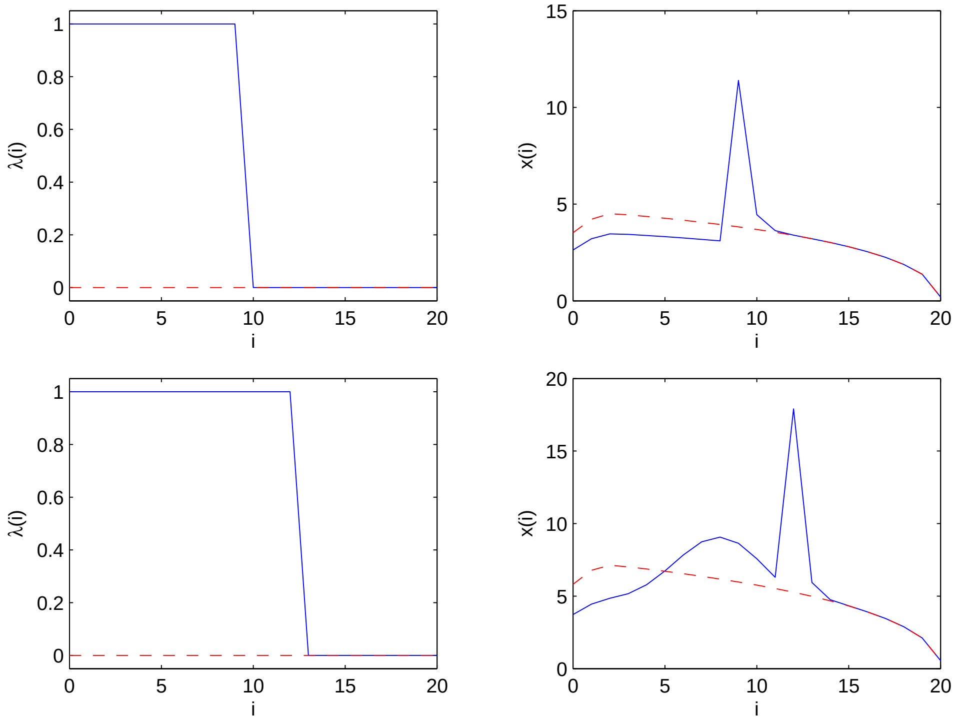

Figure 10: Entry deterrence and accommodation. Entry and advertising policy functions $\lambda(i)$ and $x(i)$ with entry ($\phi^e < \infty$, solid line) and without entry ($\phi^e = \infty$, dashed). Awareness advertising with $v = 4$ and $\phi^e = 375$ or ∞ (top panels) and $v = 8$ and $\phi^e = 675$ or ∞ (bottom panels).

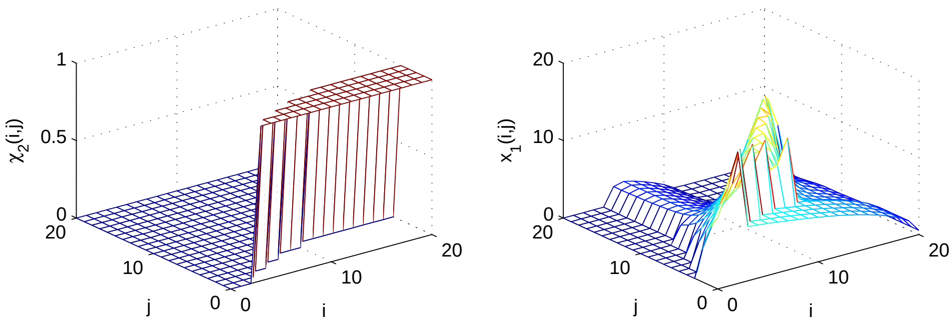

Figure 11: Exit inducement. Firm 2's exit policy function $\chi_2(i, j)$ and firm 1's advertising policy function $x_1(i, j)$. Awareness advertising with $v = 8$ and $\phi = 650$.