On the Competitive Effects of Multimarket Contact☆

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Abstract

Changes in the extent of multi-market contact (MMC) between firms often affect market outcomes – quantities and prices. We show that a strategic but purely competitive effect of changes in MMC can change the quantity provided in a market by a firm by as much as 50%, and the prices a firm sets by as much as 20%. This may have important welfare implications, specifically with regards to horizontal mergers. Studying mergers that span several markets, we show that a myopic merger policy may thwart a surplus-increasing merger wave. The analysis does not rely on any tacit or explicit collusive behavior by the firms.

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1. Introduction

Large firms are often active in more than one market and commonly compete with each other in many, but not necessarily all, markets. While inconclusive, most empirical work finds a positive correlation between increases in firms' MMC and prices. A common microeconomic explanation for MMC effects on market outcomes is that MMC facilitates mutual forbearance (i.e., tacit collusion). Alternatively, **?** show that when one firm has access to markets not served by its rival in the overlapping market then the type of competition (complements or substitutes) and the cost structure

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of the firms (economies of scale or diseconomies of scale) affect the firm's behavior in the overlapping market.

This paper extends the approach in **?** to study an alternative, purely competitive, microeconomic foundation for the relation between MMC and market outcomes. Whenever a firm makes investments that can be then used to serve multiple markets, changes in MMC will affect prices and outcomes by competitive responses of the firms' rivals. We call this the *competitive effect of MMC (*henceforth C-MMC effect*).*

Competition in our framework takes place in two stages. In the first, each firm makes a non-reversible investment (e.g., production capacity) that is transferable across its different markets. In the second stage, firms compete in either prices or quantities in several different markets. However, the total quantity sold by a firm over all of its markets cannot exceed the firm's production in the first stage. We model MMC by letting firms serve *overlapping markets* that are also served by rivals and *private markets* that are only served by the firm. We characterize the effect for general differentiated demand functions. For linear demand, we show that changes in MMC can, through the C-MMC effect, change the quantity provided in a market by a firm by as much as 50%, the prices a firm sets by as much as 20%, and the firm's profits by over 10%.

This two stage dynamic, where a sunk transferable investment is shared across the firm's different markets, is common in many industries.[1] Airlines, which have been the subject of MMC studies we discuss below, have the two-stage dynamic where fleet scheduling decisions are made months before the first seat on the flight is offered for sale (see e.g. **?** and **?**). However, seats on a direct flight, e.g. from NYC to Miami, FL are sold on multiple markets – all markets from the US Northeast to/from Miami, many markets from Europe to/from Miami, markets to/from South America and the US, etc. The airline faces different rivals in all these different markets. Gate purchasing decisions have exactly the same dynamics.

The two-stage structure also fits settings where firms make a sunk capacity decision for an intermediate input that is then used by the firm in the second stage for one of several different finished products. For example, our setting fits nicely with firms' internal capital market where firms first set a capital budget and then allocate resources across geographical units or product

[1]The formalization of 'invest then produce' dates back to at least **?** that originally considered capital investment followed labor production costs. **?** and **?** study this dynamic in a single market setting. **?** and **?** formalized the economic use of committed investments in capacity or in cost reduction to deter entry and help incumbents achieve a Stackelberg type leadership position in a market.

markets. Alternatively, the first stage may capture the firm's decision of the number and size of all production facilities, and subsequently allocate production from all facilities across product lines and geographies, a practice that is common in manufacturing: e.g., automobiles, appliances, cement, steel, etc. The same rationale holds for any specialized input that can be utilized by different units within the firm where the cost of increasing or decreasing the use of the specialized input is prohibitively high.[2]

The key characteristic connecting all these is that once investment is sunk, multi-market firms have the *flexibility* to reallocate the sunk investment across their different markets. If market conditions deteriorate for American Airlines on the route NYC-Miami, for example, it can reallocate more of the seats on the flight from NYC to Miami to the route NYC-Bogota connecting in Miami; the more destinations American Airlines serves in/out of NYC or Miami, the higher its flexibility in reallocating the seats on the NYC-Miami flight.

The C-MMC effect arises when a rival can take advantage of this flexibility. If a rival is aggressive in an overlapping market, a multi-market firm can reallocate a larger share of the sunk investment into markets in which the rival does not operate. Thus, a firm's flexibility to reallocate the sunk investment to non-overlapping markets increases the rival's profit from aggressive deviations.

This strategy requires the rival to *commit* to an aggressive behavior: increase the share of its sunk investments that will be used in the overlapping markets. The fewer markets the firm serves, the stronger its commitment power. We show that in industries with MMC, equilibrium outcome can be defined in terms of the firms' *flexibility* and *commitment power*.

The main welfare implication of the C-MMC effect is that asymmetry in scope between firms improves welfare. Welfare is typically maximized when a large multi-market firm competes with smaller local firms, subject to potential scope-related cost savings. This result is most relevant for horizontal mergers, which we further discuss below.

We derive comparative statics describing the C-MMC effect when second stage competition is either in prices or in quantities, independent of any specific demand form. We show that when firms compete in prices, any increase in MMC increases prices for all firms in the overlapping markets and in *all* other markets served by these firms. That is, an increase in MMC between firms A and B increases equilibrium prices also in markets served *only* by A or B. When firms compete in

[2] For examples, see, e.g., **?** for car manufacturing, **?** for the pulp and paper industry, and **?** for a follow-up analysis considering merger effects.

quantities, we show that an increase in a firm's MMC increases its own quantity and decreases its rival's quantity.

The model allows us to also characterize the implications of an increase in overlap between the firms. This describes the changes in market outcomes as two firms gradually enter each-other's markets, possibly to the point that all markets are served by both firms. The C-MMC effect in this case is *non-monotonic* for industries in which firms compete in quantities. If firms overlap in just a few of the markets they serve, an increase in MMC increases quantities (and decreases prices). The effect is reversed if firms overlap in most markets. In industries characterized by price competition, an increase in overlap always increases prices.

There is by now a large empirical literature documenting the relation between the extent of firms' multi-market contact (MMC) and market outcomes. **?** surveys the earlier studies. Earlier and recent examples include **???????????** for MMC studies on the airline, banking, telecommunication, cement, software, hotel, and cooking oil industries.

Our competitive MMC effect complements the mutual forbearance, or collusive, effect of MMC suggested by **?**, which states that as firms interact over more markets, the long run returns from collusion are higher, increasing the feasibility of tacit collusion. **?** formalize this and show, however, that the increase in the number of markets also implies an increase in the returns for deviating from any collusive agreement. Therefore, some additional conditions must be met for the collusive effect of MMC.[3] Our model complements the mutual forbearance literature and shows that MMC can alter firms' strategies and market outcomes absent any long term collusive strategies. Whether firms adopt collusive strategies when facing MMC or other competitive strategies is then an empirical question.

While the comparative statics of the competitive and collusive MMC effects may be similar in some settings, there is an important qualitative difference between the two effects. Mutual forbearance implies that MMC causes multi-market firms to produce less than single-market firms. However, the C-MMC effect elicits *higher* production rates than in the no-MMC benchmark. Therefore, simple pre vs. post merger quantity or price evaluations can mislead regulators (or policy makers) to

[3]**?** show that asymmetries between markets can generate the collusive MMC effect. **?** obtains the collusive MMC effect if firms aggregate profits over the multiple markets are a concave function of their profit in each market (due, e.g. to managers' risk averseness or compensation structures or to the firms' tax incentives). **?** shows that MMC can improve collusion in the presence of imperfect monitoring. See also **?** for a formulation of mutual forbearance using conjectural variations.

the conclusion that MMC is enabling firms to produce at higher margins and lower surplus while in fact it is the absence of any MMC that strictly decreases surplus.

MMC is closely related to horizontal merger evaluation. Mergers often increase the number of markets a firm operates in, and by this drastically change the extent of MMC between the merged firm and its competitors.[4] Section 4 shows that through the C-MMC effect, a merger can increase or decrease consumer surplus and total welfare without changing market power, production efficiency nor facilitating collusion.

Our analysis provides a simple policy recommendation: regulators should strive for asymmetry. Industries will be more competitive if large multi-market firms have viable local small competitors. If production economies favor scope, a small overlap between firms should increase competition, while significant overlap would hinder competition, regardless of any mutual forbearance concerns.

The competitive and the mutual forbearance effects may have qualitatively different welfare predictions and therefore understanding which effect applies has significant policy implications. For example, if firms compete in quantities, the competitive effect implies that a merger that increases the number of overlapping markets between firms may increase welfare, *if it increases* the number of non-overlapping markets *even more*. In contrast, the mutual forbearance effect implies that any increase in overlap can only increase the potential for tacit collusion.

Another implication pertains to dynamic merger policy. The C-MMC effect implies that when considering mergers that span several markets, it may well be that one merger is surplus increasing while a second merger is not, even if both create the same cost efficiencies and have no implications on the within-market structure. We show in section 4 how this may cause a myopic regulator that is committed to reject surplus-decreasing mergers to thwart a surplus-increasing merger wave. This result complements **?** who, considering mergers within a specific market, find that a myopic merger policy is sufficient.

The different microeconomics foundations of C-MMC compared to the mutual forbearance effect of MMC provide possible empirical approaches to distinguish between the two effects. In particular, the main empirical prediction of our paper is that the C-MMC effect is present *only* across markets that share a sunk transferable investment. In other markets, a change in MMC should not affect

[4] In two recent large horizontal mergers the merging firms had little overlap prior to the merger. Delta and Northwest argued that they do not compete directly in most markets. Comcast and Time Warner Cable, by law, did not compete directly in any market.

competitive behavior.

To the best of our knowledge, this empirical hypothesis was tested only by **?**. The authors there find that in the airline industry, changes in MMC only affected markets that used the same hub, consistent with our model's prediction. Section 5 discusses this result.

Our paper is closest to the contemporaneously developed work by **?**. There, firms also compete in two stages with a capacity decision followed by quantity competition in two markets. **?** finds that the solution for each firm is between Cournot and Stackelberg. Our proposition 2.1 generalizes **?**'s formal results. Our analysis provides several important additional results. First, we formalize the comparative statics, while **?** only identifies the deviation from the standard Cournot and provides the limit result. The comparative statics analysis provides the non-monotonic results that are not present in **?**. Second, we also analyze the case where the second stage game is in strategic complements (prices), which **?** does not. Third, our comparative statics analysis obtains formal market level (i.e., welfare) results in addition to firm level effects. Fourth, our analysis is not limited to two markets. Finally, our more general analysis allows us to consider merger implications.

Our analysis is closely related to the seminal work of **?** (hereafter BGK), which show that when one firm has access to private markets market outcomes depend on whether competition is in strategic complements or strategic substitutes and whether firms' cost structure across the different markets exhibit economies or diseconomies of scale. Our analysis differs from that of BGK in two important ways. First, we allow firms to make a single investment decision that applies equally to both markets, which in turn determine the only extent of diseconomies across markets. Second, we allow for both firms to have access to private markets. The discussion in section 2.4 expands on the subtle but important modeling differences and illustrates their implication in the BGK two-market setting.

The analysis's results depart from the results in BGK in three main ways: (i) The effect of a change in the rival's MMC in our model has the same sign whether firms compete in strategic substitutes or complements, while in BGK the two cases always have opposing signs; (ii) We find a *non-monotonic effec*t for a joint change in MMC when firms compete in strategic substitutes, while in BGK all effects are monotonic; and (iii) When both firms have private markets, diseconomies across markets creates strategic links between the firms' private markets which are otherwise unrelated, a setting that is not considered in BGK.

In another related paper, **?** also allow firms to have overlapping and private markets. However, they force the firms to have uniform price in all markets and do not consider other multi-market effects. Finally, note that firms may mitigate competitive effects, including the C-MMC effect, by designing commitment mechanisms either between firms (e.g. tacit collusive) or within firms (e.g., managerial contracts as in **?**) .

Section 2 presents and solves the model based on quantity competition. Section 3 analyzes price competition. Section 4 analyzes horizontal mergers and discusses the implications for merger policy, section 5 provides suggestive empirical evidence, and section 6 concludes.

2. Quantity Competition

2.1. Setup

Consider an industry with two firms, identified by $i \in \{A, B\}$ and three types of markets: *overlapping* markets in which both firms are active, and *private* markets for each firm – markets in which firm A operates but firm B does not, and markets in which firm B operates but firm A does not. All markets of each type are identical in terms of demand. However the number of markets of each type may vary. In particular, we denote by M, m_A, and m_B the unit measure, or number, of the overlapping markets, A's private markets, and B's private markets, correspondingly.

The model has two stages. In the first stage firms simultaneously make a sunk transferable investment, denoted k_A, k_B. Firm i pays a constant marginal cost c_i per unit of investment.[5] To fix ideas, we refer to this investment as production capacity. However, it may be interpreted as any investment that is sunk, transferable across markets, and can be utilized to increase production.

After both capacities are fixed, in the second stage, firms *compete in quantities* subject to their installed capacity. Each firm chooses the number of units to provide to each of the overlapping markets and the number to provide to each of its private markets. A firm's total output across all markets cannot be larger than its installed capacity. Markets clear accordingly.

We denote by q_A and q_B the quantity sold by each firm in each of the overlapping markets. Since markets are identical in terms of demand, each firm will offer identical quantities in all markets of a certain type. That is, the total quantity offered in all overlapping markets is $M(q_A + q_B)$. Similarly, denote by $\hat{q}_A$ and $\hat{q}_B$ the quantity sold in each private market so that the total quantity firm i offers in its private markets is $m_i \cdot \hat{q}_i$.

[5]The results are qualitatively unaffected when allowing for weakly convex costs. See also the discussion following proposition 2.1.

Firm i's inverse demand in each overlapping market is $p_i(q_i, q_{-i})$. Similarly, firm i's inverse demand in a private market is $\hat{p}_i(\hat{q}_i)$. For simplicity, we ignore second stage costs. However, the price functions can be seen as net of any second stage per-unit costs and therefore all the results apply without loss of generality to any linear second stage cost that may differ between firms and market type (overlapping/private).

A pure strategy *equilibrium* $(\mathbf{k}^\star, \mathbf{q}_A^\star, \mathbf{q}_B^\star)$ is a pair of scalars $\mathbf{k}^\star = (k_A^\star, k_B^\star)$ indicating the capacity set by each firm, and two pairs $\mathbf{q}_A^\star = (q_A^\star, \hat{q}_A^\star)$ and $\mathbf{q}_B^\star = (q_B^\star, \hat{q}_B^\star)$ indicating the quantity allocation chosen by each firm in each market type. The game is solved through backward induction. In the second stage, given capacities k_A and k_B, firms set quantities for each market type to maximize profit. Costs depend only on aggregate capacity, :

$$\begin{aligned} \pi_i(k_A, k_B) = \max_{q_i, \hat{q}_i \geq 0} \quad & M q_i p_i(q_i, q_{-i}) + m_i \hat{q}_i \hat{p}_i(\hat{q}_i) - k_i c_i \\ \text{s.t.} \quad & M q_i + m_i \hat{q}_i \leq k_i \end{aligned} \tag{2.1}$$

A sub game equilibrium $(\mathbf{q}_A, \mathbf{q}_B; \mathbf{k})$ is the set of quantity allocations that form an equilibrium given the first stage capacities $\mathbf{k}$. Throughout, we use η_i and $\hat{\eta}_i$ to denote firm i's marginal revenue curve in the overlapping and private markets, respectively:

$$\begin{aligned} \eta_i &= q_i \frac{\partial p_i(q_i, q_{-i})}{\partial q_i} + p_i(q_i, q_{-i}) \\ \hat{\eta}_i &= \hat{q}_i \hat{p}_i'(\hat{q}_i) + \hat{p}_i(\hat{q}_i) \ . \end{aligned}$$

We assume the following standard regularity conditions on the inverse demand functions:

Assumption 1. *All inverse demand functions are twice differentiable and:*

1. *Downward sloping and quasi concave in own quantity;*
2. *Goods are (imperfect) substitutes:* $\frac{\partial p_i}{\partial q_i} \leq \frac{\partial p_i}{\partial q_{-i}} < 0$
3. $\hat{p}_i(0) > c_i$ *and for any* q_{-i} *such that* $p_{-i}(q_{-i}, 0) > c_{-i}$, $p_i(0, q_{-i}) > c_i$.
4. *Marginal revenue is decreasing in own and rival quantity:* $\frac{\partial \eta_i(q_i, q_j)}{\partial q_i} \leq \frac{\partial \eta_i(q_i, q_j)}{\partial q_j} < 0$

The first two statements of the assumption are self explanatory. The third guarantees that no firm optimally stays out of any market. The fourth is the weak decreasing differences condition used to guarantee equilibrium existence for quantity competition (see, e.g., **?** pg. 151). The assumption always holds if demand is concave and goods are perfect substitutes but is weaker.

If there are second stage per-unit cost, assumption 2 simply implies that if i's rival is profitable in the overlapping market at a given quantity, then there is some quantity i can sell at a net profit as well.

Suppose for now that the second stage sub-game equilibrium exists and is unique for each first stage capacity pair k_A, k_B. The first stage problem for each firm is then given by:

$$\Pi_i = \max_{k_i \geq 0} \pi_i(k_A, k_B) \quad . \tag{2.2}$$

Below we show that a subgame perfect equilibrium in pure strategies exists, and characterize it. However, we first introduce the important concepts of flexibility and commitment in MMC.

2.2. Measuring MMC - Flexibility and Commitment

Most empirical studies consider M as the extent of MMC. Our analysis suggests instead to define the firm's extent of MMC to be the ratio between the number of overlapping markets and the firm's private markets:[6]

Definition 2.1. The extent of MMC for firm i is $\lambda_i = \frac{M}{m_i}$

Using λ_i allows the measure of MMC to be asymmetric: for example, if $m_A \gg M \gg m_B$ then MMC is small for firm A and large for firm B. The degree of MMC is large when M is large relative to both m_A and m_B.

In addition, we introduce two measures, flexibility and commitment power, that depend on the equilibrium outcomes similar to, for example, elasticity. As common for elasticity, we specify the relevant variables in the definition of flexibility and commitment but omit them for brevity in the discussion.

Definition 2.2. Firm i's *flexibility*, ϕ_i, is the firm's second stage reaction to a change in its rival's quantity allocation given any first stage capacities k_i, k_{-i} and rival quantity in the overlapping market. Formally: $\phi_i \equiv \frac{dq_i}{dq_{-i}}$.

Flexibility measures A's best response to B's second stage deviation in a possible equilibrium neighborhood. Because an increase in rival quantity decreases profits, firms' choices in quantity

[6] More recently, **?** uses both our relative measure and the standard M measure.

competition tend to be strategic substitutes and thus we expect, and prove below, that $\phi_i \leq 0$. Flexibility differs from the standard best response, which we denote $\overline{\phi}_i$,[7] because the economic cost of increasing q_i varies. In particular, as all production costs have been paid for in the previous stage, the cost of an increase in quantity in the overlapping markets is the opportunity cost of the required decrease in the private markets quantity.

When A's extent of MMC (λ_A) is large, A has a relatively small number of private markets. In these settings, moving quantity from the many overlapping markets to the few private markets will overwhelm the private markets and thus drastically harm profitability. The firm therefore behaves in the second stage as if it is in a Cournot game with little flexibility ($\phi_A \to 0$). In contrast, when A's extent of MMC is small, its private markets can absorb excess capacity without much effect. The analytic form of ϕ_i, derived in the appendix, confirms this intuition.

While in general flexibility may be profitable when dealing with uncertainty, in our setting, flexibility in the second stage makes the firm vulnerable. If firm A is flexible (ϕ_A is large in absolute terms) while firm B is not ($\phi_B \to 0$), B can make its second stage decision in the first stage. The resulting dynamics are as if the flexible firm is a Stackelberg follower and its rival a Stackelberg leader. However, for this to work, firm B needs to be able to make a credible first stage commitment:

Definition 2.3. Firm i's *commitment power*, $\sigma_i \equiv M \frac{dq_i^\star}{dk_i}$, is the change in the firm's second stage subgame equilibrium quantity in the overlapping markets following an additional unit of capacity

Commitment power is the fraction of the additional unit of capacity that would be allocated to the overlapping markets. For example, suppose both firms have some flexibility and both set capacity so that each can set the monopoly quantity in its private markets and the duopoly quantity in the overlapping markets. With these allocations in the second stage, A's marginal revenue equals to cost in both market types. A now can try to take advantage of its rival's flexibility and increase capacity and quantity in the overlapping markets. If A could commit to allocating all additional units to the overlapping market, A would indeed behave as a Stackelberg leader. However, the additional units reduce A's marginal revenue in the overlapping markets. As a result, it becomes strictly optimal for A to shift some of the new quantity to its *private* markets.

In any equilibrium, an increase in quantity reduces the marginal revenue and therefore σ_i is

[7] Using marginal revenues, the result for the standard Cournot model is $\overline{\phi}_i = -\left(\frac{\partial \eta_i}{\partial q_{-i}} / \frac{\partial \eta_i}{\partial q_i}\right)$.

positive but less than one. At the extreme, if firm i has no private markets ($\lambda_i \to \infty$), any first stage investment will be used in the overlapping markets and $\sigma_i \to 1$. In contrast, if the number of its private markets is large relative to the number of overlapping markets ($\lambda_i \to 0$), the bulk of i's capacity-increase must be used in its private markets, and $\sigma_i \to 0$.

2.3. MMC Equilibrium

With the definitions at hand, we first establish the second stage subgame equilibrium behavior. All proofs are in the appendix.

Lemma 2.1. *In any Subgame Perfect Nash Equilibrium:*

1. *The subgame has a unique equilibrium for any first stage capacities.*
2. *Firm i's marginal revenue is identical in both types of markets:* $\eta_i = \hat{\eta}_i$.
3. *If the first stage has an equilibrium in pure strategies then the capacity constraint binds:* $k_i = Mq_i + m_i\hat{q}_i$ *and all markets are served by all relevant firms* ($q_i\hat{q}_i > 0$).

To understand why Lemma 2.1 holds, consider A's best response to any q_B. First observe that A's optimal response must equalize marginal revenues across all of its markets. Otherwise, shifting quantity from low marginal revenue markets to high marginal revenue markets will increase profits. Next, observe that A's capacity constraint does not bind only if $\eta_A = 0$. Otherwise, profits are increased by simply allocating some unutilized capacity to either market. However, if $\eta_A = 0$ in any pure strategy equilibrium, A is strictly better off by producing less capacity to begin with.

With both of the previous results at hand, it is clear that in essence A's problem in the second stage is to allocate units across its overlapping and private markets. In other words, the second stage "effective" marginal cost of quantity in the overlapping market is the marginal revenue in the private market. This effective marginal cost is increasing and convex and as marginal revenue is strictly decreasing in own and rival quantity, the subgame equilibrium is unique:

Proposition 2.1. *The game has a unique SPNE in pure strategies,* $\phi_i \in \left[\overline{\phi}_i, 0\right]$, $\sigma_i \in (0,1)$ *and the following equation holds*

$$c_i - \eta_i = c_i - \hat{\eta}_i = q_i \frac{\partial p_i\left(q_i, q_{-i}\right)}{\partial q_{-i}} \phi_{-i}\sigma_i > 0 \qquad i \in \{A, B\} \ . \tag{2.3}$$

Firms' first stage capacities are strategic substitutes.

In the standard Cournot equilibrium, firms set marginal revenue at marginal cost ($\eta_i = c$). We refer to the difference between cost and marginal revenue in proposition 2.1 as firm *$i's$ competitive effect of MMC*, or C-MMC effect. Proposition 2.1 clarifies that in any SPNE a firm's competitive effect of MMC depends on three components: the per-market revenue change from its rival quantity change $\left(q_i \frac{\partial p_i}{\partial q_{-i}}\right)$, the rival's flexibility, ϕ_{-i}, and the firm's commitment power σ_i. That is, the extent of MMC, λ_i, affects firms' behavior *only* through flexibility and commitment.

Both flexibility and commitment can be derived as functions of the extent of MMC (λ_i) and the effect of quantities on marginal revenues in equilibrium. These are provided in the appendix (Lemma AppendixA.1). The derivation in Lemma AppendixA.1, while technical, confirms that, σ_i and ϕ_i depend on the MMC measurement λ_i and λ_{-i} rather than on the absolute values $M, m_i,$ and m_{-i}, and therefore so does the C-MMC effect. In other words, the correct comparative statics with respect to MMC use λ. Comparative statics with respect to specific market sizes (M,m) can be derived from how market size affects λ, and how λ affects outcomes. The former is straightforward using $\lambda_i \equiv \frac{M}{m_i}$, and the latter is derived below.

2.4. Model Discussion and Relation to **?**

The following example clarifies the model's main strategic interaction and compares it to the seminal **?** (i.e. BGK) model. Suppose demand in each market is $P = 1 - Q$ and the cost of production is zero per unit. If there are no private markets and both firms only serve one market, the standard Cournot equilibrium is obtained. Both firms equalize marginal revenue $\left(P - q^i\right)$ and marginal costs (0), resulting in quantities of $\frac{1}{3}$ each. Note that the best response for each firm is $\frac{\partial q_1^A}{\partial q_1^B} = -\frac{1}{2}$.

Splitting the Cournot game to two stages where both firms can set capacity in the first stage and then decide how much to sell in the second stage has no effect because no firm would produce unsold quantities. The first stage problem is identical to the above and the result is unchanged.

In contrast, if B can commit and set capacity in the first stage and quantity in the second stage while A decides on both capacity and quantity in the second stage, we have the familiar Stackelberg dynamic. B's first order condition accounts for A's second stage flexibility. Indeed as B fully commits all first stage capacity to market 1 and A has the standard Cournot best response, applying our proposition 2.1 letting $\phi_A = -\frac{1}{2}$ and $\sigma_B = 1$ obtains the familiar Stackelberg equilibrium solution ($q_A = \frac{1}{4}$, $q_B = \frac{1}{2}$), an increase of 12.5% in total quantity.

However, suppose both firms can decide on capacity in the first stage but A also serves an additional identical market, market 2, in which A is a monopolist. In this case, A can shift quantity from the overlapping market to its private market after B has effectively set its overlapping market quantity when choosing its first stage capacity. Because the slope of the marginal revenue curves is identical in both of A's markets, A's second stage best response now is $\phi_A \equiv \frac{\partial q_1^A}{\partial q_1^B} = -\frac{1}{4}$. That is, A's flexibility implies a second stage response that is exactly half of the one-stage Cournot game. Since B still has no flexibility in the second stage ($\phi_B = 0$), the outcome, in a sense, is a game that is "half way" between Cournot and Stackelberg. In equilibrium, total quantity in market 1 increases by 5%, from $\frac{2}{3}$ to $\frac{7}{10}$, a little less than half of the Stackelberg increase of 12.5%. Note that because A still equates marginal revenue to marginal cost, A's quantity in market 2 is the same a monopolist would sell absent any MMC considerations.

What will happen if both firms enter an additional market, market 3, that is also identical to market 1? B still has zero flexibility and full commitment. In fact, the only change is in A's flexibility which is now $-\frac{1}{6}$ instead of $-\frac{1}{4}$. Therefore the equilibrium condition for B draws closer to Cournot, changing to $P - q^B + \frac{1}{6}q^B = 0$ and so $q^B = \frac{3}{8}$. In general, letting M be the number of overlapping markets, we have $q^B = \frac{1+M}{2+3M}$ and $q^A = \frac{1+2M}{4+6M}$. It is immediate that as overlap increases, the equilibrium approaches Cournot and total quantity decreases (note that $\lambda_B = \infty$ in this example as $m_B = 0$).

Let us compare this example to the BGK analysis. BGK let A choose quantity (or prices, in the case of strategic complements) for each market and B choose its quantity in the overlapping market. BGK consider a first game in which all choices are simultaneous and a second game in which A chooses the private market quantity first and then both firms choose the overlapping market quantity. In both cases, BGK explicitly assume economies or diseconomies across A's two markets, observing there is no effect otherwise. Although there are no explicit connections between A's two markets in our model, the two stage setting with earlier capacity choice creates diseconomies across markets. While it may at first seem that our joint capacity setting can be interpreted as a micro-foundation for BGK's results under diseconomies across A's two markets, this is only partly accurate.

To see the main difference, compare to BGK's analysis that assumes cost diseconomies for A. *The key observation is that, in both of BGK's games, the effect is for A to deviate from equating marginal revenue to marginal cost, while B best responds.* In the simultaneous move game no firm considers any strategic effects but rather A simply adjusts to its cost disadvantage. In the sequential move game, A considers B's response to A's cost diseconomies. However, in neither cases does B take strategic advantage of A's diseconomies. Formally, in BGK B's equilibrium condition still equates marginal revenue to marginal cost while A's equilibrium condition deviates. This is the exact opposite of our analysis, in which B's equilibrium condition *deviates* while A still equates marginal revenue to marginal cost.

To summarize, in both the simultaneous and the sequential games studied by BGK, their framework only allows the firm with private markets (the more flexible firm) to commit in its private markets; i.e., letting A commit to a larger than optimal quantity in the private market. BGK do not capture the less flexible firm's ability to commit to the overlapping market, which is the catalyst of our competitive MMC effect.

2.5. Comparative Statics

Underlying the C-MMC effect is the observation that the existence of a private market affects the firm's reaction in the second stage, and that the firm's rival may take advantage of this. We start with generalizing the example above:

Proposition 2.2. *C-MMC effect at extremes:*

- *If MMC is large for one firm, $\lambda_i \to \infty$, and small for the other, $\lambda_{-i} \to 0$, then the equilibrium in the overlapping markets converges to the Stackelberg equilibrium with firm i as the Stackelberg leader.*
- *If both firms' MMC is large, $\lambda_i \to \infty$, or small, $\lambda_i \to 0$, the equilibrium in the overlapping markets converges to the Cournot equilibrium.*

The Stackelberg outcome would occur, for example, if firm B is a local firm while firm A serves many localities including B's. To illustrate this, assume $c_A = c_B = c$ and firms set quantities ignoring the C-MMC effect; i.e., in all markets marginal revenue equals marginal cost ($= c$). Firm A is flexible and can absorb any excess capacity from the overlapping markets in its private markets. At the extreme ($\lambda_B \to \infty$ and $\lambda_A \to 0$), firm B effectively sets its quantity in the overlapping markets in the first stage while firm A sets it only in the second stage – the Stackelberg result is obtained.

The C-MMC effect is positive because it provides an extra incentive for firms to be aggressive: the rival has the flexibility to accommodate aggressive behavior by hurting other markets in which the aggressive firm is not active. Formally, the revenue change $q_i \frac{\partial p_i}{\partial q_{-i}}$ is negative, and as discussed above (and proved in the appendix) flexibility is also negative (second stage quantities are strategic substitutes) and commitment is positive. Therefore, an increase in absolute terms in any element in the C-MMC only pushes both firms' marginal revenues below first stage cost. Because without MMC, $\eta_i = c_i$, we have that marginal revenue is lower for all firms compared to regular Cournot and therefore total industry profits are lower.

The private markets are also affected by MMC. In order to commit to being more aggressive in the overlapping markets, a firm must increase its private markets' quantity. Formally, this follows from the firm equating marginal revenues in the private and overlapping markets. As quantities never exceed the surplus maximizing level, the welfare effect is unambiguous. Total market quantity, total welfare and consumer surplus are always higher in all markets as a result of MMC, while total industry profit in equilibrium is always lower.

Corollary 2.1. *Consumer and total surplus is at least as high in markets with MMC as without. Total market profit is at most as high in markets with MMC as without.*

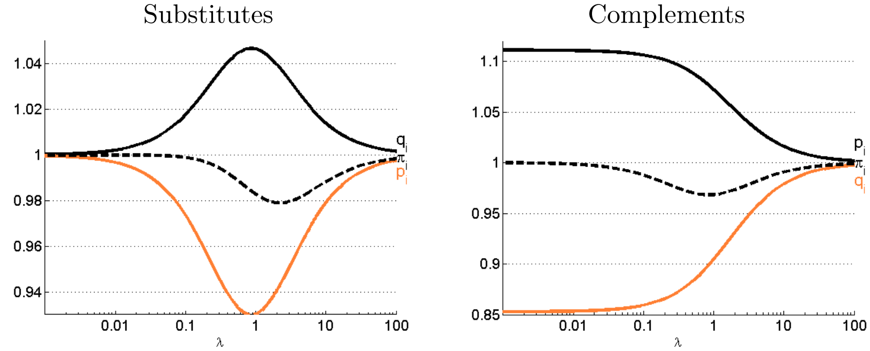
Proposition 2.4 provides the key comparative statics result, namely that as firms gradually enter each-other's markets, the increase in MMC may increase or decrease prices, depending on the initial level of MMC. Figure 2.1 illustrates this for symmetric firms. The figure, however, considers two simultaneous changes in the model's parameters (λ_A and λ_B), which is a-typical of comparative static analysis. In order to disentangle the two effects, we first consider the effect of a change in only one firm's MMC which essentially corresponds to cases in which the number of private markets of one firm changes.

Proposition 2.3. *Holding one firm's MMC fixed, a firm's equilibrium per-market-quantity in both types of markets* ***increases*** *with the firm's MMC and* ***decreases*** *with its rival's MMC*

$$\frac{dq_i}{d\lambda_i} \geq 0, \quad \frac{d\hat{q}_i}{d\lambda_i} \geq 0, \quad \frac{dq_i}{d\lambda_{-i}} \leq 0, \quad \frac{d\hat{q}_i}{d\lambda_{-i}} \leq 0$$

The top left panel in figure 2.2 provides intuition. The figure shows firm A's and B's quantities in each overlapping market as a function of A's extent of MMC, while keeping B's MMC at a fixed

Figure 2.1: Market Outcomes - Symmetric MMC



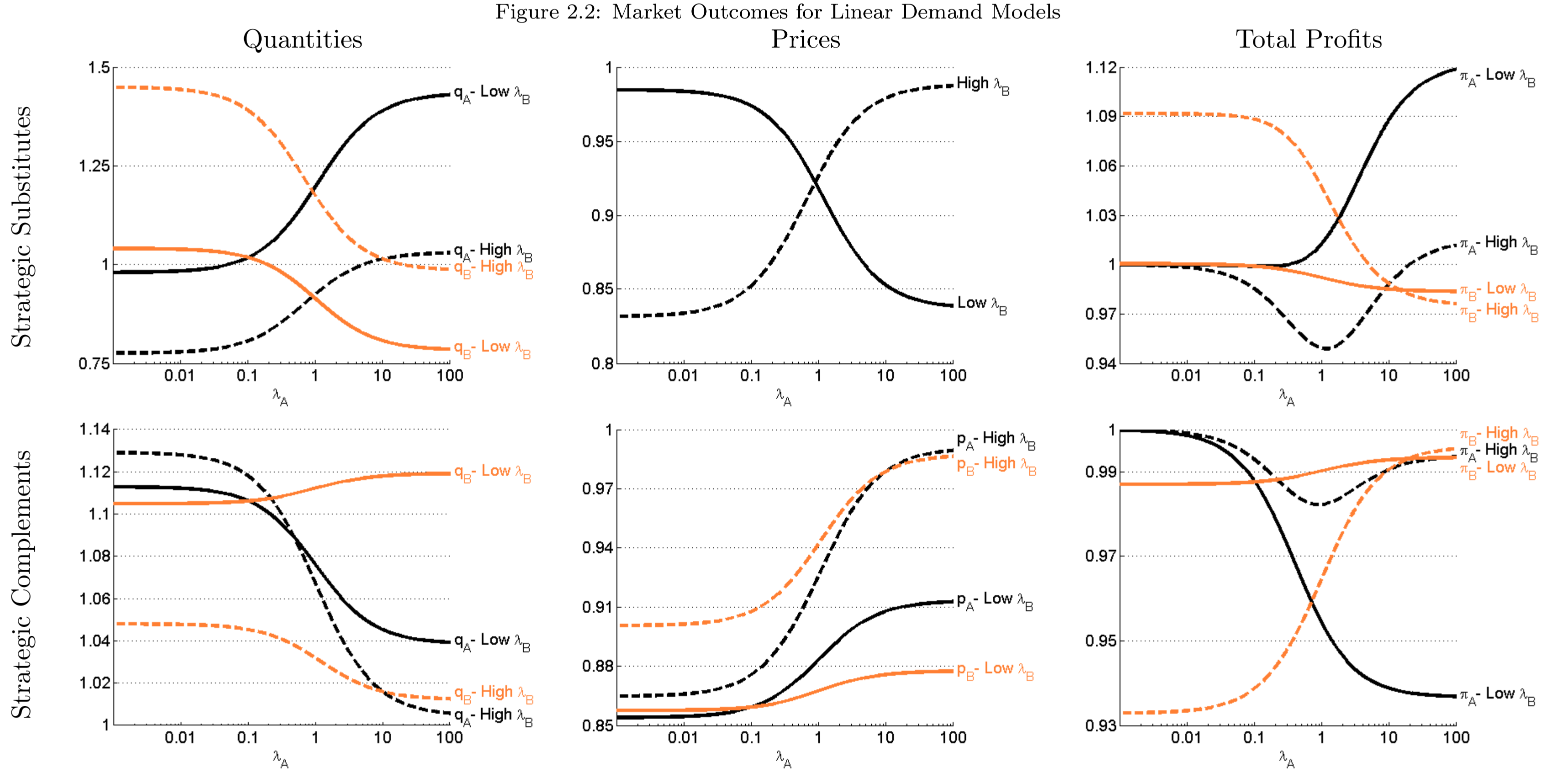
Graph is for symmetric MMC: $\lambda_A = \lambda_B = \lambda$. The *Substitutes* figure assumes linear inverse demand: $P(q_i, q_{-i}) = a - bq_i - bq_{-i}$ and $\hat{P}_i(\hat{q}) = P(\hat{q}, 0)$. The *Complements* figure uses linear demand $q_i(p_i, p_{-i}) = a - bp_i + \frac{b}{2}p_{-i}$ and $\hat{q}_i(\hat{p}_i) = q_i(\hat{p}_i, 0)$. Both plots show a firm's quantity (black) and price (orange) in the overlapping markets and profits (dotted) across all markets. As firms are symmetric, only values of one firm are shown. All values are normalized (i.e. $q_i/q^\star$) by the appropriate benchmark. For *Substitutes* the benchmark is the standard Cournot: $q^\star = \frac{a-c}{3b}$, $p^\star = \frac{a+2c}{3}$, $\pi^\star = M\frac{1}{b}\left(\frac{a-c}{3}\right)^2 + m_i\frac{1}{b}\left(\frac{a-c}{2}\right)^2$. For *Complements* it is the two-shot game with no cross market spillovers: $q^\star = \frac{3}{10}(2a - bc)$, $p^\star = \frac{1}{5b}(4a + 3c)$, $\pi^\star = M\frac{3}{b}\left(\frac{2a-bc}{5}\right)^2 + m_i\frac{1}{b}\left(\frac{a-bc}{2}\right)^2$. Given the normalization, the *Substitutes* figure is independent of the demand and cost constants. The *Complements* figure assumes $a = 10$, $b = 1$, and $c = 1$.

high or low level. The top panel uses an inverse linear demand model and the outcomes are presented relative to the standard Cournot outcomes for the same demand functions. Both prices and total profits can either decrease or increase with MMC as a function of the rival's MMC level. This leaves little room for general comparative statics.

Intuitively, facing a low-MMC rival, as A's MMC increases it becomes a Stackelberg leader - setting higher quantity at lower prices for higher total profits. However, facing a high-MMC rival, A is a Stackelberg follower with low MMC. As A's MMC increases, the change in quantity or price is slower than the increase in the share of A's markets in which it is the Stackelberg follower, which causes the dip in profits seen in the dotted line in the top right panel. At $\lambda_A = 10$, A's MMC level matches B's and the firms' quantities and profits are close to the Cournot benchmark. However, holding B's MMC fixed at $\lambda_B = 10$ (i.e., 10 times more overlapping markets than private), B still has some flexibility. A's strategic effect therefore remains strictly positive even at very high λ_A, and the equilibrium does not converge to Cournot.

Proposition 2.3 identifies three implications of the model. First, a change in MMC has the same qualitative effect on the firm's *private and overlapping* markets. This is because MMC affects the firm's optimal marginal revenue in all markets in which it operates. Second, an increase in firm i's MMC makes firm i more aggressive in all markets. This is an implication of the intuition developed

Figure 2.2: Market Outcomes for Linear Demand Models



The figure shows firms' quantities and prices in the overlapping markets and firms' total profits across both market types. As in figure 2.1, values normalized (i.e. $q_i/q^{\star}$) by the appropriate benchmark. The x-axis is A's MMC ($\lambda_i = \frac{M}{m_i}$). Two lines are plotted for each firm: solid lines for $\lambda_B = 0.1$ (Low) and dotted lines for $\lambda_B = 10$ (High). The *Substitutes* panels are independent of the demand and cost constants and the *Complements* panels assume $a = 10$, $b = 1$, and $c = 1$.

above – an increase in MMC shifts the firm towards a 'Stackelberg leader' strategy. Finally, an increase in firm i's MMC makes its rival less aggressive in all markets. This last result $\left(\frac{d\hat{q}_i}{d\lambda_{-i}} \geq 0\right)$ implies that a change in one of the firm's private markets may affect outcomes in rivals' private markets–markets in which the firm does not operate at all. That is, firm A may decrease its quantities in markets it is serving as a monopolist as a result of a decrease in demand in markets monopolized by firm B.

Proposition 2.3 allows for a straightforward assessment of MMC on welfare in the private markets:

Corollary 2.2. *As a firm's MMC increases, total output and consumer surplus in its private markets increase. Total output and consumer surplus in the rival's private markets decrease.*

Proposition 2.3 states that an increase in one of the firms' MMC has opposing effects on firms' equilibrium quantity in the overlapping markets. It is therefore not surprising that the effect on total output in the overlapping markets is unclear:

Proposition 2.4. *When competition is in strategic substitutes there is a non-monotonic relationship between total output and the extent of MMC. In particular, for any m_A, m_B, there are $M^0 \in (0, \infty)$ and $M^1 \in \left(M^0, \infty\right)$ such that for $M \in \left[0, M^0\right]$, overlapping markets' quantity increases in M and for $M \in \left(M^1, \infty\right)$, overlapping markets' quantity decreases in M.*

Proposition 2.4 states the direction of the effect of MMC on total quantity in the overlapping markets only in the extremes. The proposition thus implicitly implies that in order to know the direction of the effect elsewhere, one must make strong assumptions about the shape of the demand function. Figure 2.1 provides this for the standard linear demand case. The symmetric increase in λ – the share of overlapping markets – describes the case that the firms gradually enter each-other's market. We see that initially the increase in MMC intensifies competition because each firm understands its rival has significant flexibility. The increase in MMC then mainly increases each firm's ability to commit quantity to the overlapping market, driving an increase in quantities. However, the increase in MMC also decreases flexibility, which reduces competition. Eventually, as MMC passes a threshold, the decrease in flexibility has a stronger effect and any increase in MMC decrease overall quantities.

For example, suppose both firms had 100 private markets with demand $P = 1 - Q$ and there was just one overlapping market. Equilibrium should be almost unaffected by MMC. Indeed, equilibrium quantities will be only about 0.2% larger than the Cournot benchmark in the overlapping market and 0.3% larger than monopoly optimum in each private market.[8] If each firm enters 30 of its rival's markets, there are only 70 private markets per firm and 61 overlapping markets, $\lambda = .87$ and quantities exceed Cournot by about 5% in each overlapping market and 7% in each monopoly market. In comparison, recall that shifting to Stackelberg increases quantities by 12.5%. An increase in MMC intensified competition as each firm wants to take advantage of its rival's flexibility. However, any further increase in MMC will decrease quantities. If firms enter 60 more markets served by its rivals, $\lambda = \frac{181}{10} \approx 18$ and quantities in each market are only about 1% larger than without overlap.

Our discussion so far has focused on the effect of MMC on firms' output. The top right panel of figure 2.2 and the dotted line in the left panel of figure 2.1 present the effect of MMC on each firm's total profits. Since we are looking at firms' total profits across their overlapping and private markets, we take the benchmark profits to be the profits earned if the firm acts monopolistically in its private markets and according to the standard Cournot in the overlapping markets, denoted π^*.[9]

As expected, as the firm becomes more aggressive in the overlapping markets the rival responds softly and profits increase with MMC (see e.g., the case of low λ_B in the right panel of figure 2.2). Interestingly, profits may be non-monotonic in MMC. Consider, for example, the case where $\lambda_B \gg \lambda_A$ (dotted line in the right panel of figure 2.2). If $\lambda_A \to 0$ then $\pi_A \to \pi^\star$: A's profits are driven predominantly by its monopolistic position in its private markets, and while A is soft in the overlapping markets, these markets correspond to a very small share of A's profits. As A's MMC increases, the overlapping markets become a more important driver for A's profitability. Since A's MMC is still smaller than B's MMC, firm A remains the "Stackelberg follower" in the overlapping markets. That is, a larger share of A's profits comes now from markets where A's profitability is relatively low. As A's MMC increases, its flexibility decreases, and B's ability to behave aggressively in the overlapping markets diminishes. As a result, the price in the overlapping markets increases (see top middle panel of figure 2.2), and A's output in the overlapping markets increases as well (top left panel of figure 2.2). The combined effects improve A's profitability, and indeed as depicted in figure 2.2, A's profits start increasing. Once $\lambda_A \gg \lambda_B$, firm A becomes the "Stackelberg leader"

[8] If A and B have the same number of private and overlapping markets and demand in each market is $P = 1 - Q$, equilibrium condition is $P - q = q\phi\sigma$. By symmetry, $P = 1 - 2q$ in the overlapping market. In the monopoly market $P = 1 - q$. Firm level quantity in each overlapping market is $q = \frac{1}{3+\phi\sigma}$ and in each private market $\hat{q} = \frac{1}{2+\phi\sigma}$ where $\phi\sigma = -\frac{2\lambda}{4(1+\lambda)^2-1}$ is derived using the formulas in the appendix, recalling that the partial derivative of marginal revenues in this case with respect to own quantity is -2 and with respect to rival quantity is -1.

[9] Note that because we plot profit relative to the no-MMC benchmark, the graph cannot be used to interpret total industry profits. The precise definition of $\pi^\star$ can be found in the caption of figure 2.1.

in the overlapping markets. Since in this case A's profits are mainly driven from the overlapping markets, A's total profits are higher than the benchmark $\pi^\star$.

3. Price Competition

We now turn to the case where firms compete in prices in the second stage. Using the same three market-types as before, the only change in our setting is that in the second stage, firms choose prices rather than quantities. Demand curves in overlapping markets are $q_i(p_i, p_{-i})$ and in private markets are $\hat{q}_i(\hat{p}_i)$. The second stage problem for firm i is:

$$\begin{aligned} \pi_i(k_A, p_{-i}) = \max_{p_i, \hat{p}_i} \quad & M q_i\left(p_i, p_{-i}\right) \cdot p_i + m_i \cdot \hat{q}_i\left(\hat{p}_i\right) \cdot \hat{p}_i - k_i c_i \\ \text{s.t.} \quad & M q_i\left(p_i, p_{-i}\right) + m_i \hat{q}_i\left(\hat{p}_i\right) \leq k_i \\ & q_i(p_i, p_{-i}) \geq 0 \quad ; \quad \hat{q}_i(\hat{p}_i) \geq 0 \end{aligned} \tag{3.1}$$

The second stage game is now in strategic complements. The economics of a capacity choice followed by a pricing game requires additional structure. **?** first solved this model for homogeneous goods and showed that the game has the same solution as Cournot if the residual demand maintains the same distribution as the original demand. **?**, however, showed that the resulting equilibrium depends on the assumptions on residual demand if production costs are sufficiently cheap and is independent of residual demand assumptions if costs are large.

We instead assume aggregate demand can be described by a smooth demand function that is unaffected by residual demand. That is, the quantity demanded from i in the overlapping market, $q_i(p_i, p_{-i})$, depends only on the posted prices and not on whether the rival can or cannot supply the quantity demanded.[10]

The MMC setting and analysis are similar to the case where the second stage is in strategic substitutes. We therefore only present in this section the definitions, results, and any new or different intuition. Detailed analysis can be found in the appendix.

[10] Such demand can arise, for example, if consumers have imperfect information on firms' capacities, and incur a search cost for soliciting multiple suppliers. In such settings, individuals would expect that firms' prices and consumer choices are such that no one is rationed and firms sell at capacity. A consumer that were to unexpectedly be rationed would not seek out an alternative supplier, expecting to be also rationed by the alternative supplier. **?** and **?** show that if the capacity then pricing game assumes differentiated linear demand functions, it is without loss to assume smoothness around the equilibrium.

We first state the standard assumptions:

Assumption 2. *All demand curves are positive, twice differentiable and:*

1. *The goods are normal and substitutes:* $\frac{\partial q_i(\cdot)}{\partial p_i} \leq 0 \leq \frac{\partial q_i(\cdot)}{\partial p_{-i}}$
2. *Private and overlapping markets are always relevant for both firms:* $\hat{q}_i(c_i) > 0$ *and* $q_i(c_i, c_{-i}) > 0$.

As in the quantity setting second-stage, adding a firm-market-type second stage per-unit cost is equivalent to interpreting the price as the margin on top of the cost. Under this interpretation, assumption 2 only requires that if no firm demands any margin in the second stage, both firms have strictly positive demand. Note it is not assumed that prices (or margins) are positive.

Price setting games require additional assumptions for equilibrium to be "well behaved". Instead of imposing additional specific conditions on the demand function, we state the most common condition and a simple regularity condition. Consider the standard one-shot pricing game in which each firm's profit is given by $\vec{\pi}_i(p_i, p_{-i}) = q_i(p_i, p_{-i})\, p_i - C_i(q_i(p_i, p_{-i}))$ where $C_i(q)$ is any arbitrary cost function that is positive, strictly increasing and strictly convex for any $q > 0$ with $C_i(0) = 0$.

Assumption 3. *The overlapping market demand functions* $q_i()$ *are such that in any one shot game defined by* $\vec{\pi}_i(p_i, p_{-i})$ *using any cost functions* $C_i(q)$ *as defined above:*

1. *There is a unique equilibrium that is continuous in all of the demand and cost parameters.*
2. *In this equilibrium* $\frac{\partial p_i^*}{\partial p_{-i}} > 0$, $\frac{dq_i^*}{dp_{-i}} > 0$.

See **?** for a discussion of assumption 3. Existence and uniqueness are satisfied if $q_i()$ is linear, Logit, CES or log-concave (and down-ward sloping) in own price. A common sufficient assumption for $\frac{\partial p_i^*}{\partial p_{-i}} > 0$ (i.e. prices are strategic complements) is $\frac{\partial^2 q_i}{\partial p_i \partial p_{-i}} \geq 0$. Because $\frac{\partial q_i}{\partial p_{-i}} > 0$, $\frac{dq_i^*}{dp_{-i}} > 0$ (i.e. quantity increases in rival price) usually holds but can be violated by some specifically constructed demand functions and therefore we explicitly assume this as well.

The first stage problem is formally the same as in the quantity game, with the second stage profit π_i referring to the pricing game 3.1 instead: $\max_{k_i \geq 0} \pi_i(k_i, k_{-i})$.

As in the quantity game, the marginal revenue from a unit of capacity holding the rival's price fixed plays a key role in determining the equilibrium. As marginal revenue here is a function of price, we denote it by $\zeta_i(p_i, p_{-i})$.[11] As in the quantity game, the capacity constraint binds and each

[11] Marginal revenue (with respect to quantity) in the pricing game accounts for the change in price. Smoothness

firm's marginal revenue is identical in all its markets. Because the formal statements are similar to those of the quantity game, all Lemmas are relegated to the appendix.

We define *flexibility* and *commitment power* similarly to the quantity game.

Definition 3.1. In the pricing game, firm i's *flexibility*, $\phi_i \equiv \frac{\partial p_i}{\partial p_{-i}}$, is the firm's second stage reaction to a change in its rival's overlapping markets' price. Commitment power, $\sigma_i = \frac{dp_i^\star}{dk_i}$, is the change in the firm's second stage price in each overlapping market following an additional unit of capacity.

Consistent with the basic intuitions, flexibility is positive (prices are strategic complements) and commitment power is negative (selling more requires lower prices). The appendix provides ϕ_i and σ_i in terms of marginal revenue and MMC. We now characterize the equilibrium:

Proposition 3.1. *The game has a unique SPNE in pure strategies, first stage capacities are strategic substitutes, $\phi_i \in \left(0, \frac{1}{2}\right)$, $\sigma_i < 0$ and the following equation holds*

$$\zeta_i - c_i = \hat{\zeta}_i - c_i = \left(\frac{\partial q_i}{\partial p_{-i}} / \frac{\partial q_i}{\partial p_i}\right) \cdot q_i \cdot \phi_{-i} \cdot \sigma_i > 0 \qquad i \in \{A, B\} \; . \tag{3.2}$$

Proposition 3.1 shows that the underlying fundamentals of the C-MMC effect (difference between cost and marginal revenue) are the same whether firms compete in prices or quantities. The difference between the two cases are the economic forces that drive firms' behavior, which change significantly as a result of the move from quantity to price competition. In particular, under price competition, *MMC makes all firms less aggressive*, as indicated by the sign of $\zeta_i - c_i$ in (3.2).

The intuition is as follows: commitment power (from high MMC) allows the firm to credibly commit to increasing prices in the overlapping markets. To this end, MMC allows firms to be less aggressive in the first stage and choose a lower capacity level. This strategy is profitable as long as the rival is limited to reacting with prices rather than shifting capacity from its private markets, which happens when the rival also has high MMC.

Proposition 3.2 presents the comparative statics for price competition:

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of the demand functions implies that $\frac{dq_i}{dp_i} = 1/\frac{dp_i}{dq_i}$ and therefore MR is given by $\hat{\zeta}_i = \hat{p}_i + \hat{q}_i(p_i)/\hat{q}_i'(\hat{p}_i)$ and $\zeta_i = p_i + q_i(p_i, p_{-i})/\frac{dq_i(p_i, p_{-i})}{dp_i}$.

Proposition 3.2. *Any increase in MMC increases prices for both firms:*

$$\frac{dp_i}{dm_i} < 0 \; ; \; \frac{dp_i}{dm_{-i}} \leq 0 \; ; \; \frac{dp_i}{dM} \geq 0$$
$$\frac{d\hat{p}_i}{dm_i} < 0 \; ; \; \frac{d\hat{p}_i}{dm_{-i}} \leq 0 \; ; \; \frac{d\hat{p}_i}{dM} \geq 0$$

The results in proposition 3.2 are depicted in the middle panel of figure 2.2.[12] The main implication of proposition 3.2 is that when firms compete in prices, increases in MMC *always* reduce competition both in the overlapping *and* the private markets.

Corollary 3.1. *An increase in MMC decreases consumer surplus and total welfare in all private and overlapping markets.*

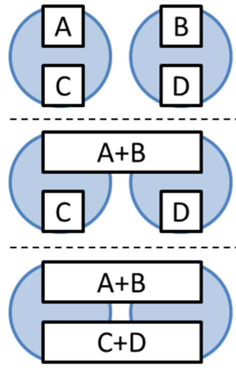
Increasing a firm's MMC increases its commitment power and reduces its flexibility. The increase in commitment power allows the firm to hold back capacity in the first stage and set a high price in the second. The decreased flexibility incentivizes the rival to also hold back additional capacity and increase price in the second stage, knowing that the other firm will follow. In contrast to the quantity game, both effects go in the same direction. Hence, *any* increase in MMC allows the firms to commit to competing softer.

Furthermore, while in the quantity game the C-MMC effect is minimal when MMC is large, in the case of strategic complements large MMC produces the strongest C-MMC effect. Commitment power increases in absolute terms with MMC because the firm must sell a larger share of its additional capacity in the overlapping markets, while flexibility increases with MMC as the game is in strategic complements. That is, in the case of strategic complements both ϕ_{-i} and σ_i increase in absolute terms with MMC, resulting in a stronger price response when MMC is large.

The effect of MMC on firm profits cannot be generally signed. Profits for a linear demand curve are provided in the bottom right panel of figure 2.2 and the dotted line in the right panel of figure 2.1. The case where λ_B is high demonstrates that as A's MMC increases, its profits increase as a result of both firms' soft behavior in the overlapping markets. When λ_B is low, however, the result may be reversed. If $\lambda_A \to 0$ then A's profits are driven predominantly by its monopolistic position in its private markets, and thus as in the quantity game $\pi_A \to \pi^\star$. As A's MMC increases, a larger

[12] We take the benchmark prices to be the prices set if the firm acts monopolistically in its private markets and according to a two-stage game in the overlapping markets. Closed form solutions for prices and quantities are provided in the appendix.

Figure 4.1: Merger Example Diagram



Two markets (circles) and four firms (boxes). The first merger is between firms A and B. After the merger, firm C considers the second market as a private market of firm 'A+B'. The second merger is between firms C and D.

portion of its profits comes from the overlapping markets. Since B's commitment power is low, the firms cannot commit to a large enough increase in price (relative to the benchmark), negatively affecting A's profits.

4. Merger Analysis

The analysis above suggests that the C-MMC effect could potentially affect the profitability and welfare implications of a merger. Specifically, MMC considerations may discourage firms from otherwise profitable and consumer surplus increasing mergers as they try to avoid the "Stackelberg follower" outcome. In this case, mergers that do not provide additional benefits, such as cost-savings, would not be pursued.

In this section we explore how mergers and MMC interact. In particular, we extend the basic two firm model to allow for mergers by considering four firms (labeled A,B,C, and D) where A only overlaps with C, and B only overlaps with D, as depicted in figure 4.1. We study two mergers: a merger between firms A and B, which forms firm AB, and a follow-up merger between C and D–forming firm CD. This last merger allows us to provide insight into merger waves.

In each case we study the effect of the merger on consumer surplus and profitability. If the merger is not profitable absent any cost efficiencies, we examine the cost efficiencies required to make such merger profitable. As both mergers keep the number of firms in each market constant, the analysis abstracts from reductions in the number of competitors. We also abstract from any effects due to mutual forbearance (which can arise in the second merger) as these are orthogonal to the C-MMC effect. The analysis in the previous sections provides general comparative statics for the MMC effect. These can be used to derive the following important characterization of the effect of horizontal mergers on welfare and profits:

Proposition 4.1. *Considering only the C-MMC effect, the first (A+B) merger increases consumer surplus and total welfare. The second (C+D) merger decreases both. If the second stage competition is in prices, the first (resp. second) merger reduces (resp. increases) per-market profits for all firms. If the second stage competition is in quantities, both mergers reduce per-market profits for the merging firms and increase per-market profits for the non-merging firms.*

The first part of the proposition illustrates an important intuition about the C-MMC effect: asymmetry across firms increases overall capacity in the market. The first merger creates a large multi-market firm and thus increases the asymmetry across firms. This, in turn, increases overall capacity and welfare. The second merger evens the firms' MMC – both firms overlap with each other in all markets – and therefore has the opposite effect.

The remaining parts of the proposition illustrate the importance of the type of second stage competition. If competition is in prices, both firms lose from a reduction in MMC and gain from an increase in it. In contrast, if competition is in quantities, asymmetry allows the smaller firm to gain "Stackelberg-like" leadership. As a result, the first merger increases per-market profitability for the non-merging firm, in part at the expense of the merging firm. Similarly, the second merger returns the market to the standard duopoly outcome, reducing the per-market profit of the (second) merging firm, in part to the benefit of firm AB that merged first.

The proposition highlights the contrast between merger profitability and welfare effects. Only the first merger increases surplus, but absent other gains (such as cost efficiencies), this merger reduces per-market profits for the merging firm, regardless of the type of second stage competition. To evaluate the relationship between merger related cost efficiencies and MMC, in the following subsections we impose some specific assumptions on the model.

4.1. Merger Profitability with Cost Reduction and MMC

We use the same setting as in section 3: firms simultaneously set capacities in the first stage and compete on prices in the second stage. Results for a parallel version in which second stage competition is in quantities are qualitatively similar to those provided here. We assume that all firms have a marginal cost of $c > 0$ and that demand is given by the linear form:[13] $q_i = a - b \cdot p_i + \frac{b}{2} p_{-i} \qquad (a, b) > 0$.

? shows this would be the demand function for a representative consumer model with utility $U(q_1, q_2) = \frac{2}{b} \cdot \left[a \cdot (q_1 + q_2) - \frac{1}{3}\left(q_1^2 + q_2^2 + q_1 q_2\right)\right]$. As in figure 4.1, there are two separate markets: one served by firms A and C and one served by B and D.[14] We study the equilibrium of this game under three different ownership structures. First we analyze the case where all firms operate independently. We then study the case where firms A and B merge and operate jointly. Finally, in order to allow for the analysis of a merger wave, we study the structure where firms A and B are merged, as well as firms C and D, and contrast the outcomes to a setting where only A and B have merged . We allow mergers to increase production efficiency. In particular, a merger reduces the unit cost to $(1 - \gamma) \cdot c$ for $\gamma \in [0, 1)$.

4.2. Benchmark Equilibrium

Before the first merger, the extent of MMC of all firms is 1: A and C fully overlap as do B and D. Each firm's market quantity, price, and profits are:

$$k_i = q_i = \frac{3}{10}(2a - bc) \qquad p_i = \frac{4a + 3bc}{5b} \qquad \pi_i = \frac{3}{b}\left(\frac{2a - bc}{5}\right)^2$$

Total welfare and consumer surplus per market are calculated using the representative consumer utility $U()$:

$$W = \frac{21}{50b}(2a - bc)^2 \qquad CS = \frac{9}{50b}(2a - bc)^2$$

As in section 3, the C-MMC effect induces quantities and welfare that are lower than in a game without MMC considerations.

To avoid corner-solutions, we assume that demand, costs and cost savings are such that profits (and quantities) are positive for all firms.

[13]Results for a rival price effect of $b_2 \neq \frac{b}{2}$ are qualitatively the same but require more notation without providing additional insights.

[14]The analysis requires only notational adjustment to allow each firm to serve any number of markets, as long as A only overlaps with C and B only overlaps with D before the merger. See appendix AppendixB for a discussion on more than two firms per market in the basic model.

Assumption 4. *Benchmark profits are positive. In addition, cost efficiencies generated by a merger are small enough such that rivals would produce, in equilibrium, a positive amount absent any MMC considerations:* $\gamma < 3\left(\frac{2a-bc}{bc}\right)$.[15]

To simplify notation, we denote $\underline{\pi} \equiv \frac{2a-bc}{bc}$.

4.3. Evaluating the First Merger

We first analyze the case where firms A and B merge to form AB. Post-merger, AB serves both markets, while the nonmerged firms serve a single market each.

Based on lemma 2.1, firms choose capacities in the first stage such that their installed capacity is fully utilized. Since the nonmerged firms serve a single market, they have no flexibility in the second stage and thus commit to a price-quantity pair when making their first stage capacity decision. The merged firm cannot make this same commitment. Serving two markets gives it the flexibility to reallocate quantity across its markets in the second stage. The nonmerged firms' strong commitment power combined with the merged firm's flexibility allows the nonmerging firms to be more aggressive in their capacity choice. This aggressiveness, however, may be offset by the cost efficiencies the merged firm may enjoy. Below we discuss the circumstances under which cost efficiencies compensate the merged firm for the soft behavior imposed by the C-MMC effect.

The following lemma provides the equilibrium quantities for the post-merger game. Interestingly, equilibrium quantities are such that the constraint required for the nonmerged firms to remain active post-merger is the same as the upper bound on cost savings imposed by assumption 4–which is based on the benchmark model and does not incorporate the C-MMC effect. That is, MMC does not by itself make it easier for mergers to induce exit.

Lemma 4.1. *If firms A and B merge, the resulting market quantities for the merged- (AB) and nonmerged (C, D) firms are*

$$\begin{aligned} q_{AB} \equiv \frac{k_{AB}}{2} &= \frac{3}{202} bc\left(20\underline{\pi} + 27\gamma\right) \\ q_D = q_C &= \frac{3}{202} bc\left(21\underline{\pi} - 7\gamma\right) \end{aligned}$$

[15] This cutoff for cost efficiencies is driven from a single market duopoly, in which *wlog*, firm A's cost is $(1-\gamma)c$ and firm C's cost is c. Equilibrium quantities and profits can be found in the appendix.

The cost savings from the merger (γ) and the C-MMC effect have two conflicting effects on the merged- and nonmerged firms' capacity decision. For the merged firm, the cost savings increase the merged firm's quantity. In contrast, the C-MMC effect makes the nonmerged firms more aggressive and thus has a negative effect on the merged firm's quantity (cf proposition 3.2). Nevertheless, the first order effect of both cost savings and the C-MMC effect is an increase in output. Any decrease in quantity is only a reaction to the rival's quantity increase. Therefore, the merger increases total quantity and consumer surplus. Since pre-merger firms were producing less than socially efficient, total welfare increases as well.

Given the opposing effects costs saving and the C-MMC effect have on firms' output decision, whether post-merger firms increase or decrease output depends on which effect dominates. Proposition 4.2 identifies the relevant conditions:

Proposition 4.2. *If firms A and B merge, total market quantity, total welfare and consumer surplus increases. However, per market:*

- *The merged firm produces more post-merger iff cost savings (γ) satisfy $\gamma > \frac{1}{135}\underline{\pi}$*
- *The merged firm produces more than the nonmerged firm iff cost savings (γ) satisfy $\gamma > \frac{1}{34}\underline{\pi}$*
- *Each nonmerged firm produces **less** post-merger iff cost savings (γ) satisfy $\gamma > \frac{4}{35}\underline{\pi}$*

The first result in proposition 4.2 identifies the cost savings needed for the merged firm to increase output, i.e., the conditions under which the cost-savings effect outweighs the C-MMC effect. Larger cost-savings, identified in the second condition, are required for the merged firm to offset the C-MMC effect and dominate the market in shares. The final condition identifies the significant cost-savings required to completely overturn the C-MMC effect and decrease the rival's quantity to below the pre-merger levels. Note that the final condition requires cost savings roughly 16 times larger than the first. As γ is bounded above by one, if $a > 5bc$ cost savings may never be large enough to decrease rivals' quantities.

All conditions require a non-trivial cost saving. For example, if we set $a = 80$, $b = 2$, and $c = 10$, (so $\underline{\pi} = 7/2$) the required cost savings are roughly 5%, 20% and 80% for the merged firm to increase production , increase market share, and for the nonmerged firm to decrease production, respectively. Note that ignoring any MMC considerations, the merger would increase the merged firm's (and overall market) quantity for any cost saving.

The same forces that affect quantities also affect profits.[16] Post merger, the merged firm is flexible and thus more accommodating. Consequently, absent any cost savings, the merged firm's joint profits are lower than pre-merger. However, this negative effect may be offset by cost savings. That is the merger is profitable only under large enough cost savings:

Proposition 4.3. *If firms A and B merge:*

- *The merger increases the merged firm's profits (relative to the pre-merger joint profits) iff cost savings satisfy* $\gamma > \frac{1}{135}\underline{\pi}$
- *The non merged firms' profits increase iff* $\gamma < .00535\underline{\pi}$

Corollary 4.1. *Any profitable merger* ***reduces*** *profits for the nonmerged firms.*

Taking propositions 4.2 and 4.3 together we see that the merger is profitable if, and only if, the merged firm produces more after the merger. Nonetheless, there is a large range of cost savings for which the C-MMC effect is strong enough to increase the nonmerged firms' output beyond the merged firm's increase and consequently dominate the market in shares.

Corollary 4.2. *For moderate cost savings:* $\gamma \in \left[\frac{1}{135}\underline{\pi}, \frac{1}{34}\underline{\pi}\right]$*, the merger is profitable, both firms produce more* ***and*** *the nonmerged firms increase their market share relative to the merged firm.*

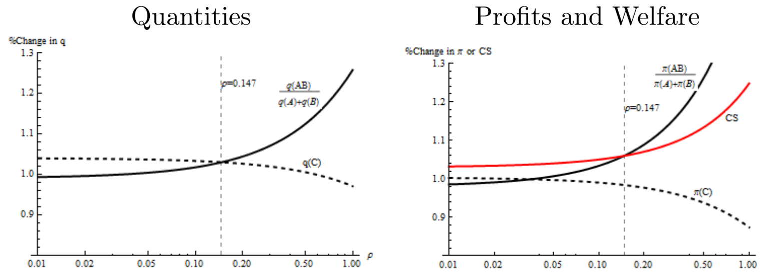
Since costs cannot be lower than zero, in industries with low production costs, cost efficiencies from mergers will be such that either corollary 4.2 applies or the merger isn't profitable: :

Corollary 4.3. *If costs are extremely low relative to demand* $\left(c \leq \frac{2}{136}\frac{a}{b}\right)$*, the merger cannot be profitable. If costs are sufficiently low relative to demand* $\left(c \leq \frac{2}{35}\frac{a}{b}\right)$, ***any*** *profitable merger provides the outcome described in corollary 4.2.*

We have just established that there is a non-trivial range of cost-saving, profitable, and surplus-increasing mergers in which *the merged firm's market share decreases post-merger.* This implies that one should be cautious when evaluating mergers' welfare-gains based on firms' market share (see for example, **?** and **?**). Again, to illustrate, if $a = 80, \quad b = 2$ and $c = 10$, a merger that generates cost savings between 5% to 20% could be mistaken for decreasing consumer surplus if the estimation assumes that the rival's quantities reflect market fundamentals without accounting for MMC effects.

[16]Firm profits per market post-merger are: $\pi_m^M = 3bc^2\left(\frac{20\underline{\pi}+27\gamma}{101}\right)^2$ and $\pi_m^N = 3bc^2 \cdot \frac{91}{2}\left(\frac{3\underline{\pi}-\gamma}{101}\right)^2$

Figure 4.2: Market Outcomes - Merger with MMC



The change in firms' market quantities (left) and profits (right) when firms A and B merge as a function of the cost-savings generated by the merger (γ). The dark solid line and dashed line represent the merged and nonmerged firms, respectively. The CS line in the right plot identifies the change in consumer surplus. The parameters for the plots are $a = 300$, $b = 10$, $c = 10$. The merger is profitable whenever $\gamma > .037$. The merged firm produces more than its rivals only when $\gamma > .147$, which is the exact point in which the profit increase (in %) is greater than the increase in consumer surplus. CS increases by about 3% without any cost savings ($\gamma = 0$) and about 25% if cost savings are almost full ($\gamma \to 1$).

Figure 4.2 illustrates the effects of the merger on quantities, profits, and welfare. Absent cost reductions, the merger reduces profits for the merged firm. A 3.7% reduction in marginal cost makes the merger profitable, and a 14.7% reduction in marginal cost is required for the merged firm to increase market share. In other words, for modest cost savings, the main determinant of the market outcomes of a merger is the C-MMC effect.

In conclusion, this section finds that MMC considerations reduce the profitability of a merger by increasing the aggressiveness of the nonmerged firms. The main implication is that under MMC, some cost-saving mergers may not be pursued. Furthermore, using market outcomes to estimate the cost-savings implied by a merger may provide qualitatively wrong conclusions if the C-MMC effect is not accounted for. Specifically, cost-*decreasing* mergers that meet the criteria in corollary 4.2 may be estimated as cost-*increasing*. Finally, note that the predicted effects above are consistent with the predictions generated by the mutual forbearance theories of MMC. However, no firm in our model implemented any cooperative practices. This suggests caution when analyzing mergers so as to not confound mutual forbearance with the purely competitive effects of MMC

4.4. Dynamic Merger Analysis

Having established the effect of MMC on a single merger, we now consider the policy implications for merger evaluation of a follow-up merger, and the resulting overall dynamic merger policy. In particular, suppose that the nonmerged firms (C and D) now consider merging too; expecting the same cost savings (γ) as in the first merger. If the second merger is carried out, the degree of MMC for both merged firms will again be 1. The equilibrium would be the same as the benchmark described in section 4.2 with costs $c \cdot (1-\gamma)$ for each firm.

The second merger has two effects on quantities and profits. First, consistent with proposition 3.2, the increase in MMC decreases quantities and increases prices for both firms. Second, the cost-savings for C and D decrease overall prices, increase CD's profits and decrease AB's profits. The next proposition summarizes these results:

Proposition 4.4. *The second merger is profitable iff* $\gamma \geq 0.00134\underline{\pi}$. *In particular, if cost efficiencies are large enough such that the first merger is profitable, then so is the second merger. Moreover, the firm that merged first benefits from the second merger iff the first merger induced a decrease in the merged firm's market share; i.e.:* $\gamma < \frac{1}{34}\underline{\pi}$.

As in the first merger, the second merger has two opposing effects on each firms' profits. The second merged firm (CD) loses the "Stackelberg-leader" advantage it enjoyed due to the C-MMC effect, but gains from the cost savings. The first merged firm (AB) gains as the increase in its rival's MMC decreases the rival's aggressiveness, but loses from having a more efficient rival.

If cost savings were sufficient to make the first merger profitable, they are sufficient to make the second one profitable. As shown in corollary 4.2, there is a range of cost savings for which the merger is profitable yet the merged firm's market share is smaller than the nonmerged firms'. Proposition 4.4 shows that it is for this range of cost savings that the second merger increases firm AB's profits, such that both firms gain from the second merger. If, however, cost savings are large enough such that AB dominates the market when competing against firms C and D, then the second merger hurts AB's profitability.

Finally, proposition 4.5 considers the welfare effects of a merger wave.

Proposition 4.5. *The second merger increases consumer surplus iff it decreases profits for the first merged firm* ($\gamma > \frac{1}{34}\underline{\pi}$). *If* $\gamma < 0.0082\underline{\pi}$ *the second merger decreases total welfare as well as consumer surplus.*

The second merger brings the degree of MMC back to the pre-merger MMC levels yet with lower costs, suggesting that consumer surplus and total firm profits should increase. However, the decrease in MMC following the second merger makes firm CD a softer competitor and consequently decreases welfare. The second merger thus increases consumer (or total) surplus, in addition to the increase delivered by the first merger, iff the cost-savings effect is large enough to outweigh the C-MMC effect. Note that the cutoff for consumer surplus was already identified in proposition 4.4 as the cutoff that determines the effect of the second merger on AB's profitability.

Proposition 4.5 implies that a regulator that is committed to approving only CS-increasing mergers would approve the first merger but reject the second merger whenever $\gamma < \frac{\pi}{34}$. This dynamic dependency can potentially distort merger decisions. For example, suppose that $\gamma = \frac{\pi}{200}$. A single merger is not profitable (proposition 4.3). Nevertheless, if AB merges, the second merger is profitable for CD (proposition 4.4) and results in higher profits for AB relative to the pre-merger equilibrium. Thus, if firms are forward looking, the industry will consolidate; increasing consumer surplus by the cost reduction.

Note, however, that the second merger is surplus decreasing compared to the first (proposition 4.5) and would thus be blocked by a regulator committed to blocking CS-decreasing mergers.[17] If the firms expect the second merger to be blocked, no merger would take place. The regulator's myopic policy would thus result in lower consumer surplus than otherwise. The potential for such distortions increases if mergers require a fixed cost, as this increases the profit requirement in propositions 4.3 and 4.4 without affecting the consumer surplus cutoff in proposition 4.5.

In this example, a regulator that is expected to approve only surplus-enhancing mergers obtains lower surplus than a regulator that approves the second, surplus decreasing merger. It is not clear to us if the regulator has tools to commit to eventually approving myopically surplus-decreasing mergers, or whether regulators can approve only "packages" of mergers.[18] However, this suggests caution when using estimated post-merger quantities, prices, or even welfare levels to evaluate the merits of a regulator's decision to approve a merger.

5. Competition or collusion? the US Airline Industry as an example

If firms are price-setting, increases in MMC always reduce competition and therefore decrease both consumer surplus and, since the market is below the perfectly competitive quantity, total welfare. This implies that the competitive MMC effect can be difficult to distinguish from the mutual forbearance effect of MMC. Based on **?** and **?**, this section considers an example from the US domestic airline industry to illustrate the difficulty of determining whether the MMC effect is

[17] The regulator objection could be expressed, for example, as a limit on the number of "mega firms" in the industry.
[18] We thank a referee for suggesting this.

competitive or collusive and how empirical researchers can attempt to distinguish the two. Both studies use route pricing data for the years 1984 to 1988 and find that MMC is positively correlated with firm prices.[19]

The airline industry is an ideal industry to apply our model of competitive MMC. Airlines schedule flights from spokes to their hub (or between spokes) well in advance of actually selling tickets on the routes using these flights.[20] Moreover, the prevalence of the Hub and Spoke system, which dominated the US airline market since deregulation (with the relatively recent exception of Southwest) implies that virtually all domestic flights serve many routes (markets). For example, a flight between New-York City (NYC) to Chicago can be used to fly passengers from NYC to Chicago as well passengers from NYC to Denver, Austin, and dozens of other destinations; all connecting in Chicago. However, a flight between NYC to Chicago cannot be used to fly passengers from San Diego to Seattle. Thus, for example, United Airline's NYC-Chicago flights provide it with capacity that is sunk (the specific plane will takeoff, regardless of any marginal sale) and *transferable* across the different routes that utilize the NYC-Chicago flight leg. When United makes market level decisions (i.e., prices per market), the total number of seats available for all markets that use the same flight is fixed, and the majority of the cost is already committed.

The main empirical prediction of our model is that the competitive MMC effect is present *only* in markets that share a sunk transferable investment. This allows us to separate between MMC effects that could be competitive or collusive and MMC effects that could only be collusive. For example, suppose that two carriers have three hubs each. One for travel on the east coast, one for travel on the west coast and one for all other travel. The competitive effect applies only for sunk, transferable investment. Since it is impossible to transfer any sunk investment between the hubs (e.g., an East-coast flight cannot be used to serve West-coast routes), following changes in MMC on the East coast, the competitive effect would only affect East coast prices. In contrast, for mutual forbearance analysis, the location of the MMC change makes no difference. Therefore, an empirical approach that distinguishes between MMC changes on the same hub and MMC changes on different hubs can differentiate between the two effects. In particular, finding that that MMC changes in a

[19] **?** provide a more current and expanded analysis of MMC in the airline industry. Their analysis finds the same price effects. However, they do not perform the specific test we are interested in.

[20] See e.g. **?**. Roughly speaking, an airline scheduling process includes four steps. The airline first allocates to each city-pair planes by type (e.g., two 747 flights a day between Chicago and Cleveland). Then allocates specific planes (i.e., tail numbers), then determines a maintenance schedule, and finally assigns crews. The procedure is done using demand forecasts and flights are typically scheduled more than six months before takeoff.

remote hub affect prices in *all* shared routes supports the existence of mutual forbearance effects. Finding that prices changed only on routes that connect at the remote hub supports the competitive effect.

The study by **?** (hereafter, GW) performs exactly this empirical test. For every given route, GW define two distinct measures of MMC between airlines i and j on that market (route): (i) the "standard" MMC measure–counting **all** routes served by both airlines; and (ii) an MMC measure that counts all routes served by both airlines **and** share a common endpoint with the current market. This second measure of MMC corresponds to our definition of MMC. That is, if United enters one of American's markets in the East coast (e.g., Portland, ME – Durham, NC) the standard MMC measure for United and American increases. However, the second MMC measure for these carriers increases on East coast routes from either Portland or Durham (e.g., Boston, MA – Durham, SC), but not on the West coast routes (e.g., San Diego, CA – Seattle, WA).

Running a regression with airlines' yield (price per mile) as the dependent variable and the two measures of MMC as the explaining variables, GW find that an increase in MMC on routes that share a common endpoint was correlated with higher yield but an increase in MMC on routes that do not share an endpoint was not.

GW interpret their results as suggesting that common endpoints are required for MMC to facilitate collusion. In contrast, our model posits that the positive relationship between MMC and prices may in fact be the outcome of a competitive, rather than a collusive, behavior driven by shared sunk, transferable investment. That is, GW's results provide compelling evidence that the possible relationship between MMC and price in the US airline industry may be due to competitive rather than collusive effects.[21]

The distinction has significant policy implications when considering mergers in the industry. The three recent mergers (Delta-Northwest, United-Continental and American-US Airways) all significantly affect MMC. If MMC in the industry facilitates tacit collusion, the mergers may well decrease welfare. However, if the main effect of MMC is through the competitive effect, regulatory intervention is not required.

[21] Our conclusions are reserved mainly because of the significant changes in empirical methods, in particular for establishing causality, in the last two decades.

6. Conclusion

Analysis of MMC is more relevant than ever. While there is rich empirical research, the theoretical foundation focused on collusive considerations. Our analysis shows that the basic microeconomics forces in our model are just as important as the collusive ones. Empirical research on MMC would benefit from accounting for these effects.

Our framework is especially important for the evaluation of horizontal mergers. The current horizontal mergers literature assumes away any MMC considerations. Our analysis shows that while for a single merger the C-MMC effect reduces the merged firm's profitability, a follow-up merger may increase the first merged firm's profits to a point above the initial profitability level. More importantly, such a follow-up merger will also increase welfare above the initial level–before any merger took place. This suggests that when considering multi-market mergers, a policy that rejects all surplus-decreasing mergers may in fact harm consumers. Multi-market mergers should not be considered at the merger-by-merger level. Rather, the regulator should account for the dynamic effect of each merger on subsequent mergers, and thereby on the ultimate market structure.

Another implication of the analysis regards the interpretation of quantity differences as indicative of efficiency differences for international firms. There is now considerable evidence that exporting firms produce more and are more efficient and profitable than firms that sell domestically. See e.g., **????**. This is consistent with the theoretical results established in **?**, which shows that exporting firms tend to be more efficient than those limited to the domestic market. Our paper suggests caution, in particular when *empirically* inferring the *extent* of efficiency differences. The C-MMC effect implies that a firm that does not have private markets (a domestic firm) may strategically reduce output and increase price. A rival firm that does have private markets (exporter) would then follow with a reduction in output and a price increase, albeit of smaller magnitudes. As a result, even if the firms enjoy the same efficiency, the exporting firm would have a larger market share, higher profits, and lower prices than the domestic firm.

Previous literature on MMC focused on perceived differences between large and small firms in terms of productivity or mutual forbearance. The C-MMC effect can be confounded with either of the two. Thus, an important empirical challenge is to distinguish between the competitive effect and the mutual forbearance effect of MMC. A key difference between the two effects is that while the former relies on specific physical diseconomies across markets (i.e., sunk, transferable investments), the latter does not. This opens possibilities for future research to empirically distinguish between the two hypotheses by separating the effect of changes in MMC that use the same capacity (i.e., use of a common distribution center or airline hub) and changes in MMC that do not.

Our formal analysis considered only the case of two firms in a specific market. As exemplified in section 4, the qualitative results easily extend to more complicated models. Appendix AppendixB extends the model to the case where the overlapping markets are served by more than two firms and shows that results stay qualitatively unchanged.

AppendixA. Proofs of Propositions and Lemmas

We present the formal proofs to all Lemmas, Propositions, and claims in the text.

AppendixA.1. Quantity Competition

Since variables c, k, q, m and function p are indexed by i, for brevity, throughout this section, we omit the i subscript (e.g. use c for c_i) and use a $'-'$ subscript to identify the rival (e.g. use c_- for c_{-i}).

All numbered subscripts denote partial derivatives with respect to own (1) and rival (2) quantities. That is $\eta_1 \equiv \frac{\partial \eta_i}{\partial q_i}$ $\eta_2 \equiv \frac{\partial \eta_i}{\partial q_{-i}}$, $\eta_{12} \equiv \frac{\partial^2 \eta_i}{\partial q_i \partial q_{-i}}$ and $\eta_{-,1} \equiv \frac{\partial \eta_{-i}()}{\partial q_{-i}}$.

It is useful to recall that prices decrease in own and rival quantity, and η is marginal revenue: $\eta \equiv p_1 q + p < 0$, $\eta_1 = p_{11} + 2p_1 < 0$, $\eta_2 = p_{12} q + p_2 < 0$ and $\eta_1 < \eta_2 < 0$

Lemma. *2.1 In any Subgame Perfect Nash Equilibrium:*

1. *The subgame has a unique equilibrium for any first stage capacities.*
2. *Firm i's marginal revenue is identical in both types of markets:* $\eta = \hat{\eta}$.
3. *If the first stage has an equilibrium in pure strategies then the capacity constraint binds:* $k = Mq + m\hat{q}$ *and all markets are served by all relevant firms* ($q\hat{q} > 0$)*.*

Proof. The subgame problem is given in eq. 2.1. Let $\hat{q}^0$ be the monopoly quantity in i's private market when costs are zero, i.e. $\hat{\eta}(\hat{q}^0) = 0$. Consider the following alternative second stage problem:

$$\tilde{\pi}(q_-, k) = \max_{q \geq 0} \qquad Mqp(q, q_-) - mC(q) + m\hat{p}\left(\hat{q}^0\right) - ck$$

$$C'(q) \equiv \begin{cases} 0 & if\ Mq < k - m\hat{q}^0 \\ \hat{\eta}\left((k - Mq)/m\right) & if\ Mq \in [k - m\hat{q}^0, k) \\ \hat{\eta}(0) + \frac{Mq-k}{m} & if\ Mq \geq k \end{cases}$$

To see the relation with the original second stage problem:

- For $Mq < k - m\hat{q}^0$, the quantity allocated to the overlapping market does not change the quantity allocated to the private market and therefore, has no second stage cost.
- For $Mq \in [k - m\hat{q}^0, k)$, any quantity not allocated to the overlapping market would be allocated to the private market. Therefore, the marginal cost of quantity is the marginal revenue in the private market.
- For $Mq \geq k$ the marginal cost in the alternative problem is finite at $\hat{\eta}(0)$ which is a relaxation of the original problem in which the cost of production for such output choice is infinite under the true problem.

If $Mq \in [k - m\hat{q}^0, k)$ in $\tilde{\pi}$ then i's payoff is equivalent to the payoff in π with the same q and the capacity constraint binding, such that $\hat{q} = (k - Mq)/m$. If $Mq < k - m\hat{q}^0$ then i's payoff in $\tilde{\pi}$ is equivalent to the payoff in π for the same q and $\hat{q} = \hat{q}^0$. If $Mq > k$, $\tilde{\pi}$ specifies a very high marginal cost while for π the capacity constraint is violated. Therefore, the second stage game defined by $\tilde{\pi}$ is a relaxation of π for the case that $Mq > k$.

At $Mq = k - m\hat{q}^0$, $(k - Mq)/m = \hat{q}^0$ and at $Mq = k$, $(k - Mq)/m = 0$. This together with $\hat{\eta}$ differentiable and strictly decreasing implies $C'(q)$ is continuous and strictly increasing. $\tilde{\pi}$ defines a standard differentiated Cournot game. Because assumption 1 part 4 implies the second stage game is strictly submodular, Tarski's fixed point theorem applies (cf. **?** pg. 151) and a unique second stage equilibrium in pure strategies (q_A, q_B) exists for any capacity pair. The equilibrium pair (q_A, q_B) is thus continuous in the first stage capacities (k_A, k_B) and best responses are decreasing in rival quantities: $\frac{dq}{dq_-} < 0$.

As long as neither firm ever chooses $Mq > k$ in $\tilde{\pi}$ in equilibrium, the equilibrium in $\tilde{\pi}$ for k_A, k_B

is the unique equilibrium in π. It therefore remains to show that (a) the capacity constraint binds, (b) firms never choose $Mq \geq k$ in $\tilde{\pi}$, (c) both q and $\hat{q}$ are strictly positive and (d) $\eta = \hat{\eta}$ in any pure strategy SPNE.

For the capacity constraint, the same arguments as in **?** imply this for both $\tilde{\pi}$ and π, for any pure-strategy SPNE: Assume A has slack capacity. Firm A can reduce k_A by half the slack capacity. Neither firms' second stage problems are affected and so the second stage is still an equilibrium. This saved A a strictly positive cost, and thus A's total profit increased, contradicting the original equilibrium.

The **?** argument also implies $Mq < k$. Suppose $Mq \geq k$. Then by optimality $C'(q)$ implies $\eta = \hat{\eta}(0) > c$ by assumption 1 part 3. Then i can increase k by some $M\varepsilon > 0$ and increase q by ε. If q_- is unchanged, this will strictly increase A's profits because $\eta > c$. Because $\frac{dq_-}{dq} < 0$ and $\eta_2 < 0$, profits increase even more.

That $Mq < k$ in $\tilde{\pi}$ implies that $\hat{q} > 0$ in π. This is sufficient for the set of equilibria for the two second stage games defined by π and $\tilde{\pi}$ to be identical.

To prove that $q > 0$, we first show that for any equilibrium q_-, $p(0, q_{-i}) > c$. By assumption 1, this follows if $p_-(q_-, 0) \geq c_-$. Suppose that in equilibrium $p_-(q_-, 0) < c_-$. i's rival can improve its profits by setting $k_- = \hat{m}\hat{q}_-^m$ and setting $q_- = 0$. Since this strategy strictly dominates the rival's current strategy, the current strategy cannot be part of a pure strategy SPNE.

That $p(0, q_-) > c$ implies, $k > m\hat{q}^m$: Suppose $k < m\hat{q}^m$, i can increase profits by increasing k by $\hat{q} - \frac{k}{m} > 0$ and allocating all new capacity to it's private markets. The same is true if $k = m\hat{q}^m$ and $q > 0$. Finally, if $k = m\hat{q}^m$ and $q = 0$ then $p(0, q_-) > c$ implies $\eta_i > c = \hat{\eta}_i$ and this cannot be an equilibrium.

Finally, $k > m\hat{q}^m$ obtains that in the second stage game $\tilde{\pi}$, $\hat{\eta}\left(\frac{k}{m}\right) < c$. Therefore because $p(0, q_-) > c$ at $q = 0$: $\eta > c > \hat{\eta}\left(\frac{k}{m}\right) = C'(0)$. Therefore, $q > 0$.

That $\eta = \hat{\eta}$ is now immediate from the first order condition of π ignoring the now-redundant non-negativity constraints. This can also be shown from the first order condition for $\tilde{\pi}$: $M\eta = mC'\frac{M}{m}$ and so $\eta = C' = \hat{\eta}$. □

Following from Lemma 2.1, the next intermediate results are useful:

Corollary AppendixA.1. *In any pure SPNE, the following second stage profit function is equiv-*

alent to 2.1 without loss of generality:

$$\pi(k, k_{-i}) = \max_{q \geq 0} \quad Mqp\left(q, q_{-}\right) + m\frac{k-Mq}{m}\hat{p}\left(\frac{k-Mq}{m}\right) - ck \tag{A.1}$$

Lemma AppendixA.1. *In any SPNE:*

$$\phi = -\frac{\eta_2}{\eta_1 + \lambda\hat{\eta}_1} \in \left[\overline{\phi}, 0\right] \;\; ; \;\; and \;\; \sigma = \frac{\lambda\hat{\eta}_1}{\eta_1 + \eta_2\phi_{-} + \lambda\hat{\eta}_1} \in (0, 1)$$

Proof. ϕ is derived using the implicit function theorem and applying Lemma 2.1:

$$\phi = \frac{\partial q^*}{\partial q_{-}^*} = -\frac{\pi_{12}}{\pi_{11}} = -\frac{\eta_2}{\eta_1 + \lambda\hat{\eta}_1} < 0$$

The signs follow from all marginal revenues strictly decreasing in own and rival quantities (assumption 1).

For σ, recall that by definition $\sigma \equiv M\frac{dq^*}{dk}$. Using **?**, $\frac{dq^*}{dk} = \frac{\partial q^*}{\partial k} + \phi\frac{dq_{-}^*}{dk}$ and $\frac{dq_{-}^*}{dk} = \phi_{-}\frac{dq^*}{dk}$ so $\frac{dq^*}{dk} = \frac{\frac{\partial q^*}{\partial k}}{1-\phi\phi_{-}}$. $\frac{\partial q^*}{\partial k}$ and $1 - \phi\phi_{-}$ are given by:

$$\frac{\partial q^*}{\partial k} = -\frac{\pi_{1k}}{\pi_{11}} = \frac{\hat{\eta}_1}{m\left(\eta_1 + \lambda\hat{\eta}_1\right)} > 0 \quad .$$
$$1 - \phi\phi_{-} = 1 + \phi_{-}\frac{\eta_2}{\eta_1 + \lambda\hat{\eta}_1} = \frac{\eta_1 + \lambda\hat{\eta}_1 + \phi_{-}\eta_2}{\eta_1 + \lambda\hat{\eta}_1}$$

Plugging in the values for $\frac{\partial q^*}{\partial k}$ and $1 - \phi\phi_{-}$ obtains the stated result.

To determine bounds, assumption 1 part 4 provides that all marginal revenue curves are decreasing in both own and rival quantity and $|\eta_2| \leq |\eta_1|$ and therefore

$$0 > \phi \geq -\frac{\eta_2}{\eta_1} = \overline{\phi} > -1 \; .$$

$\sigma > 0$ follows from $\frac{\partial q^*}{\partial k} > 0$ and ϕ, ϕ_{-} both $\in (-1, 0)$. From $0 < -\eta_2 \leq -\eta_1$ and $\phi_{-} > -1$ we have $\eta_1 + \phi_{-}\eta_2 < 0$ which implies $\sigma < 1$.

The following will be used in later proofs. All signs follow from $\eta_2 < 0$, $\eta_1 < 0$, $\hat{\eta}_1 < 0$ and $\eta_1 + \phi_{-}\eta_2 < 0$:

- $\frac{\partial \phi}{\partial \lambda} = \frac{\eta_2\hat{\eta}_1}{(\eta_1+\lambda\hat{\eta}_1)^2} \geq 0$ and $\frac{\partial \phi}{\partial \lambda_{-}} = 0$.

- $\frac{\partial\sigma}{\partial\lambda} = \frac{\hat{\eta}_1(\eta_1+\phi_-\eta_2)}{(\eta_1+\eta_2\phi_-+\lambda\hat{\eta}_1)^2} \geq 0$.
- $\frac{\partial\sigma}{\partial\lambda_-} = -\frac{\partial\phi_-}{\partial\lambda_-}\frac{\hat{\eta}_1\eta_2}{(\eta_1+\eta_2\phi_-+\lambda\hat{\eta}_1)^2} \leq 0$ (recall that $\frac{\partial\phi}{\partial\lambda} \geq 0$).

□

Proposition. *2.1 The game has a unique SPNE in pure strategies, $\phi_i \in \left[\overline{\phi}_i, 0\right]$, $\sigma_i \in (0,1)$ and the following equation holds*

$$c_i - \eta_i = c_i - \hat{\eta}_i = q_i \frac{\partial p_i\left(q_i, q_{-i}\right)}{\partial q_{-i}}\phi_{-i}\sigma_i > 0 \qquad i \in \{A, B\}\,. \tag{A.2}$$

Firms' first stage capacities are strategic substitutes.

Proof. **?** derives the envelope theorem parallel and comparative statics of a Nash Equilibria for a general model. We utilize the results in **?** to characterize the second stage subgame equilibrium, σ, and $\frac{dq}{dk_-}$. We then apply Tarski's fixed point theorem as in **?** to prove existence and uniqueness of the pure strategy equilibrium.

For brevity below, we let $R\left(k_i, k_j, q_i, q_j\right)$ denote the objective's value for any second stage allocation.

We first note that assumptions A.1 through A.5 in **?** hold:

1. The objective is in $C^{(2)}$and there are no constraints. Thus assumptions A.1 and A.2 hold.
2. Following Lemma 2.1 for every k_i, k_j, q_j in the neighborhood of any equilibrium allocation, i's optimal response is unique. Thus assumption A.3 holds.
3. Following Lemma 2.1 for every k_i, k_j in the neighborhood of any equilibrium k_i, k_j, the second stage equilibrium is unique and the correspondence $q_i\left(k_i, k_j\right)$ is in $C^{(2)}$ (to maintain equal marginal revenues). Thus, assumptions A.4 and A.5 hold.

Now, for any k_-, interpret k as α in **?**. Theorem 1 in **?** then implies

$$\frac{\partial\pi}{\partial k} = m\left[\frac{\partial R}{\partial k} + \frac{\partial R}{\partial q_-}\frac{\partial q}{\partial k}\right] = \left[m\frac{\hat{\eta}}{m} - c + Mq\frac{\partial p}{\partial q_-}\frac{\partial q_-}{\partial k}\right]$$

Because any equilibrium must satisfy, $\frac{\partial\pi}{\partial k} = 0$, we have that in every equilibrium $c - \hat{\eta} = Mq\frac{\partial p}{\partial q_-}\frac{\partial q_-}{\partial k}$. Next, because k_- does not directly affect R: $\frac{\partial q_-}{\partial k} = \frac{dq}{dk}\frac{dq_-}{dq} = \frac{\sigma}{M}\phi_-$.

The last equality follows from definitions. This provides the equilibrium equation stated in the proposition. $\eta = \hat{\eta}$ provides the result for $c - \eta$. The sign follows from $q > 0$ (Lemma 2.1), $\sigma > 0$ and $\phi_- < 0$ (Lemma AppendixA.1) and $\frac{\partial p}{\partial q_-} < 0$ (assumption 1).

That $\frac{\partial q_-}{\partial k} < 0$ ($\sigma > 0$ and $\phi_- < 0$) together with $\frac{\partial q_-}{\partial k_-} > 0$ obtains that the first stage best response function $k(k_-)$ is strictly decreasing. That is, first stage capacities are strict strategic substitutes. The feasible set is a lattice ($q \geq 0$) and thus Tarski's fixed point theorem applies as in **?** and the first stage game has a unique equilibrium. □

Proposition. *2.2 C-MMC effect at extremes:*

- *If MMC is large for one firm, $\lambda_i \to \infty$, and small for the other, $\lambda_{-i} \to 0$, then the equilibrium in the overlapping markets is the Stackelberg equilibrium with firm i as the Stackelberg leader.*
- *If both firms' MMC is large, $\lambda_i \to \infty$, or small, $\lambda_i \to 0$, the equilibrium in the overlapping markets is the Cournot equilibrium.*

Proof. Observe that $\lim_{\lambda\to\infty} \phi = \overline{\phi}$, $\lim_{\lambda\to\infty} \sigma = 0$, $\lim_{\lambda\to 0} \phi = 0$, and $\lim_{\lambda\to 0} \sigma = 1$. The FOCs of the two players when $\lambda \to \infty$ (MMC is very large) and $\lambda_- \to 0$ (rival has many more private markets than overlapping) are then:

$$c = \eta \qquad \text{and} \qquad c_- = \eta_- + q_- \frac{\partial p_-}{\partial q} \overline{\phi}$$

which are exactly the same FOC as the Stackelberg game with firm i as the follower. □

Proposition. 2.3 *Holding one firm's MMC fixed, a firm's equilibrium per-market quantity in both types of markets* ***increases*** *with the firm's MMC and* ***decreases*** *with the rival's MMC*

$$\frac{dq_i}{d\lambda_i} > 0 \quad \frac{d\hat{q}_i}{d\lambda_i} > 0 \qquad\qquad \frac{dq_i}{d\lambda_{-i}} < 0 \quad \frac{d\hat{q}_i}{d\lambda_{-i}} < 0$$

Proof. Start at any equilibrium and consider an increase in λ holding λ_- fixed. To show the comparative statics as above, we proceed as follows:

1. Assume second stage quantities are unaffected and adjust k accordingly so that the capacity constraint still holds with equality. Because q,$\hat{q}$ and q_- are unchanged, $\eta, \hat{\eta}$ are unchanged. Evaluate $\frac{\partial \pi}{\partial k}$ and $\frac{\partial \pi_-}{\partial k_-}$ at this changed λ and unchanged second stage quantities.

2. Observe that if $\frac{\partial \pi}{\partial k} = \frac{\partial \pi_-}{\partial k_-} = 0$ then $\frac{dq}{d\lambda} = 0$ and $\frac{d\hat{q}}{d\lambda} = 0$. Similarly, because $\sigma > 0$, if $\frac{\partial \pi}{\partial k} > 0$ then i's optimal response is to increase quantity per market not accounting yet for rival response. We show this is indeed the case by showing that $\partial \frac{\partial \pi}{\partial k} / \partial \lambda > 0$ holding q's fixed (as described in step 1).
3. Similarly, we show that $\partial \frac{\partial \pi_-}{\partial k_-} / \partial \lambda < 0$ holding q's fixed (as described in step 1) . Because $\sigma_- > 0$, i's rival's optimal response is to decrease quantity per market not accounting yet for i's action.
4. Finally, because of strategic substitution, the two first-order responses in steps 2,3 above reinforce each other and the proof is complete.

We next show the claims from steps 2 and 3 hold:

$$\partial \frac{\partial \pi}{\partial k} / \partial \lambda = \partial \left[\sigma \phi_- \left(q \frac{\partial p}{\partial q_-} \right) \right] / \partial \lambda = q \frac{\partial p}{\partial q_-} \left(\frac{\partial \sigma}{\partial \lambda} \phi_- + \frac{\partial \phi_-}{\partial \lambda} \sigma \right) = q \frac{\partial p}{\partial q_-} \frac{\partial \sigma}{\partial \lambda} \phi_- > 0$$

$q \frac{\partial p}{\partial q_-}$ is unchanged with λ and negative. From lemma AppendixA.1: $\phi_- < 0$, $\frac{\partial \phi_-}{\partial \lambda} = 0$ and $\frac{\partial \sigma}{\partial \lambda} \geq 0$. Similarly:

$$\partial \frac{\partial \pi_-}{\partial k_-} / \partial \lambda = \partial \left[\sigma_- \phi q_- \frac{\partial p_-}{\partial q} \right] / \partial \lambda = q_- \frac{\partial p_-}{\partial q} \left(\frac{\partial \sigma_-}{\partial \lambda} \phi + \frac{\partial \phi}{\partial \lambda} \sigma_- \right) < 0$$

From lemma AppendixA.1: $\frac{\partial \phi}{\partial \lambda} > 0$ and $\frac{\partial \sigma_-}{\partial \lambda} < 0$. Therefore, as $\phi < 0$, $\sigma_- > 0$, and $q_- \frac{\partial p_-}{\partial q} < 0$, the sign is obtained. □

Proposition. 2.4 *When competition is in strategic substitutes there is a non-monotonic relationship between total output and the extent of MMC. In particular, for any m_A, m_B, there are $M^0 \in (0, \infty)$ and $M^1 \in \left(M^0, \infty\right)$ such that for $M \in \left[0, M^0\right]$, overlapping markets' quantity increases in M and for $M \in \left(M^1, \infty\right)$, overlapping markets' quantity decreases in M.*

Proof. It is sufficient to show that an increase in M decreases (for the first case) or increases (for the second case) the equilibrium marginal revenue for both firms. Starting from the FOC

$$\eta = c - q \frac{\partial p}{\partial q_-} \sigma \phi_-$$

As c is fixed, it is equivalent to show that $q \frac{\partial p}{\partial q_-} \sigma \phi_-$increases (for the first case) and decreases (for the second). For any $m_A, m_B < \infty$, if $M = 0$ then $\sigma = 0$ and if $M \to \infty$ then $\phi_- \to 0$. Thus, at both limits for M, $\eta = c$. However, away from the limits, for any $M \in (0, \infty)$, $\sigma_i > 0$, $\phi_- < 0$

and $\eta < c$. Continuity of all elements implies the result. □

AppendixA.2. Price Competition

As in the quantity competition proofs, we omit the i subscript in the proofs from c, p, k ,q, $\hat{q}$, and $\hat{\eta}$. Number subscripts indicate partial derivative with respect to own (1) and rival (2) price.

AppendixA.2.1. Regularity Result

Recall the one-shot standard game defined before assumption 3. Let α be a cost parameter such that $\frac{\partial C}{\partial \alpha \partial q} > 0$ and denote marginal cost by C': i.e. $C' \equiv \frac{\partial C}{\partial q}$.

Lemma AppendixA.2. *In the one-shot standard game defined before assumption 3, let q^* be i's equilibrium quantity. An increase in i's marginal costs increases both firms' prices, decreases i's quantity and increases the rival's quantity:* $\frac{dp_i^*}{d\alpha_i} > 0$, $\frac{dq_i^*}{d\alpha_i} < 0$, $\frac{dp_{-i}^*}{d\alpha_i} > 0$ *and* $\frac{dq_{-i}^*}{d\alpha_i} > 0$.

Proof. Applying **?** equation (16): $\frac{dp^*}{d\alpha} = \frac{\partial p^*}{\partial p_-^*}\frac{dp_-^*}{d\alpha} + \frac{\partial p^*}{\partial \alpha}$ and $\frac{dp_-^*}{d\alpha} = \frac{\partial p_-^*}{\partial p^*}\frac{dp^*}{d\alpha}$. We use the full derivative to denote the full effect and the partial derivative to denote the effect holding all other variables at the equilibrium levels. Thus $\frac{dp^*}{d\alpha} = \frac{\partial p^*}{\partial p_-^*}\frac{\partial p_-^*}{\partial p^*}\frac{dp^*}{d\alpha} + \frac{\partial p^*}{\partial \alpha}$ and therefore $\frac{dp^*}{d\alpha}\left(1 - \frac{\partial p^*}{\partial p_-^*}\frac{\partial p_-^*}{\partial p^*}\right) = \frac{\partial p^*}{\partial \alpha}$.

By assumption 3, $\frac{\partial p^*}{\partial p_-^*}\frac{\partial p_-^*}{\partial p^*} \in (0,1)$. The FOC for the standard game is $\frac{\partial \vec{\pi}}{\partial p} = q_1\left(p - C'\right) + q = 0$. By construction, the SOC is negative. Observe that $\frac{\partial^2 \vec{\pi}}{\partial p \partial \alpha} = -q_1 \frac{\partial^2 C}{\partial q \partial \alpha} > 0$. Applying the implicit function theorem, $\frac{\partial p^*}{\partial \alpha} = -\frac{\frac{\partial^2 \vec{\pi}}{\partial p \partial \alpha}}{\frac{\partial^2 \vec{\pi}}{\partial p \partial p}} > 0$ and therefore $\frac{dp^*}{d\alpha} > \frac{\partial p^*}{\partial \alpha} > 0$ and $\frac{dp_-^*}{d\alpha} \in \left(0, \frac{dp^*}{d\alpha}\right)$.

Letting p^*, p_-^* denote the equilibrium prices, at any equilibrium: $\frac{dq^*}{d\alpha} = \frac{dp^*}{d\alpha}q_1 + q_2\frac{dp_-^*}{d\alpha}$. Using the above, $\frac{dq^*}{d\alpha} = \frac{dp^*}{d\alpha}\left(q_1 + q_2\frac{\partial p_-^*}{\partial p}\right)$. By assumption 3, $\frac{\partial p_-^*}{\partial p} \in (0,1)$ and by assumption 2, $q_1 < -q_2 < 0$. Therefore, $q_1 + q_2\frac{dp_-^*}{dp} < 0$ and thus $\frac{dq^*}{d\alpha} < 0$.

Finally, $\frac{dq_-^*}{d\alpha} = \frac{dp^*}{d\alpha}q_{-,2} + q_{-,1}\frac{dp_-^*}{d\alpha} = \frac{dp^*}{d\alpha}\left(q_{-,2} + q_{-,1}\frac{\partial p_-^*}{\partial p}\right) = \frac{dp^*}{d\alpha}\frac{dq_-^*}{dp}$. By assumption 3, $\frac{dq_-^*}{dp} > 0$ and so $\frac{dq_-^*}{d\alpha} > 0$. □

Lemma AppendixA.3. $\frac{\partial p}{\partial p_-} \in \left(0, \frac{1}{2}\right)$ *and* $\frac{dq^*}{dp_-} \in \left(\frac{q_2}{2}, q_2\right)$. *In addition, holding fixed prices and* C'*,* $\frac{\partial p}{\partial p_-}$ *is increasing in* C'' *(i.e. price mimicking is stronger if the responder's cost function is more convex).*

Proof. First order effect is $\frac{\partial \vec{\pi}}{\partial p} = q_1\left(p - C'\right) + q$. The second order effects are

$$\begin{aligned}\frac{\partial^2 \vec{\pi}}{\partial p \partial p} &= q_{11}\left(p - C'\right) + q_1\left(2 - C'' q_1\right)\\ \frac{\partial^2 \vec{\pi}}{\partial p \partial p_-} &= q_{12}\left(p - C'\right) + q_2\left(1 - C'' q_1\right)\end{aligned}\quad .$$

Using the FOC, $p - C^{'} = -\frac{q}{q_1}$ and so

$$\frac{\partial^2 \vec{\pi}}{\partial p \partial p} = q_1 \left(2 - C^{''} q_1\right) - q_{11} \frac{q}{q_1}$$
$$\frac{\partial^2 \vec{\pi}}{\partial p \partial p_-} = q_2 \left(1 - C^{''} q_1\right) - q_{12} \frac{q}{q_1} \quad .$$

Let $z \equiv C^{''} q_1 \geq 0$ (recall $C'' \geq 0$ and $q_1 < 0$). Then $\frac{\partial p}{\partial p_-} = -\frac{\frac{\partial^2 \vec{\pi}}{\partial p \partial p_-}}{\frac{\partial^2 \vec{\pi}}{\partial p \partial p}} = -\frac{q_2(1+z) - q_{12}\frac{q}{q_1}}{q_1(2+z) - q_{11}\frac{q}{q_1}}$ and:

$$\frac{\partial \frac{\partial p}{\partial p_-}}{\partial z} = -\frac{q_2}{q_1(2+z) - q_{11}\frac{q}{q_1}} - q_1 \frac{\frac{\partial p}{\partial p_-}}{q_1(2+z) - q_{11}\frac{q}{q_1}} = -\left[\frac{q_2 + q_1 \frac{\partial p}{\partial p_-}}{q_1(2+z) - q_{11}\frac{q}{q_1}}\right] \geq 0$$

The sign follows from $\frac{dq^*}{dp_-} = q_2 + \frac{\partial p^*}{\partial p_-} q_1 \geq 0$ and the the denominator is the SOC, which is negative in equilibrium.

Because $z \in [0, \infty)$, we have that $\frac{\partial p}{\partial p_-} \in \left(-\frac{q_2 - q_{12}\frac{q}{q_1}}{2q_1 - q_{11}\frac{q}{q_1}}, -\frac{q_2}{2q_1}\right)$ which by $q_2 < -q_1$ and $\frac{\partial p}{\partial p_-} \geq 0$ (or $q_{12} \geq 0$) also implies $\frac{\partial p}{\partial p_-} \in \left(0, \frac{1}{2}\right)$ and therefore $\frac{dq^*}{dp_-} \in \left(\frac{q_2}{2}, q_2\right)$. □

Lemma AppendixA.4. *In any Subgame Perfect Nash Equilibrium:*

1. *The second stage subgame has a unique equilibrium.*
2. *Firm i's marginal revenue is identical in both types of markets.*
3. *If the first stage equilibrium is in pure strategies:*
   (a) *The capacity constraint binds:* $k_i = M q_i + m_i \hat{q}_i$.
   (b) *All markets are served by all relevant firms* ($q_i \hat{q}_i > 0$)
   (c) $\frac{\partial p}{\partial p_-} \in \left(0, \frac{1}{2}\right)$ *and* $\frac{dq^*}{dp_-} \in \left(\frac{q_2}{2}, q_2\right)$
   (d) *Holding rival capacity fixed, an increase in own capacity decreases all prices, increases own quantity in both private and overlapping markets, and decreases rival quantity in the overlapping market:* $\frac{dq_i^*}{dk_i} > 0, \frac{d\hat{q}_i^*}{dk_i} > 0, \frac{dp_i^*}{dk_i} < 0, \frac{d\hat{p}_i^*}{dk_i} < 0, \frac{dp_{-i}^*}{dk_i} < 0,$ *and* $\frac{dq_{-i}^*}{dk_i} < 0$.

Proof. In the private market, considering quality or price setting behavior by the firm is identical. Invert $\hat{q}(p)$ to obtain $\hat{p}(q)$ and let $\hat{\eta}(q)$ be the marginal revenue at quantity q. Note that for every price, by definition, $\hat{\eta}\left(\frac{k - Mq(p,p_-)}{m}\right) = \hat{\zeta}(p, p_-)$. Consider the same alternative second stage problem as for the quantity setting game except for the overlapping market revenue term:

$$\tilde{\pi}\left(p_{-},k\right)=\max_{q\geq 0} \qquad Mpq\left(p,p_{-}\right)-mC\left(q\left(p,p_{-}\right)\right)+m\hat{p}\left(\hat{q}^{0}\right)-ck$$

$$C'(q) \quad \equiv \quad \begin{cases} 0 & if\ Mq < k - m\hat{q}^{0} \\ \hat{\eta}\left(\left(k-Mq\right)/m\right) & if\ Mq \in [k-m\hat{q}^{0},k) \\ \hat{\eta}\left(0\right)+\frac{k-Mq}{m} & if\ Mq \geq k \end{cases}$$

Recall that $\hat{q}^{0}$ is the monopoly quantity in i's private market when costs are zero: $\hat{\eta}(\hat{q}^{0})=0$. $C\left(\right)$ is convex because $\hat{\eta}_{1}<0$. By assumption 3, this subgame game has a unique equilibrium for any k_{A},k_{B} with the functions $p_{i}\left(k\right)$ continuous. The same arguments as in the quantity game (Lemma 2.1) now apply.

Since the second stage game is equivalent to a one shot game with the cost function $C\left(\right)$ given above, assumption 3 implies that for any first stage capacities $\frac{\partial p^{*}}{\partial p_{-}}\in(0,1)$. Moreover, because an increase in k is equivalent to a decrease in marginal cost for the second stage game, Lemma AppendixA.4 then implies the last claim.

Finally, the equivalent second stage problem to be considered in the rest of the appendix is

$$\pi(k,k_{-})=\max_{p} \quad Mpq\left(p,p_{-}\right)+m\tfrac{k-Mq(\cdot)}{m}\hat{p}\left(\tfrac{k-Mq(\cdot)}{m}\right)-ck$$

With the second stage FOC:

$$\frac{\partial\pi}{\partial p}=M\left[q\left(\right)+pq_{1}-q_{1}\hat{\eta}\left(\frac{k-Mq\left(\right)}{m}\right)\right]=0$$

Because $\hat{\eta}\left(\frac{k-Mq(p,p_{-})}{m}\right)=\hat{\zeta}\left(p,p_{-}\right)=\zeta\left(p,p_{-}\right)$, the second stage FOC can be used to write

$$p-\hat{\eta}=-\frac{q}{q_{1}} \quad .$$

The second stage SOC is

$$\begin{aligned}\frac{\partial^2 \pi}{\partial p \partial p} &= M\left[2q_1 + pq_{11} - \hat{\eta}q_{11} + q_1\frac{M}{m}q_1\hat{\eta}_1\right] \\ &= M\left[q_1\left(2 + \lambda q\hat{\eta}_1\right) + q_{11}\left(p - \hat{\eta}\right)\right] \\ &= M\left[q_1\left(2 + \lambda q\hat{\eta}_1\right) - q_{11}\frac{q}{q_1}\right] < 0\end{aligned}$$

The sign follows from the second stage equilibrium existence. In addition

$$\frac{\partial^2 \pi}{\partial p \partial p_-} = M\left[q_2\left(1 + \lambda q\hat{\eta}_1\right) - q_{12}\frac{q}{q_1}\right] > 0$$

□

Proposition. 3.1 *The game has a unique SPNE in pure strategies, first stage capacities are strategic substitutes, $\phi_i \in \left(0, \frac{1}{2}\right)$, $\sigma_i < 0$ and the following equation holds*

$$\zeta_i - c_i = \hat{\zeta}_i - c_i = M\left(\frac{\partial q_i}{\partial p_{-i}} / \frac{\partial q_i}{\partial p_i}\right) \cdot q_i \cdot \phi_{-i} \cdot \sigma_i > 0 \qquad i \in \{A, B\}\ .$$

Proof. The proof is very similar to the quantity setting case. note that assumptions A.1 through A.5 in **?** hold as in the quantity setting game. Let

$$R\left(p, p_-; k, k_-\right) \equiv Mpq\left(p, p_-\right) + m\frac{k - Mq\left(\cdot\right)}{m}\hat{p}\left(\frac{k - Mq\left(\cdot\right)}{m}\right) - ck$$

Where p and p_- are functions of k, k_-. For any k_-, interpret k as α in **?**. Applying the envelope theorem (Theorem 1 in **?**):

$$\frac{\partial \pi}{\partial k} = \frac{\partial R}{\partial k} + \frac{\partial R}{\partial p_-}\frac{\partial p_-}{\partial k} = m\frac{\hat{\eta}}{m} - c + \frac{\partial p_-}{\partial k}\frac{\partial q}{\partial p_-}\left(Mp - m\frac{M}{m}\hat{\eta}\right)$$

Observe that $\frac{\partial R}{\partial p_-}$ has two components (holding p fixed), the reduction in quantity in the overlapping market and the increase in quantity in the private market. By construction $\frac{\partial p_-}{\partial k} = \sigma\phi_-$ and optimality in the second stage implies $p - \hat{\eta} = -\frac{q}{q_1}$ and $\hat{\eta} = \zeta = \hat{\zeta}$. Thus we can simplify

$$\frac{\partial \pi}{\partial k} = \zeta - c - Mq\frac{q_2}{q_1}\sigma\phi_-$$

That first stage capacities are strict strategic substitutes follows from $\frac{dq^*}{dk} > 0$ and $\frac{dq^*_-}{dk} < 0$, both proved in Lemma AppendixA.4. As the space for k is a lattice, this also proves the equilibrium exists and is unique. The derivations and signs for ϕ and σ are provided in Lemmas AppendixA.5 and AppendixA.6 ☐

Lemma AppendixA.5. *In equilibrium of the price setting game,* $\phi = -\frac{q_2(1+\lambda q_1\hat{\eta}_1)-q_{12}\frac{q}{q_1}}{q_1(2+\lambda q_1\hat{\eta}_1)-q_{11}\frac{q}{q_1}} \in \left(0, \frac{1}{2}\right)$. *Moreover,* $\frac{\partial \phi}{\partial \lambda} > 0$.

Proof. By definition, $\phi = \frac{\partial p^*}{\partial p_-}$ in the second stage game, holding capacities fixed. The implicit function theorem applies and so $\phi = -\frac{\frac{\partial^2 \pi}{\partial p \partial p_-}}{\frac{\partial^2 \pi}{\partial p \partial p}}$ Using the second stage from Lemma AppendixA.4 obtains the result. As the same equilibrium can be obtained in the one-shot game with the cost function in Lemma AppendixA.4, the results follow from Lemma AppendixA.5. Note that $\lim_{\lambda \to 0} \phi = \vec{\phi}$ (reaction in a one-shot game with $c = c_i$ but current prices) and $\lim_{\lambda \to \infty} \phi = -\frac{q_2}{q_1}$. ☐

Lemma AppendixA.6. *In the equilibrium of the pricing game:*

$$M\sigma_i = \frac{\lambda q_1 \hat{\eta}_1}{\left(q_1\left(2 + \lambda q \hat{\eta}_1\right) - q_{11}\frac{q}{q_1}\right)\left(1 - \phi\phi_-\right)} < 0$$

Proof. $\sigma \equiv \frac{dp}{dk}$. Using **?** equation (16):

$$\frac{dp}{dk} = \frac{\partial p}{\partial p_-}\frac{dp_-}{dk} + \frac{\partial p}{\partial k}$$
$$\frac{dp_-}{dk} = \frac{\partial p_-}{\partial p}\frac{dp}{dk}$$

Plugging the second equation in the first, using ϕ and σ and isolating σ obtains $\sigma = \frac{\partial p/\partial k}{1-\phi\phi_-}$. Apply the implicit function theorem $\frac{\partial p}{\partial k} = -\frac{\partial^2 \pi}{\partial p \partial k} / \frac{\partial^2 \pi}{\partial p \partial p}$. $\frac{\partial^2 \pi}{\partial p \partial p} = M\left[q_1\left(2 + \lambda q \hat{\eta}_1\right) - q_{11}\frac{q}{q_1}\right]$ was derived above and $\frac{\partial^2 \pi}{\partial p \partial k} = -\lambda q_1 \hat{\eta}_1$. Therefore

$$\frac{\partial p}{\partial k} = \frac{\lambda q_1 \hat{\eta}_1}{M\left[q_1\left(2 + \lambda q \hat{\eta}_1\right) - q_{11}\frac{q}{q_1}\right]}$$

The sign follows from $q_1\hat{\eta}_1 > 0$, $\frac{\partial^2\pi}{\partial p \partial p} < 0$ and $\phi\phi_- \in (0,1)$. □

Proposition. 3.2 *Any increase in MMC increases prices for both firms.*

$$\tfrac{dp_i}{dm_i} < 0 \quad \tfrac{dp_i}{dm_{-i}} \leq 0 \quad \frac{dp_i}{dM} \geq 0$$

$$\tfrac{d\hat{p}_i}{dm_i} < 0 \quad \tfrac{d\hat{p}_i}{dm_{-i}} \leq 0 \quad \frac{d\hat{p}_i}{dM} \geq 0$$

Proof. The proof is similar to that of proposition 2.3. First, observe that if the comparative static of the MMC effect for each firm holding the rival choices unchanged obtains the price reaction stated in the proposition, then $\frac{\partial p}{\partial p_-} > 0$ implies the result when accounting for the rival's response.

Next, using $\hat{\eta} = c + \frac{q_2}{q_1} q\phi_- \cdot \sigma M$, we have that, letting capacities adjust according to the change in m with current prices, $\frac{\partial\hat{\eta}}{\partial m} = \frac{q_2}{q_1} q \frac{\partial(M\phi_-\sigma)}{\partial m}$. Since we only care about the sign, observe that $\frac{\partial p}{\partial m}$ has the same sign as $\frac{\partial\hat{\eta}}{\partial m}$ and that $\frac{q_2}{q_1} q < 0$. Therefore, we need to show that

$$\frac{\partial\left(M\phi_-\sigma\right)}{\partial m} \geq 0 \quad , \quad \frac{\partial\left(M\phi_-\sigma\right)}{\partial m_-} \geq 0 \quad \frac{\partial\left(M\phi_-\sigma\right)}{\partial M} \leq 0$$

Let $z \equiv \frac{\lambda q_1 \hat{\eta}_1}{q_1(2+\lambda q\hat{\eta}_1) - q_{11}\frac{q}{q_1}}$ and $y \equiv \frac{\phi_-}{1-\phi\phi_-}$. By the previous derivations of ϕ and σ, $M\phi_-\sigma = yz$. Both y and z depend on λ and λ_- rather than m, m_- and M. Because $\lambda = \frac{M}{m}$ and $\lambda_- = \frac{M}{m_-}$ and all are strictly positive, it is enough to prove that $\frac{\partial(yz)}{\partial\lambda} \leq 0$ and $\frac{\partial(yz)}{\partial\lambda_-} \leq 0$.

It was proved above that $\phi \in \left(0, \frac{1}{2}\right)$, $\phi_- \in \left(0, \frac{1}{2}\right)$, $\frac{\partial\phi}{\partial\lambda} \geq 0$, and by observation, $\frac{\partial\phi_-}{\partial\lambda} = \frac{\partial\phi}{\partial\lambda_-} = 0$. Therefore $y \geq 0$ and

$$\frac{\partial y}{\partial\lambda} = \frac{\phi_-^2}{\left(1-\phi\phi_-\right)^2}\frac{\partial\phi}{\partial\lambda} \geq 0$$
$$\frac{\partial y}{\partial\lambda_-} = \frac{\partial\phi_-}{\partial\lambda_-}\frac{1-\phi\phi_- + \phi\phi_-}{1-\phi\phi_-} \geq 0$$

Recall that $q_1, \hat{\eta}_1$ and the denominator in z is the SOC which has to be negative. Then $z < 0$, $\frac{\partial z_-}{\partial\lambda} = \frac{\partial z}{\partial\lambda_-} = 0$, and

$$\frac{\partial z}{\partial\lambda} = q_1\hat{\eta}_1 \left(\frac{1-q_1 z}{q_1\left(2+\lambda q\hat{\eta}_1\right) - q_{11}\frac{q}{q_1}}\right) < 0 \quad .$$

Finally, $\frac{\partial(yz)}{\partial\lambda} = \frac{\partial y}{\partial\lambda} z + \frac{\partial z}{\partial\lambda} y < 0$ as both z elements are negative and both y elements are positive.

Similarly $\frac{\partial(yz)}{\partial\lambda_-} = \frac{\partial y}{\partial\lambda_-} z < 0$. □

AppendixA.3. Merger Model (section 4)

AppendixA.3.1. Premerger Benchmarks

- The pre-merger benchmark is as derived by setting $m_i = 0$ for both firms in the linear solution.
- If one firm (A) has a cost of $(1-\gamma)\, c$, re-solve the linear solution with the different costs. The resulting equilibrium quantities and profits are

$$k_A = \frac{3}{10}\left(2a - bc + \frac{4}{3}bc\gamma\right) \quad ; \quad k_C = \frac{3}{10}\left(2a - bc - \frac{1}{3}bc\gamma\right)$$
$$\pi_A = \frac{3}{b}\left(\frac{2a - bc + \frac{4}{3}bc\gamma}{5}\right)^2 \quad ; \quad \pi_C = \frac{3}{b}\left(\frac{2a - bc - \frac{1}{3}bc\gamma}{5}\right)^2$$

- If both firms obtain the lower cost, simply replace c in the pre-merger benchmark with $(1-\gamma)\, c$.

Proposition. *4.1 Considering only the C-MMC effect, the first (A+B) merger increases consumer surplus and total welfare. The second (C+D) merger decreases both. If the second stage competition is in prices, the first (resp. second) merger reduces (resp. increases) per-market profits for all firms. If the second stage competition is in quantities, both mergers reduce per-market profits for the merging firms and increase per-market profits for the non-merging firms.*

Proof. The first merger increases λ_i (fraction of markets not served by the rival) for the merging firm (AB) and has no implication on λ_i of the non-merging firm. The first merger creates a private market for the merged firm that is also served by either firm C or D. That fact that the private market is not monopolistic and also served by another firm has no effect on the equilibrium with either firm. The private market second stage demand is determined holding the other firm's quantity or price fixed. The second merger eliminates the private markets.

Proposition 2.3 then implies the statements for quantity competition. Proposition 3.2 implies the same for price competition. □

Proposition. *4.2 If firms A and B merge, total market quantity, total welfare and consumer surplus increases. However, per market:*

- *The merged-firm produces more post-merger iff cost savings (γ) satisfy $\gamma > \frac{1}{135}\underline{\pi}$*
- *The merged-firm produces more than the non-merged firm iff cost savings (γ) satisfy $\gamma > \frac{1}{34}\underline{\pi}$*

- *Each non-merged firm produces **less** post-merger iff cost savings* (γ) *satisfy* $\gamma > \frac{4}{35}\underline{\pi}$

Proof. Premerger market quantity per firm is $\frac{3}{10}bc\underline{\pi}$. Market shares are $\frac{1}{2}$ per firm. Post merger quantities are given in Lemma 4.1.

Total market quantity is $q_{AB} + q_C = \frac{3}{202}bc\left(41\underline{\pi} + 20\gamma\right)$. As quantity increases with γ it is sufficient to show for $\gamma = 0 : \frac{123}{202} - \frac{6}{10} > 0$. Total welfare and CS are

$$\begin{aligned} W\left(q_{AB}, q_C\right) &= U\left(q_{AB}, q_C\right) - c\left(1-\gamma\right)q_{AB} - cq_C \\ CS\left(q_{AB}, q_C, p_{AB}, p_C\right) &= U\left(q_{AB}, q_C\right) - p_{AB}q_{AB} - p_C q_C \ . \end{aligned}$$

Prices are

$$p_{AB} = \frac{80a + 61bc - 47bc\gamma}{101b} \qquad ; \qquad p_C = \frac{78a + 62bc - 13bc\gamma}{101b} \quad .$$

Plugging in all the values shows that both W and CS increase in γ and are larger at $\gamma = 0$ than the values pre-merger. The values were obtained using the Mathematica algebra solver. We reproduce the values here for reference:

$$\begin{aligned} CS^{OneMerger} &= \frac{3}{20402b}\left(5044a^2 - 2abc\left(2522 - 1213\gamma\right) + (bc)^2\left(1261 - 1213\gamma + 589\gamma^2\right)\right) \\ W^{OneMerger} &= \frac{3}{20402b}\left(11520a^2 - 2abc\left(5760 - 2827\gamma\right) + (bc)^2\left(2880 - 2827\gamma + 2138\gamma^2\right)\right) \end{aligned}$$

$$\begin{aligned} CS^{OneMerger}\left(\gamma = 0\right) &= \frac{3783}{2b}\left(\frac{2a - bc}{101}\right)^2 \\ W^{OneMerger}\left(\gamma = 0\right) &= \frac{4320}{b}\left(\frac{2a - bc}{101}\right)^2 \end{aligned}$$

$$\begin{aligned} CS^{NoMerger} &= \frac{9}{2b}\left(\frac{2a - bc}{5}\right)^2 \\ W^{NoMerger} &= \frac{21}{2b}\left(\frac{2a - bc}{5}\right)^2 \end{aligned}$$

It is easy to verify that

$$CS^{OneMerger}\left(\gamma = 0\right) - CS^{NoMerger} = \frac{1383}{255025b}\left(2a - bc\right)^2 > 0$$

and

$$W^{OneMerger}(\gamma=0)-W^{NoMerger}=\frac{1779}{510050b}(2a-bc)^2>0$$

- Merged firm produces more post-merger iff $\gamma \geq \frac{\underline{\pi}}{135}$. Simplify $\frac{3}{202}bc\left(20\underline{\pi}+27\gamma\right)\geq\frac{3}{10}bc\underline{\pi}$.
- Merged firm's market share increases iff $\gamma \geq \frac{\underline{\pi}}{34}$. Simplify $\frac{3}{202}bc\left(20\underline{\pi}+27\gamma\right)\geq\frac{3}{202}bc\left(21\underline{\pi}-7\gamma\right)$.
- Nonmerged firm produces less iff $\gamma \geq \underline{\pi}\frac{4}{35}$. Simplify $\frac{3}{202}bc\left(21\underline{\pi}-7\gamma\right)\geq\frac{3}{10}bc\underline{\pi}$.

□

Proposition. *4.3 If firms A and B merge:*

- *The merger increases the merged-firm's profits (relative to the pre-merger joint profits) iff cost savings satisfy* $\gamma > \frac{1}{135}\underline{\pi}$
- *The non-merged firms' profits increase iff* $.00535\underline{\pi}$

Proof. Pre-merger profits are $\pi=\frac{3}{b}(bc)^2\left(\frac{\underline{\pi}}{5}\right)^2$. Post merger profits are $\pi_{AB}=\frac{3}{b}(bc)^2\left(\frac{20\underline{\pi}+27\gamma}{101}\right)^2$ and $\pi_C=\frac{3}{b}\cdot(bc)^2\cdot\frac{91}{2}\left(\frac{3\underline{\pi}-\gamma}{101}\right)^2$. Comparing the terms obtains the results. □

Proposition. *4.4 The second merger is profitable iff* $\gamma \geq 0.00134\underline{\pi}$*. In particular, if cost efficiencies are large enough such that the first merger is profitable, then so is the second merger. Moreover, the firm that merged first benefits from the second merger iff the first merger induced a decrease in the merged-firm's market share; i.e.:*$\gamma<\frac{1}{34}\underline{\pi}$.

Proof. The condition for $\pi_C<\pi_{CD}$ is:

$$\frac{3}{b}\cdot(bc)^2\cdot\frac{91}{2}\left(\frac{3\underline{\pi}-\gamma}{101}\right)^2<\frac{3}{b}(bc)^2\left(\frac{\underline{\pi}+\gamma}{5}\right)^2.$$

The condition for $\pi_{AB}^{OneMerger}<\pi_{AB}^{TwoMergers}$ is:

$$\frac{3}{b}(bc)^2\left(\frac{20\underline{\pi}+27\gamma}{101}\right)^2<\frac{3}{b}(bc)^2\left(\frac{\underline{\pi}+\gamma}{5}\right)^2.$$

Isolating γ in each term obtains the result. □

Proposition. *4.5 The second merger increases consumer surplus iff it decreases profits for the first merged firm (*$\gamma>\frac{1}{34}\underline{\pi}$*). If* $\gamma<0.0082\underline{\pi}$ *the second merger decreases total welfare as well as consumer surplus.*

Proof. The values for CS and total welfare after the second merger are the same as for no merger, with c replaced by $(1-\gamma)\,c$. These are provided in the proof for proposition 4.2. Comparing to the values after one merger (also provided in the same proof) obtains two rather complicated terms. The values were obtained using the Mathematica algebra solver. For both CS and W, the difference from the single merger is increasing in γ and is negative for $\gamma = 0$. Thus, the proposition is obtained by finding the γ values for which CS or total welfare exactly equal before and after the second merger. □

AppendixB. *N* Firms

Consider an industry with many markets in which all firms are active (overlapping markets) and in addition each firm has some markets that it dominates (private markets). As in the two firm case, we let m_i denote the number of i's private markets and M the number of overlapping markets. For simplicity we assume that each firm has a private market $\left(m^i > 0\right)$ and all markets have identical inverse demand $P(Q)$ where Q is the total quantity in the market, and $P\left(\right)$ satisfies the demand regularity assumption (1). As in the two firm case, we use q_i,$\hat{q}_i$, ζ_i and $\hat{\zeta}_i$ to denote i's quantities in the private and overlapping markets and it's marginal revenues.

For the results, it is sufficient to focus on *rival* flexibility and *own* commitment. Thus, we let ϕ_{-i} denote the second stage reaction curve of all of i's rivals for a marginal change in i's quantity in all overlapping markets:

$$\phi_{-i} \equiv \sum_{n \neq i} \frac{\partial q_n}{\partial q_i}$$

Commitment is the same as in the two firm case:

$$\sigma_i = M \frac{dq_i}{dk_i}$$

The intermediate results established for the two firm case extend directly, as the arguments do not depend on the two firm structure. These are stated without proof:

Lemma AppendixB.1. *For any I-firm equilibrium, the following hold:*

- *Capacity binds for each firm.*

- *Marginal revenue for i is identical in all markets i serves.* $\eta_i = \eta_i$.

- $\frac{d\phi_{-i}}{dm_{j\neq i}} \leq 0$. *That is, as i's rivals have better outside options for their extra capacity, they become more accommodating (flexible).*

- $\frac{d\sigma_i}{dm_i} \leq 0$ *and* $\frac{d\sigma_i}{dm_{j\neq i}} \geq 0$. *That is, as i's private markets can accommodate more quantity, i's commitment to all of its overlapping markets decreases and i's rivals' commitment power in the overlapping markets increases.*

For the comparative static result, we also directly assume that, as in the two firm case, strategic substitution is maintained – if firm i increases quantity in all of its markets, its rivals weakly decrease their quantity. The proof of proposition 2.3 can now be replicated directly.