Reputation Turnaround, Negotiated Block Trade, and Endogenous Cost of Corporate Control

Pak Hung Au
Division of Economics
School of Humanities and Social Sciences
Nanyang Technological University
aupakhung@gmail.com

Yuk-fai Fong
Department of Economics
Hong Kong University of Science & Technology
yfong@ust.hk

Jin Li
Kellogg School of Management
Northwestern University
jin-li@kellogg.northwestern.edu

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Abstract
Reputation is a valuable asset to firms, yet the impact of corporate governance of reputation-reliant firms is underexplored. This paper investigates how a firm’s reputation in the product market responds to a change in its controlling shareholder, and derives the optimal firm ownership and control structure. We consider a dynamic model of an experience-goods firm, in which a controlling shareholder actively engages in management, and the controlling share block can be traded through private negotiation. In the optimal equilibrium, the firm’s reputation in the product market is linked to its behavior in the market for corporate control to provide proper incentive for the controlling shareholder to maintain a good firm reputation. Our analysis also identifies an endogenous cost of corporate control, and provides a rationale for the separation of ownership and control. We derive the optimal ownership structure and draw implications on the dynamics of control premium.

JEL Classifications: L14, L15, D86

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1 Introduction

Reputation is a valuable asset to a firm, because consumer’s purchasing decisions are often based on their perception of the firm’s reputation. Consumer’s faith in the firm is particularly important if some dimensions of the firm’s products or services are difficult to monitor and contract upon. A firm with good reputation can make more sales, charge a price premium, thus enjoy a higher profit. When its reputation turns bad, it loses customers and profit. To build and maintain a good reputation, the firm’s top executives must devote personal effort and spend the company’s valuable resources into establishing a history of offering high-quality products. In this paper, we consider firms that rely on their reputation in the product market, and study how turnover of ownership and the design of corporate governance structure can facilitate the firm’s management of its reputation in the product market and through which enhances its profit. To the best of our knowledge, this is the first study of implications of reputation concern in the product market on the firm’s corporate governance structure.

It is standard in the literature to capture reputation concern in the product market by considering a firm as a producer of experience goods. A product is an experience good if consumers cannot observe its product quality at the time of purchase, but their consumption experience provides public signals about the product’s quality.\(^1\) High-quality production is costly to the firm and therefore, without proper incentive, the firm will shirk on its quality and rational consumers correctly anticipated this. Klein and Leffler (1981) proposed a reputation mechanism according to which if a firm continues to produce at high quality, it carries a good reputation and customers will pay a price premium for its products; once the firm produces bad quality, its reputation turns bad, and, believing that the firm will produce at low quality, customers punish the firm by either asking for a large discount on its product or not purchasing from the firm at least for some time. One implicit assumption of the reputation mechanism is that the firm is owned by a single shareholder throughout the lifetime of the firm. When the firm does not have perfect control of its quality, such punishment necessarily occurs with a positive probability on the equilibrium path, leading to destruction of shareholder value.

In this paper, we revisit the reputation mechanism in a setting which recognizes two commonly observed features of corporate governance: (i) in many companies there are two types of shareholders: controlling shareholders who make managerial decisions, and noncontrolling shareholders who do not; and (ii) the controlling-share block can be traded in the market for corporate control. By simultaneously studying the firm’s strategy for reputation management in the product market and the design of its dynamic ownership structure, we obtain the following main findings: (i) voluntary turnover of controlling-share block can help turnaround a firm’s damaged reputation and enhance

\(^1\)Some examples of experience goods are: movies, red wines newly introduced to the market, some consumer durables such as a new generation of smartphone.
the firm’s long-run shareholder value, and (ii) the optimal ownership structure is the outcome of the tradeoff between managing the controlling shareholder’s moral hazard and mitigating the punishment from the product market.

Formally, our model is a repeated game with imperfect public monitoring augmented with voluntary player turnover, and we solve for the perfect Bayesian equilibrium that maximizes the total shareholder value. In the optimal equilibrium, following a bad outcome, the controlling shareholder voluntarily sells her block of shares to a new entrepreneur through negotiation. As long as the endogenously determined negotiated price for the shares is sufficiently low, the controlling shareholder is sufficiently punished and the fear of punishment provide him enough incentive to exert personal effort and spend company’s valuable resources to improve the product’s quality. The low negotiated price can be sustained in equilibrium because if the controlling shareholder fails to sell her shares, consumers would believe that the firm engages in high-quality production only if a huge discount is offered. Since the new entrepreneur and the noncontrolling shareholders are not responsible for the bad outcome, once the control block changes hands, customers no longer have to punish the firm severely and will continue to pay a premium for the firm’s product. Consumers’ preferential treatment of the new controlling shareholder allows the firm’s damaged reputation to be repaired through the turnover of control block. It is interesting to note that in the optimal equilibrium, the gain from trade of the control block arises endogenously from the product market’s differential treatments of the incumbent and new controlling shareholders. Another noteworthy feature is that the incumbent controlling shareholder exits the firm voluntarily and she receives a price for the share block in excess of what she could get if she stay in control of the company.

In our setting, the controlling shareholder enjoys a positive private benefit in excess of the effort cost of managing the company and there is no other exogenous cost of corporate control. This means that the control premium, defined as the difference in values between controlling shares and noncontrolling shares, is positive as long as the controlling shareholder and minority shareholders receive the same stream of income per share. However, in equilibria with turnover of controlling share block, the control premium is not necessarily positive. The reason is that in the optimal equilibrium with turnover of control rights, the controlling shareholder has to bear an endogenous cost of corporate control: they have to be punished for bad outcomes on the equilibrium path while minority shareholders do not. More specifically, following a bad outcome, the controlling shareholder has to voluntarily sell her shares at a low price while the firm’s high profit is preserved. The noncontrolling shareholders are entitled to the entire stream of high profits, while the controlling shareholder is not. This creates a wedge between the income streams offered by each controlling share and noncontrolling share. When this wedge exceeds the net private benefit of control, the control premium becomes negative. This theoretical possibility of negative control premium is contrary to the conventional wisdom that shares with more control rights are valued weakly higher than shares with less or no control rights. Our finding is also empirically relevant because there are
well documented examples of negative control premiums.\textsuperscript{2} Moreover, our model predicts that the control premium is lower and more likely to be negative when a firm is performing poorly. This prediction is also consistent with empirical findings.\textsuperscript{3}

Our model provides a rationale for partial separation of ownership and control, and sheds light on the optimal firm ownership structure. In the optimal equilibrium, controlling shares are subject to punishment in the product market when the product quality fails, but noncontrolling shares are not. This implies that the total shareholder value can be raised by converting some of the controlling shares into noncontrolling shares. In other words, the founder of the company can benefit by issuing noncontrolling shares after setting up the company. Note that despite this benefit of issuing noncontrolling shares, the total shareholder value does not monotonically increase in the fraction of noncontrolling shares. As more shares are converted into noncontrolling shares, the controlling shareholder’s incentive to exert effort weakens because she now receives a smaller share of the profit but is required to put forth the same amount of managerial effort to maintain high-quality production. In our framework, the optimal share structure is the outcome of a tradeoff between managing the controlling shareholder’s moral hazard problem and preserving firm value from product market’s punishment.

The structure of the paper is as follows. Below, we discuss the literature in economics and corporate finance pertinent to our study. In Section 2, we set up our model. Section 3 analyzes the benchmark case in which the market for corporate control is shut down. In Section 4, we study the effect of negotiated block trade on the firm’s reputation and profit, and show how the product market and the market for corporate control interact with each other in equilibrium. Section 5 discusses how our theory accounts for an endogenous cost of corporate control and implications on control premium. In Section 6, we solve for the optimal ownership structure. Section 7 discusses some modelling issues and generalizations. The final section concludes.

1.1 Related Literature

The insight that profits from future sales incentivize sellers to engage in good behavior dates back to Klein and Leffler (1981). In their model of repeated game with perfect monitoring, when the seller sufficiently cares about the future, there exists a (subgame perfect) equilibrium in which buyers pay a premium to purchase from the seller if and only if the seller has always provided high-quality goods in the past. The fear of losing the future profits deters the seller from cheating\textsuperscript{4}

\textsuperscript{2}While many empirical studies have found that shares with voting rights are traded at a positive premium (see Zingales (1994) for a notable example), there are plenty of examples of negative control premium. See, for example, Chen (2004), Dyck and Zingales (2004), Kruse, Kyono, and Suzuki (2006), Lease, McConnell, and Mikkelsen (1983), Pinegar and Ravichandran (2003), and Valero, Gomez, and Reyes (2008). These studies are further discussed in Section 5.

\textsuperscript{3}See, for example, Barclay and Holderness (1989) and Kruse, Kyono, and Suzuki (2006).
(by offering low-quality goods). If the seller does not have perfect control of quality, as it is the case in reality, when she fails to provide high-quality goods, the reputation is sullied and she loses the profits associated with good reputation.

The notion of firm instead of its owner as a reputation-bearer is first proposed by Kreps (1990). He points out that even though a firm owner has a finite lifetime, she is motivated to maintain a good firm reputation because when she retires and has to sell the firm, the buyer is willing to pay a higher price if the firm has a good reputation. The theory of firm ownership turnover has later been further developed by Tadelis (1999, 2002, 2003) and Mailath and Samuelson (2001) who study secret ownership transfers of firms with good reputation. Contrary to Tadelis (1999, 2002, 2003) and Mailath and Samuelson (2001), our complementary theory is about publicly observable ownership turnover of firms with damaged reputation. Our theory also differs from the theories of firm ownership turnover mentioned above in that our model allows us to endogenize the timing of turnover, demonstrate how ownership turnover can help turnaround a firm’s bad reputation, derive the optimal ownership structure, and draw implications on the dynamics of control premium. For a comprehensive survey of the literature on seller reputation, see Bar-Isaac and Tadelis (2008).

Our theory considers firms in which a dominant shareholder holds a significant fraction of total shares and the rest of shares are widely held by small investors. Empirical research on corporate ownership concentration shows that the existence of controlling shareholders in modern corporations, even in large and publicly-listed ones, is prevalent (see, La Porta, Lopez-de-Silane, and Shleifer (1999) for an international study, and Gadhoun, Lang, and Young (2005) and Anderson and Reeb (2003) for studies on U.S. firms). It is common for these controlling shareholders to have control rights in excess of their cash flow rights and to actively participate in management. Closely related are empirical studies on insider ownership. Holderness, Kroszner, and Sheehan (1999) find that insiders (firm’s main officers and directors) on average owned 21 percent of the common stock of a typical firm. A motivation for large block ownership is that it is effective in overcoming a free-rider problem: while overseeing firm operations and improving firm performance benefit all shareholders, these activities involve high personal cost. By assigning substantial cash-flow right to one or a few large shareholders, they have strong incentives to engage in value-improving activities. This is the view put forth by Shleifer and Vishny (1986), and Demsetz and Lehn (1985).

Building on the notion that share ownership mitigates the controlling shareholder’s moral hazard,

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4 There are other notable differences between their theories and ours. In terms of modelling, these theories rely heavily on adverse selection, i.e., firm owners abilities are hidden information. Also, the owners’ exit decisions are exogenous, and firm’s performance does not improve following ownership transfer. In sharp contrast, our model is a purely one of moral hazard. Moreover, in our model, a controlling shareholder exits following bad performance and the firm’s performance is expected to improve under the new controlling shareholder.

5 See also Mikkelson and Partch (1989) for related findings.

6 In this paper, we abstract away from the strategic interaction among multiple blockholders by assuming there is a single controlling shareholder. This allows us to simplify the analysis and focus on the relationship between firm reputation and turnover of the control block.
we introduce a novel benefit of separation of ownership and control and derive the optimal ownership structure in our setting.

In his seminal article, Manne (1965) suggests that the market for corporate control can incentivize incumbent managers by threatening them with the prospect of losing their job in case the firm is acquired following poor performance. Jensen and Ruback (1983) argue that control transactions such as tender offers, and proxy contests are best viewed as relatively passive shareholders choosing among competing managerial teams. In their framework, poorly performing managers are involuntarily forced out of the firm, and deprived of the rent and/or private benefit associated with their job. Barclay and Holderness (1991) identify negotiated block trade as an important class of corporate control transactions by presenting empirical evidence that negotiated block trade is very often associated with extensive post-trade managerial and board turnover. They point out that negotiated block trade is best viewed as corporate control events in which “activist stockholders.... buy control of a company and hire and fire management to achieve a better resource utilization”. In these studies, turnover of ownership and control improves the firm’s performance by making punishment for bad outcome more severe and/or replacing poor managers with more capable ones. While we also show that negotiated block trade can improve firm profit, turnover enhances firm profit through very different forces in our setting. In our model, the incumbent controlling shareholder exits the firm voluntarily following bad performance, and the entrepreneur replacing her is no more capable at running the business. Barclay and Holderness (1989, 1991) are the first systematic empirical studies of negotiated block trades. We discuss the connection between their empirical findings and our model’s predictions in Section 5. For a comprehensive survey of block ownership and transaction, see Holderness (2003).

The most frequently cited benefits of separation of ownership and control include management specialization, risk-sharing, and liquidity constraint (Fama and Jensen (1983) and Shleifer and Vishny (1997)). By taking into account the management of reputation in the product market, we identify an underexplored benefit in separating ownership and control. The benefit arises because turnover of control allows punishment to be targeted on the controlling shareholder and the firm’s damaged reputation to recover in a less costly way. As a result, noncontrolling shares can enjoy a higher profit stream, and the total shareholder value can be higher if a firm has a larger fraction of outstanding noncontrolling shares. Finally, our analysis of optimal ownership structure is related to Zingales (1995), which investigates how the original firm owner can maximize proceeds from the sale of the firm by first selling a fraction of cash-flow rights in a competitive IPO market, and then directly bargaining with a new owner. While his model and ours share some common features, the objectives of the analyses are markedly different. His model studies how the original owner can best utilize the difference in his bargaining powers in the IPO market and the corporate control market in order to extract the increase in firm profit and/or private benefit of control by the new owner. In contrast, our model’s objective is to analyze the ownership structure that minimizes the impact
of product market punishment on firm value while providing proper incentive for the controlling shareholder in maintaining a good firm reputation.

2 Model

Players Time is discrete and infinite, \( t = 1, 2, \ldots \). There are three kinds of players in the game: entrepreneurs, customers, and investors. All players share the same discount factor, \( \delta \in (0, 1) \), across periods. There is a continuum of anonymous customers of measure one. The market is served by a monopoly firm possessing a technology of producing an experience goods, i.e., goods of which the quality cannot be observed at the time of purchase. The monopoly firm is owned by one entrepreneur who has full control rights over the firm’s business decisions, and a continuum of anonymous investors who own the company’s shares but have no control rights. We call the entrepreneur with control rights the controlling shareholder and the other investors the noncontrolling shareholders. Denote the fraction of shares owned by the controlling shareholder by \( \theta \), and the remaining fraction, \( 1 - \theta \), is owned by the non-controlling shareholders. For now, \( \theta \) is assumed to be fixed and exogenous, but in Section 6, we will endogenize \( \theta \) by considering it as optimally chosen by the founder of the company. The share structure \( \theta \) and the identity of the controlling shareholder are perfectly observable to all players.

Production Technology In every period, \( t \), the production technology may yield two possible outcomes, \( y_t \in \{0, 1\} \), with each outcome representing the utility received by customers upon consumption. The realization of the outcome is publicly observable and perfectly correlated among customers consuming the goods in period \( t \). The probability of each outcome depends on both the monetary production cost the firm incurs, \( c_t \in \{c^H, c^L\} \), and the controlling shareholder’s effort choice in monitoring and managing, \( e_t \in \{e^H, e^L\} \), and we assume\(^7\)

\[
1 > \Pr (y_t = 1 | e_t = e^H \land c_t = c^H) \equiv p \equiv \Pr (y_t = 1 | e_t \neq e^H \lor c_t \neq c^H) > 0.
\]

While \( c_t \) is born by all shareholders, both \( c_t \) and \( e_t \) are chosen by the controlling shareholder. Both \( c_t \) and \( e_t \) are unobservable by consumers. Since \( p < 1 \) and \( q > 0 \), this is a game of imperfect public monitoring. Effort and monetary costs are perfect complements in the sense that both have to be high to result in a high likelihood of a good outcome; neither \( e^H \) nor \( c^H \) alone will result in high likelihood of good outcome. When \( e_t = e^H \) and \( c_t = c^H \), we say the firm engage in high-quality production, even though doing so does not guarantee high quality; otherwise, we say it engages in low-quality production. We assume that quality improvement is socially efficient:

\[
e^H + c^H - (e^L + c^L) < p - q.
\]

\(^7\)We do not include both effort cost and monetary cost purely for realism. Corollary 3 and the discussion preceding that make it clear that \( c^H - c^L \) and \( e^H - e^L \) affect the firm’s optimal ownership structure differently.
The interpretation of the production technology is that quality improvement requires purchasing expensive production inputs and providing incentives for workers (who are not explicitly modelled here). To implement high-quality production, it is also necessary for the controlling shareholder to engage in effortful management and monitoring. The assumption of perfect complementarity is made for simplicity.

**Payoffs** Denote the price the firm charges by $P_t$. Denote the (normalized) values of each unit of controlling shares and noncontrolling shares in period $t$ by $V_t$ and $U_t$, respectively. A customer buying from the firm receives an instantaneous payoff of

$$Pr(y_t = 1) - P_t.$$  

The controlling shareholder receives fraction $\theta$ of the firm’s profit and incurs effort cost $e_t$. We assume she also receives an exogenous private benefit of control, $B$.\(^8\) Our assumption that the private benefit is independent of $\theta$ is in line with the model in Zingales (1995). When all customers buy from the firm, her total payoff in period $t$ is

$$\theta V_t = B - e_t + \theta (P_t - c_t).$$

Noncontrolling shareholders simply receive fraction $(1 - \theta)$ of the firm’s profit:

$$(1 - \theta) U_t = (1 - \theta) (P_t - c_t).$$

We assume that $B - e^H > B - e^L > 0$ so that when the controlling shareholder and noncontrolling shareholders receive the same stream of income per share, the net benefit of controlling the company is positive regardless of the controlling shareholder’s effort.\(^9\) Furthermore, we assume $p - c^H > q - c^L \geq 0$. This assumption ensures that even if customers hold the most pessimistic belief that the firm engages in low-quality production forever, the firm can still earn a nonnegative profit, thus the values of both kinds of shares remain nonnegative.\(^10\)

**Turnover of Controlling Shareholder** We model negotiated trading of the controlling share

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\(^8\)If the private benefit is partly derived from appropriating shareholders’ profit, then we have

$$\theta V_t = B - e_t + \theta (P_t - c_t - b); \ U_t = P_t - c_t - b$$

for some $b \leq B$. The analysis will not be qualitatively affected in this alternative setup. To see this, define $\hat{c}_t \equiv c_t + b$. Then we can express the payoffs in the form as in our model:

$$\theta V_t = B - e_t + \theta (P_t - \hat{c}_t); \ U_t = P_t - c_t.$$

\(^9\)It will be clear that in the context of our model, these two types of shareholders will receive the same stream of income when ownership turnover is not allowed or when consumers treat the incumbent controlling shareholder and new controlling shareholder symmetrically. However, when ownership turnover is allowed, they may not receive the same stream of income.

\(^10\)This assumption simplifies the analysis without affecting the main message.
block in the following way: every period an entrepreneur arrives and may purchase the entire block of shares from the incumbent controlling shareholder. After the purchase, the entrepreneur becomes the new controlling shareholder of the firm. If acquisition does not take place in a period, the potential acquirer exits forever. When acquisition takes place, it is publicly observable. However, the actual transfer price can neither be publicly observed nor credibly disclosed by the transacting parties.\textsuperscript{11} We assume that the transaction price is determined by Nash bargaining, and we denote the incumbent’s bargaining power by $\beta \in (0, 1)$.\textsuperscript{12}

**Timeline** The following figure illustrates the timeline within each period:

![Timeline Diagram]

Finally, we assume that if there are transfers between the controlling shareholder and noncontrolling shareholders, such transfers can neither be publicly observed nor credibly disclosed by the transacting parties. If any transfer between the controlling shareholder and any noncontrolling shareholder is creating any value, we assume that the value will be fully captured by the controlling shareholder. In other words, the controlling shareholder has 100% bargaining power over the noncontrolling shareholders.\textsuperscript{13}

The main objective of our analysis is to characterize the *optimal equilibrium* of the game. We define the optimal equilibrium as the perfect public equilibrium (PPE) that maximizes the total

\textsuperscript{11}If the transaction price of the controlling share block is observable, then a low equilibrium price can be easily enforced by consumers’ belief that the new owner will engage in high quality production if and only if the transaction price is sufficiently low.

\textsuperscript{12}It is quite natural to assume that the incumbent’s bargaining power is less than 1. Zingales (1995) also makes a similar assumption.

\textsuperscript{13}The role of this assumption is to rule out the use of transfer between controlling and noncontrolling shareholders as a punishment device for bad outcome.
(normalized) shareholder value:

\[ S = \theta V + (1 - \theta) U. \]

It will become clear in Section 6 that \( S \) is also the value of the company to the founder if she can sell noncontrolling shares to perfectly competitive investors. Note that the total shareholder value is bounded from above by \( S := B + p - (e^H + c^H) \), which is achieved when the firm engages in high-quality production every period and consumers pay \( p \) every period. The lower bound of the total shareholder value is \( S := B + q - (e^L + c^L) \), which is achieved when the firm engages in low-quality production every period and consumers pay \( q \) every period. We are particularly interested in the condition under which the firm can achieve the highest possible total shareholder value. Before proceeding with our characterization, we consider the benchmark in which the market for corporate control is shut down, i.e., transfer of the controlling share block is not feasible.

### 3 Benchmark Case: No Transfer of Controlling Share Block

In this section, we consider the case in which there is only one player who can be the controlling shareholder, i.e., the firm’s control rights cannot be transferred. The purpose of this section is to show that any equilibrium in which the firm engages in high-quality production necessarily entails the destruction of the firm’s profit. We show that when ownership turnover is not allowed, the optimal equilibrium yields a total shareholder value strictly less than the theoretical upper bound, i.e. \( S < \overline{S} \), the firm profit (which is also the value per noncontrolling share) is strictly less than \( p - c^H \), and the value of each controlling share is strictly less than \( p - c^H + (B - e^H) / \theta \).

First note that the controlling shareholder has the option of perpetually engaging in low-quality production and selling the product at \( P_t = q \). When the controlling shareholder exercises this option, each noncontrolling share receives a flow payoff of \( q - c^L \) and each controlling share receives \( (B - e^L) / \theta + (q - c^L) > 0 \). Also note that our assumption that each individual consumer has zero measure and is anonymous implies that the firm may not charge higher than the expected value of the product, i.e., \( P_t \leq p \). We do not impose a lower bound on \( P_t \) in the formal analysis.

Recall that \( V \) and \( U \) are the per-unit market values of the controlling and noncontrolling shares, respectively. When the controlling shares cannot be traded, both the controlling and noncontrolling shareholders receive the present discounted value of the firm’s profit stream and the values of the two classes of shares differ only due to the private benefits and effort costs:

\[ V = U + \frac{B - e^H}{\theta} > U. \]

In other words, there is a positive control premium of \( (B - e^H) / \theta \).

Let \( W \) be the (normalized) market value per controlling share following a bad outcome. If
W is strictly less than V, it is attained by the firm giving a sufficiently deep discount for one period.\textsuperscript{14} To ensure that the controlling shareholder is willing to offer the required discount, any deviation will trigger the off-the-equilibrium-path on which consumers believe the firm perpetually engages in low-quality production. Notice that every time the controlling shareholder is punished, the noncontrolling shareholders are punished to the same extent on a per-share basis.

The value of each controlling share, \( V \), is given by

\[
\theta V = (1 - \delta) \left[ B - e^H + \theta (P - c^H) \right] + \delta (p \theta V + (1 - p) \theta W). 
\]

Since \( U \) and \( V \) differ only by a constant, both \( U \) and \( V \) increase in \( W \). To induce effort, the following incentive constraint is needed:

\[
\theta V \geq (1 - \delta) \left[ B - e^L + \theta (P - c^L) \right] + \delta (q \theta V + (1 - q) \theta W). \quad (1)
\]

Combining the two to eliminate \( V \), we obtain

\[
\frac{(1 - \delta) \left[ B - e^H + \theta (P - c^H) \right] + \delta (1 - p) \theta W}{\theta (1 - \delta p)} \geq \frac{(1 - \delta) \left[ B - e^L + \theta (P - c^L) \right] + \delta (1 - q) \theta W}{\theta (1 - \delta q)}. \quad (2)
\]

One immediate observation is that since \( 1/(1 - \delta p) \geq 1/(1 - \delta q) \), for any given \( W \), the incentive constraint is easier to be satisfied with a higher \( P \). Setting a higher \( P \) also raises both \( U \) and \( V \). Therefore, in the optimal equilibrium, \( P = p \).

For the analysis to be nontrivial, it is necessary that the moral hazard problem is not too severe. Specifically, we need

\[
e^H - e^L + c^H - c^L \leq \frac{(p - q)^2}{1 - q}. \quad (3)
\]

We will adopt this assumption throughout the paper. Since \( (p - q)^2 / (1 - q) < p - q \), (3) implies our earlier assumption that quality improvement is socially efficient. In fact, it means that the efficiency gain from quality improvement must be large enough for high-quality production to be sustainable.

The proposition below states the result of this section formally.

**Proposition 1** Let \( \bar{V}^0 (\theta) \) be the maximum equilibrium value per controlling share, \( \bar{U}^0 (\theta) \) be the maximum equilibrium value per noncontrolling share, and \( \bar{S}^0 (\theta) \) be the maximum equilibrium total

\textsuperscript{14} An alternative punishment is that with a certain probability consumers coordinate to believe that the firm forever engage in low-quality production in the future.
shareholder value. Suppose (3) holds and

\[ \theta > \theta \equiv \frac{(1-q)(e^H - e^L)}{(p-q)^2 - (1-q)(e^H - e^L)}. \]

(i) If

\[ \delta \geq \hat{\delta}(\theta) \equiv \frac{e^H - e^L + \theta (e^H - e^L)}{q(e^H - e^L + \theta (e^H - e^L)) + \theta (p-q)^2}, \]

then the equilibrium share values are given by:

\[ \bar{V}^0(\theta) = B - e^H + \frac{1-p}{p-q}(\frac{e^H - e^L}{\theta} + c^H - c^L), \]

\[ \bar{U}^0(\theta) = p - c^H - \frac{1-p}{p-q}(\frac{e^H - e^L}{\theta} + c^H - c^L), \]

\[ \bar{S}^0(\theta) = B - e^H + p - c^H - \frac{1-p}{p-q}(\frac{e^H - e^L}{\theta} + c^H - c^L). \]

(ii) If \( \delta < \hat{\delta}(\theta) \), then

\[ \bar{V}^0(\theta) = B - e^L + q - c^L, \]

\[ \bar{U}^0(\theta) = q - c^L, \]

\[ \bar{S}^0(\theta) = B - e^L + q - c^L. \]

Clearly, when the discount factor is too low, i.e., when \( \delta < \hat{\delta}(\theta) \), high-quality production will not be sustainable. Proposition 1 points out that even when the discount factor is high enough, i.e., when \( \delta \geq \hat{\delta}(\theta) \), the monopolist is still unable to charge consumers the expected value of its product every period. This is due to the fact that the firm can only charge consumers their reservation value \( p \) during the normal phase; whenever the firm has produced at the low quality, which happens with a positive probability, it has to offer consumers a discount even if they continue to produce at high quality. This loss in profit is similar in nature to the loss in profits of collusive oligopolies under imperfect public monitoring identified by Green and Porter (1984).

Focusing on the case of \( \delta \geq \hat{\delta}(\theta) \), the first term \( (p - c^H) \) in \( \bar{U}^0(\theta) \), is the expected accounting profit of the firm if the firm always operates in the absence of an agency problem. Similarly, the first term in \( \bar{V}^0(\theta) \) is the sum of the same expected accounting profit and the net private benefit per share. The second terms in \( \bar{U}^0(\theta) \) and \( \bar{V}^0(\theta) \) are the profits that must be destroyed to provide incentives for the controlling shareholder to improve output quality. Notice that the noncontrolling shareholders suffer the same loss in profits as does the controlling shareholder. Following a bad outcome, the controlling shareholder must be punished or she will have no incentive to exert
high effort and incur high monetary costs to increase the chance of producing high-quality goods. However, the punishment imposes a *negative externality* on the noncontrolling shareholders, who are also punished despite the fact that they are not responsible for the bad outcome. Perhaps more importantly, noncontrolling shareholders do not suffer from a moral hazard so it is wasteful in terms of shareholder value to punish them for a bad outcome.

Another point worth noting is that the severity of the agency problem is related to the share structure of the firm. It can be verified that the cutoff discount factor $\hat{\delta}$ is decreasing in the fraction of controlling shares, $\theta$. Figure 2 depicts how $\hat{\delta}$ changes in $\theta$.

![Figure 2](image)

When the controlling shareholder owns too few shares, i.e., when $\theta \leq \theta^*$ there is no discount factor at which high-quality production is sustainable. Define $\theta(\delta)$ as the solution to $\delta = \hat{\delta}(\theta)$ for $\delta \in [\hat{\delta}(1), 1)$.

It is easy to see from Proposition 1 that $\bar{S}^0(\theta) = \theta V^0(\theta) + (1 - \theta) U^0(\theta)$ is the lowest for $\theta < \theta(\delta)$ and increases in $\theta$ for $\theta \geq \theta(\delta)$. The analysis so far implies that the optimal share structure is to set $\theta = 1$. In other words, all the shares should be owned by the controlling shareholder. Doing so maximizes $\bar{S}^0(\theta)$ and minimizes $\hat{\delta}(\theta)$. Note that in the optimal equilibrium, $U$ and $V$ are also maximized respectively.

## 4 The Effect of Negotiated Block Trade

In this section, we investigate the interplay between the product market and the market for corporate control in providing incentives for reputation maintenance. Our focus is on negotiated block trade, and in our model, the market for corporate control works as follows: in each period, an entrepreneur arrives and engage in private negotiation with the incumbent controlling shareholder

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\(^{15}\theta(\delta)\) is well-defined on the domain $[\hat{\delta}(1), 1)$ as $\hat{\delta}(\theta)$ is strictly decreasing in $\theta$ for $\theta \in (0, 1]$.
in acquiring the entire block of controlling shares. If negotiation yields trade, the entrepreneur becomes the new controlling shareholder; otherwise, she exits the game forever.

Consider the following equilibrium, which consists of four phases: a normal phase, an on-the-equilibrium-path punishment phase, and two off-the-equilibrium-path punishment phases.

- The game begins in the normal phase. In the normal phase, the controlling shareholder sets the price at $p$ and engages in high-quality production, i.e., exerts effort $e^H$ and incurs monetary cost $c^H$ on the firm’s behalf. If the outcome is good, there will be no turnover of controlling shareholder and the game stays in the normal phase. If the outcome is bad, the game switches to the on-the-equilibrium-path punishment phase.

- In the on-the-equilibrium-path punishment phase, the controlling shareholder sells the entire block of controlling shares to a new entrepreneur at the transaction price $T$ through Nash bargaining. The firm under the new controlling shareholder may or may not have to offer the good at a discounted price. The new controlling shareholder engages in high-quality production in the on-the-equilibrium-path punishment phase and receives a continuation payoff $\hat{W}$. The game switches back to the normal phase if the outcome is good but stays in the on-the-equilibrium-path punishment phase if the outcome is bad.

- If the Nash bargaining breaks down, then the game switches to the first off-the-equilibrium-path punishment phase in which the incumbent controlling shareholder continues to engage in high-quality production and offers a one-period discount to customers for the experience good, receiving a continuation payoff $W$.

- Any other publicly observable deviations, including a deviation from the above-mentioned punishment phases, will trigger the second off-the-equilibrium-path punishment phase in which the controlling shareholder forever engages in low-quality production and sets price equal to $q$.

In the search of the optimal equilibrium, it is without loss of generality to focus on the class of equilibria outlined above. In our model, there are only two feasible ways of imposing punishment on the controlling shareholder for bad outcomes: (i) a cut in its product price (or equivalently coordinating on a certain probability of forever reverting to the low-quality-low-price equilibrium); and (ii) an outright sale of controlling shares to the newly arrived entrepreneur at a discounted

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16 In Section 5, when we analyze the company’s control premium, we will discuss a payoff equivalent equilibrium in which turnover also takes place following a good outcome.
price.\textsuperscript{17,18} In the benchmark analysis in the section above, we focus only on the product market punishment (i). In what follows, we shall show that as long as high-quality production can be supported, the total shareholder value can be improved relative to the benchmark case when both the product market and the market for corporate control (i.e., punishment (ii) above) are involved.

We first explain how the transaction price of the control block is determined. Recall that $W$ is the value of a controlling share in the first off-the-equilibrium-path punishment phase, i.e., when the control block is retained by the incumbent controlling shareholder, and that $\hat{W}$ is the corresponding value when ownership is transferred to the new entrepreneur. The incumbent and the new entrepreneur engages in Nash bargaining in which the bargaining power of the incumbent is $\beta$. Thus, the transaction price per share, $T$, is given by

$$T = W + \beta(\hat{W} - W).$$

To account for the value of a controlling share $V$, a shortcut is to imagine hypothetically that every time a bad outcome arises, the controlling shareholder, instead of realizing the loss of $\theta (V - T)$ by selling her block of shares, realized the loss of $\theta (V - T)$ but then continued to hold on to the controlling shares. With this interpretation, the value per controlling share can be expressed as

$$V = \frac{B - e^H}{\theta} + (p - c^H) \frac{V - T}{1 - \delta}. \tag{5}$$

To account for the value of a noncontrolling share $U$, notice that the company’s profit per share loses the amount $V - \hat{W}$ every time a bad outcome is realized and the control block subsequently changes hands. Both the new controlling shareholder and the noncontrolling shareholders suffer the same loss. Therefore,

$$U = (p - c^H) - \delta (1 - p) \frac{V - \hat{W}}{1 - \delta}. \tag{6}$$

We show in the following proposition that by allowing the turnover of the controlling shares, the value of the noncontrolling shares can be increased and the highest possible value of the noncontrolling shares, $p - c^H$, can be attained if the discount factor is large enough. Recall we focus on the optimal equilibrium, i.e., the equilibrium that maximizes the total shareholder value, $S = \theta V + (1 - \theta) U$. Let $\bar{U}$ and $\bar{V}$ be the values of noncontrolling shares and the controlling shares in the optimal equilibrium, respectively.

\textsuperscript{17}Here, the sale of controlling shares must be outright simply because $\theta$ is assumed to be fixed. If we do not assume $\theta$ is fixed, the optimal relational contract may require only a partial sale of controlling shares to the newly arrived entrepreneur while the remaining controlling shares are sold as non-controlling shares to outside investors. However, the optimal relational contract always requires an outright sale of controlling shares when $\theta$ is chosen optimally by the founder of the company, the case we analyze in Section 6.

\textsuperscript{18}Transfers between controlling shareholder and noncontrolling shareholders cannot be used to punish the incumbent controlling shareholder for bad outcome because it is assumed that the amount of transfer cannot be credibly disclosed publicly and that the incumbent controlling shareholder has full bargaining power over the noncontrolling shareholders.
Proposition 2 Suppose (3) holds, \( \theta > \theta \), and \( \beta \in (0, 1) \). For each \( \beta \), there exists \( \tilde{\delta}(\beta, \theta) \) such that

(i) if \( \delta \in [0, \tilde{\delta}(\theta)) \), then

\[
\bar{U} = q - c^L \text{ and } \bar{V} = \bar{U} + \frac{B - e^L}{\theta};
\]

(ii) if \( \delta = \tilde{\delta}(\theta) \), then

\[
\bar{U} = \bar{U}^0(\theta) \text{ and } \bar{V} = \bar{V}^0(\theta);
\]

(iii) if \( \delta \in (\tilde{\delta}(\theta), \tilde{\delta}(\beta, \theta)) \), then

\[
\bar{U} \in (\bar{U}^0(\theta), p - c^H) \text{ and } \bar{V} = \bar{V}^0(\theta);
\]

(iv) if \( \delta \in [\tilde{\delta}(\beta, \theta), 1) \), then

\[
\bar{U} = p - c^H \text{ and } \bar{V} = \bar{V}^0(\theta).
\]

According to Part (i) of Proposition 2, if high-quality production is not sustainable in the absence of the market for corporate control, then allowing turnover of control block cannot increase firm profits. This is because turnover of control block cannot change the fact that the worst possible punishment payoff to the controlling shareholder is \( B - e^L + \theta (q - c^L) \) and that such punishment is not enough to incentivize her. However, Parts (ii)-(iv) of the proposition suggest that as long as high-quality production is sustainable in the original game without turnover, then turnover can improve the noncontrolling shareholders’ value and such improvement is increasing in \( \delta \). When \( \delta \) is sufficiently high, specifically when \( \delta \geq \tilde{\delta}(\beta, \theta) \), noncontrolling shares attain the maximum value of \( p - c^H \). Figure 3 depicts what the cutoff \( \tilde{\delta}(\beta, \theta) \) looks like.
Several remarks about the proposition and the optimal equilibrium are in order. First, although the controlling shareholder’s equilibrium payoff remains unchanged, and in both cases she earns less than $B - c^H + \theta (p - c^H)$, there is a notable difference in the way she earns that payoff. When the turnover of control block is not allowed, the controlling shareholder earns the net private benefit and her share of the firm’s stream of profits, which is less than $p - c^H$ per period, because the firm has to offer a price discount to customers in the period following every bad outcome. On the other hand, with a turnover of control block, the controlling shareholder does not capture the entire stream of firm profits because, once a bad outcome is observed, she is prescribed to sell her controlling shares at the discounted price. This observation is closely connected to the discussion on endogenous cost of corporate control discussed in the next section.

Second, in the optimal equilibrium, the incumbent controlling shareholder exits voluntarily. Were she stays in the firm with a damaged reputation, the product market would impose a very severe punishment on the firm, leading to a very low continuation payoff to her. She is thus willing to sell her control block as long as the transaction price $\theta T$ exceeds the continuation payoff $\theta W$. This is possible because the new entrepreneur is not responsible for the bad outcome of the company, and is not going to be subject to the product market punishment. As a result, her value for the control block exceeds that of the incumbent. In our terminology, we have $\theta \hat{W} > \theta W$. It is important to note that the product market’s reaction to the change in controlling shareholder is NOT exogenously imposed. It is an equilibrium response: customers take optimal action given their belief, which is consistent with strategy employed by the controlling shareholders.

Third, both the product market and the market for corporate control impose punishment on a poorly performing controlling shareholder when $\delta \in (\hat{\delta} (\theta), \tilde{\delta} (\beta, \theta))$. In this case, because the incumbent enjoys a relatively high bargaining power over the new entrepreneur, she is able to capture a larger fraction of the gain from trading the block. In order to make the magnitude of punishment on the incumbent sufficiently large, it is necessary that the new entrepreneur receives a low enough continuation payoff after taking over the firm. Thus, the firm will still suffer a phase of low profit under the new leadership. However, the magnitude of this profit loss is lower than in the benchmark case where the market for corporate control is absent.

Last but not least, our theory predicts that turnover of top management is more likely to occur when the firm has performed poorly and the turnover leads to an improvement in firm performance. This prediction is consistent with the empirical findings that CEO/ownership turnover is more frequently preceded by poor company performance, and that oftentimes the replacement of the CEO and/or a change in the ownership and the board directors results in successful turnarounds.

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19 As we will show in Section 6 below, this is sometimes the relevant parameter case when the ownership structure, i.e., the value $\theta$, is chosen optimally to maximize total shareholder value.

Our model is also able to account for the empirical studies that have found that strategic change is often not an integral part of turnaround (for example, see Hambrick and Schecter (1983), Robbins and Pearce II (1992), and Kanter (2003)). Another noteworthy feature of the equilibrium, distinct from existing management theories, is that even though the new controlling shareholder is no better at running the company than the incumbent controlling shareholder, we still see an improvement in the firm’s performance following an ownership turnover. The theory proposed here therefore provides an explanation for change in control even in those cases in which “the potential benefits from changing blockholders are less apparent” (Barclay and Holderness (1991)).

In the remainder of this section, we discuss the model’s implication on the relationship between the acquisition premium and the post-acquisition performance of the company. Acquisition premiums involved in mergers and acquisitions are often sizeable, so those involved in these activities are naturally interested to know whether a higher acquisition premium is associated with a better post-acquisition performance. Traditional management theories suggest a positive relationship between the acquisition premium and the post-acquisition performance. This is because a manager who is more capable or has identified a higher value in the target company is willing to pay a higher acquisition premium and the company is also expected to perform better. However, this view cannot account for the negative relationships between the acquisition premium and the post-acquisition performance identified in some empirical studies (see for example, Sirower (1994), and Krishnan, Hitt, and Park (2007)).

Here, we hold fixed other parameters and study how changes in the bargaining power of the incumbent, $\beta$, affects the transfer price and the firm’s profit margin immediately after the acquisition. Recall that in the optimal equilibrium constructed above, following a bad outcome of production, the block of controlling shares is sold to a new entrepreneur at a (per-share) price of $T$ through Nash bargaining. If $\delta \in (\hat{\delta}(\theta), \tilde{\delta}(\beta, \theta))$, the firm has to offer a price discount to customers after the new entrepreneur takes control. Denote the associated discount by $D(\beta, \theta)$. It is immediate that the value of a noncontrolling share following a bad outcome, denoted by $\bar{U}$, is below its value during the normal phase, $U$:

$$p - c^H - (1 - \delta p) D(\beta, \theta) = \bar{U} < U = p - c^H - \delta (1 - p) D(\beta, \theta).$$

Define the acquisition premium as the (per-share) transaction price of the block of controlling shares minus the value of the noncontrolling shares during a downturn, i.e., $T - \underline{U}$. If $\delta \geq \tilde{\delta}(\beta, \theta)$, then the new owner does not have to offer any discount to customers and thus $\bar{U} = \underline{U}$.

\footnote{In our model, because a turnover of controlling shareholders always occurs after a bad outcome, on the equilibrium path, the incumbent owner never runs the company after a bad outcome. So this is an improvement over the off-equilibrium path on which takeover does not occur. If we introduce some friction during takeover so that ownership does not change hands immediately following a bad outcome, we will see low on-the-equilibrium-path profit following a bad outcome and subsequent improvement following the takeover.}

\footnote{The exact formula for the discount can be found in the proof of Proposition 2.}
Corollary 1  An increase in the bargaining power of the incumbent controlling shareholder β has the following effect on the firm’s optimal equilibrium:

(i) The acquisition premium, \( T - U_c \), weakly increases.

(ii) The firm’s accounting profit in the period after the turnover of the controlling shares, given by \( p - c^H - D(\beta, \theta) \), weakly decreases.

Moreover, if \( \delta > \hat{\delta}(\theta) \), the above relations are strict when \( \beta \) is sufficiently large.

According to the corollary above, if the source of variation is different allocations of the bargaining power between the incumbent and new controlling shareholders, then our model predicts a (weakly) negative relationship between the acquisition premium paid by the acquirer and the firm’s post-acquisition performance. One descriptive argument used by the authors who empirically identified this negative relationship is that a high acquisition premium is an indication of bad managerial decision or managerial hubris so the manager also tends to make bad decisions when running the company (for example, see Hayward and Hambrick (1997)). Our explanation is different and has more structure; in our model all managers have the same managerial ability. The negative relationship is a necessary part of the equilibrium to ensure the incumbent controlling shareholder with a higher bargaining power will not be overpaid so that she has the proper incentive to maintain the company’s reputation.

5 Endogenous Cost of Corporate Control

In this section, we explore the endogenous cost of corporate control in the model. The classical theory in asset pricing suggests that share value is determined by the present value of the company’s profit stream. In our model, the role played by the holders of the shares affects their value. In particular, controlling shareholder and noncontrolling shareholders are not entitled to the same profit stream. An endogenous cost of control arises in our model because the controlling shareholder must be punished following a bad outcome, while the noncontrolling shareholders either do not have to be punished (if \( \delta \geq \overline{\delta}(\beta, \theta) \)), or they are punished less severely than the controlling shareholder when they have to be punished (if \( \delta \in (\overline{\delta}(\theta), \overline{\delta}(\beta, \theta)) \)). We will show that because of this cost of control, although the net private benefit per share, \( (B - c^H)/\theta \), is positive, the control premium, defined as the difference between the market value of a controlling share and the market value of a noncontrolling share, may be negative. Moreover, because the punishment targeted at the controlling shareholder takes place during difficult times, the control premium is lower and more likely to be negative when the firm is performing poorly.

For ease of exposition, in the previous section, we focused on equilibria in which controlling shares are traded only following a bad outcome and noncontrolling shares are never traded. One can easily construct payoff-equivalent equilibria in which both the controlling and noncontrolling shares are traded following a good outcome. If controlling shares were traded following a good
outcome, the market price would be $\bar{V}$. If noncontrolling shares were traded, the market price would be $\bar{U}$ following a good outcome and $\check{U}$ following a bad outcome. Denote the control premium following a good and bad outcome by $\Delta^H$ and $\Delta^L$ respectively. Therefore, we have $\Delta^H \equiv \bar{V} - \bar{U}$ and $\Delta^L \equiv \check{T} - \check{U}$. It is clear that in our simple setting, the measurable control premium defined here and acquisition premium defined in the previous section are identical.

**Corollary 2** (i) If $\delta > \hat{\delta}(\theta)$, then the control premium following bad outcome is lower than that following a good outcome, i.e., $\Delta^H > \Delta^L$.

(ii) If $\delta > \hat{\delta}(\theta)$, then there exists a $B^* > e^H$ such that the control premium following good outcome is negative, i.e., $\Delta^H < 0$ if and only if $B \in [e^H, B^*)$.

The control premium can be negative if the private benefit of control is too small relative to the endogenous cost of corporate control. Negative control premiums have been identified empirically. Holthausen, Leftwich, and Mayers (1987) found that in large seller-initiated block transactions, buyers received price concessions. Barclay and Holderness (1989) found that 20% of their sample block trades were priced at a discount. Relatedly, Lease, McConnell, and Mikkelson (1983), Pinegar and Ravichandran (2003), Chen (2004), Kruse, Kyono, and Suzuki (2006), and Valero, Gomez, and Reyes (2008) found that some companies’ shares with superior voting rights were traded at a discount compared to the shares with inferior voting rights. Some informal arguments for the observed negative control premiums are that shares with inferior control rights are more liquid and that the controlling shareholder may have to bear legal liabilities (Dyck and Zingales (2004)). Nevertheless, the empirical observation of negative control premiums is considered by some to be puzzling because there is no formal theory that rationalizes it.\(^{23}\)

Furthermore, our theory’s specific prediction that the control premium is lower and more likely to be negative during downturns is also consistent with the empirical finding of Barclay and Holderness (1989, 1991) that the average premium is lower following poor performance, and with Kruse, Kyono, and Suzuki (2006) that the estimated private benefits of control in their data are the most negative when the target firm is financially distressed.

## 6 Optimal Ownership Structure

In this section, we endogenize the firm’s ownership structure by finding the optimal value of $\theta$ from the company founder’s perspective. The total payoff of the founder consists of two components, the value of the shares that she retains and the proceeds from the sales of the noncontrolling shares. We assume there is perfect competition for noncontrolling shares among investors which allows the company founder to fully capture the value of the noncontrolling shares. Therefore, the total payoff

of the founder is $S = \theta V + (1 - \theta) U$. Note that this is the total shareholder value of the company at the time when the ownership structure is determined. However, it is less than the sum of the net private benefit of control and the company’s profit. This is because part of the value of the company is captured through Nash bargaining by future controlling shareholders who take over the company’s control.

The basic tradeoff here is about managing the controlling shareholder’s moral hazard problem and preserving firm profit from product market punishment. In the presence of a corporate control market, a small value of $\theta$ helps mitigate the moral hazard problem, as the cost of high-quality production is lower for the controlling shareholder. This increases the value of the controlling share block. On the other hand, if $\theta$ is small, the turnover mechanism is less effective in generating punishment on the controlling shareholders, resulting in a lower firm revenue. To see this, note that the smaller the value of $\theta$, the more severe the punishment the turnover mechanism must impose on each unit of controlling share to preserve incentives. However, because of the Nash bargaining between the incoming and the incumbent controlling shareholders, the effectiveness of punishment is limited, and a product price discount is called for when $\theta$ is sufficiently small (recall case (iii) of Proposition 2). Moreover, this price discount is decreasing in $\theta$.\(^{24}\)

Recall $\theta(\delta)$ is the solution to $\delta = \hat{\delta}(\theta)$. Define $\tilde{\delta}(\delta, \beta)$ as the solution to $\delta = \tilde{\delta}(\delta, \theta)$.\(^{25}\) It is easy to verify that for $\delta \in (\hat{\delta}(1), 1)$, $\theta(\delta) < \tilde{\theta}(\delta) < \hat{\theta}(\delta, \beta)$ (see Figure 4).

![Figure 4](image)

**Proposition 3** Let $\theta^*$ be the optimal fraction of the controlling shares. If $\delta \in [\hat{\delta}(1), 1)$, then

(i) $\theta^*$ is unique and is in the interval $(\theta(\delta), \min\{\tilde{\theta}(\delta, \beta), 1\}]$. Moreover, $\theta^* \rightarrow \theta$ as $\delta \rightarrow 1$.

(ii) $\theta^*$ is weakly increasing in $c^H - c^L$, $c^H - e^L$, and $\beta$; and it is weakly decreasing in $\delta$. Furthermore, the above relations are strict if $\delta \in (\tilde{\delta}(\beta, 1), 1)$.

\(^{24}\)For the explicit formula, see (15) in the proof of Proposition 2.

\(^{25}\) $\tilde{\theta}(\delta, \beta)$ is well-defined because $\tilde{\delta}(\beta, \theta)$ is strictly decreasing as a function of $\theta$.\(\)
The main message of part (i) of Proposition 3 is that the optimal share structure is to convert some controlling shares, but not as many of them as possible, into noncontrolling shares. The comparative statics results of part (ii) arise from the tradeoff discussed above. More specifically, we can break down the marginal effect of a reduction in $\theta$ below $\bar{\theta}(\delta, \beta)$ into (a) the marginal benefit that arises from mitigating moral hazard; and (b) the marginal cost that arises from more severe product market punishment. The marginal benefit is given by

$$\frac{\partial}{\partial \theta} (\theta \bar{V}^0(\theta)) = \frac{1}{p} - \frac{p}{q} (c^H - c^L);$$

whereas the marginal cost is given by

$$\frac{\partial}{\partial \theta} ((1 - \theta) \bar{U}) = \frac{1 - p}{p - q} \frac{\delta}{1 - \delta} \left[ \left( \beta^{-1} (\delta^{-1} - q) - (1 - q) \right) \left( \frac{c^H - c^L}{\delta^2} + c^H - c^L \right) - (p - q)^2 \left( \beta^{-1} - 1 \right) \right].$$

First, it is straightforward to see that an increase in $e^H - e^L$ and $\beta$ raise the marginal cost without any effect on the marginal benefit, so $\theta^*$ goes up. Second, if $c^H - c^L$ increases, the marginal cost goes up by more than the marginal benefit, and $\theta^*$ becomes larger. Finally, if $\delta$ increases, the marginal benefit is unaffected, whereas the marginal cost goes down, so $\theta^*$ becomes smaller. These comparative statics results hold strictly whenever $\theta^* < 1$, or equivalently, $\delta \in \left( \hat{\delta}(\beta, 1), 1 \right)$.

The analysis in this section is related to Zingales (1995), who derives the practice of selling cash-flow rights to disperse shareholders and selling control rights through direct bargaining as the outcome of maximization of total proceeds from the sale of a company. Our model has the similar feature that disperse shareholders are perfectly competitive and the acquirer of control rights has substantial bargaining power. Other than that, our analysis is different in several important ways. In Zingales’s model, the incumbent owner goes public if and only if the incoming controlling shareholder generates a higher profit for the firm. Furthermore, the sale of cash-flow rights is partial only if the incumbent derives a higher level of private benefit than the new owner. On the contrary, in our model, the company founder goes public and chooses selling cash-flow rights partially even if every potential controlling shareholder generates the same level of private benefit for the firm and derives the same level of private benefit. Our results emerge from the consideration of controlling shareholder’s moral hazard problem and mitigating the impact of product market punishment on firm value, while these factors are absent in Zingales’s analysis.

Finally, we remark that the tradeoff discussed above highlights conditions under which the optimal ownership structure $\theta^*$ lies strictly below $\min \{ \bar{\theta}(\delta, \beta), 1 \}$, the case in which the turnover mechanism does not perfectly restore the firm’s profit. Whether $\theta^* < \min \{ \bar{\theta}(\delta, \beta), 1 \}$ holds or not

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is determined by whether the marginal benefit of cutting $\theta$ exceeds the associated marginal cost evaluated at $\theta = \min \{ \bar{\theta}(\delta, \beta), 1 \}$. In particular, observe that if $c^H - c^L = 0$, there is no benefit in reducing $\theta$ below $\bar{\theta}(\delta, \beta)$, whereas the marginal cost is positive, resulting in $\theta^* = \min \{ \bar{\theta}(\delta, \beta), 1 \}$. On the other hand, if $e^H - e^L$ and/or $c^H - c^L$ is too large, the marginal cost of cutting $\theta$ always exceeds that of the marginal benefit, and $\theta^* = \min \{ \bar{\theta}(\delta, \beta), 1 \}$. Only if the difference in monetary costs $c^H - c^L$ is moderate, and the difference in effort cost $e^H - e^L$ is sufficiently small, would $\theta^*$ be strictly interior.

**Corollary 3** Suppose $e^H - e^L < \frac{\delta(1-\beta)(p-q)^2}{1 - \delta q - \beta(1-q)}$. Then there exists a pair of positive numbers $C$, and $\bar{C}$ with $C < \bar{C}$ such that the optimal structure $\theta^* \in (\bar{\theta}(\delta), \min \{ \bar{\theta}(\delta, \beta), 1 \})$ if and only if $e^H - c^L < (C, \bar{C})$. If $e^H - e^L \geq \frac{\delta(1-\beta)(p-q)^2}{1 - \delta q - \beta(1-q)}$, then $\theta^* = \min \{ \bar{\theta}(\delta, \beta), 1 \}$ for all $c^H, c^L$.

### 7 Discussion

For tractability, we have abstracted away from many issues in our analysis. Below, we discuss some of them.

**Competition Among Potential Owners** We have ignored the issue of competition among potential acquirers by assuming that every period only one potential acquirer enters the game. A simple way to capture the impact of competition is to assume the bargaining power of the incumbent controlling shareholder increases with the intensity of the competition for the control rights. In the extreme case where competition is so fierce that the incumbent has all the bargaining power, i.e., $\beta = 1$, negotiated block trade fails to act as a disciplinary device. We take the view that it is unlikely that the incumbent has $100\%$ of the bargaining power. Even when there are simultaneously multiple buyers seeking control of the firm, as long as the incumbent owner cannot commit to a grand mechanism (say by holding an auction), but instead has to sequentially bargain with one buyer at a time, there are bargaining protocols with which the seller only receives a fraction of the total surplus. Moreover, oftentimes the incumbent owner has to face competition from owners of other companies trying to sell control rights in the market for corporate control, which limits their bargaining power. We find it comforting that for any interior split of bargaining power, i.e., for $\beta \in (0, 1)$, allowing turnover of controlling shareholders increases the maximum firm profit and total shareholder value for a range of sufficiently high discount factors.

**Alternative Turnaround Mechanisms** We have focused on the effect of negotiated block trade on turning around a damaged reputation on a firm’s profit. We have done so not because...
this is the only possible way to preserve firm value from product market punishment. Rather, our focus is partially motivated by the fact that turnover of ownership and control is common and empirical findings that ownership and management turnover is an integral part of a successful turnaround. In fact, if we allow the controlling shareholder to credibly burn money (for example, by making payments to a third party), then there exist equilibria in which consumers forgive the firm’s bad outcome if and only if the controlling shareholder have burnt a large enough amount of money. Requiring the controlling shareholder to burn money following every bad outcome may cause her to eventually run into her liquidity constraint because bad outcomes are associated with low (possibly negative) profits. Even if a few bad outcomes may not cause any trouble, a sufficiently long streak of bad outcomes, which always happens with a positive probability, will cause the controlling shareholder’s liquidity constraint to fail and the equilibrium to unravel. By contrast, under the turnover mechanism proposed here, the controlling shareholder will be receiving a payment for selling the controlling shares so she will not run into her liquidity constraint.

**Probabilistic Availability of New Owners and Costly Turnover** In our formal analysis, the market for corporate control is frictionless in the sense that there is a potential controlling shareholder available to take over the firm’s control every period. In a more realistic setting, following a bad outcome, there may not be any potential buyers immediately available. In this case, on-the-equilibrium-path punishment of the firm will continue to take place until ownership changes hands. This will give rise to a more natural empirical implication that when a firm’s reputation is tarnished, the firm’s profitability will decrease and stay low until a new controlling shareholder takes over.

8 Conclusion

This paper studies the corporate finance of firms whose sales and profit are sensitive to its reputation in the product market. It analyzes how the product market and the market for corporate control interact to provide the incentives for the firm’s key decision makers to put in effort into maintaining a good firm reputation. We find that turnover in the controlling share block can help a firm turnaround its reputation even if all controlling shareholders has the same ability and their departure from the firm is voluntary. The equilibrium property that the controlling shareholder must be punished for bad outcomes but the noncontrolling shareholders can be spared gives rise to an endogenous cost of corporate control. The theory’s prediction that the firm’s control premium can be negative, and that it is lower after the company’s reputation is damaged, are consistent with empirical evidence. Finally, we show that the firm’s founder resolves the tradeoff between managing moral hazard and preserving firm value by partially selling cash-flow rights as noncontrolling shares.

Appendix
Proof of Proposition 1. Plugging $P = p$ into the incentive constraint (2) gives us

$$W \leq \bar{W}(\delta, \theta) = \frac{B - e^H}{\theta} + (p - c^H) - \frac{(1 - \delta)p}{\delta}(\frac{e^H - e^L}{\theta} + c^H - c^L).$$

(8)

A necessary condition for the sustainability of high effort, i.e., $\bar{W}(\delta, \theta) \geq (B - e^L)/(\theta + (q - c^L))$, is given by

$$\frac{B - e^L}{\theta} + (q - c^L) \leq \frac{B - e^H}{\theta} + (p - c^H) - \frac{(1 - \delta)p}{\delta}(\frac{e^H - e^L}{\theta} + c^H - c^L).$$

(9)

As long as (9) is satisfied, there exists $W$ such that (8) is satisfied. One immediate result is that $B$ does not affect the sustainability of high effort. This is because the controlling shareholder receives $B$ regardless. This inequality can be rewritten as

$$\delta \geq \hat{\delta}(\theta) = \frac{e^H - e^L + \theta(c^H - c^L)}{q(e^H - e^L + \theta(c^H - c^L)) + \theta(p - q)^2}.$$

Note that $\hat{\delta}(\theta)$ is decreasing in $\theta$, $\hat{\delta}(0) = 1/q > 1$ and

$$\hat{\delta}(1) = \frac{e^H - e^L + c^H - c^L}{q(e^H - e^L + c^H - c^L) + (p - q)^2}.$$

If $\hat{\delta}(1) > 1$, then $\hat{\delta}(\theta) > 1$ for all $\theta$ and high effort is unsustainable for discount factors. Therefore, for the analysis to be nontrivial, it is necessary that $\hat{\delta}(1) \leq 1$, which is equivalent to (3). When (3) holds, there exists $\theta$ such that $\hat{\delta}(\theta) \leq 1$ if and only if

$$\theta \geq \theta \equiv \frac{(1 - q)(e^H - e^L)}{(p - q)^2 - (1 - q)(c^H - c^L)}.$$

For $\delta \geq \hat{\delta}(\theta)$, the maximum value of the controlling share can be obtained by plugging $W = \bar{W}(\delta, \theta)$ from (8) into (1). Since $W$ is enforced by a discounted price, setting $W = \bar{W}(\delta, \theta)$ means the firm gives a discount just large enough to support the incentive to engage in high-quality production. In other words, setting $W = \bar{W}(\delta, \theta)$ maximizes $U$ and $V$, which in turn maximizes $S$. ■

Proof of Proposition 2. (i) If $\delta < \hat{\delta}(\theta)$, then $\bar{W}(\delta, \theta) < (B - e^L)/(\theta + (q - c^H))$. Therefore, there does not exists $T \in [(B - e^L)/(\theta + (q - c^H)) , \bar{W}(\delta, \theta)]$. In other words, when high-quality
production is not sustainable in the absence of transfer of controlling shares, allowing transfer of these shares will not improve the performance of the firm because the possibility of ownership turnover cannot lower the controlling shareholder’s continuation payoff below $B - e^L + \theta (q - c^L)$. Therefore, if $\delta < \delta (\theta)$, only low effort can be supported in equilibrium even when turnover is allowed. Consequently, $\tilde{V} = \frac{B - e^L}{\theta} + \tilde{U}$ and $\tilde{U} = q - c^L$.

(ii) When $\delta = \tilde{\delta}(\theta)$, in order to support high effort, the continuation payoff to the controlling shareholder must be set at $\theta (q - c^L) + B - e^L$ following a bad outcome, for incentive provision. When controlling shares turnover is allowed, it requires a transaction price of $\theta (q - c^L) + B - e^L$ to sustain effort. When the incumbent has positive bargaining power, this is possible only if the surplus from the trade of controlling shares is zero, i.e. the newly arrived entrepreneur has to be punished as severely as the incumbent after he takeovers the firm. Thus, the firm’s profit cannot be increased by turnover. Consequently, $\tilde{U}$ and $\tilde{V}$ stay at $\tilde{U}^0(\theta)$ and $\tilde{V}^0(\theta)$ respectively.

(iii) – (iv) In order to construct the optimal equilibrium, we look for PPE that maximizes the total shareholder value $S = \theta V + (1 - \theta) U$.

An upper bound on the equilibrium value of $U$ in any PPE is $p - c^H$.

Equation (5), which determines the value of controlling share, $V$, in those equilibria with ownership turnover and high-quality production, can be rewritten as

$$V = \frac{1 - \delta}{1 - \delta p} \left[ \frac{B - e^H}{\theta} + (p - c^H) \right] + \frac{\delta (1 - p)}{1 - \delta p} T$$  \hspace{1cm} (10)

Thus, the equilibrium value of $V$ is increasing in the transaction price of controlling shares $T$, as long as $T$ is small enough to sustain incentive for high-quality production. More precisely, the following incentive constraint must be satisfied to motivate high-quality production

$$T \leq \tilde{W}(\delta, \theta) \equiv \frac{B - e^H}{\theta} + (p - c^H) - \frac{(1 - \delta p) \left( \frac{e^H - c^L}{\theta} + c^H - c^L \right)}{\delta (p - q)}$$  \hspace{1cm} (11)

Therefore, the maximum possible value of $V$ in any PPE with turnover of control block and high-quality production is given by equation (10) with $T = \tilde{W}(\delta, \theta)$. Consequently, an upper bound on the equilibrium value of $V$ in any PPE is $\tilde{V}^0(\theta)$.

Now, we show the upper bounds on $U$ and $V$ can be achieved if the discount factor is large enough. This then implies that when the discount factor is large enough, the optimal equilibrium yields $\tilde{U} = p - c^H$ and $\tilde{V} = \tilde{V}^0(\theta)$.

First, in order for the controlling share to achieve the value $\tilde{V}^0(\theta)$, we have to set $T = \tilde{W}(\delta, \theta)$. Next, recall $\tilde{W}$ is the value of controlling shares to the new controlling shareholder following an ownership turnover. Suppose high-quality production can be supported with $\tilde{W} = \tilde{V}^0(\theta)$. Then no price cut is needed in the equilibrium punishment phase and from equation (6), the noncontrolling shares achieve the value $\tilde{U} = p - c^H$. Substituting $\tilde{W} = \tilde{V}^0(\theta)$ and $T = \tilde{W}(\delta, \theta)$ into equation (4) and rearrange, we can express the value of controlling shares to the incumbent following an
off-equilibrium negotiation breakdown, $W$, as

$$W = \frac{\bar{W}(\delta, \theta) - \beta V^0(\theta)}{1 - \beta}$$

This off-equilibrium value of controlling share can be supported by requiring the incumbent to offer a price discount to customers for one period. The discount is given by

$$\bar{V}^0(\theta) - W = \bar{V}^0(\theta) - W(\delta, \theta)$$

The equilibrium described above is feasible if and only if

$$W \geq B - e^L \theta + (q - c^L)$$ (12)

This translates into the following condition:

$$\delta \geq \tilde{\delta}(\theta) \equiv \frac{e^H - e^L + \theta(e^H - c^L)}{(1 - q) \beta + q)}(e^H - e^L + \theta(e^H - c^L)) + (1 - \beta) \theta(p - q)^2.$$ (13)

In sum, if $\delta \in \tilde{\delta}(\theta), 1$, $\bar{U} = p - e^H$ and $\bar{V} = V^0(\theta)$ in the optimal equilibrium. The equilibrium takes the form described in the text, with $T = W(\delta, \theta)$. On the off-equilibrium path punishment phase in which the controlling shares are retained by the incumbent, a price cut $\frac{V^0(\theta) - W(\delta, \theta)}{(1 - \delta)(1 - \beta)}$ is offered to customers for one period. This concludes the proof for part (iv) of the proposition.

Next, suppose $\delta \in \tilde{\delta}(\theta), \tilde{\delta}(\theta)$. In the optimal equilibrium, $\bar{V} > \bar{W}$, and a price discount is offered in the on-the-equilibrium punishment phase. It is therefore immediate that $\bar{U} < p - e^H$.

We now proceed to construct the optimal equilibrium.

Equation (4) can be written as

$$\bar{W} = \frac{T - (1 - \beta)W}{\beta}$$ (14)

Using (14) and (10), the price cut, denoted by $D(\beta, \delta)$, can be written as

$$D(\beta, \delta) \equiv \frac{V - \bar{W}}{1 - \delta} = \frac{1}{1 - \delta p} \left[ \frac{B - e^H}{\theta} + (p - c^H) \right] - \frac{1}{1 - \delta} \left[ \frac{1 - \delta (1 - p)}{1 - \delta p} \right] T + \frac{1 - \beta}{\beta (1 - \delta) W}$$ (15)

Note that because $\delta \in \tilde{\delta}(\theta), \tilde{\delta}(\theta)$, the price cut $D(\beta, \delta)$ is positive for all $T$ and $W$ such that $T \in [0, \bar{W}(\delta, \theta)]$ (by (11)) and $W \geq (B - e^L) / \theta + (q - c^L)$ (by (12)). The price cut is therefore increasing in $W$ and decreasing in $T$ (since $1/\beta > 1 > \delta (1 - p)/1 - \delta p$). Therefore, to get the minimum equilibrium price cut, we should set $W = (B - e^L) / \theta + (q - c^L)$ and $T = \bar{W}(\delta, \theta)$. Note that by setting $T = \bar{W}(\delta, \theta)$, we also achieve the upper bound on the equilibrium value of controlling shares $V = V^0(\theta)$. Thus, $\bar{V} = V^0(\theta)$. 26
By setting \( W = (B - e^L) / \theta + (q - c^L) \) and \( T = \tilde{W} (\delta, \theta) \), the value of noncontrolling shares in the optimal equilibrium is

\[
\tilde{U} = (p - c^H) - \delta \frac{1 - p}{1 - \delta} \left\{ \frac{e^H - e^L}{p - q} + \frac{c^H - c^L}{p - q} \left[ \beta^{-1} (\delta^{-1} - q) - (1 - q) \right] - (p - q) (\beta^{-1} - 1) \right\}
\] (16)

In sum, if \( \delta \in [\hat{\delta} (\theta), \hat{\delta} (\beta, \theta)] \), \( \tilde{U} \) is given by (16) and \( \tilde{V} = \tilde{V}_0 (\theta) \) in the optimal equilibrium. The equilibrium takes the form described in the text, with \( T = \tilde{W} (\delta, \theta) \). On the off-equilibrium path in which the controlling shares are retained by the incumbent, a price cut of \( \frac{V_0 (\theta) - \left[ (B - e^L) / \theta + (q - c^L) \right]}{1 - \delta (1 - \beta)} \) is offered to customers for one period.

Finally, because \( \delta \in (\hat{\delta} (\theta), \hat{\delta} (\beta, \theta)) \), it can readily verified that

\[
\tilde{U} \in \left( p - c^H \right) - (1 - p) \left\{ \left( \frac{e^H - e^L}{p - q} + \frac{c^H - c^L}{p - q} \right) \right\}, p - c^H = (\tilde{U}_0 (\theta), p - c^H).
\]

\[\blacksquare\]

**Proof of Corollary 1.** Fix a \( \theta \geq \theta \). If \( \delta \leq \hat{\delta} (\theta) \), then \( \beta \) has no effect on \( \tilde{W} \) and hence no effect on the acquisition premium and the post-acquisition accounting profit. For the rest of the proof, we consider the case \( \delta > \hat{\delta} (\theta) \).

Recall that \( \hat{\delta} (0, \theta) = \tilde{\delta} (\theta) \) and that \( \hat{\delta} (\beta, \theta) \) is strictly increasing in \( \beta \), we can thus define its inverse: let \( \hat{\beta} (\delta, \theta) \) be the solution to \( \delta = \hat{\delta} (\beta, \theta) \). When \( \beta \leq \hat{\beta} (\delta, \theta) \), we have \( \delta \geq \hat{\delta} (\beta, \theta) \).

Therefore, according to the proof of Proposition 2, \( \tilde{W} = \tilde{V}_0 (\theta) \) and \( T = \tilde{W} (\delta, \theta) \). Both the acquisition premium, \( T - \tilde{U} \), and post-acquisition profit, \( p - c^H \), are locally invariant to \( \beta \).

When \( \beta > \hat{\beta} (\delta, \theta) \), we have \( \delta < \hat{\delta} (\beta, \theta) \). According to the proof of Proposition 2, \( T \) remains at \( \tilde{W} (\delta, \theta) \) and

\[
\tilde{W} = \frac{\tilde{W} (\delta, \theta) - (1 - \beta) \left( \frac{B - e^L}{\theta} + (q - c^L) \right)}{\beta}
\]

Therefore, \( \tilde{W} \) is equal to \( \tilde{V}_0 (\theta) \) when \( \beta \leq \hat{\beta} (\delta, \theta) \) and is strictly decreasing in \( \beta \) for \( \beta \in (\hat{\beta} (\delta, \theta), 1) \). The result then follows because the acquisition premium varies inversely with \( \tilde{W} \) while the post-acquisition accounting profit varies positively with \( \tilde{W} \). \[\blacksquare\]

**Proof of Corollary 2.** Based on Proposition 2, if \( \delta \geq \hat{\delta} (\beta, \theta) \), then \( \tilde{V} = \tilde{V}_0 (\theta) \), \( \tilde{U} = \tilde{U}_0 = p - c^H \), and \( T = \tilde{W} (\delta, \theta) \). The expressions for \( \Delta^H \) and \( \Delta^L \) then follow by direct substitution. It follows immediately that \( \Delta^H > \Delta^L \). Also, \( \Delta^H < 0 \) if \( \frac{B - e^H}{\theta} \) is sufficiently small.

Next, if \( \delta \in (\hat{\delta} (\theta), \hat{\delta} (\beta, \theta)) \), then \( \tilde{V} \) and \( T \) remain at \( \tilde{V}_0 (\theta) \) and \( \tilde{W} (\delta, \theta) \), respectively. The expressions for \( \tilde{U} \) and \( \tilde{U}_0 \) are given by (7). According to the proof of Proposition 1,

\[
\hat{W} = \frac{\tilde{W} (\delta, \theta) - (1 - \beta) \left( \frac{B - e^L}{\theta} + (q - c^L) \right)}{\beta}.
\]
Direct substitution gives the expressions for $\Delta^H$ and $\Delta^L$. Note that the term in the brackets is negative if and only if

$$\delta > \delta_1(\theta) = \frac{c^H - c^L + \frac{e^H - e^L}{\theta}}{(p - q)^2 + q \left( c^H - c^L + \frac{e^H - e^L}{\theta} \right)}.$$

It can be readily verified that $\delta_1(\theta) < \tilde{\delta}(\beta, \theta)$ under (3). Thus, if $\delta \in \left( \delta_1(\theta), \tilde{\delta}(\beta, \theta) \right)$ and $\frac{B - e^H}{\theta}$ is sufficiently small, then $\Delta^H$ can be negative. Finally, note that

$$\Delta^H - \Delta^L = (V^0(\theta) - \bar{U}) - (W(\delta, \theta) - U) = \bar{W} - W(\delta, \theta).$$

Because $\bar{W} - W(\delta, \theta) > 0$, we obtain that $\Delta^H > \Delta^L$ as in the former case. Straightforward calculation yields the following expressions for $\Delta^H$:

$$\Delta^H = \begin{cases} \frac{B - e^H}{\theta} - \frac{1}{(p - q)} \left( \frac{e^H - e^L}{\theta} + c^H - c^L \right) & \text{if } \delta \geq \tilde{\delta}(\beta, \theta) \\ \left( \frac{B - e^H}{\theta} - \frac{1}{(p - q)} \left( \delta(p - q) - \frac{1}{p - q} \left( \frac{e^H - e^L}{\theta} + c^H - c^L \right) \right) \right) & \text{if } \delta \in (\tilde{\delta}(\theta), \tilde{\delta}(\beta, \theta)) \end{cases}$$

It is immediate to see that in both cases, $\Delta^H < 0$ if $B$ is sufficiently small. ■

**Proof of Proposition 3.** (i) First, it is immediately apparent that when $\delta = \tilde{\delta}(1)$, $\theta^*$ is unique and equal to one as any lower $\theta$ cannot support high-quality production. Similarly, for any $\delta \in \left( \tilde{\delta}(1), 1 \right)$, it is suboptimal to set $\theta$ below $\bar{\theta}(\delta)$.

Next, when $\theta = \bar{\theta}(\delta)$ or equivalently, $\delta = \tilde{\delta}(\theta)$, we have that $\bar{U} = U^0(\bar{\theta}(\delta))$ and $\bar{V} = V^0(\bar{\theta}(\delta))$, according to Proposition 1. Therefore, $\bar{U} - \bar{V} = -\frac{B - e^H}{\theta} < 0$ and non-controlling shares is less valuable than controlling shares. Since the value of each kind of shares is strictly increasing in $\theta$ when $\theta \in \left[ \bar{\theta}(\delta), \min \left\{ \bar{\theta}(\delta, \beta), 1 \right\} \right]$ and $\delta \in \left( \tilde{\delta}(1), 1 \right)$, the optimal ownership structure $\theta^*$ strictly exceeds $\bar{\theta}(\delta)$.

Moreover, if $\tilde{\theta}(\delta, \beta) < 1$ and $\theta \geq \tilde{\theta}(\delta, \beta)$, then according to Proposition 1, $U(\theta) = p - e^H$ and $V(\theta) = V^0(\theta)$. This gives

$$S(\theta) = B - e^H + (p - c^H) - \frac{1}{p - q} [e^H - e^L + \theta (c^H - c^L)].$$

Therefore, $S$ is strictly decreasing in $\theta$ for $\theta \geq \tilde{\theta}(\delta, \beta)$ and it is suboptimal to set $\theta$ above $\bar{\theta}(\delta, \beta)$. We have thus established that $\theta^*(\delta) \in \left[ \bar{\theta}(\delta), \min \left\{ \bar{\theta}(\delta, \beta), 1 \right\} \right]$.

To see why $\theta^*(\delta)$ is unique when when $\delta \in \left( \tilde{\delta}(1), 1 \right)$, recall by definitions, $\theta \in \left[ \bar{\theta}(\delta), \min \left\{ \bar{\theta}(\delta, \beta), 1 \right\} \right]$ if and only if $\delta \in (\tilde{\delta}(\theta), \tilde{\delta}(\beta, \theta)]$. Using part (iii) of Proposition 1, the total shareholder value
\[ S(\theta) = \theta \bar{V}(\theta) + (1 - \theta) \bar{U}(\theta) \] for this range of \( \theta \) is given by

\[
S(\theta) = \theta \left[ \frac{B - e^H}{\theta} + (p - e^H) - \frac{1 - p}{p - q} \left( \frac{e^H - e^L}{\theta} + c^H - c^L \right) \right] + (1 - \theta) \left[ (p - c^H) - \delta \left( \frac{1 - p}{1 - \delta} \right) \left( \frac{e^H - e^L}{p - q} \right) \right].
\]

Direct computation gives the second derivative:

\[
S''(\theta) = -2(1 - \delta \beta)^{-3} \left( \frac{1 - p}{1 - \delta} \frac{e^H - e^L}{p - q} \right) (\beta^{-1} (\delta^{-1} - q) - (1 - q)) < 0.
\]

Since \( S \) is strictly concave in \( \theta \) in the interval \( (\hat{\theta}(\delta), \min \{ \tilde{\theta}(\delta, \beta), 1 \} \) \), \( \theta^* \) is unique when \( \delta \in \left( \hat{\delta}(1), 1 \right) \).

Finally, it is easy to verify that both \( \theta(\delta) \rightarrow \bar{\theta} \) and \( \tilde{\theta}(\delta, \beta) \rightarrow \hat{\theta} \) as \( \delta \rightarrow 1 \). Therefore, \( \theta^* \) converges to \( \bar{\theta} \).

(ii) By the strict concavity of \( S(\theta) \) in the interval \( (\hat{\theta}, \min \{ \tilde{\theta}, 1 \} \), \( \theta^* \) is characterized by the first order condition \( S'(\theta^*) = 0 \), which can be simplified into

\[
\hat{\theta} = \frac{(e^H - e^L) (1 - \delta q - \beta \delta (1 - q))}{(1 - \beta) (p - q)^2 - (1 - \delta (q + \beta (1 - q)) (e^H - e^L))}.
\]

From the proof of part (i), we know that the optimal ownership structure \( \theta^* \) is given by

\[
\theta^* = \min \left\{ 1, \bar{\theta}, \hat{\theta} \right\},
\]

where

\[
\bar{\theta} \equiv \frac{(e^H - e^L) ((1 - q) \delta) - \beta \delta (1 - q))}{\delta (1 - \beta) (p - q)^2 - (1 - \delta (q + \beta (1 - q)) (e^H - e^L))}.
\]

Note here, \( \bar{\theta} \) is the inverse of \( \hat{\theta} \). Furthermore, if \( \delta \in \left( \hat{\delta}(\beta, 1), 1 \right) \), then \( \theta^* < 1 \).

It is obvious that both \( \theta \) and \( \tilde{\theta} \) are strictly increasing in \( e^H - e^L \) and \( c^H - c^L \). Thus, \( \theta^* \) is weakly increasing in \( e^H - e^L \) and \( c^H - c^L \) if \( \delta \in (0, 1) \). It is strictly decreasing in \( e^H - e^L \) and \( c^H - c^L \) if \( \delta \in \left( \hat{\delta}(\beta, 1), 1 \right) \).

Direct computation shows that \( \frac{\partial \bar{\theta}}{\partial \delta}, \frac{\partial \hat{\theta}}{\partial \delta} < 0 \). Thus, \( \theta^* \) is weakly decreasing in \( \delta \) for \( \delta \in (0, 1) \),
and strictly decreasing in $\delta$ if $\delta \in \left( \bar{\delta} (\beta, 1), 1 \right)$.

$$
\frac{\partial \bar{\theta}}{\partial \delta} = \frac{- (c^H - c^L) (1 - \beta) (p - q)^2}{\left( \delta (1 - \beta) (p - q)^2 - (1 - \delta (q + \beta (1 - q))) (c^H - c^L) \right)^2} < 0,
$$

$$
\frac{\partial \bar{\theta}}{\partial \beta} = \frac{1}{2 \bar{\theta}^*} \frac{c^H - c^L - (p - q)^2}{1 - \beta} \frac{\delta (p - q)^2 - (e^H - c^L)(1 - q - \delta)}{2 \bar{\theta}^*} < 0.
$$

Direct computation shows that $\frac{\partial \bar{\theta}}{\partial \beta}, \frac{\partial \bar{\theta}}{\partial \delta} > 0$. Thus, $\bar{\theta}^*$ is weakly increasing in $\beta$ for $\delta \in (0, 1)$, and strictly increasing in $\beta$ if $\delta \in \left( \bar{\delta} (\beta, 1), 1 \right)$.

$$
\frac{\partial \bar{\theta}}{\partial \beta} = \frac{\delta (1 - \delta) (c^H - c^L)(p - q)^2}{\left( \delta (1 - \beta) (p - q)^2 - (1 - \delta (q + \beta (1 - q))) (c^H - c^L) \right)^2} > 0,
$$

$$
\frac{\partial \bar{\theta}}{\partial \delta} = \frac{1}{2 \bar{\theta}^*} \frac{c^H - c^L}{\delta (p - q)^2 - (c^H - c^L)(1 - \delta q)(1 - \beta)^2} > 0.
$$

**Proof of Corollary 3:** Using the definitions in the proof of Proposition 3, the optimal ownership structure is strictly interior if and only if $\bar{\theta} < 1$ and $\bar{\theta} < \bar{\theta}$. Upon straightforward algebra manipulation, the inequalities can be written as

$$
c^H - c^L < \frac{1}{(1 - \delta q)} \left[ \delta (p - q)^2 - (c^H - c^L) \left( \frac{1 - \delta q - \beta \delta (1 - q)}{1 - \beta} \right) \right] \equiv \bar{C}, \quad \text{and}
$$

$$
0 > \delta (1 - \beta) (p - q)^2 \left[ \delta (1 - \beta) (p - q)^2 - (c^H - c^L) \left( (1 - q\delta) - \beta \delta (1 - q) \right) \right]
$$

$$
+ (1 - \beta) (1 - q\delta - \beta \delta (1 - q)) \left[ (1 - q\delta) (c^H - c^L) - 2 \delta (p - q)^2 \right](c^H - c^L)
$$

$$
+ (1 - q\delta - \beta \delta (1 - q))^2 \left( c^H - c^L \right)^2.
$$

(17)

The condition $c^H - c^L < \frac{\delta (1 - \beta) (p - q)^2}{1 - \delta q - \beta \delta (1 - q)}$ ensures that $\bar{C}$ is positive. It also ensures that inequality (17) holds at $c^H - c^L = \bar{C}$. Combined with the observation that inequality (17) does not hold at $c^H - c^L = 0$, the system of inequalities above hold for some $c^H - c^L \in (\bar{C}, \bar{C})$, where $\bar{C} > 0$. Finally, direct computation shows that the efficiency requirement (3) holds for all $c^H - c^L < \bar{C}$. ■

**References**


