Information Revelation in Relational Contracts*

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Abstract

We explore subjective performance reviews in long-term employment relationships. We show that firms benefit from separating the task of evaluating the worker from the task of paying him. The separation allows the reviewer to better manage the review process, and can therefore reward the worker for his good performance with not only a good review contemporaneously, but also a promise of better review in the future. Such reviews spread the reward for the worker's good performance across time and lower the firm's maximal temptation to renege on the reward. The manner in which information is managed exhibits patterns consistent with a number of well-documented biases in performance reviews.

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1 Introduction

Performance reviews are pervasive in modern labor markets.[1] While these reviews help collect information about worker performance, they are typically subjective, and consequently, contain inaccuracies and biases. A small but growing literature has studied the difficulties of using subjective evaluations, and shown that they constrain the efficiency of relationships.[2] A feature of this literature is that the organizational structure is taken as given: the entity that carries out the review—the principal—also incurs the cost of compensation. In practice, however, "in most organizations agency relationships are multi-layered" (Prendergast and Topel (1993)) and performance reviews are typically carried out by supervisors, and the compensation decisions are instead made by the top of the organizations, using the reviews as an input.

Motivated by this observation, we study the organizational response to subjective performance reviews in long-term employment relationships. We show that the firm benefits from separating the task of evaluating the worker from the task of paying him. The separation allows the reviewer to better manage the information flow and increase the efficiency of the organization. Moreover, by managing information strategically, the reviewer exhibits review patterns that are consistent with a number of well-documented biases in performance reviews.

In particular, we follow the literature on subjective performance evaluations by modeling long-term employment relationships as relational contracts, where firms motivate workers using discretionary bonuses; see Malcomson (2013) for a review of relational contracting models. To sustain a relational contract, the key condition is that the firm's maximal reneging temptation, the maximal bonus it needs to pay, cannot exceed the future surplus of the relationship.

Our central result is that by managing the review process, the firm can more effectively motivate the worker by easing the tension between the need to motivate the worker by offering a bonus and the temptation to renege on it once the performance is delivered. Specifically, the entity that pays the bonus—the owners of the firm—should be different from the entity that provides performance review—the supervisors or the

[1]According to a 2013 survey conducted by the Society for Human Resource Management, 94% of organizations conduct performance appraisal. (Source: http://blog.blr.com/2013/07/2013-performance-management-survey-infographic.html)

[2]See, for example, Levin (2003), MacLeod (2003), Fuchs (2007), Chan and Zheng (2011), and Maestri (2012).

human resource management departments. Such separation allows the supervisor to strategically manage information available to the owner of the firm.

The review process we consider has the following features. As one might expect, the supervisor sends a good review in every period if the worker has performed well. Crucially, however, the supervisor may be lenient and also send a good review even if the worker has not performed well. We refer to the probability of the supervisor doing so as his leniency level. The supervisor's leniency level changes over time and depends on the past history. In particular, if the worker performed well last period, the supervisor becomes more lenient. If the worker did not perform well last period, yet the review was good, the supervisor becomes less lenient.

One property of our review process is that the supervisor spreads the reward for the worker's good performance across periods. Specifically, if a worker performs well, the supervisor rewards him in two ways. He rewards the worker with a good review, and therefore, with a bonus payment in the current period. He also rewards the worker by being more lenient in the future, leading to a higher payoff for the worker in the future. By spreading the reward across periods, the supervisor keeps the worker motivated, while reducing the maximal bonus the firm must pay in any given period.

Reducing the maximal bonus alone, however, does not necessarily reduce the firm's reneging temptation because the firm's future payoff may also be reduced. Importantly, under our review process, the firm's future payoff is independent of the supervisor's review today. Notice that the firm's future payoff is lower when the worker has performed well since in this case, the supervisor will be more lenient and more likely to give a good review in the future. But the supervisor may also give a good review when the worker has not performed well. In this case, the supervisor will be less lenient in the future, giving the firm a higher future payoff. By averaging these two cases, the supervisor's review does not change the firm's future payoff even though the performance of the worker does. By maintaining the firm's future payoff constant while reducing the maximal bonus, our review process relaxes the firm's non-reneging constraint.

An implication of our result is that when relational contracts are used, the celebrated Informativeness Principle (Holmstrom 1979) fails. In particular, it is crucial that the firm does not observe the worker's performance. Otherwise, following a good performance of the worker, the firm anticipates that more bonus will be paid in the future, and would therefore renege on the bonus. Similarly, if the worker receives a good review yet knows that his performance is bad, he anticipates that less bonus will be paid in the future, and may prefer to exit the relationship.

Several literatures have documented a variety of biases in performance evaluations.[3] One of the most common form of biases is leniency bias, in which good reviews can be given for poor performance (e.g., Holzbach, 1978). Another frequently documented bias is the spillover effect, in which the worker's current period performance is evaluated in part based on his past performances (e.g., Bol and Smith, 2011). The existing literature has assumed that these biases are detrimental (see Rynes et al. 2005 for a review). Our analysis suggests, however, these "biases" may reflect strategic HR management practice that enhances efficiency.

Recall that a key feature of our review process is that the supervisor promises good reviews in the future for past good performance, giving rise directly to the spillover effect. This feature also implies that the frequency of good reviews is higher than that of good performance, leading to leniency bias. Viewed in isolation, these biases weaken the worker's incentives and hurt the relationship. But viewed in a longer horizon, these "biases" may be efficiency-enhancing. Incidentally, in the only empirical analysis of effects of performance appraisal biases that we are aware of, Bol (2011) found that leniency bias has a positive effect on the employee's future performance.

Our paper contributes to two strands of the literature. First, it contributes to the theoretical works that explore the relationship between the information structure and efficiency. Within this literature, Kandori (1992) shows that garbling signals within periods weakly decreases efficiency in repeated games with imperfect public monitoring. Abreu, Milgrom, and Pearce (1992) and Fuchs (2007) have shown that reducing the frequency of information release can benefit the relationship. Kandori and Obara (2006) show that when signals do not have full support, the use of private (mixed) strategies can give rise to equilibria that are more efficient. In our model, the principal's action is publicly observed, so the use of mixed strategy does not help relax the incentive constraints of the players by better detecting deviations.

[3] See, for example, Murphy et al. (1985) and Nisbett et al (1977) in psychology, Blanchard et al. (1986) and Bol and Smith (2011) in accounting, Bretz et al. (1992) and Jacobs et al. (1985) in management, and Prendergast (1999) and Prendergast and Topel (1993) in economics.

More relatedly, Fuchs (2007) has shown that the principal reduces the amount of surplus destroyed in a relationship by withholding her private information from the agent. In our model, surplus needs not to be destroyed to motivate the agent because output is publicly observed. In addition, the supervisor withholds her information from both the principal and the agent. We show that by revealing his information properly, the supervisor can lower the discount factor for sustaining an efficient relational contract.

Second, this paper contributes to the literature that studies how to use external instruments to increase the efficiency of relational contracts. Baker, Gibbons, and Murphy (1994) show that explicit contracts can enhance the efficiency of relationships by reducing the gain from reneging, but can also crowd out relational contracts by improving players' outside options. On the role of ownership structure, Rayo (2007) shows that when the actions of players are unobservable (and the First Order Approach is valid), the optimal ownership shares should be concentrated; otherwise, the optimal ownership shares should be diffused. The external instrument explored in our paper is revelation of information. We show that the efficiency of the relationship can be enhanced by reducing information revealed through intertemporal garbling of signals. This points to a benefit of using intermediaries to manipulate information. Finally, Deb, Li, and Mukherjee (forthcoming) show that peer evaluations can improve efficiency in relational contracts, but should be used sparingly. In contrast to our paper, peer evaluations do not have spillover effects on future compensations.

The rest of the paper is organized as follows: We provide a parametric example in Section 2 to illustrate the main idea of our paper. The model is set up in Section 3 and we present our main results in Section 4. Section 5 discusses the robustness of our results and examines properties of general reporting rules. Section 6 concludes.

2 A Parametric Example

We begin with the following example. There is one principal and one agent. Both are risk-neutral, infinitely lived, and share a discount factor δ. The outside options of both parties are 0. In each period, the agent privately chooses to work or shirk. If he works, output is y (high) with probability p and is 0 (low) with probability $1-p$. If he shirks, output is always 0. The cost of working is c, and we assume that $py-c>0$, so it is socially efficient for the agent to work.

Suppose first output is publicly observable but non-contractible, and the principal motivates the agent through relational contracts. Then this setup is a special case of Levin (2003), who shows that the efficient relational contract is stationary. The principal offers a base wage w in each period and a discretionary bonus b if output is y. To sustain an efficient relational contract, the agent must be willing to work and the principal must not renege on the bonus. To motivate the agent to work, his expected gain from working must be no lower than the cost of effort, i.e., $pb \geq c$. For the principal not to renege on the bonus, her future loss from reneging must be no lower than the bonus amount. Without loss of generality, we may assume that the principal and the agent set $w = c - pb$ so that the principal captures all surplus of the relationship, and take their outside options forever if the principal reneges. This implies that the principal will not renege if the bonus amount is smaller than the future surplus of the relationship, i.e., $b \leq \delta\,(py - c)\,/\,(1 - \delta)$.

In summary, an efficient relational contract is sustainable if and only if

$$\frac{\delta}{1-\delta}\,(py - c) \geq b \geq \frac{c}{p}.$$

It follows that there exists a cutoff discount factor δ^* such that an efficient relational contract is sustainable if and only if $\delta \geq \delta^*$. For illustrative purposes, let $c = 1$, $y = 26$, and $p = 1/13$. Then the minimal bonus to motivate the agent is 13, and the cutoff discount factor δ^* solves $\delta^*\,(py - c)\,/\,(1 - \delta^*) = 13$ and is equal to 13/14.

Now suppose that rather than being publicly observed, outputs are observed only by a disinterested supervisor. In each period, the supervisor sends a public report. If the supervisor reports truthfully—his report perfectly reveals output in each period—the cutoff discount factor for sustaining an efficient relational contract is again 13/14. We show below that the supervisor can lower the cutoff discount factor if he does not report truthfully.

Before describing our reporting rule, we note that the commonly studied T-period reviews—the supervisor reveals outputs every T periods and the principal pays a discretionary bonus B_T at the end of each *reporting cycle* if output is y in at least one of the T periods—cannot lower the cutoff discount factor. This is because to motivate the agent to work in the last of every T periods, B_T must be at least 13. Because of discounting, B_T must be even greater to motivate the agent to work in earlier periods. Yet for $\delta < \delta^* = 13/14$, the maximal bonus the principal is willing to pay, or her future payoff from not reneging, is less than 13. This implies that for all $\delta < \delta^*$, the agent cannot be motivated to work. In general, this argument implies that any reporting rule with a definitive ending date for each reporting cycle cannot lower the discount factor for sustaining an efficient relational contract.

Now consider the following reporting rule and the associated relational contract. The supervisor sends either G or B in each period, and the principal pays the agent a fixed wage w and, if G is reported, a bonus b. In each period, the supervisor sends G if output is y. If output is 0, however, the supervisor does not always send B. Instead, he may be "lenient" and report G. The probability of doing so depends on output and the report in the previous period. (1) If output was y, he always reports G. (2) If output was 0 yet previous report was G, he always reports B. (3) If output was 0 and previous report was B, or if this is the first period of the game, he reports G with probability 1/4.

An implication of (3) is that whenever the supervisor reports B, the reporting cycle restarts in the next period. Unlike T-period reviews, the restarting date of each reporting cycle is stochastic. When the reporting cycle restarts, the supervisor is at his baseline leniency level—he forgives a low output with probability 1/4. Afterwards, the supervisor forgives if output last period was y and does not forgive if it was 0. Since neither the principal nor the agent observes output, they also do not know the exact leniency level of the supervisor. Using Bayes rule, one can show that both the principal and the agent believe that the supervisor is lenient with probability 1/4 in each period in equilibrium.

To see how the reporting rule helps sustain an efficient relational contract, let $b^* = 13$ and $w^* = c - (4/13)\, b^*$. Suppose for now that the principal is willing to pay this bonus, and consider the agent's incentive to work. By exerting effort, the agent increases the probability that output is y, which leads to a bonus. But a high output also gives the agent a higher continuation payoff since the supervisor will send G in the next period regardless. Let the agent's continuation payoff be V_y if output is y, and let it be V_0^G (V_0^B) if output is 0 and the report is G (B). Recall that when output is 0, the supervisor sends a G report with probability 1/4. This implies that the gain from output being y rather than 0 is

$$b^* + \delta V_y - \left(\frac{1}{4} \left(b^* + \delta V_0^G \right) + \frac{3}{4} \delta V_0^B \right) = \frac{3}{4} b^* + \delta \left(V_y - V_0^B + \frac{1}{4} \left(V_0^B - V_0^G \right) \right). \quad (1)$$

Using the reporting rule, routine calculation shows that $V_0^B - V_0^G = 3b^* / (3\delta + 13)$ and $V_y - V_0^B = 9b^* / (3\delta + 13)$.[4] At $\delta^* = 13/14$, according to (1), the gain from having a high output is $(45/34)\, b^* > b^*$. For a fixed bonus amount, our reporting rule therefore provides a stronger incentive for the agent to exert effort than truthful reporting. This is precisely because the gain from a high output spills over to future periods.

Now we show that the principal will not renege on the bonus. This follows because the choice of w^* ensures that the agent's expected payoff in equilibrium is always 0, and therefore the principal captures the entire expected future surplus of the relationship. At $\delta^* = 13/14$, the principal's expected future payoff is $13 \geq b^*$, so she will not renege. Notice, however, it is crucial that the principal does not observe output. If she knew output is y, she would know that the supervisor would always send G next period and that her future payoff is less than 13, leading her to renege. By keeping the principal uncertain about output, our reporting rule ensures that the principal's future payoffs following a G- and B-report are the same.

In summary, the supervisor can help sustain the relationship by revealing less information. The reporting rule spreads the gain from a high output across periods, so that the same bonus amount—since they are paid out more frequently—provides stronger incentive for the agent than truthful reporting. In addition, it keeps the principal uncertain about outputs so she will not renege. These two features combined imply that an efficient relational contract is sustainable for a smaller discount factor.

Formalizing and generalizing this intuition is involved, however, because in contrast to truthful reporting, the agent might benefit from multistage deviations under our reporting rule. Recall that the agent is uninformed about output if he works. But if he shirks and a good report is sent out, he knows that output is 0. This information

[4] To see this, note that $V_0^B - V_0^G = \frac{12}{13}\frac{1}{4}\left(b^* + \delta\left(V_0^G - V_0^B\right)\right)$ since V_0^B and V_0^G differ only when both (i) output is zero and (ii) the agent is forgiven under V_0^G, which happens with probability $\frac{12}{13}\frac{1}{4}$. This equation gives that $V_0^B - V_0^G = 3b^* / (3\delta + 13)$.

Next, note that $V_y - V_0^B = \frac{12}{13}\frac{3}{4}\left(b^* + \delta\left(V_0^G - V_0^B\right)\right)$ since V_y and V_0^L differ only when both (i) output is zero and (ii) the agent is not forgiven under V_0^B, which happens with probability $\frac{12}{13}\frac{3}{4}$. This implies that $V_y - V_0^B = 9b^* / (3\delta + 13)$.

may induce him to shirk again or even exit the relationship. The possibility of multistage deviations implies that it is no longer straightforward to calculate the agent's deviation payoff. We tackle this issue in the general analysis. For interested readers, we provide the details of the calculation for this example in an online appendix.

3 Setup

Time is discrete and indexed by $t \in \{1, 2, ..., \infty\}$.

3.1 Players and Production

There is a principal, an agent, and a supervisor. All players are risk-neutral, infinitely lived, and share a discount factor δ. The agent's and the principal's respective per-period outside options are $\underline{u}$ and $\underline{\pi}$. To focus on the effect of information revelation, we assume that the supervisor is a nonstrategic player whose payoff is normalized to 0 whether he stays in or exits the relationship.

If the principal and the agent engage in production together in period t, the agent chooses effort $e_t \in \{0, 1\}$. If the agent works, his effort cost is $c(1) = c$. If he shirks, the effort cost is $c(0) = 0$. The agent's effort choice generates a stochastic output $y_t \in \{0, y\}$ for the principal. The output is more likely to be high if the agent works:

$$\Pr\{y_t = y|e_t = 1\} = p_0 > \Pr\{y_t = y|e_t = 0\} = q_0.$$

Let $\gamma(1) = p_0 y$ be the expected output if the agent works and $\gamma(0) = q_0 y$.

The production function is commonly used in the literature. We can extend the model to allow for multiple outputs with MLRP. In this case, there is a cutoff such that the bonus is paid either output is above the cutoff or when the supervisor sends a good report (to be described below). This cutoff divides outputs into two groups so that the production function is essentially binary. The binary-effort assumption, however, is more restrictive and is made for analytical convenience. In Subsection 5.1, we show that the main result of the paper continues to hold with three effort levels when effort costs are sufficiently convex, and we discuss how the model can be generalized. Define $s(1) \equiv \gamma(1) - c - \underline{\pi} - \underline{u}$ as the per-period joint surplus when the agent works, and similarly, define $s(0) \equiv \gamma(0) - \underline{\pi} - \underline{u}$. We assume that the relationship has a positive surplus if and only if the agent works: $s(1) > 0 > s(0)$.

3.2 Timing and Information Structure

At the beginning of period t, the principal offers to the agent a history-dependent compensation package consisting of a base wage w_t and a nonnegative *end-of-period bonus* b_t. The agent chooses whether to accept the offer: $d_t \in \{0, 1\}$. If the agent rejects it ($d_t = 0$), all players take their outside options for the period. If the agent accepts, he receives w_t and chooses a privately observed e_t. The supervisor then obtains a private signal $y_t^s \in \{L, H\}$, where the superscript s indicates that the signal can be subjective. The signal is independent of output y_t conditional on effort, and is more likely to be high if the agent works:

$$\Pr\{y_t^s = H | e_t = 1\} = p > \Pr\{y_t^s = H | e_t = 0\} = q.$$

The supervisor sends a public report $s_t \in \mathbf{S}$, $\mathbf{S}$ being the set of possible reports, once he receives the signal. Following the report, output y_t is realized and publicly observed. Denote $\phi_t = (s_t, y_t)$ as the publicly observable performance outcomes in period t and Φ_t as the set of ϕ_t. After observing ϕ_t, the principal decides whether to pay b_t. Denote $W_t = w_t + b_t$ as the agent's total compensation for the period.

Before describing the strategy and equilibrium concepts, we comment on the information structure and the form of compensation. One key part of the information structure is the supervisor's reports. Since the supervisor's signals are private, he does not need to report them truthfully. He can delay reporting his signals and can randomize his reports. Another part of the information structure is the publicly observed outputs. While outputs determine the principal's payoff, their informational roles are not essential. The main message of the paper is unchanged, for example, when the principal realizes his benefits with sufficient delay, so that essentially his only source of information is the supervisor's reports. For the compensation form, the end-of-period bonus b_t is the difference between the base wage w_t and total compensation W_t. We include b_t in our description to help the exposition, but since w_t and W_t completely determine the player's payoffs, we omit b_t in our description of the players' strategy below.

3.3 Strategies and Equilibrium Concept

Since the supervisor is nonstrategic, we only describe the strategies of the principal and the agent. Denote $h_t = \{w_t, d_t, \phi_t, W_t\}$ as the public events that occur in period

t, and $h^t = \{h_n\}_{n=0}^{t-1}$ as the public history at the beginning of period t, where $h^1 = \emptyset$. Let $H^t = \{h^t\}$. The principal observes only the public history. The agent observes his past actions $e^t = \{e_n\}_{n=1}^{t-1}$ in addition to the public history. Denote $H_A^t = H^t \cup \{e^t\}$ as the set of the agent's private history (h_A^t) at the beginning of period t.

Denote s^P as the principal's strategy, which specifies the wage w_t and the total compensation W_t for each period t. Notice that w_t and W_t both depend on the the available public history. Denote s^A as the agent's strategy, which specifies his acceptance decision d_t and his effort decision e_t for each period t. The agent's decisions depend on both the public history and his private past efforts. Next, denote the principal's belief as μ^P, which assigns to every information set of the principal, i.e., every element in the public history, a probability measure on the set of histories in the information set. Define the agent's belief μ^A analogously.

Note that the principal or the agent do not play mixed strategies in our model. As will be clear from our analysis below, an efficient relational contract can be sustained only if the maximal bonus the principal pays is no larger than the expected discounted surplus of the relationship. When the principal randomizes, she makes bonus payments more volatile and (weakly) increases the maximal bonus. When the agent randomizes, he lowers the expected discounted surplus of the relationship. Mixed strategies therefore do not help sustain an efficient relational contract.

A similar reasoning implies that adding a public randomization device does not help sustain an efficient relational contract. Since public randomization adds fluctuation to the total expected bonus to the agent, the conditional total expected bonus following some realization of public randomization must be (weakly) higher than the total expected bonus prior to the public randomization. This makes the principal more likely to renege on the bonus following this particular realization.

To define our solution concept, Perfect Bayesian Equilibrium (PBE), first let the agent's expected payoff following private history h_A^t and w_t be

$$\widehat{U}(h_A^t, w_t, s^A, s^P) = E[\sum_{\tau=t}^{\infty} \delta^{\tau-t}\{\underline{u} + 1_{\{d_\tau=1\}}(-ce_\tau + W_\tau - \underline{u})\}|h_A^t, w_t, s^A, s^P].$$

Define $\widetilde{U}(h_A^t, w_t, d_t, s^A, s^P)$ accordingly as the agent's expected payoff following his acceptance decision in period t. Next, let the principal's expected payoff following the agent's private history h_A^t be

$$\widehat{\pi}(h_A^t, s^A, s^P) = E[\sum_{\tau=t}^{\infty} \delta^{\tau-t}\{\underline{\pi} + 1_{\{d_\tau=1\}}(\gamma(e_\tau) - W_\tau - \underline{\pi})\}|h_A^t, s^A, s^P],$$

where recall that $\gamma(e_\tau)$ is the expected output for effort e_τ. Define the principal's expected payoff following public history h^t as

$$\widehat{\Pi}(h^t, s^A, s^P) = E_{\mu^P}[\widehat{\pi}(h_A^t, s^A, s^P)|h^t],$$

where the expectation is taken over the agent's possible private histories (h_A^t) according to the principal's belief (μ^P) conditional on public history h^t. Finally, denote $\widetilde{\pi}(h_A^t, w_t, d_t, \phi_t, s^A, s^P)$ as the principal's expected payoff in period t following the agent's private history h_A^t, the principal's wage offer w_t, the agent's acceptance decision d_t, and the performance outcomes ϕ_t. Define $\widetilde{\Pi}(h^t, w_t, d_t, \phi_t, s^A, s^P)$ accordingly.

A PBE in this model consists of the principal's strategy (s^{*P}), the agent's strategy (s^{*A}), the principal's belief (μ^P), and the agent's belief (μ^A), such that the following are satisfied. First, following any history $\{h_A^t, w_t\}$ and $\{h_A^t, w_t, d_t\}$, and for any $\widetilde{s}^A$,

$$\begin{aligned} \widehat{U}(h_A^t, w_t, s^{*A}, s^{*P}) &\geq \widehat{U}(h_A^t, w_t, \widetilde{s}^A, s^{*P}); \\ \widetilde{U}(h_A^t, w_t, d_t, s^{*A}, s^{*P}) &\geq \widetilde{U}(h_A^t, w_t, d_t, \widetilde{s}^A, s^{*P}). \end{aligned}$$

Second, following any history h^t and $\{h^t, w_t, d_t, \phi_t\}$, and for any $\widetilde{s}^P$,

$$\begin{aligned} \widehat{\Pi}(h^t, s^{*A}, s^{*P}) &\geq \widehat{\Pi}(h^t, s^{*A}, \widetilde{s}^P); \\ \widetilde{\Pi}(h^t, w_t, d_t, s_t, s^{*A}, s^{*P}) &\geq \widetilde{\Pi}(h^t, w_t, d_t, s_t, s^{*A}, \widetilde{s}^P). \end{aligned}$$

Third, the beliefs are consistent with (s^{*P}, s^{*A}) and are updated with the Bayes rule whenever possible. Note that the agent has private information about his effort. So the agent's belief about the past history depends on his *actual effort levels*. In contrast, the principal's belief about the past history depends only on the agent's *equilibrium effort levels* as long as the agent has not publicly deviated. If the agent publicly deviates by not entering the relationship in any period, we assume the principal believes that the agent has never put in effort in periods with low public output.

When the signals are also publicly observed, a commonly used equilibrium concept is Perfect Public Equilibrium (PPE). PPE requires the strategies to depend only on the public history. This restriction is inappropriate when the supervisor's reports (and thus the agent's payoff) depend on the past history of signals. When the agent's effort affects future reports, his private history contains payoff-relevant information and should be used to his advantage.

4 Analysis

In this section, we study how information structures affect the efficiency of the relational contract. Subsection 4.1 reviews the condition for sustaining an efficient relational contract under full revelation of signals. Subsection 4.2 presents our main result that the supervisor can help sustain an efficient relational contract by revealing less information.

Below, we restrict our analysis to the case that $q_0 = 0$ and $q = 0$, i.e., both output and the supervisor's signal are low when the agent shirks. We assume $q_0 = 0$ for ease of exposition and can relax it. The assumption that $q = 0$, however, is important for the analysis to be tractable. We discuss the $q > 0$ case in Section 5.1.

4.1 Benchmark: Fully Revealing Reports

Let $P(\phi_t|e_t)$ be the probability that performance outcome $\phi_t = (s_t, y_t)$ happens when the agent's effort is e_t. Suppose the reports fully reveal the signals, i.e., $s_t = y_t^s$ for all t, then $P(\phi_t|0) = 1$ for $\phi_t = (L, 0)$ and $P(\phi_t|0) = 0$ otherwise. This implies that the likelihood ratio that the agent shirks, $P(\phi_t|0)/P(\phi_t|1) = 0$ unless $\phi_t = (L, 0)$. In other words, the agent must have exerted effort unless both the report is L and output is 0. Ordering the performance outcomes according to the likelihood ratio, we can then apply the argument in Levin (2003) to show that the optimal relational contract is stationary.[5] In each period, the principal offers the agent a base wage w and gives him a bonus b if either output is high ($y_t = y$) or the report is good ($s_t = H$).

We now provide the condition for sustaining an efficient relational contract with fully revealing reports. First, if the agent works, the probability of receiving a bonus

[5] Levin (2003) requires CDFC for his analysis. Given that effort is binary, CDFC is not needed for our analysis since there is no difference between a local deviation and a global deviation.

is $p_s \equiv p_0 + p - pp_0$. If he shirks, he does not receive a bonus. To motivate the agent to work, his expected gain from working must exceed the cost of effort:

$$p_s b \geq c. \tag{2}$$

Next, we assume the principal can set the base wage to capture the entire surplus of the relationship. Recall that $s(1)$ is the per-period surplus if the agent works, so for the principal not to renege on the bonus, the following must be satisfied:

$$\frac{\delta}{1-\delta} s(1) \geq b. \tag{3}$$

Combining (2) and (3) shows that a relational contract can induce effort when

$$\frac{\delta}{1-\delta} s(1) \geq \frac{c}{p_s}. \tag{4}$$

In other words, the incentive cost should be smaller than the discounted expected future surplus. Inequality (4) implies that the sustainability of the relational contract depends on the extremes. In other words, the set of discount factors that allow for efficiency is completely determined once the value of the maximal reneging temptation (c/p_s) and the expected per-period surplus in the relationship $(s(1))$ are given. When (4) is satisfied, the optimal relational contract can be achieved by setting $b = \delta s(1) / (1-\delta)$ and $w = \underline{u} + c - p_s b$.

4.2 Main Results

In this section, we show that the supervisor can help sustain the relational contract by sending out less-informative reports. We consider a class of efficient equilibrium where essentially the principal pays the agent a fixed base wage and a bonus on top of that if either the output is high or the supervisor's report good. Our main result is that when the supervisor ties the reports to past signals, he can lower the discount factor necessary for supporting the efficient relational contract.

4.2.1 Spillover Reporting

The key to our result is the supervisor's reporting rule. In particular, we consider the following class of reporting rules. The supervisor reports either G (good) or B

(bad) in each period. These reports are partitioned into reporting cycles that end stochastically. The first reporting cycle starts in period 1, and each new reporting cycle starts if the supervisor reports B in the previous period.

Within each reporting cycle, the supervisor reports G if his signal is high. If the signal is low, he reports G with probability f (and B with probability $1-f$) if the signal in the previous period within the reporting cycle is high. If that signal is again low, the supervisor reports B. When the supervisor observes a low signal at the beginning of a reporting cycle (so that there is no within-cycle previous period), the supervisor reports G with probability $\rho_f^* f$ and B with probability $1-\rho_f^* f$, where ρ_f^* is the unique positive value satisfying $\rho_f^* = p/\left(p + (1-p)\,\rho_f^* f\right)$.

Figure 1a illustrates the reporting rule when the current period is not the first period of a reporting cycle, and Figure 1b illustrates the reporting rule when the current period is the first period of a reporting cycle.

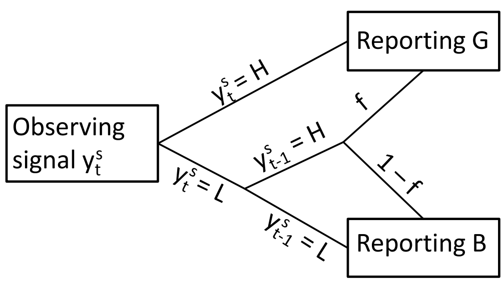

Figure 1a: Reporting rule (*not* in first period of reporting cycle)

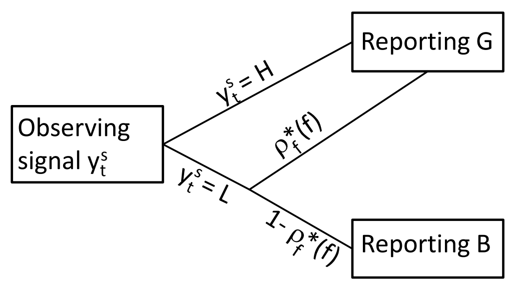

Figure 1b: Reporting rule (in first period of reporting cycle)

We denote this class of reporting rules as f*-spillover reporting* because following a high signal, with probability f the supervisor will send out a good report in the next period even if his signal is low. Following a high output, f is the probability that the supervisor is lenient next period. When $f = 0$, there is no spillover: G is reported if and only if the signal in the current period is high. This is the benchmark case in Subsection 4.1 where the reports fully reveal the signals. When $f = 1$, a high signal means that the supervisor reports G both for the current- and the next period. This is the reporting rule considered in the parametric example in Section 2. Finally, we choose ρ_f^* so that when a good report is sent, the conditional probability of a high signal is always equal to ρ_f^* in equilibrium. The choice of ρ_f^* ensures that in each period, the principal is lenient with probability $\rho_f^* f$ and sends out a good report with probability $p + (1-p)\,\rho_f^* f$.

4.2.2 Perfect Bayesian Equilibrium (PBE)

We now show that f-spillover reporting can help sustain efficient relational contracts. In particular, consider the following class of strategies.

The principal offers $w_1 = w$ in period 1. If the agent has always accepted the contract, the principal offers for $t > 1$

$$w_t = \begin{cases} w & \text{if } s_{t-1} = B \text{ and } y_{t-1} = 0, \\ w + b/\delta & \text{otherwise,} \end{cases}$$

where $w - c + \left(p_s + (1-p_s)\,\rho_f^* f\right) b = \underline{u}$. If the agent has ever rejected the offer, the principal offers $w_t = \underline{u} - 1$. The agent's total compensation at the end of period is given by $W_t = w_t$ for all t.

The agent accepts the principal's contract offer if and only if $w_t > \underline{u}$ or the principal has never deviated. The agent puts in effort if and only if there is no public deviation and the probability of a low signal in the previous period is smaller than ρ_f^*.

An important feature of the PBE is that $w_t = W_t$ for each t; so no end-of-period bonus is paid out. However, when a higher wage of $w + b/\delta$ is paid out, it should be interpreted as containing a *deferred bonus* of amount b being rewarded for the agent's good performance, measured by either $s_{t-1} = G$ or $y_{t-1} = y$, in the pervious period. We therefore denote this class of strategies as stationary strategies with a deferred bonus. We discuss how postponement of bonus helps prevent the agent's multistage deviation at the end of the session.

Also note that we choose w and b so that from period 2 on the principal captures the entire surplus of the relationship. Moreover, in our analysis of the principal's optimal relational contract, w_1 is also set to w so that the principal captures the entire surplus of the relationship. To span the entire set of payoffs of efficient equilibria, one can choose any $w_1 \in [w, w + s\,(1)\,/\,(1-\delta)]$ to arbitrarily divide the surplus between the principal and the agent. Finally, note that when the agent deviates, the principal's choice of $w_t = \underline{u} - 1$ is made for convenience. One can also choose any $w_t < \underline{u}$ since it will again lead the agent to choose his outside option.

Our main result shows that relative to full revelation of outputs, f-spillover reporting reduces the discount factor necessary to support the efficient relational contract. Denote $\delta^*(f)$ as the smallest discount factor such that there exists a PBE supported by a stationary strategy with a deferred bonus. Note that when $f = 0$, the supervisor reveals his signals fully, so $\delta^*(0)$ is determined by Eq (4) in the benchmark case.

Proposition 1: *Suppose* $1 - p_0 - \delta\rho^* > 0$*, where* $\rho^* = p/\left(p + (1-p)\,\rho^*\right)$; *then the following holds:*
(i): $\delta^*(1) < \delta^*(0)$ *if and only if* $p_s/\left(1-p_s\right) < \rho^*\delta - p\left(1-\rho^*\right)\delta^2$.
(ii): *If* $\delta^*(f) < \delta^*(0)$ *for some* $f \in (0,1)$, *then* $\delta^*(1) < \delta^*(f)$.

Part (i) provides the condition for 1-spillover reporting to lower the cutoff discount factor that sustains the efficient relational contract. Part (ii) shows that when full revelation of signals is not optimal, 1-spillover reporting has the lowest cutoff discount factor within the class. This allows us to focus below on 1-spillover reporting, which we refer to as spillover reporting for convenience.

To see why and when spillover reporting helps, we take a three-step approach. First, we introduce a value function for the agent to help describe when an efficient relational contract can be sustained under f-spillover reporting. Notice that the agent's payoff depends on the supervisor's signal in the previous period. Since the agent does not observe the signal, denote ρ as the probability of a high signal in the previous period, and let $\rho = \rho_f^*$ if the period is at the beginning of a reporting cycle. In this case, ρ becomes a state variable that summarizes the agent's payoff. Denote $V(\rho)$ as the agent's value function, and let $V_e(\rho)$ ($V_s(\rho)$) be the agent's maximum expected payoff if he works (shirks) this period. It follows that $V(\rho) = \max\{V_e(\rho), V_s(\rho)\}$.

For ease of exposition, let the agent's outside option $\underline{u}$ be 0, and define $p_f^g(\rho) = p + (1-p)\,f\rho$ as the probability of a good report if the agent works. For convenience, we assume for now that the agent never takes his outside option. We will return to this assumption later. Under these assumptions,

$$\begin{aligned} V_e(\rho) &= w - c + p_f^g(\rho)\left(b + \delta V(p/p_f^g(\rho))\right) + (1 - p_f^g(\rho))p_0 b; \\ V_s(\rho) &= w + \rho f\left(b + \delta V(0)\right). \end{aligned}$$

To see the expression for $V_e(\rho)$, notice that if the agent works, a good report is sent with probability $p_f^g(\rho)$. The agent is rewarded with a deferred bonus b and infers that the probability of a high signal is $p/p_f^g(\rho)$. With probability $1-p_f^g(\rho)$, a bad report is sent. The agent still receives a deferred bonus if the output is high, which happens with probability p_0. When a bad report is sent, a new reporting cycle starts next period and the agent's continuation payoff is given by $\delta V(\rho_f^*)=0$ (since the principal captures all of the surplus). Similarly, when the agent shirks, a good report is sent with probability ρf. In this case, the agent is rewarded with a deferred bonus b, but he infers that the probability of a high signal is 0. Note that $V(0)$ is negative since $V(\rho)$ is increasing in ρ and $V(\rho_f^*)=0$ by construction. As a result, the agent prefers to exit the relationship if he knows $\rho=0$. This possibility complicates the analysis, and we discuss this point in detail at the end of the section.

We now take our second step by deriving the condition for sustaining an efficient relational contract. As in the benchmark case, we start with the agent's incentive constraint. Since $\rho=\rho_f^*$ along the equilibrium path, the agent puts in effort as long as $V_e\left(\rho_f^*\right)\geq V_s\left(\rho_f^*\right)$. Recall that $p_s\equiv p_0+p-pp_0$ is the probability that either the output or the signal is high. From the expression for V_e and V_s above (and noting that $p/p_f^g\left(\rho_f^*\right)=\rho_f^*$ and $\delta V(\rho_f^*)=0$), we can rewrite this inequality as

$$\left(1-\rho_f^* f\right)p_s b+\rho_f^* f\delta\left(V(\rho_f^*)-V\left(0\right)\right)\geq c. \tag{IC-g}$$

Relative to the agent's IC in the benchmark case, spillover reporting has two effects on the agent's incentive constraints. The first one is an *information-loss effect*: when reports are less informative of the agent's effort, they reduce the incentive of the agent. This is captured by the factor $1-\rho_f^* f$, where $\rho_f^* f$ is the probability that the supervisor sends out a good report even if his signal is low. The information-loss effect therefore makes the bonus less sensitive to effort and makes the agent's incentive constraint more difficult to satisfy. This effect is well known in the literature on the garbling of signals; see, for example, Kandori (1992).

The second is a *smoothing effect*, captured by $\rho_f^* f\delta\left(V(\rho_f^*)-V\left(0\right)\right)$. Under f-spillover reporting, the agent has a higher continuation payoff by working $\left(\delta V(\rho_f^*)\right)$ than shirking $(\delta V\left(0\right))$, so the rewards for working are partly paid in the future and smoothed across periods. This allows for the same contemporaneous reward (the bonus) to provide stronger motivation for the agent, making the incentive constraint easier to satisfy. Notice that under f-spillover reporting, bonus is paid more frequently. When $f=0$ (so that the information is fully revealed), the supervisor sends out a good report with probability p per period. When $f = 1$, the supervisor sends out a good report with probability $p + (1-p)\rho^*$. We can interpret ρ^* as the equilibrium amount of spillover in reporting since it is the probability that a good report is sent out even if the signal is low and $(1-p)\rho^*$ as the equilibrium amount of leniency since it is the increased frequency of bonus.

Before evaluating the overall effect from spillover reporting, we next list the principal's non-reneging constraint. Since the principal captures all of the surplus, his non-reneging constraint is again

$$\frac{\delta}{1-\delta} s(1) \geq b. \tag{NR-g}$$

Combining this with the agent's incentive constraint, we obtain the condition for sustaining an efficient relational contract:

$$\frac{\delta}{1-\delta} s(1) \geq \frac{c - \rho_f^* f \delta \left(V(\rho_f^*) - V(0)\right)}{p_s \left(1 - \rho_f^* f\right)}, \tag{NSC-g}$$

which corresponds to Eq (4) in the benchmark case. When $f = 0$, the right-hand side (RHS) is equal to c/p_s, so Eq (NSC-g) includes the benchmark as a special case. In general, f-spillover reporting can help sustain efficient relational contracts when the RHS is smaller than c/p_s. When $f > 0$, both the denominator and the numerator decrease. In the denominator, $1 - \rho_f^* f$ reflects the information-loss effect. In the numerator, $\rho_f^* f \delta \left(V(\rho_f^*) - V(0)\right)$ reflects the smoothing effect. Note that the smoothing effect is absent in within-period garbling, which is why within-period garbling does not help sustain relational contracts.

Next, we take our third step to discuss when the overall effect of spillover reporting is positive. By Eq (NSC-g), spillover reporting helps if and only if

$$\delta \left(V(\rho_f^*) - V(0)\right) > c. \tag{Gain}$$

This condition is easier to satisfy for a larger discount factor δ. Under spillover reporting, part of the reward is paid out in the future, so the size of bonus has to increase to account for the "interest payment" associated with the postponement of bonus payment. When the agent is more patient, this increase is smaller, making spillover reporting more effective. This condition is also easier to satisfy if $V(\rho_f^*) - V(0)$ is larger. When $f = 0$, $V(\rho_f^*) = V(0)$ since a high signal last period gives no extra benefit. This suggests that the smoothing effect is dominated by the information effect when f is small. When f is larger, a high signal this period leads to a larger probability of a good report next period. This suggests that the smoothing effect increases with f and that $f = 1$ is optimal within this class of reporting rules.

To obtain the exact condition for when the smoothing effect dominates, one must calculate $V(0)$. This calculation, however, requires knowing the agent's effort choice for $\rho = 0$, and moreover, his effort choice for all future realizations of ρs that are associated with the continuation payoffs originating from $V(0)$. Specifically, let $\rho_t = 0$ in period t and suppose the agent puts in effort. Now if the supervisor sends a good report, the agent then infers that the signal must be high, i.e., $\rho_{t+1} = 1$. This calculation then requires knowing the agent's choice of e_{t+1} given $\rho_{t+1} = 1$ and so on.

When $\delta = \delta^*(1)$, we compute in the proof the value function and therefore $V(0)$. This leads to the condition in Proposition 1 for when spillover reporting can help. The key to this calculation is to show that the agent will put in effort if and only if $\rho \leq \rho_f^*$. When ρ is larger, the agent is more likely to receive a good report even if the signal is low. This lowers the agent's incentive to put in effort. This suggests that the agent's expected gain for effort is decreasing in ρ, so he puts in effort if and only if ρ is below some cutoff. The details of this argument are provided in the proof.

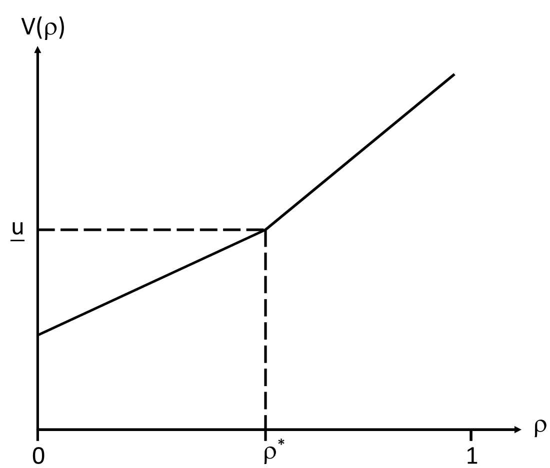

Figure 2: Agent's value function

Figure 2 illustrates the value function of the agent at the cutoff discount factor with $f = 1$, and define $\rho^* \equiv \rho_1^*$. The value function is piecewise linear in ρ and has a kink at ρ^*. In particular, the agent prefers to work if and only if $\rho < \rho^*$.

We now conclude the section by highlighting some properties of the relational contract. First, it is important that neither principal nor the agent knows the supervisor's exact signal. If the principal knows the supervisor's signal, when she sees $\rho = 1$, she will renege on the bonus since $\rho = 1$ implies that the agent's continuation payoff is high, and therefore, the principal's continuation from not reneging is low. Similarly, if the agent knows the supervisor's signal, when he knows that $\rho = 1$, he prefers to shirk since $(V(1) = V_s(1))$. In general, for a reporting rule to improve over fully revealing reporting, the continuation payoffs of the players cannot always be common knowledge.

Second, one must check for multistage deviations to ensure that the agent puts in effort. Under f-spillover reporting, when the agent shirks, his belief about his future payoffs will be different from that of the principal, and the one-stage-deviation principle does not apply. To see multi-stage deviation matters, suppose the agent shirks and receives a good report. He then infers that $\rho = 0$. Since $V(0) = V_e(0)$, the agent the prefers to exert effort when $\rho = 0$. But once the agent works and the supervisor again sends a good report, the agent then infers that the signal must be high ($\rho = 1$). It follows that the agent's optimal effort is then to shirk ($V(1) = V_s(1)$). In summary, if an agent has just shirked but received a good report, his subsequent optimal effort choice is to work and then to shirk.

We formally check in the proof that there are no profitable multistage deviations, but finish the section by noting the following example, which explains why the bonus must be deferred. Again suppose the agent shirks and a good report is sent out. If the bonus for the good report is paid out at the end of the period, the agent would strictly prefer to exit since he knows the signal is low ($\rho = 0$) and that $V(0)$ is less than his outside option (see Figure 2). In other words, this type of shirk-then-exit behavior can be profitable if the bonus payment is not deferred. However, if the bonus is deferred and paid through a higher base wage in the following period, the agent must accept next period's contract to receive the bonus and this ensures that he always stays. The importance of deferring of bonus payment in our analysis stands in contrast to Levin (2003) where the timing of bonus is irrelevant so one can assume that the bonus is paid out at the end of the period.

5 Discussion

In the previous section, we show that spillover reporting can help sustain the efficient relational contract. In this section, we show that the mechanism behind spillover reporting is robust to a number of extensions in Subsection 5.1, and discuss general properties of reporting rules in Subsection 5.2. All formal descriptions of the setups, results, and proofs are relegated to the online appendix.

With the exception of Subsection 5.1.4, we assume in this section that only the supervisor has an informative signal about the agent's effort. This corresponds to the case in which there are no informative public signals of the output, i.e., y_t is unobservable. This simplification allows us to focus on the main mechanism of the paper without providing superfluous details. All formal results can be adapted to allow for observable outputs.

5.1 Robustness

5.1.1 q>0

In Section 4, we assume for tractability that $q = 0$, so the supervisor never receives a high signal when the agent shirks. When $q > 0$, the value function $V(\rho)$ no longer has an analytical solution, so checking multistage deviation becomes very difficult. In the online appendix, we compute $V(\rho)$ numerically for $q > 0$ and show that spillover reporting can lower the bonus amount necessary for motivating the agent. We also prove formally that spillover reporting can help for a sufficiently small q.

5.1.2 Multiple Effort Levels

The agent's effort level is binary in Section 4. The online appendix considers a model with three levels of effort, $e \in \{0, 1, 2\}$, and efficiency requires $e = 2$. We show that as long as the effort costs are sufficiently convex ($(c(2) - c(1)) / (c(1) - c(0))$ is large enough), spillover reporting can help sustain the efficient relational contract.

When the cost function is sufficiently convex, the gain of deviating from $e = 1$ to 0 is small relative to that from $e = 2$ to 1. If the agent does not gain from deviating to $e = 1$, he will not gain from deviating to $e = 0$. More generally, a sufficiently convex effort-cost structure implies that ruling out profitable local deviations is enough for ruling out global deviations. This suggests that spillover reporting can improve the sustainability of relational contracts when the cost structure is sufficiently convex.

5.1.3 Multiple Agents

Our next extension considers $n > 1$ identical agents. When the principal maintains n independent relationships with the agents, our result is unchanged since both the total surplus and the maximal reneging temptation increase n-fold and therefore cancel each other out. The optimal relational contract with n agents, however, is not independent. When the signals are fully revealed, the optimal relational contract specifies a fixed bonus pool that is paid out whenever at least one agent has a high signal, and it is shared equally among agents with high signals (Levin, 2002).

In the online appendix, we construct a corresponding relational contract with spillover reporting. There, the bonus pool is paid out at the beginning of the following period whenever at least one agent has a good report, where the report is obtained through spillover reporting as in Section 4. We provide conditions for when spillover reporting helps and establish a limit result for the gain of spillover reporting. Specifically, the minimal surplus for sustaining the efficient relational contract under full revelation of signals is δ times more than that under spillover reporting as p goes to 0, and the limit is independent of n.

5.1.4 Collusion

In the main model, we abstract away from incentive issues of the supervisor to focus on the gain from spillover reporting. Since the agent's pay depends on the supervisor's reports, one issue is that the supervisor may collude with the agent; see, for example, Tirole (1986) and the large literature that followed. In the online appendix, we study how the principal may design compensation for the supervisor to prevent collusion.

We show that by offering a fraction of the output to the supervisor, the principal can prevent the supervisor from always sending good reports about the agent. The key observation is that if the supervisor colludes with the agent by always sending a good report, the agent will shirk. This results in low outputs and therefore reduces the supervisor's pay. In the online appendix, we provide the formal conditions under which the principal can prevent this type of collusion by giving a large enough share of the output to the supervisor.

Another possibility is that the principal may collude with the supervisor. This type of collusion may be prevented if the agent can catch the collusion with some probability. In this case, the agent takes his outside options forever if he discovers that the supervisor deviates from the reporting rule. When the supervisor has rent in the relationship, it can be shown that the fear of losing future rent in the relationship can deter the supervisor from colluding with the principal.

5.2 General Reporting Rules

In Subsection 4.2, we show how f-spillover reporting helps sustain efficient relational contracts. There are many other reporting rules the supervisor can use. In this subsection, we explore the necessary conditions for a reporting rule to help and how much a reporting rule can reduce the surplus needed for an efficient relational contract.

5.2.1 Necessary Conditions

A necessary condition for any reporting rule to help is that the supervisor's memory must be *persistent*. To explain this condition, we first consider T-period reviews, where the agent is evaluated and compensated every T periods. Radner (1985), Abreu, Milgrom, and Pearce (1991), and Fuchs (2007) show that by revealing information less frequently, T-period reviews can reduce the surplus destroyed in a number of settings. In our environment, however, no surplus is destroyed, and T-period reviews cannot help sustain an efficient relational contract. In the parametric example in Section 2, we considered T-period reviews in which the principal pays the agent a discretionary bonus B_T if output in any of the T periods is high. Proposition 2 shows that this result holds for all types of bonus payments.

Proposition 2: *Let $s(1)$ be the expected future of the relationship per period. Suppose the supervisor reports his signals truthfully every T periods. A necessary condition to sustain the efficient relational contract is*

$$\frac{\delta}{1-\delta}s(1) \geq \frac{c}{p}.$$

While we consider more general types of bonus payments in Proposition 2, the intuition for the result is the same as in the parametric example. Specifically, denote $B_T(h)$ $(B_T(l))$ as the agent's expected bonus if the signal in the T-th period is high (low). First, to induce the agent to exert effort in the T-th period, we must have $B_T(h) - B_T(l) \geq c/p$. Because of discounting, $B_T(h) - B_T(l)$ must be even greater to motivate the agent to work in earlier periods. Second, for the principal not to renege, the discounted future surplus $(\delta s(1) / (1-\delta))$ must be greater than the maximal bonus, which is in turn bigger than the difference in the expected bonus $(B_T(h) - B_T(l))$. Combining the the two arguments above, we obtain the condition in Proposition 2.

The argument above also shows, more generally, why any reporting rule that restarts on predetermined dates cannot improve efficiency over the full revelation of signals. Therefore, for any reporting rule to help, the supervisor's memory must be persistent in the sense that there cannot be a predetermined date after which all past histories become irrelevant. We discuss this property more formally in the online appendix. Notice that under f-spillover reporting, although the supervisor neglects all past histories following a B report, the date of the B report is not predetermined.

For a reporting rule to help, the persistence of memory is necessary but not sufficient. Recall from the discussion in Section 4.2 that an important feature for spillover reporting is that the agent's continuation payoffs are not common knowledge. The lack of common knowledge is an essential feature for a reporting rule to improve efficiency, and this observation applies to other contexts as well. In a related paper, Ekmekci (2011) examines a product-choice game between a long-run seller and a sequence of short-run buyers. Related to our reporting rule, he defines a rating system as a mapping from past outputs to signals. Ekmekci (2011) considers a Markovian rating system, i.e., the latest rating depends on the previous rating and the latest output. As a result, the seller's continuation payoff is common knowledge, unless the seller has privately known types (such as commitment types). In contrast, the spillover reporting rule we propose is not Markovian, but rather *hidden Markovian*; the supervisor's report depends on a hidden state variable, namely, the realization of the signals. This difference explains why the rating system cannot enhance efficiency in Ekmekci (2011) unless the seller has privately known types, while spillover reporting rule helps here even if the agent only has a single publicly known type.

5.2.2 Limits of Gain from Reporting

Under spillover reporting, a high signal in the current period guarantees a good report in the next period. Many other reporting rules can also defer the reward and may help sustain the efficient relational contract. The supervisor, for example, can send a good report as long as any of the past n signals is high. For a given level of surplus, one would like to know whether there are reporting rules that can sustain the efficient relational contract. This problem, however, is difficult because the set of reporting rules is large and checking multistage deviations is hard. Nevertheless, the next proposition partially addresses the problem by establishing a lower bound on the minimal surplus necessary to sustain efficiency.

Proposition 3: *Let $s(1)$ be the expected future surplus of the relationship per period. For any reporting rule to sustain the efficient relational contract, one must have*

$$\frac{\delta}{1-\delta}s(1) \geq \sqrt{4p(1-p)}\frac{c}{p}.$$

In particular, the full revelation of signals is the optimal reporting rule when $p = 1/2$.

When the signals are fully revealed, recall that the smallest discounted future surplus to sustain efficiency must satisfy $S \geq c/p$, where $S = \delta s(1)/(1-\delta)$. Proposition 3 shows that the smallest surplus to sustain efficiency under any reporting rule must satisfy $S \geq \sqrt{4p(1-p)}c/p$. In other words, the optimal reporting rule can lower the surplus necessary for sustaining efficiency by at most a factor of $1 - \sqrt{4p(1-p)}$. Notice that $\sqrt{4p(1-p)} = 1$ when $p = 1/2$. Proposition 3 therefore implies that the full revelation of signals is optimal when $p = 1/2$.

We now provide an intuition for the condition in Proposition 3, which arises through an argument on the variance of the agent's payoff. Note that we can rewrite the condition as

$$\frac{S^2}{4} \geq p(1-p)\left(\frac{c}{p}\right)^2.$$

Here, the left-hand side can be viewed as an upper bound on the variance of the agent's per-period pay that is allowed by the surplus, and the right-hand side is a lower bound necessary for effort. For the left-hand side, notice that the principal's non-reneging constraint implies that the maximal bonus is bounded by S. Since any nonnegative random variable bounded by S cannot have a variance above $S^2/4$, the left-hand side provides an upper bound on the variance of the agent's realized pay per period, calculated from bonus payments.[6]

[6] The maximal variance is obtained by a binary random variable that is equal to 0 half of the time and equal to S the other half of the time.

To see that the right-hand side provides a lower bound, notice that the agent's *actual pay* from period t is a binary random variable whose value is u_h (for a high signal) with probability p and u_l (for a low signal) with probability $1-p$. To induce effort from the agent, we have $u_h - u_l \geq c/p$. This implies that the variance of the agent's actual pay per period must be greater than $p(1-p)(c/p)^2$.

Under a general reporting rule, the agent's realized pay needs not to be the same as his actual pay each period since some of the actual pay can be deferred to the future. But for sufficiently long periods of time, the agent's average realized pay per period will be close to his average actual pay. It follows that $S^2/4$ is the maximal feasible variance of average pay, and $p(1-p)(c/p)^2$ is the minimal variance necessary to induce effort. This leads to the condition in Proposition 3. Finally, when $p = 1/2$ (and setting the bonus equal to S), full revelation of signals is optimal because it generates the maximal variance of pay per period ($S^2/4$). No other reporting rule can generate enough variance to induce effort if full revelation fails to do so.

6 Conclusion

In this paper, we show that supervisors can improve the sustainability of relational contracts by revealing less information. We study f-spillover reporting rules in which the reward for an agent's good performance is spread over periods. This reduces the required bonus size necessary for inducing effort and therefore reduces the principal's temptation to renege on the bonus. We show that 1-spillover reporting is optimal within this class of reporting rules.

More generally, we provide a necessary condition for an arbitrary reporting rule to lower the surplus necessary for sustaining the efficient relational contract. To improve over the full revelation of signals, a reporting rule must allow the memory to persist. Finally, we establish an upper bound for the gain from reporting in terms of how much it can lower the amount of surplus required to sustain efficient relational contracts. The upper bound implies that when the probability of a high signal is a half (under effort), full revelation of signals is optimal.

Our results highlight two features of the reporting rule that help strengthen the credibility of the organization. First, the reporting rule is lenient in the sense that the supervisor sometimes sends a good report even when he observes a bad performance from the worker. Second, the reporting rule exhibits the spillover effect: the supervisor rewards past good performance by sending a good report today. Scholars in various literatures, and management scholars in particular, have documented both of these biases, and have suggested several possible rationales for why they are so prevalent.

Leniency bias can arise, for instance, if supervisors are averse to communicating negative evaluations to employees either due to psychological cost or the desire to avoid confrontation and limit criticism (McGregor, 1957; Bernardin & Buckley, 1981; Prendergast, 1999). In addition, when accurate performance measures are not readily available and the supervisor does not want to incur the cost of acquiring the information, he may simply give more positive ratings (Harris, 1994; Bol, 2011). In terms of spillover effect, Bol and Smith (2011) hypothesize that it can result from cognitive distortion, according to which individuals tend to unknowingly process information in a way that is consistent with a previously held belief—in this case formed on the basis of past performance. Also, when the supervisor considers the current performance measure to be deficient, he may try to make adjustment to the current rating to be consistent with previous ratings, giving rise to the spillover effect (Woods, 2012).

Our results do not rule out these alternative mechanisms, and in fact our forces could easily reinforce and coexist with other explanations. However, the organizational implications of our model are quite different. In particular, our results suggest that both of these biases may be features of reporting rules that enhance an organization's performance, rather than undermining it. Therefore, a firm might condone such biases among her supervisors in order to strengthen the credibility of the organization's incentive scheme. Conversely, if a firm insists on transparency or rigid reporting standards, it might eliminate the reporting flexibility that is the foundation of this mechanism, resulting in lower effort and productivity.

In the main section of the paper, we consider a disinterested supervisor. In the extension section, we also briefly consider the case in which the supervisor is self-interested and can collude with the agent. We show that the principal can prevent stationary collusion by giving a share of the output to the supervisor. In general, there may be more sophisticated types of collusion between the supervisor and the agent. For example, the supervisor's collusive reports can depend on past realizations of outputs, and if too many past outputs are low, the supervisor can send out bad reports. This type of collusion may induce the worker to exert effort while extracting extra bonus from the principal. The principal, of course, may also offer more complicated contracts to the supervisor or use other tools, such as turnover and rotation, to prevent these types of collusion. Formal study of how collusion affects relational contracts is an interesting line of future research.

References

[1] Abreu, Dilip, Paul Milgrom, and David Pearce (1992). "Information and Timing in Repeated Partnerships." *Econometrica*, 59(6), 1713-1733.

[2] Baker, George, Robert Gibbons, and Kevin J. Murphy (1994). "Subjective Performance Measures in Optimal Incentive Contracts." *Quarterly Journal of Economics*, 109(4), 1125-1156.

[3] Bernardin, J. H. and R. M. Buckley (1981). "Strategies in Rater Training." *Academy of Management Review*, 6(2): 205-12.

[4] Bol, Jasmijn C. (2011). "The Determinants and Performance Effects of Managers' Performance Evaluation Biases." *The Accounting Review*, 86(5), 1549-1575.

[5] ——— and Steven D. Smith (2011). "Spillover Effects in Subjective Performance Evaluation: Bias and the Asymmetric Influence of Controllability." *The Accounting Review*, 86(4), 1213-1230.

[6] Blanchard, Garth A., Chee W. Chow, and Eric Noreen (1986). "Information asymmetry, incentive schemes, and information biasing: The case of hospital budgeting under rate regulation." *The Accounting Review*, 61(1), 1–15.

[7] Bretz, Robert D., George T. Milkovich, and Walter Read (1992). "The current state of performance appraisal research and practice: Concerns, directions, and implications." *Journal of Management*, 18(2), 321–352.

[8] Chan, Jimmy and Bingyong Zheng (2011). "Rewarding Improvements: Optimal Dynamic Contracts with Subjective Evaluation," *Rand Journal of Economics*, 42(4), 758-775.

[9] Deb, Joyee, Jin Li, and Arijit Mukherjee (2014). "Relational contracts with subjective peer evaluations." Working Paper.

[10] Ekmekci, Mehmet (2011). "Sustainable Reputations with Rating Systems." *Journal of Economic Theory.* 146(2), 479-503.

[11] Fuchs, William (2007). "Contracting with Repeated Moral Hazard and Private Evaluations." *American Economic Review*, 97(4), 1432-1448.

[12] Harris, M. M. (1994). "Rater Motivation in the Performance Appraisal Context: A Theoretical Framework." *Journal of Management*, 20(4): 737-56.

[13] Holmstrom, Bengt (1979). "Moral hazard and observability." *The Bell Journal of Economics*, 74-91.

[14] Holzbach, Robert L (1978). "Rater bias in performance ratings: Superior, self-, and peer ratings." *Journal of Applied Psychology*, 63(5), 579.

[15] Jacobs, Rick and Steve WJ Kozlowski (1985). "A closer look at halo error in performance ratings." *The Academy of Management Journal*, 28(1), 201-212.

[16] Kandori, Michihiro (1992). "The Use of Information in Repeated Games with Imperfect Monitoring." *Review of Economic Studies*, 59, 581–594.

[17] ——— and Ichiro Obara (2006). "Less is More: an Observability Paradox in Repeated Games." *International Journal of Game Theory*, 34, 475–493.

[18] Levin, Jonathan (2002). "Multilateral Contracting and the Employment Relationship." *Quarterly Journal of Economics*, 117(3), 1075-1103.

[19] ——— (2003). "Relational Incentive Contracts." *American Economic Review*, 93(3), 835-857.

[20] MacLeod, Bentley W. (2003) "Optimal contracting with subjective evaluation." *The American Economic Review*, 93(1), 216-240.

[21] Maestri, Lucas (2012). "Bonus Payments Versus Efficiency Wages in the Repeated Principal Principal-Agent Model with Subjective Evaluations," *American Economic Journal: Microeconomics*, 4, 34-56.

[22] Malcomson, James M. (2013). "Relational Incentive Contracts." Chapter 25 in *Handbook of Organizational Economics,* edited by Robert Gibbons and John Roberts,1014-1065.

[23] McGregor, D. M. (1957), "An Uneasy Look at Performance Appraisal." *Harvard Business Review*, 35(3): 89-94.

[24] Murphy, Kevin R., William K. Balzer, Maura C. Lockhart, and Elaine J. Eisenman (1985). "Effects of previous performance on evaluations of present performance." *Journal of Applied Psychology,* 70(1), 72-84.

[25] Nisbett, Richard E. and Timothy D. Wilson (1977). "The halo effect: Evidence for unconscious alteration of judgments." *Journal of Personality and Social Psychology*, 35(4), 250-256.

[26] Prendergast, Canice (1999). "The provision of incentives in firms." *Journal of Economic Literature*, 37(1), 7-63.

[27] ——— and Robert Topel (1993). "Discretion and bias in performance evaluation." *European Economic Review*, 37(2), 355-365.

[28] Radner, Roy (1985). "Repeated Principal-Agent Models with Discounting." *Econometrica*, 53(4), 1173-1198.

[29] Rayo, Luis (2007). "Relational Incentives and Moral Hazard in Teams." *Review of Economic Studies*, 74(3), 937-963.

[30] Rynes, Sara L., Barry Gerhart, and Laura Parks (2005). "Personnel psychology: Performance evaluation and pay for performance." *Annual Review of Psychology*, 56, 571–600.

[31] Tirole, Jean (1986). "Hierarchies and Bureaucracies: On the Role of Collusion in Organizations." *Journal of Law, Economics, and Organization*, 2(2), 181-214.

[32] Woods, Alexander (2012). "Subjective adjustments to objective performance measures: The influence of prior performance." Accounting, Organizations and Society, 37, 403–425.

Appendix A: Proof of Proposition 1

Proof. Part (i). To simplify the exposition, we set $\underline{u} = 0$, and recall that $\rho^* = \rho_1^*$ (since $f = 1$). In an efficient relational contract, the agent always puts in effort and generates surplus $s(1) = p_0 y - c - \underline{v}$ per period. Given the choice of w and b the surplus is captured entirely by the principal, so she will not deviate as long as $b \leq \delta s(1) / (1 - \delta)$. This implies that finding the cutoff discount factor $\delta^*(1)$ is equivalent to finding the smallest deferred bonus b^* such that the agent always puts in effort. To do this, we take the following steps.

Step 1: *Recursive Formulation.* Given the supervisor's reporting strategy, the agent's payoff is completely determined by the probability that the signal is high in the last period ρ (and $\rho = \rho^*$ when the report in the last period is bad). Let $V(\rho)$ be the agent's value function at the beginning of a period assuming that he has accepted the principal's offer. Let $V_e(\rho)$ ($V_s(\rho)$) be the agent's value function if he works (shirks) this period, then $V(\rho) = \max\{V_e(\rho), V_s(\rho)\}$, and the following holds:

$$\begin{aligned}
V_e(\rho) &= w^* - c + (p + (1-p)\rho) \max\left\{0, b^* + \delta V\left(\frac{p}{p + (1-p)\rho}\right)\right\}) \\
&\quad + (1 - p - (1-p)\rho)\left(p_0 \max\{0, b^* + \delta V(\rho^*)\} + (1 - p_0)\max\{0, \delta V(\rho^*)\}\right); \\
V_s(\rho) &= w^* + \rho \max\{0, b^* + \delta V(0)\} + (1 - \rho)\max\{0, \delta V(\rho^*)\}.
\end{aligned}$$

The max operators capture the possibility that the agent can take his outside option at the beginning of next period. In addition, recall that if the agent starts at $\rho = \rho^*$ and works, the probability of a high signal last period is again ρ^*. This implies that to check the agent has the incentive to work every period, it suffices to check $V_e(\rho^*) \geq V_s(\rho^*)$. Moreover, at the smallest deferred bonus b^* we must have $V_e(\rho^*) = V_s(\rho^*)$. Note that by substituting out V_e and V_s, the resulting functional equation for V can be written as $V = T(V)$, where T is a monotone and nonexpansive operator that maps bounded functions on $[0, 1]$ to the same space, and thus, has a unique solution. Below, we construct the unique value function that this holds.

Step 2: *The value function.* Consider the following candidate value function:

$$V(\rho) = \begin{cases} \alpha(\rho - \rho^*) & \text{for } \rho \leq \rho^* \\ \beta(\rho - \rho^*) & \text{for } \rho > \rho^* \end{cases},$$

where

$$\alpha=\frac{(1-p)\,(1-p_0-\delta\rho^*)}{1-\delta^2p\,(1-\rho^*)}b^*,\ \ \beta=\frac{1-(1-p_0)\,(1-p)\,\delta\rho^*}{1-\delta^2p\,(1-\rho^*)}b^*,$$

and

$$c=\left(p_s+\rho^*\left((1-p_s)-\frac{1-(1-p_s)\,\delta\rho^*}{1-\delta^2p\,(1-\rho^*)}\right)\right)b^*,$$

where recall $p_s=p_0+(1-p_0)\,p$. In addition, $V(\rho)=V_e(\rho)$ for $\rho\leq\rho^*$ and $V(\rho)=V_s(\rho)$ for $\rho>\rho^*$ so that the agent puts in effort if and only if $\rho\leq\rho^*$.

Before proceeding, we note the following. First, it can be checked that the choice of α and β satisfy the following two equations that will be used later:

$$\begin{aligned}(1-p)\,((1-p_0)\,b^*-\delta\beta\rho^*) &= \alpha; &\text{(alpha)}\\ b^*-\delta\alpha\rho^* &= \beta. &\text{(beta)}\end{aligned}$$

Second, $\beta>0$, and given $1-p_0-\delta\rho^*>0$, we then also have $\alpha>0$. Third, the choice of b^* gives that

$$w^*+\rho^*\,(b^*-\delta\alpha\rho^*)=0, \qquad \text{(b*)}$$

where $w^*=c-(p_s+(1-p_s)\,\rho^*)\,b^*$ (given $f=1$).

For the candidate value function to be the value function, the following must hold:

$$\begin{aligned}V(\rho) &= V_e(\rho)\ \text{ for } \rho\leq\rho^*; &\text{(PK-e)}\\ V(\rho) &= V_s(\rho) \text{ for } \rho>\rho^*; &\text{(PK-s)}\end{aligned}$$

$$\begin{aligned}V_e(\rho) &\geq V_s(\rho)\ \text{ for } \rho\leq\rho^*; &\text{(IC-e)}\\ V_s(\rho) &\geq V_e(\rho)\ \text{ for } \rho>\rho^*. &\text{(IC-s)}\end{aligned}$$

To check these constraints, we first derive new expressions for $V_e(\rho)$ and $V_s(\rho)$ using the functional equations. The functional equations for $V_e(\rho)$ and $V_s(\rho)$ can be simplified by noting that $V(\rho^*)=0$ and that for all ρ, we have $b^*+\delta V\,(\rho)>0$. This inequality follows because $V\,(\rho)$ is increasing in ρ and $b^*+\delta V\,(0)=b^*-\delta\alpha\rho^*=\beta$, where we use Eq(beta) for the equality. Given the simplification, we have

$$\begin{aligned} V_e(\rho) &= w^* - c + (p + (1-p)\rho)(b^* + \delta V(\frac{p}{p+(1-p)\rho})) \\ &\quad +(1 - p - (1-p)\rho)p_0 b^*; \\ V_s(\rho) &= w^* + \rho\left(b^* + \delta V(0)\right). \end{aligned}$$

To obtain the new expression for $V_s(\rho)$, the functional equation implies $\frac{dV_s(\rho)}{d\rho} = b^* + \delta V(0) = b^* - \delta\alpha\rho^* = \beta$, where the second equality uses the expression of $V(0)$ and the last equality uses Eq(beta). We also note that $V_s(\rho^*) = 0$ because of Eq(b*). Together, this gives that

$$V_s(\rho) = \beta(\rho - \rho^*). \tag{Vs-alt}$$

To obtain the new expression for $V_e(\rho)$, note that $\frac{p}{p+(1-p)\rho} \geq \rho^*$ if and only if $\rho \leq \rho^*$. Substituting for the expression of $V(\frac{p}{p+(1-p)\rho})$ in the functional equation, we have

$$\frac{dV_e(\rho)}{d\rho} = \begin{cases} (1-p)\left((1-p_0)\,b^* - \delta\beta\rho^*\right) & \text{for } \rho \leq \rho^* \\ (1-p)\left((1-p_0)\,b^* - \delta\alpha\rho^*\right) & \text{for } \rho > \rho^* \end{cases}.$$

Noting that $V_e(\rho^*) = 0$ (by using the expression of w^*), we get

$$V_e(\rho) = \begin{cases} (1-p)\left((1-p_0)\,b^* - \delta\beta\rho^*\right)(\rho - \rho^*) & \text{for } \rho \leq \rho^* \\ (1-p)\left((1-p_0)\,b^* - \delta\alpha\rho^*\right)(\rho - \rho^*) & \text{for } \rho > \rho^* \end{cases}. \tag{Ve-alt}$$

From the expressions of $V_s(\rho)$ and $V_e(\rho)$, we see immediately that $V(\rho) = V_s(\rho)$ for $\rho > \rho^*$. Also note that $(1-p)\left((1-p_0)\,b^* - \delta\beta\rho^*\right) = \alpha$ by Eq(beta), so $V(\rho) = V_e(\rho)$ for $\rho \leq \rho^*$. As a result, the promise-keeping constraints are satisfied.

Finally, we check the IC constraints. First, we check IC-e. Given Eq(Ve-alt), checking $V_e(\rho) = V(\rho) > V_s(\rho)$ for $\rho < \rho^*$ is equivalent to checking that $\alpha < \beta$. Notice that

$$\beta = \frac{1 - (1-p_0)(1-p)\,\delta\rho^*}{(1-p)(1-p_0-\delta\rho^*)}\alpha > \alpha.$$

where the inequality follows because

$$\begin{aligned} & 1 - (1-p_0)(1-p)\,\delta\rho^* - (1-p)(1-p_0-\delta\rho^*) \\ = \; & 1 - (1-p)(1-p_0-p_0\delta\rho^*) \\ > \; & 0. \end{aligned}$$

Next, we check IC-s. From Eq(Ve-alt), IC-s ($V(\rho) > V_e(\rho)$ for $\rho > \rho^*$) is equivalent to $(1-p)((1-p_0)b^* - \delta\alpha\rho^*) < \beta$. This inequality is implied by $\alpha < \beta$ because $(1-p)((1-p_0)b^* - \delta\alpha\rho^*) - \alpha = (1-p)\delta\rho^*(\beta-\alpha) < \beta - \alpha$. This finishes checking IC-s, and therefore, the IC constraints. The candidate value function is therefore the value function.

Step 3: *Condition for Improvement.* Using the expression of b^* in the value function, we obtain that the condition for sustaining the relational contract under spillover reporting is given by

$$\frac{\delta}{1-\delta}s(1) \geq \frac{c}{p_s + \rho^*\left(1 - p_s - \frac{1-(1-p_s)\delta\rho^*}{1-\delta^2 p(1-\rho^*)}\right)}. \tag{NSC-1}$$

It follows that spillover reporting improves the condition for sustaining the relational contract if and only if

$$(p_s + (1-p_s)\rho^*) - \rho^*\frac{1-(1-p_s)\delta\rho^*}{1-\delta^2 p(1-\rho^*)} > p_s,$$

or equivalently,

$$-\frac{p_s}{1-p_s} + \rho^*\delta - p(1-\rho^*)\delta^2 > 0.$$

This finishes the proof of Part (i).

Part (ii). To find $\delta^*(f)$, it is equivalent to find the minimal deferred bonus $b^*(f)$ necessary for sustaining an efficient relational contract. Instead of finding $b^*(f)$ directly, we provide a lower bound $b(f) \leq b^*(f)$ for each f. We show that if the lower bound is smaller than c/p_s for some f, then the lower bounds $b(f)$ are minimized at $f = 1$. In this case, this lower bound $b(1) = b^*(1)$ found in Part (i). This proves Part (ii).

Step 1: *A Necessary Condition.* We use $V(\rho)$ again to denote the value of the agent when the probability of a high signal is ρ under f-spillover reporting, and define $V_e(\rho)$ and $V_s(\rho)$ accordingly. Suppressing the dependence of w^* and b^* on f, we have

$$\begin{aligned}
V(\rho) &= \max\{V_e(\rho), V_s(\rho)\}; \\
V_e(\rho) &= w^* - c + (p + (1-p)\rho f)\max\left\{0, b^* + \delta V\left(\frac{p}{p+(1-p)\rho f}\right)\right\}) \\
&\quad + (1 - p - (1-p)\rho f)\left(p_0 \max\left\{0, b^* + \delta V(\rho_f^*)\right\} + (1-p_0)\max\left\{0, \delta V(\rho_f^*)\right\}\right); \\
V_s(\rho) &= w^* + \rho f \max\{0, b^* + \delta V(0)\} + (1-\rho f)\max\left\{0, \delta V(\rho_f^*)\right\}.
\end{aligned}$$

Under the efficient relational contract, $V_e\left(\rho_f^*\right) \geq V_s\left(\rho_f^*\right)$. Since $V(\rho_f^*) = 0$ and $\max\{0, b^* + \delta V(0)\} \geq b^* + \delta V(0)$, $V_e\left(\rho_f^*\right) \geq V_s\left(\rho_f^*\right)$ implies that

$$\left(1 - \rho_f^* f\right) p_s b^* \geq c + \rho_f^* f \delta \left(V(0) - V(\rho_f^*)\right). \tag{IC-f}$$

Next, since $V(0) \geq V_e(0)$, we have

$$V(0) - V(\rho_f^*) \geq \delta p \left(V(1) - V\left(\rho_f^*\right)\right) - (1 - p_s) f \rho_f^* b^*. \tag{L1}$$

Moreover, since $V(1) \geq V_s(1)$, it follows that

$$V(1) \geq w^* + f(b^* + \delta V(0)) + (1 - f)\delta V(\rho_f^*).$$

Note that $V(\rho_f^*) = w^* - c + (p_s + (1 - p_s)\rho_f^* f)b^* + \delta V(\rho_f^*)$, so

$$V(1) - V\left(\rho_f^*\right) \geq \left(f - p_s - (1 - p_s)\rho_f^* f\right) b^* + \delta f \left(V(0) - V\left(\rho_f^*\right)\right). \tag{L2}$$

Combining Eq(L1) and Eq(L2), we obtain a lower bound $V(0) - V\left(\rho_f^*\right)$, and using this lower bound in Eq(IC-f), we get, after some algebra,

$$\frac{c}{b^*} \leq p_s + \rho_f^* f \left(1 - p_s - \frac{1 - \delta \rho_f^* f (1 - p_s)}{1 - \delta^2 p f \left(1 - \rho_f^*\right)}\right) \equiv A(f).$$

It follows that a necessary condition to sustain the relationship is

$$\frac{\delta}{1-\delta} s(1) \geq b^* \geq \frac{c}{A(f)}. \tag{NSC-f}$$

Step 2. *The optimality of* $f = 1$. Notice that when $f = 1$, Eq(NSC-f) is exactly the same as the necessary and sufficient condition in Part (i) (Eq(NSC-1)). Since $c/A(f)$ provides a lower bound for the surplus for sustaining the relational contract, it follows that choosing $f = 1$ is optimal as long as $c/A(f)$ is minimized at $f = 1$, or alternatively, $A(f)$ is maximized at $f = 1$.

To show this, let $\rho_f^* f = x$ and note the following. First, x is increasing in f. This follows because $\rho_f^* = p/(p + (1 - p)x)$ and if $dx/df \leq 0$ for some f, we must have $d\rho_f^*/df \geq 0$, which then contradicts $dx/df \leq 0$ since $dx/df = \rho_f^* + f d\rho_f^*/df > 0$.

Second, $2x(1 - p)\delta < 1$ for all x. To see this, notice that given x is increasing in f, and at $f = 1$ we have $x = \rho^*$, it suffices to show that $2(1 - p)\rho^*\delta < 1$. Recall

$\rho^* = p/\left(p + (1-p)\rho^*\right)$, and using ρ^* to substitute for p, we have

$$\rho^*(1-p) = \frac{\rho^* - (\rho^*)^2}{1 - \rho^* + (\rho^*)^2} < \frac{1}{2},$$

where the inequality follows because it is equivalent to $1 - 3\rho^* + 3(\rho^*)^2 > 0$, which holds for all value of ρ^*. This shows that $2x(1-p)\delta < 1$ for all x.

Now notice that

$$A(f) = p_s + x\left(1 - p_s - \frac{1 - \delta(1-p_s)x}{1 - \delta^2(1-p)x^2}\right) \equiv p_s + xB(x),$$

where the equality uses $pf = \rho_f^* f\left(p + (1-p)\rho_f^* f\right) = x(p + (1-p)x)$. If f-spillover reporting improves on full revelation of signals for some f, then $A(f) > p_s$, and the corresponding $B(x) > 0$. In this case,

$$\begin{aligned}
\frac{d(xB(x))}{dx} &= B(x) + x\delta\frac{(1-p_s)\left(1-\delta^2(1-p)x^2\right) - 2x(1-p)\delta(1-\delta(1-p_s)x)}{\left(1-\delta^2(1-p)x^2\right)^2} \\
&> B(x) + x\delta\frac{(1-p_s)\left(1-\delta^2(1-p)x^2\right) - (1-\delta(1-p_s)x)}{\left(1-\delta^2(1-p)x^2\right)^2} \\
&= B(x) + \frac{x\delta B(x)}{1-\delta^2(1-p)x^2} \\
&> 0,
\end{aligned}$$

where the first inequality follows because $2x(1-p)\delta < 0$, which we showed above. This implies that if $B(x) > 0$, then $xB(x)$ is increasing in x, and therefore, $xB(x)$ is maximized at $x_{\max} \equiv \rho^*$, which corresponds to $f = 1$. ■

7 Online Appendix (Not for Publication)

7.1 Omitted Calculation for Parametric Example

In this part of the appendix, we provide the detailed calculation behind how our reporting rule helps sustain efficient relational contract in the numerical example in Section 2. Recall that in the example, we set $c = 1$, $y = 26$, and $p = 1/13$. In addition, the minimum bonus to motivate the agent under truthful reporting is 13, and the cutoff discount factor δ^* is $13/14$.

Under the relational contract considered in the example, the principal pays the agent a fixed wage w^* and an additional b^* if the supervisor reports G in each period. Specifically, we set $b^* = 13$ and $w^* = c - (4/13)\, b^*$. This implies that at $\delta^* = 13/14$, the principal's discounted future surplus is equal to 13, so she is willing to pay the bonus. To show that our reporting rule lowers the cutoff discount factor, it therefore suffices to show that the agent has a strict incentive to exert effort, or equivalently, the agent's payoff if he shirks is strictly lower than his equilibrium payoff.

As mentioned in the example, calculating the agent's payoff once he shirks requires us to take into account the possibility of multistage deviations by the agent. Specifically, if the agent shirks and a good report is sent out, he knows that output is 0. This information may induce him to shirk again or even exit the relationship. To tackle the possibility of multi-stage deviation, notice that under our reporting rule, the agent's expected discounted payoff is determined by the probability that output was y in the previous period—and if this is the first period of the game or if the report in the previous report was L, we can set this probability to be $1/4$. Now denote $V(\rho)$ as the agent's expected discounted payoff if output in the previous period was y with probability ρ.

Along the equilibrium path, this probability is always $1/4$, so the agent's equilibrium payoff is given by

$$V(\frac{1}{4}) = w^* - c + \frac{4}{13}b^* + \delta^* V(\frac{1}{4}). \qquad \text{(V-A)}$$

Now if the agent shirks, his payoff is given by

$$w^* + \frac{1}{4}\left(b^* + \delta^* V(0)\right) + \frac{3}{4}\delta^* V(\frac{1}{4}),$$

where notice if the agent shirks, the supervisor still sends a good report with probability $1/4$, but when this occurs, the agent knows that output is y with probability 0, so his continuation payoff is $V(0)$. Comparing these two expressions, we then obtain that the agent will not shirk if

$$\frac{3}{52}b^* + \frac{1}{4}\delta^*\left(V(\frac{1}{4}) - V(0)\right) \geq c.$$

Using $b^* = 13$, $\delta^* = 13/14$, $c = 1$, we can write the agent's incentive compatibility condition as

$$V(\frac{1}{4}) - V(0) \geq \frac{14}{13}. \tag{IC-A}$$

We now show that this inequality is strict. For now, suppose that the agent will not exit the relationship following a deviation, and we will return to this issue later. Given that the agent will not exit, we only need to consider the agent's subsequent effort choices. In this case, we have

$$V(0) = \max\{V_e(0), V_s(0)\},$$

where $V_e(0)$ is the agent's payoff if he works and $V_s(0)$ is his payoff if he shirks.

Now

$$V_e(0) = w^* - c + \frac{1}{13}(b^* + \delta^* V(1)) + \frac{12}{13}\delta^* V(\frac{1}{4}),$$

where notice that if the supervisor sends G, the agent knows that output is y for sure, and therefore, his continuation payoff is $V(1)$.

Similarly,

$$V_s(0) = w^* + \delta^* V(\frac{1}{4})$$

since the supervisor sends B for sure in this case.

Notice that if $V(0) = V_s(0)$, we can then use (V-A) to show that

$$V(\frac{1}{4}) - V(0) = \frac{4}{13}b^* - c = 3.$$

The inequality in (IC-A) is therefore strict, and we are done.

Now suppose $V(0) = V_e(0)$ so that

$$V(0) = w^* - c + \frac{1}{13}(b^* + \delta^* V(1)) + \frac{12}{13}\delta^* V(\frac{1}{4}).$$

Using (V-A), we then have

$$V(\frac{1}{4}) - V(0) = \frac{3}{13}b^* - \frac{1}{13}\delta^*\left(V(1) - V\left(\frac{1}{4}\right)\right). \tag{DIFF}$$

This equality implies that to show that $V(\frac{1}{4}) - V(0) > 14/13$, we need to evaluate $V(1) - V\left(\frac{1}{4}\right)$. Now $V(1) = \max\{V_e(1), V_s(1)\}$, so there are again two cases to consider.

First, suppose $V(1) = V_e(1)$. In this case,

$$V(1) = w^* - c + b^* + \delta^* V(\frac{1}{13}) \leq w^* - c + b^* + \delta^* V(\frac{1}{4}),$$

where the equality follows from the fact that the supervisor always reports G in this case so output is y with probability $1/13$, and the inequality follows from the fact that the agent's value is increasing in the probability that output is y. This inequality, together with (V-A), then implies that

$$V(1) - V\left(\frac{1}{4}\right) \leq \frac{9}{13}b^*.$$

Using (DIFF), we obtain that

$$V(\frac{1}{4}) - V(0) \geq \frac{3}{13}b^* - \frac{1}{13}\delta^*\frac{9}{13}b^* = \frac{33}{14}.$$

It then follows that the inequality in (IC-A) is again strict in this case.

Second, suppose that $V(1) = V_s(1)$, so that

$$V(1) = w^* + (b^* + \delta^* V(0)).$$

This expression, together with (V-A), then implies that

$$V(1) - V\left(\frac{1}{4}\right) = \frac{9}{13}b^* + c - \delta^*\left(V\left(\frac{1}{4}\right) - V(0)\right) \geq \frac{9}{13}b^* + c = 10.$$

Now using (DIFF) we have

$$V(\frac{1}{4}) - V(0) > \frac{3}{13}b^* - \frac{1}{13}\delta^* 10 = \frac{16}{7}$$

so that the inequality in (IC-A) is also strict in this case. Combining all cases, we have shown that the agent's IC constraint is slack as long as he does not exit.

Finally, we need to show the agent will not exit the relationship. To ensure this, the timing of bonus payment is important. As in our general analysis, we assume that the principal pays the bonus at the beginning of the next period as part of the fixed wage rather than at the end of the current period. Notice that the principal is indifferent between paying $b^* = 13$ at the end of current period and paying $b^*/\delta^* = 14$ at the beginning of next period. But since the bonus is postponed to after the agent accepts next period's contract, the agent is less likely to exit. To see that the agent will not exit with this timing of bonus payment, notice that if the supervisor sends B, the agent's continuation payoff is $V(1/4) = 0$, so he will not exit. And if the supervisor sends G, the agent will not exit as long as $b^*/\delta^* + V(0) \geq 0$. Our analysis above implies that

$$b^* + \delta^* V(0) \geq b^* + \delta^* V_s(0) = b^* + \delta^* \left(c - \frac{4}{13} b^* \right) = \frac{143}{14} > 0,$$

so that the agent again will not exit. This finishes the proof that $V(\frac{1}{4}) - V(0) > \frac{14}{13}$, and therefore, that the reporting rule helps lower the discount factor for sustaining an efficient relational contract.

As a postscript, we note that to make the proof above self-contained, rather than calculating the explicit value of $V(\frac{1}{4}) - V(0)$, we calculated above various lower bounds for it. Using the technique developed in our general analysis, one can show that

$$\begin{aligned} V(0) &= w^* - c + \frac{1}{13}(b^* + \delta^* V(1)) + \frac{12}{13}\delta^* V(\frac{1}{4}); \\ V(1) &= w^* + (b^* + \delta^* V(0)). \end{aligned}$$

Together with (V-A), this gives that

$$V\left(\frac{1}{4}\right) - V(0) = \frac{\frac{3}{13}\left(1 - \frac{3}{13}\delta^*\right) b^* - \frac{1}{13}\delta^* c}{1 - \frac{1}{13}(\delta^*)^2} = 448/183,$$

which is greater than 14/13.

7.2 q>0

We show that spillover reporting helps the sustainability of efficient relational contracts when $q > 0$ by presenting two results. The first one is a continuity-type of result that shows once spillover reporting helps for $q = 0$, it also helps for q close to 0. For given c, p, and δ, let S_1 be the smallest expected discounted surplus for sustaining the efficient relational contract under spillover reporting when $q = 0$.

Corollary 1: *Consider p and δ such that $-p/(1-p)+\rho^*\delta-p(1-\rho^*)\delta^2 > 0$, where $\rho^* = p/(p+(1-p)\rho^*)$. For each expected discounted surplus level $S > S_1$, there exists an associated $q^* > 0$ such that efficient relational contracts can be sustained under spillover reporting for all $q \in [0, q^*]$.*

Proof. The proof follows from standard continuity arguments. For given c, p, and δ, let b^* be the minimum deferred bonus necessary to induce effort under credit reporting. Proposition 1 implies that $b^* < c/p$ when $-p/(1-p)+\rho^*\delta-p(1-\rho^*)\delta^2 > 0$, where $\rho^* = p/(p+(1-p)\rho^*)$. Now again normalize the agent's outside option $\underline{u}$ to be 0, and for each w and b, define the agent's value functions $V(\rho), V_s(\rho)$, and $V_e(\rho)$ as in Proposition 1. Recall that at w^* and b^*, we have $V_e(\rho^*) = V_s(\rho^*)$, $V_e(\rho^*) = 0$, and $b^* + \delta V(0) > 0$. Since $V_e(\rho^*) - V_s(\rho^*)$, $V_e(\rho^*)$, and $b + \delta V(0)$ all increase in b, it follows that for each $S > b^* \equiv S_0$, there exists a small enough $\varepsilon(S)$ such that one can find w and $b \in (b^*, S)$ such that

$$\begin{aligned} V_e(\rho^*) &> V_s(\rho^*) + 3\varepsilon; \\ V_e(\rho^*) &> 2\varepsilon; \\ b + \delta V(0) &> 2\varepsilon. \end{aligned}$$

Next, or each $q > 0$, let $V^q(\rho)$ be the agent's value function at the beginning of a period assuming that he has accepted the principal's offer. Let $V_e^q(\rho)$ be the agent's value function if he works this period and $V_s^q(\rho)$ be the agent's value function if he shirks. The value functions then satisfy the following:

$$\begin{aligned}
V^q(\rho) &= \max\{V_e^q(\rho), V_s^q(\rho)\}; \\
V_e^q(\rho) &= w - c + (p + (1-p)\rho)\max\{0, b + \delta V^q(\frac{p}{p+(1-p)\rho})\} \\
&\quad +(1 - p - (1-p)\rho)\max\{0, \delta V^q(\rho^*)\}; \\
V_s^q(\rho) &= w + (q + (1-q)\rho)\max\{0, b + \delta V^q(\frac{q}{q+(1-p)\rho})\} \\
&\quad +(1-\rho)\delta V^q(\rho^*)\max\{0, \delta V^q(\rho^*)\}.
\end{aligned}$$

It is clear that there exists a q^* such that for all $q \in [0, q^*]$,

$$\max\{|V^q(\rho) - V(\rho)|, |V_e^q(\rho) - V_e(\rho)|, |V_s^q(\rho) - V_s(\rho)|\} < \varepsilon \text{ for all } \rho \in [0,1].$$

As a result,

$$\begin{aligned}
& V_e^q(\rho^*) - V_s^q(\rho^*) \\
= \; & V_e^q(\rho^*) - V_e(\rho^*) + V_e(\rho^*) - V_s(\rho^*) + V_s(\rho^*) - V_s^q(\rho^*) \\
> \; & -\varepsilon + 3\varepsilon - \varepsilon \\
= \; & \varepsilon.
\end{aligned}$$

Similarly,

$$V_e^q(\rho^*) = V_e(\rho^*) + V_e^q(\rho^*) - V_e(\rho^*) > 2\varepsilon - \varepsilon = \varepsilon,$$

and

$$b + \delta V^q(0) = b + \delta V(0) + \delta V^q(0) - \delta V(0) > 2\varepsilon - \varepsilon = \varepsilon.$$

These three inequalities imply that for the chosen w and b, it is incentive compatible for the agent to accept the contract and exerts effort. ■

Corollary 1 states that if spillover reporting improves the sustainability of efficient relational contracts when $q = 0$, it also does so for small q. To see this, note that the minimum deferred bonus necessary for effort under spillover reporting when $q = 0$ is equal to S_1. When the expected discounted surplus S is larger than S_1, the principal can set a deferred bonus slightly larger than S_1 without reneging. With the new deferred bonus, the agent strictly prefers working over shirking when $q = 0$. It follows that for small enough q the agent also prefers working since his payoff from shirking is continuous in q. This implies that spillover reporting continues to improve the sustainability of efficient relational contracts when q is small.

Beyond the continuity result above, however, it is difficult to provide general conditions for spillover reporting to improve over the full revelation of signals. The reason is that when $q > 0$, the value function $V(\rho)$ no longer has an analytical solution and it becomes difficult to check multistage deviation. As a result, our second result uses numerical methods instead. We numerically compute the minimum deferred bonus required for effort both under spillover reporting and under full revelation of signals. In particular, we compute $V(\rho)$ for each deferred-bonus level b and then find the smallest deferred bonus level that sustains efficient effort.

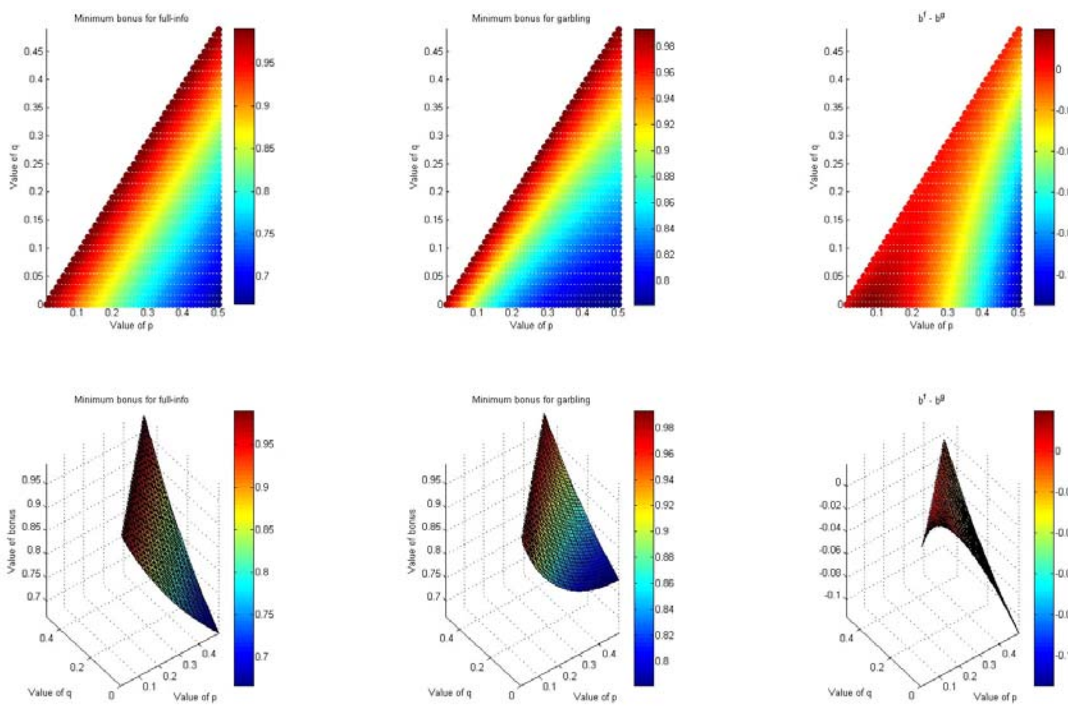

Figure 3: Numerical Simulation

Figure 3 reports our findings for discount factor equal to 0.9 and the cost of effort equal to 2. The two panels on the left (upper left and bottom left) depict the minimum deferred bonus necessary for sustaining effort under full revelation (b^f). The bottom-left panel is a 3D plot that illustrates the value of b^f for each $0 < q < p <$

0.5. The top-left panel is the corresponding thermograph that projects the 3D plot into a 2D graph by using colors to represent values. The colder colors reflect smaller values and the warmer colors reflect larger ones. Since b^f is equal to $c/(p-q)$, the colors become colder as p increases and become warmer as q increases. Moreover, all (p, q) pairs on the same negative 45-degree line has the same color.

Next, the two middle panels report the minimum deferred bonus for sustaining effort under spillover reporting (b^g), where g means that the reports are garbled from the signals. The colors again become colder as p increases and warmer as q increases, indicating that the minimum deferred bonus under spillover reporting also decreases with p and increases with q. Different from the two panels on the left, however, not all (p, q) pairs on the same negative 45 degree line has the same color. The (p, q) pairs with the same color are no longer straight lines and appear to have a slope larger than -1.

The two panels on the right report the differences in the minimum deferred bonuses required to sustain efficiency between the cases of full revelation and spillover reporting, namely, $b^f - b^g$. The thermograph in the upper-right panel makes it clear that there are values of (p, q) such that spillover reporting lowers the minimum deferred bonus to sustain effort ($b^f - b^g > 0$). The gains from spillover reporting concentrate on the bottom left region, where the values of p and q are smaller. Figure 4 marks the region of (p, g) in which spillover reporting lowers the minimum deferred bonus required to induce effort.

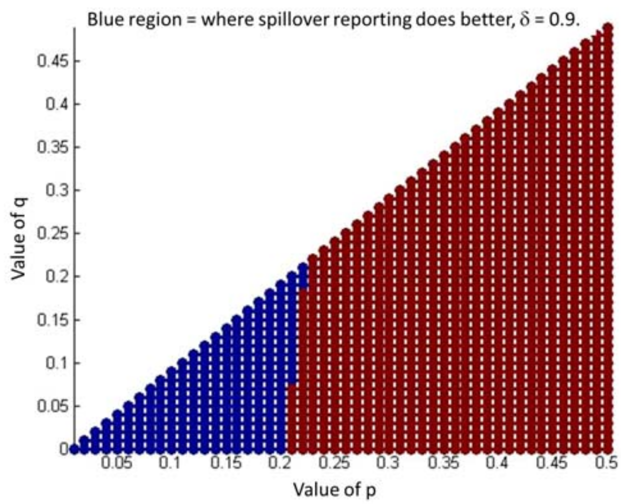

Figure 4: Region of (p,q) where spillover reporting enhances efficiency

7.3 Multiple Effort Levels

Let the agent's effort choice in period t be $e_t \in \{0, 1, 2\}$, and the associated effort cost be given by $c(0) = 0$, $c(1) = c_1 > 0$ and $c(2) = c_2 > c_1$. The output is binary: $Y_t \in \{0, y\}$, and

$$\Pr\{Y_t = y\} = \begin{cases} p & e_t = 2 \\ q & e_t = 1 \\ 0 & e_t = 0 \end{cases}.$$

where $1 > p > q > 0$.

We assume that

$$py - c_2 > qy - c_1 \geq \underline{u} + \underline{v} > 0,$$

so the joint surplus is maximized at $e = 2$, followed by $e = 1$, the outside options, and $e = 0$. Notice that we allow the relationship to be profitable even when the agent chooses $e = 1$. This assumption is not important for the analysis but as we will see below, it allows spillover reporting to help the relationship both in the extensive and intensive margins. For expositional convenience, define the discounted expected surplus as

$$S = \frac{\delta}{1-\delta}(py - c_2 - \underline{u} - \underline{\pi}).$$

We assume that the supervisor, and no other parties, observe the outputs. In other words, the supervisor's signal is exactly equal to the output.

Compared to the binary-effort case with $q > 0$, the model above essentially adds a lower level of effort ($e = 0$). As a result, if one can rule out that $e = 0$ is used in a relational contract, the result from the binary-effort model can be directly applied. The result below shows that spillover reporting can improve the efficiency of the relational contracts when effort costs are sufficiently convex.

Corollary 2: *For given c_1, c_2, p, q, and δ, let S_1 be the minimum expected discounted surplus for effort under spillover reporting in the binary-effort model. For any $S \in (S_1, \frac{c_2 - c_1}{p-q})$, efficient relational contracts are sustainable under spillover reporting when $(c_2 - c_1)/c_1 > M$ for some $M > 0$.*

Proof. Define $V(\rho)$ as the agent's value function at the beginning of a period assuming that he has accepted the principal's offer. Let $V_i(\rho)$ be the agent's value function if he puts in effort level of $i \in \{0, 1, 2\}$ this period. Normalize the agent's outside option $\underline{u}$ to be 0. For each base wage w and deferred bonus b, the value functions satisfy the following:

$$\begin{aligned}
V(\rho) &= \max\{V_0(\rho), V_1(\rho), V_2(\rho)\} \\
V_2(\rho) &= w - c_2 + (p + (1-p)\rho)\max\{0, b + \delta V(\frac{p}{p + (1-p)\rho})\} \\
&\quad + (1 - p - (1-p)\rho)\max\{0, \delta V(\rho^*)\}; \\
V_1(\rho) &= w - c_1 + (q + (1-q)\rho)\max\{0, b + \delta V(\frac{q}{q + (1-p)\rho})\} \\
&\quad + (1 - q - (1-q)\rho)\max\{0, \delta V(\rho^*)\}. \\
V_0(\rho) &= w + \rho \max\{0, b + \delta V(0)\} + (1-\rho)\max\{0, \delta V(\rho^*)\}.
\end{aligned}$$

It is clear that for each w and b, there is a unique set of value functions that satisfy the functional equations above.

Next, suppose b^* is the minimum deferred bonus for effort in the binary effort case and w^* is the associated base wage so that the agent's participation binds. Let $V_e(\rho)$, $V_s(\rho)$, and $V_b(\rho)$ be the agent's value functions associated with b^* and w^*, where we use the subscript b stands for the binary case. Now let $w = w^* + c_1$ and $b = b^*$. We show below that for sufficiently small c_1, we have $V_2(\rho) = V_e(\rho), V_1(\rho) = V_s(\rho), V(\rho) = V_b(\rho)$, and $V_0(\rho) = w + \rho(b + \delta V_b(0))$ for all ρ.

To do this, it suffices to show that the constructed value function satisfies the set of functional equations above. Given the properties of $V_e(\rho)$, $V_s(\rho)$, and $V_b(\rho)$, we only need to show that $V_1(\rho) \geq V_0(\rho)$ for all ρ. Notice that

$$\begin{aligned}
& V_1(\rho) - V_0(\rho) \\
= \; & (q + (1-q)\rho)(b + \delta V_b(\frac{q}{q + (1-p)\rho})) - \rho(b + \delta V_b(0)) - c_1 \\
> \; & (1-q)\rho(b + \delta V_b(0)) - c_1,
\end{aligned}$$

where the inequality uses the fact that V is increasing in ρ. Since $b + \delta V_b(0) > 0$ (by Proposition 1) and $b = b^*$ is independent of c_1, it is clear that for small enough c_1, $V_1(\rho) - V_0(\rho) > 0$ for all ρ.

The above implies that for small enough c_1, the agent is willing to choose $e = 2$ when $b = b^* = S_0$ and $w = w^* + c_1$. Finally, since $S > S_0$, the principal will not renege on the deferred bonus. This establishes that the stationary strategies with deferred bonus with $w = w^* + c_1$ and $b = b^*$ supports the efficient relational contracts. ■

Notice that $(c_2 - c_1)/c_1$ measures the convexity of effort cost since $c_2 - c_1$ is the marginal cost of effort from increasing $e = 1$ to $e = 2$ and c_1 is the marginal cost of effort from increasing $e = 0$ to $e = 1$. Corollary 2 therefore shows that when effort costs are sufficiently convex, spillover reporting can support the efficient relational contracts even if it is impossible to do so with perfect revelation of outputs. Notice that if the signals were revealed fully when $S \in (S_1, \frac{c_2 - c_1}{p-q})$, the agent would either choose a lower level of effort ($e = 1$) or forgo the relationship by taking his outside option. In the former case, spillover reporting improves the relationship in the intensive margin by making it more efficient. In the later case, spillover reporting improves the relationship in the extensive margin by sustaining an efficient relationship that would fail to start.

The intuition for Corollary 2 is as follows. When the cost function is sufficiently convex, the gain of deviating from $e = 1$ to $e = 0$ is small relative to the gain of deviating from $e = 2$ to $e = 1$. It follows that if the agent does not gain from deviating to $e = 1$, he will not gain from deviating to $e = 0$. Therefore, if spillover reporting helps sustain efficient relational contracts with binary effort, it can also help here when the effort cost is sufficiently convex.

This intuition suggests that spillover reporting may have wider applicability. For example, when there are multiple effort levels, ruling out profitable local deviations is sometimes enough for ruling out global deviations. Since checking local deviations requires essentially comparing payoffs from two (adjacent) effort levels, our result on spillover reporting can be applied to relax constraints that prevent local deviation. This suggests that spillover reporting can improve the sustainability of relational contracts for more general effort cost structures.

7.4 Multiple Agents

Suppose there are n identical agents, and the production is independent across them. For each agent i, his output $Y_i \in \{0, y\}$. The probability that $Y_i = y$ is p if the agent i puts in effort, and is 0 otherwise. We assume that $py - c > \underline{u} + \underline{v} > 0$, where c is the agent's cost of effort. In addition, the supervisor, and no other agents, observe the outputs. In other words, this is essentially an $n-$fold model of that in Section 4 with $p = p_0$. Below, we study two types of relationships. The first is the commonly used, but suboptimal, independent relationships. In the second one, the relationships are interdependent.

7.4.1 Independent Relationships

In this case, the principal deals with each of the agent separately. In other words, the relationships are independent of each other. To derive the condition for sustaining an efficient n-agent relationship with full signal revelations, note that agent i will put in effort if

$$b_i \geq c/p.$$

The maximal reneging temptation of the principal occurs in the case when all agents receive a bonus. The principal's gain from reneging is given by $\sum_{i=1}^{n} b_i$.

Next, the principal's surplus is given by

$$\frac{\delta n}{1-\delta}\left(py - c - \underline{u} - \underline{\pi}\right) \equiv \frac{\delta n s\left(1\right)}{1-\delta}.$$

It follows that for the principal not to renege on the bonuses, the non-reneging constraint is given by

$$\frac{\delta n s\left(1\right)}{1-\delta} \geq \sum_{i=1}^{n} b_i.$$

Combining this with the agent's incentive constraint, we obtain that the condition for sustaining the efficient relational contract with full signal revelation is given by

$$\frac{\delta ns(1)}{1-\delta} \geq \sum_{i=1}^{n} b_i \geq nc/p,$$

or equivalently,

$$\frac{\delta ns(1)}{1-\delta} \geq c/p.$$

This is the same condition as the single-agent case. Note that we showed above that this condition is necessary, but clearly this condition is also sufficient.

Next, to obtain the condition for sustaining an efficient n-agent relationship with spillover reporting, let b_g be the deferred bonus necessary to induce effort under spillover reporting (see the proof of Proposition 1, part (i) for the expression of b_g). Here, the subscript g reflects that the reports are garbled from the signals. This implies that each agent i puts in effort if

$$b_i \geq b_g.$$

As before, the non-reneging constraint of the principal is given by

$$\frac{\delta ns(1)}{1-\delta} \geq \sum_{i=1}^{n} b_i.$$

Combining this with the agent's incentive constraint, we obtain that the condition for sustaining the efficient (independent) $n-$agent relational contract with spillover reporting is

$$\frac{\delta ns(1)}{1-\delta} \geq \sum_{i=1}^{n} b_i \geq nb_g,$$

or equivalently,

$$\frac{\delta s(1)}{1-\delta} \geq b_g.$$

Again, this is the same condition as the single-agent case, and in addition, it is clear that this condition is also a sufficient one. It follows that under the independent relationships, spillover reporting helps with the $n-$agent case if and only if it helps with the single-agent case.

7.4.2 Interdependent Relationships

When the signals are fully revealed, the independent relationships considered above are suboptimal. Levin (2002) implies that the optimal relational contract takes the following form. In each period, there's a bonus pool of $B^p(n)$ to be shared by agents with good signals. With probability $(1-p)^n$, however, no agent receives a good signal, and in this case, the bonus pool is not paid out.

We now derive the agent's incentive constraint under the optimal $n-$agent relational contract (with full revelation of signals). Since the agents are symmetric under the relational contract, we suppress the subscript and let $b^p(n)$ be the expected bonus of the agent if his signal is good. The agent will put in effort if

$$b^p(n) \geq \frac{c}{p}.$$

Notice that $b^p(n)$ and $B^p(n)$ are linked through the following equation:

$$pb^p(n) = \frac{1}{n}\left(1-(1-p)^n\right) B^p(n).$$

The left-hand side is the agent's expected bonus. And the right-hand side is another way to calculate it: the expected total bonus is $\left(1-(1-p)^n\right) B^p(n)$, where $1-(1-p)^n$ is the probability that the bonus pool $B^p(n)$ is paid out, and the agent gets $1/n$ of it in expectation. The equation implies that the agent's incentive constraint can be written as

$$B^p(n) = \frac{npb^p(n)}{1-(1-p)^n} \geq \frac{np}{1-(1-p)^n}\frac{c}{p}.$$

Next, the principal's non-reneging condition is given by

$$\frac{\delta ns(1)}{1-\delta} \geq B^p(n).$$

And combining this with the agent's incentive constraint, we obtain that the necessary condition for sustaining an efficient $n-$agent relational contract with full signal revelation is

$$\frac{\delta s(1)}{1-\delta} \geq \frac{p}{1-(1-p)^n}\frac{c}{p}. \tag{n-perfect}$$

Note that this condition is also a sufficient condition; see Levin (2002).

We now consider the following $n-$agent relational contract with spillover reporting. For each agent, the supervisor sends the report according to the spillover reporting described in Section 4. As in Section 4, the principal formally pays out no end-of-period bonus. Instead, "the deferred-bonus pool" is paid out at the beginning of the next period, and it is divided equally among the agents with good reports. Each agent observes his own report at the end of each period and also a public signal for whether at least one agent has a good report. In addition, each agent observes the contracts offered by the principal to all agents at the beginning of each period. All agents take their outside options forever if any parties ever publicly deviates.

Let $\delta_0(n)$ be the smallest discount factor for an efficient $n-$agent relational contract under full revelation of signals, and $\delta_g(n)$ the corresponding discount factor under spillover reporting. Corollary 3 below provides the condition for when spillover reporting improves over full revelation of signals.

Corollary 3 *$\delta_g(n) < \delta_0(n)$ if and only if*

$$\frac{(p+(1-p)\rho^*)\left(1-(1-p)^n\right)}{\left(1-(1-(p+(1-p)\rho^*))^n\right)}\frac{1}{(p+(1-p)\rho^*)-\rho^*\frac{1-(1-p)\delta\rho^*}{1-\delta^2 p(1-\rho^*)}} < 1.$$

Proof. At $\delta = \delta_g(n)$, denote $w(n)$ and $b^g(n)$ as the agent's base wage and expected bonus (conditional on a good report) under spillover reporting. Let $B^g(n)$ be the corresponding deferred-bonus pool. Notice that $b^g(n)$ and $B^g(n)$ are linked through

$$(p+(1-p)\rho^*)\, b^g(n) = \frac{1}{n}\left(1-(1-(p+(1-p)\rho^*))^n\right) B^g(n),$$

or alternatively,

$$B^g(n) = \frac{n\,(p+(1-p)\rho^*)\, b^g(n)}{\left(1-(1-(p+(1-p)\rho^*))^n\right)}.$$

This implies that the principal's non-reneging constraint can be written as

$$\frac{\delta n s(1)}{1-\delta} \geq B^g(n) = \frac{n\,(p+(1-p)\rho^*)\, b^g(n)}{\left(1-(1-(p+(1-p)\rho^*))^n\right)}.$$

Next, we determine the value of $b^g(n)$. As in Proposition 1, each agent's expected future payoff is determined by his belief of a high signal in the previous period, and the value functions satisfy

$$\begin{aligned}
V(\rho) &= \max\{V_e(\rho), V_s(\rho)\}; \\
V_e(\rho) &= w(n) - c + (p + (1-p)\rho) \max\left\{0, b^g(n) + \delta V\left(\frac{p}{p + (1-p)\rho}\right)\right\} \\
&\quad + (1 - p - (1-p)\rho) \max\{0, \delta V(\rho^*)\}; \\
V_s(\rho) &= w(n) + \rho \max\{0, b^g(n) + \delta V(0)\} + (1-\rho) \max\{0, \delta V(\rho^*)\}.
\end{aligned}$$

Note that this is exactly the same set of functional equations as that in Proposition 1 (with $p_0 = 0$). As a result, we obtain that

$$c = \left((p + (1-p)\rho^*) - \rho^* \frac{1 - (1-p)\delta\rho^*}{1 - \delta^2 p (1-\rho^*)}\right) b^g(n).$$

Combining the above with the principal's non-reneging constraint, we obtain that the condition for sustaining the efficient relational contract is given by

$$\frac{\delta s(1)}{1-\delta} \geq \frac{(p + (1-p)\rho^*)}{(1 - (1 - (p + (1-p)\rho^*))^n)} \frac{c}{(p + (1-p)\rho^*) - \rho^* \frac{1 - (1-p)\delta\rho^*}{1 - \delta^2 p(1-\rho^*)}}. \quad \text{(n-garbling)}$$

Comparing this condition to the condition for full revelation of signals, we get that spillover reporting is an improvement if and only if

$$\begin{aligned}
&\frac{(p + (1-p)\rho^*)}{(1 - (1 - (p + (1-p)\rho^*))^n)} \frac{c}{(p + (1-p)\rho^*) - \rho^* \frac{1 - (1-p)\delta\rho^*}{1 - \delta^2 p(1-\rho^*)}} \\
\leq\ & \frac{p}{1 - (1-p)^n} \frac{c}{p}. \qquad \text{(n-agents)}
\end{aligned}$$

This finishes the proof. ∎

Notice that when $n = 1$, this condition in Corollary 3 is equivalent to that in part (i) of Proposition 1 (with $p_0 = 0$). As n increases, it can be checked that the left-hand side increases with n, so this condition becomes more difficult to satisfy. It can be shown, however, that for all n, there exists ranges of p such that spillover reporting helps to sustain relationship. Moreover, for all n, the minimal surplus for an efficient relational contract under full revelation is about δ times more as that under spillover reporting when as p goes to 0.

Corollary 4 *Let $S_0(p, \delta, n)$ be the minimal surplus necessary sustain an efficient* *relational contract with full revelation of signal. Let $S_g(p,\delta,n)$ be the corresponding surplus with spillover reporting. For all n,*

$$\lim_{p\to 0}\frac{S_0(p,\delta,n)}{S_g(p,\delta,n)}=1+\delta.$$

Proof. From the condition on the sustainability of efficient relational contracts, we have

$$\frac{S_0(p,\delta,n)}{S_g(p,\delta,n)}=\frac{(p+(1-p)\rho^*)-\rho^*\frac{1-(1-p)\delta\rho^*}{1-\delta^2 p(1-\rho^*)}}{p}\frac{A(p,n)}{B(p,n)},$$

where $A(p,n)=\frac{p}{1-(1-p)^n}$ and $B(p,n)=\frac{(p+(1-p)\rho^*)}{(1-(1-(p+(1-p)\rho^*))^n)}$, and recall that $\rho^*=p/(p+(1-p)\rho^*)$.

Now notice that we have $p=\frac{(\rho^*)^2}{1-\rho^*+(\rho^*)^2}$, so $\lim_{p\to 0}\rho^*=0$, and $\lim_{p\to 0}\frac{(\rho^*)^2}{p}=1$. Now

$$\lim_{p\to 0}\frac{S_0(p,\delta,n)}{S_g(p,\delta,n)}=\lim_{p\to 0}\frac{(p+(1-p)\rho^*)-\rho^*\frac{1-(1-p)\delta\rho^*}{1-\delta^2 p(1-\rho^*)}}{p}\frac{\lim_{p\to 0}A(p,n)}{\lim_{p\to 0}B(p,n)}.$$

And for each n, $\lim_{x\to 0}\frac{x}{1-(1-x)^n}=\frac{1}{n}$, so $\lim_{p\to 0}A(p,n)=\lim_{p\to 0}B(p,n)=\frac{1}{n}$. In addition,

$$\begin{aligned}&\lim_{p\to 0}\frac{(p+(1-p)\rho^*)-\rho^*\frac{1-(1-p)\delta\rho^*}{1-\delta^2 p(1-\rho^*)}}{p}\\ =\;&1+\lim_{p\to 0}\frac{\rho^*}{p}\frac{-p+(1-p)\delta(\rho^*-\delta p(1-\rho^*))}{1-\delta^2 p(1-\rho^*)}\\ =\;&1+\delta.\end{aligned}$$

Together, we then have

$$\lim_{p\to 0}\frac{S_0(p,\delta,n)}{S_g(p,\delta,n)}=1+\delta.$$

■

7.5 Collusion

Suppose the principal can offer the supervisor the following history-dependent contract at the beginning of each period t,

$$w_t^s+\alpha_t Y_0,$$

where α is the share of the public output that goes to the supervisor. To simplify the analysis, we assume that $w_s^t = -\alpha_t p_0 y_0$ so that the supervisor receives 0 expected payoff in equilibrium. We also assume that α share of the output directly accrues to the supervisor, and this allows us to focus on the relational contract between the principal and the agent. Now consider the following type of *stationary collusion*. The agent offers the supervisor a payment of $t_b > 0$ in each period. In return, the supervisor always sends a good report about the agent.

Now define $p_g \equiv p_s + (1-p_s)\rho^*$ be the probability that the agent receives a deferred bonus under spillover reporting (when he works and does not engage in stationary collusion). Our result below shows that when $\delta < 1/(2-p_g)$, if spillover reporting can help sustain the efficient relational contract, then the principal can prevent stationary collusion by giving enough share of the output to the supervisor.

Corollary 5: *Suppose spillover reporting helps sustain the relational contract, i.e.,* $1-p_0-\delta\rho^* > 0$ *and* $-p_s/(1-p_s)+\delta(\rho^*-\delta p(1-\rho^*)) > 0$. *Now if* $\delta < 1/(2-p_g)$, *then there exists* $\alpha^* < 1$ *such that for all* $\alpha > \alpha^*$, *the supervisor and the agent will not engage in stationary collusion.*

Proof. Let b_g be the lowest deferred bonus necessary to sustain effort under spillover reporting. From the proof of Proposition 1, we have

$$b_g = \frac{c}{p_0 + (1-p_0)(p+(1-p)\rho^*) - \rho^* \frac{1-(1-p_0)(1-p)\delta\rho^*}{1-\delta^2 p(1-\rho^*)}}.$$

To prevent stationary collusion, it suffices to show that the joint payoff between the supervisor and the agent without collusion is smaller than their joint payoff under collusion. The joint payoff without collusion is 0 by design.

Now if the agent colludes with the supervisor, he will choose to shirk. This is because if the agent always receives a good report, he always receives a deferred bonus, and therefore, does not have an incentive to put in effort. Given the agent shirks, the supervisor's payoff is given by $t_b - \alpha p_0 y_0$ per period. For the agent, his payoff under collusion is given by

$$-t_b + c + (1-p_g)\, b_g,$$

where recall that $p_g = p_s + (1-p_s)\rho^*$ is the probability that a deferred bonus is paid out to the agent under spillover reporting when the agent works.

It follows that the joint payoff of the agent and the supervisor is given by $-\alpha p_0 y_0 + c + (1 - p_g) b_g$. As a result, they will not collude if

$$\alpha p_0 y_0 - c \geq (1 - p_g) b_g. \qquad \text{(No-collusion condition)}$$

It follows that a large enough α can be chosen as long as

$$s(1) = p_0 y_0 - c \geq (1 - p_g) b_g.$$

To see that this condition is satisfied when $\delta < 1/(2 - p_g)$, recall that for a relational contract to be sustainable under spillover reporting, the principal's non-reneging condition is given by

$$\frac{\delta}{1-\delta} s(1) \geq b_g.$$

It follows that

$$s(1) \geq \frac{1-\delta}{\delta} b_g > (1 - p_g) b_g,$$

where the last inequality follows because $\delta < 1/(2 - p_g)$. ■

The main idea for why collusion will not take place is as follows. If the collusion occurs so that the supervisor always sends out good reports, the agent will shirk. This will then hurt the supervisor's payoff, and the damage is larger when the supervisor receives a bigger share of the output. For a large enough α, the cost from agent shirking is sufficiently large that the supervisor and the agent will not engage in collusion.

7.6 General Reporting Rules

Below, we show that any reporting rule with a predetermined restart date cannot help sustain efficient relational contract. Proposition 2 in the main text follows directly from it. We also provide a proof for Proposition 3.

Proposition 2': *Let (U_t, Π_t) be the expected discounted payoffs of the agent and the principal evaluated at time t. Suppose (4) fails. For all t, if there exists a predetermined $t' \geq t$ such that $(U_{t'}, \Pi_{t'})$ are independent of $h_{t'}$, then $e_t = 0$.*

Proof. Note that in our setting, the feasible surplus of the game S cannot be raised by any reporting rule. Suppose $(U_{t'}, \Pi_{t'})$ are independent of $h_{t'}$. Then $e_{t'-1}$ is solely motivated by $b_{t'-1}$, and since $b_{t'-1} \leq S < c/(p-q)$, it must hold that $e_{t'-1} = 0$. Given that $e_{t'-1} = 0$ regardless of $h_{t'-1}$ and that (4) fails, the expected sum of bonuses $E(b_{t'-2} + \delta b_{t'-1})$ should also be no larger than S. This implies that $e_{t'-2} = 0$ as well. By induction, $e_\tau = 0$ for all $\tau \in \{1, 2, ..., t'\}$. ■

Proposition 3: *Let $s(1)$ be the expected future of the relationship per period. For any reporting rule to sustain the efficient relational contract, one must have*

$$\frac{\delta s(1)}{1-\delta} \geq \sqrt{4p(1-p)}\frac{c}{p}.$$

In particular, full revelation of signals is the optimal reporting rule when $p = 1/2$.

Proof. Define $S = \delta s(1)/(1-\delta)$. When the signals are perfectly revealed, the necessary and sufficient condition for sustaining an efficient relational contract is given by

$$\frac{c}{p} \leq S.$$

We want to show that if $S < \sqrt{4p(1-p)}\frac{c}{p}$, it is impossible to construct an equilibrium in which the agent puts in effort. In particular, using standard argument as in Fuchs (2007), it suffices to show that there does not exist an equilibrium in which the agent always puts in effort (unless the relationship is terminated).

Consider an arbitrary information partition process. Pick one information set (h^t). Use x to denote the possible states within the information set. One interpretation of x is some output realizations y^t that falls into h^t.

Let $V(x)$ be the agent's continuation payoff in state x after e_{t+1} is put in but before y_{t+1} is realized and W_{t+1} is paid out. Let $V(x_i)$ be the agent's continuation payoff in state x after e_{t+1} is put in, y_{t+1} is realized but before W_{t+1} is paid out. Within each state x, we have $x_i \in \{x_y, x_0\}$, where x_y denotes that $Y_t = y$ is realized following x, and x_0 denotes that $Y_t = 0$ is realized.

Note that

$$V(x) = V(x) + p(V(x_y) - V(x)) + (1-p)(V(x_0) - V(x)).$$

And since the output Y_t is independent of the past state, we have $Cov(V(x_i) - V(x), V(x)) = 0$.

To induce effort, we need

$$E_x[V(x_y) - V(x_0)] = E_x[V(x_y) - V(x)] - E_x[V(x_0) - V(x)] \geq \frac{c}{p}.$$

This helps give a lower bound for $Var(V(x_i))$. In particular,

$$\begin{aligned}
Var(V(x_i)) &= Var(V(x)) + Var(V(x_i) - V(x)) \\
&= Var(V(x)) + E_y[Var(V(x_i) - V(x))|Y] \\
&\quad +Var(E_x[V(x_i) - V(x)|Y]) \\
&\geq Var(V(x)) + E_y[Var(V(x_i) - V(x))|Y] \\
&\quad +p(1-p)(\frac{c}{p})^2 \\
&\geq Var(V(x)) + p(1-p)(\frac{c}{p})^2,
\end{aligned}$$

where the first line follows because $Cov(V(x_i)-V(x), V(x)) = 0$, the second line uses the variance decomposition formula, the third line follows because $E_x[V(x_i)-V(x)|Y]$ is a binary value ($Y \in \{0, y\}$) such that with probability p its value is $E_x[V(x_y)-V(x)]$ and with probability $1-p$ its value is $E_x[V(x_0) - V(x)]$, and $E_x[V(x_y) - V(x)] - E_x[V(x_0) - V(x)] \geq c/p$.

Now let's provide an upper bound for $Var(V(x_i))$. Suppose a public report $s(x_i)$ will be sent out after state x_i. Let $b(s)$ be the bonus paid out to the agent (at the end of the period) following report s. This allows us to write

$$V(x_i) = b(s(x_i)) + \delta V_{s(x_i)}(x_i),$$

where $V_{s(x_i)}(x_i)$ is the continuation payoff of x_i, which goes to the information set by report $s(x_i)$.

Note that for the principal to be willing to pay the bonus, we must have

$$\max_s\{b_s + \delta E_{x_i}[V_s(x_i)|s]\} - \min_s\{b_s + \delta E_{x_i}[V_s(x_i)|s]\} \leq S.$$

Because otherwise the expected payoff of the principal following some report will be below his outside option.

Decomposing the variance on the reports, we have

$$\begin{aligned} Var(V(x_i)) &= Var(E[b_s + \delta V_s(x)|s]) + E[Var(b_s + \delta V_s(x_i)|s)] \\ &\le \frac{1}{4}S^2 + \delta^2 E[Var(V_s(x_i)|s)]. \end{aligned}$$

Now combining the upper and lower bound for $Var(V(x_i))$, we get that

$$\frac{1}{4}S^2 + \delta^2 E[Var(V_s(x_i)|s)] \ge Var(V(x)) + p(1-p)(\frac{c}{p})^2,$$

or equivalently,

$$E[Var(V_s(x_i)|s)] \ge \frac{1}{\delta^2}(Var(V(x)) + p(1-p)(\frac{c}{p})^2 - \frac{1}{4}S^2).$$

Now if $4p(1-p)(\frac{c}{p})^2 > S^2$, the inequality above implies that

$$E[Var(V_s(x_i)|s)] > \frac{1}{\delta^2}(Var(V(x)).$$

In particular, there will be one information set (associated with a signal) whose variance exceeds $\frac{1}{\delta^2}(Var(V(x))$. Now we can perform the same argument on this new information set, and we can construct a sequence of information set whose variance approaches infinity. This leads to a contradiction.

Therefore, to sustain an efficient relational contract, one must have

$$S^2 \ge 4p(1-p)(\frac{c}{p})^2.$$

It follows that when $p = 1/2$, this condition becomes $S \ge \frac{c}{p}$, which is exactly the condition for sustaining the efficient relational contract under full revelation of signals. This shows that full revelation of signals is the optimal reporting strategy when $p = 1/2$. ■