Abstract

We explore subjective performance reviews in long-term employment relationships. We show that firms benefit from separating the task of evaluating the worker from the task of paying him. The separation allows the reviewer to better manage the review process, and can therefore reward the worker for his good performance with not only a good review contemporaneously, but also a promise of better review in the future. Such reviews spread the reward for the worker’s good performance across time and lower the firm’s maximal temptation to renege on the reward. The manner in which information is managed exhibits patterns consistent with a number of well-documented behavioral biases in performance reviews.

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1 Introduction

Performance reviews are pervasive in modern labor markets.\footnote{According to a 2013 survey conducted by the Society for Human Resource Management, 94\% of organizations conduct performance appraisal. (Source: http://blog.blr.com/2013/07/2013-performance-management-survey-infographic.html)} While these reviews help collect information about worker performance, they are typically subjective, and consequently, contain inaccuracies and biases. A small but growing literature has studied the difficulties that arise when subjective evaluations are used, and shown that they constrain the efficiency of the relationship.\footnote{See, for example, Levin (2003), MacLeod (2003), Fuchs (2007), Chan and Zheng (2011), and Maestri (2012).} A feature of this literature is that the organizational structure is taken as given: the entity that carries out the review—the principal—also incurs the cost of compensation. In practice, however, “in most organizations agency relationships are multi-layered” (Prendergast and Topel (1993)) and performance reviews are typically carried out by supervisors, and the compensation decisions are instead made by the top of the organizations, using the reviews as an input.

Motivated by this observation, we study the organizational response to subjective performance reviews in long-term employment relationships. We show that the firm benefits from separating the task of evaluating the worker from the task of paying him. The separation allows the reviewer to better manage the information flow and increase the efficiency of the organization. Moreover, by managing information strategically, the reviewer exhibits review patterns that are consistent with a number of well-documented behavioral biases in performance reviews.

In particular, we follow the literature on subjective performance evaluations by modeling long-term employment relationships as relational contracts, where firms motivate workers using discretionary bonuses; see Malcomson (2013) for a review of relational contracting models. The key condition for sustaining a relational contract is that the firm’s maximal reneging temptation, the maximal bonus it needs to pay, cannot exceed the future surplus of the relationship. To better sustain the relationship, one might be tempted to reduce the maximal bonus by spreading it across periods. Simply spreading the bonus across periods, however, is ineffective because when the firm is asked to pay a bonus, it also anticipates that future bonuses will be paid, leaving the firm’s reneging temptation unchanged. When the worker’s perfor-
mances are publicly observed, Levin (2003) shows that optimal relational contracts are stationary.

Our central result is that by managing the review process, the firm can more effectively motivate the worker by easing the tension between the need to motivate the worker by offering a bonus and the temptation to renege on it once the performance is delivered. Specifically, the entity that pays the bonus—the owners of the firm—should be different from the entity that provides performance review—the supervisors or the human resource management departments. Such separation allows the supervisor to strategically manage information available to the owner of the firm.

By strategically managing the information, the supervisor rewards the worker for his good performance with not just a good review, and thus a bonus in the current period, but also a promise of better review in the future. In other words, the worker’s continuation payoff increases after a good performance. This type of review spreads the reward for the worker’s good performance across time, and therefore, makes the current bonus less sensitive to actual performance. By doing so, the supervisor helps reduce the maximal bonus the firm must pay out in any given point of time.

An implication of our result is that when relational contracts are used, the celebrated Informativeness Principle (Holmstrom 1979) fails. In particular, it is crucial that the firm does not observe the worker’s performance. Otherwise, following a good performance of the worker, the firm will again anticipate that more bonus will be paid in the future, leaving its reneging temptation unchanged. By restricting the information available to the firm, the supervisor makes the firm uninformed about whether the good review is for past or current good performance. The loss of information allows the firm’s reneging temptation to be spread across time, and therefore helps sustain the relational contract.

Several literatures have identified and documented a variety of biases in performance evaluations. One of the most common form of biases is leniency bias, in which good reviews can be given for poor performance (e.g., Holzbach, 1978). Another frequently documented bias is the spillover effect, in which the worker’s current period performance is evaluated in part based on his past performances (e.g., Bol and Smith 2011). See, for example, Murphy et al. (1985) and Nisbett et al (1977) in psychology, Blanchard et al. (1986) and Bol and Smith (2011) in accounting, Pretz et al. (1992) and Jacobs et al. (1985) in management, and Prendergast (1999) and Prendergast and Topel (1993) in economics.
Smith, 2011). The existing literature has assumed that these biases are detrimental (see Rynes et al. 2005 for a review). Our analysis suggests, however, these “biases” may reflect strategic HR management practice that enhances efficiency.

A key feature of our review process is that the supervisor promises good reviews in the future for good performance in the past, giving rise directly to the spillover effect. This feature also implies that the frequency of good reviews is higher than that of good performance, leading to leniency bias. Viewed in isolation, these biases weaken the worker’s incentives and hurt the relationship. But viewed in a longer horizon, these “biases” may enhance efficiency by spreading the reward of the agent over time. Incidentally, in the only empirical analysis of effects of performance appraisal biases that we are aware of, Bol (2011) found that leniency bias has a positive effect on the employee’s future performance.

Our paper contributes to two strands of the literature. First, it contributes to the theoretical works that explore the relationship between the information structure and efficiency. Within this literature, Kandori (1992) shows that garbling signals within periods weakly decreases efficiency in repeated games with imperfect public monitoring. Abreu, Milgrom, and Pearce (1992) and Fuchs (2007) have shown that reducing the frequency of information release can benefit the relationship. Kandori and Obara (2006) show that when signals do not have full support, the use of private (mixed) strategies can give rise to equilibria that are more efficient. In our model, the principal’s action is publicly observed, so the use of mixed strategy does not help relax the incentive constraints of the players by better detecting deviations.

More relatedly, Fuchs (2007) has shown that the principal benefits from withholding his private information from the agent. Here we emphasize that the supervisor helps the relationship by withholding information from both the principal and the agent, possibly forever. Rather than minimizing the amount of surplus burnt under subjective evaluations, we focus on how to lower the discount factor necessary for achieving efficiency. Under the review process we propose, the discount factor for sustaining the efficient relational contract is lower than that when the supervisor’s private information becomes publicly observed.

The second strand of literature this paper contributes to studies the use of external instruments to increase the efficiency of relational contracting. Baker, Gibbons, and Murphy (1994) show that the use of an explicit contract can help increase the
efficiency of the relationship by reducing the gain from reneging. They also show, however, that explicit contracts can crowd out relational contracts by improving players’ outside options. Rayo (2007) examines the role of ownership structure in sustaining relationships. He shows that when the actions of players are unobservable (and the First Order Approach is valid), the optimal ownership shares should be concentrated. When the actions are observable (so that the First Order Approach is invalid), the optimal ownership shares should be diffused. The external instrument explored in our paper is the use of information flows. Our result implies that the efficiency of the relationship can be enhanced with less information (in the sense that signals are intertemporally garbled). This suggests that strategically using intermediaries to manipulate information can increase the efficiency of the relationship. Finally, Deb, Li, and Mukherjee (2014) explore how peer evaluations can improve efficiency in relational contracts. They show that peer evaluations are useful but should be used sparingly. In contrast to our approach, they use Perfect Public Equilibrium as the solution concept, so peer evaluations do not have spillover effects on future compensations.

The rest of the paper is organized as follows: We set up the model in Section 2 and present our main results in Section 3. Section 4 discusses the robustness of our results and examines properties of general reporting rules. Section 5 concludes.

2 Setup

Time is discrete and indexed by \( t \in \{1, 2, ..., \infty\} \).

2.1 Players and Production

There is one principal, one agent, and one supervisor. The players are all risk-neutral and infinitely lived, and they all have a common discount factor \( \delta \). The agent’s per-period outside option is given by \( u \); the principal’s per-period outside option is \( \pi \). To focus on the effect of information revelation, we assume that the supervisor is a nonstrategic player whose payoff is the same whether he stays in or exits the relationship and is normalized to 0.

If the principal and the agent engage in production together in period \( t \), the agent chooses effort \( e_t \in \{0, 1\} \). If the agent works, his effort cost is \( c(1) = c \). If he shirks,
the effort cost is \( c(0) = 0 \). The agent's effort choice generates a stochastic output \( y_t \in \{0, y\} \) for the principal. The output is more likely to be high if the agent works:

\[
\Pr\{y_t = y | e_t = 1\} = p_0 > \Pr\{y_t = y | e_t = 0\} = q_0.
\]

Let \( f(1) = p_0y \) be the expected output if the agent works and \( f(0) = q_0y \).

The production function is commonly used in the literature. The binary-output assumption can be relaxed under MLRP. With multiple output levels, there will be a cutoff such that the bonus is paid only when the output is above the cutoff (Levin, 2003). This cutoff divides the output level into two groups and essentially transforms the model into one with binary output. The binary-effort assumption, however, is more restrictive and is made for analytical convenience. In Subsection 4.1, we show that the main result of the paper continues to hold with three effort levels when effort costs are sufficiently convex, and we discuss how the model can be generalized. Define \( s(1) \equiv f(1) - c - \pi - u \) as the per-period joint surplus when the agent works, and similarly, define \( s(0) \equiv f(0) - \pi - u \). We assume that the relationship has a positive surplus if and only if the agent works: \( s(1) > 0 > s(0) \).

### 2.2 Timing and Information Structure

At the beginning of period \( t \), the principal offers to the agent a history-dependent compensation package that consists of a base wage \( w_t \) and a nonnegative end-of-period bonus \( b_t \). As in Levin (2002), we restrict the end-of-period bonus to be nonnegative to simplify the exposition, and this does not affect the set of equilibrium payoffs sustainable by relational contracts. We emphasize the timing of paying \( b_t \) because reward is not necessarily paid out at the end of period and may instead be paid through a higher \( w_{t+1} \). The agent chooses whether to accept the offer: \( d_t \in \{0, 1\} \). If the agent rejects it \((d_t = 0)\), all players take their outside options for the period.

If the agent accepts, he is paid wage \( w_t \) and chooses \( e_t \). The agent’s effort is his private information. Given the agent’s effort choice, the supervisor obtains a private stochastic signal \( y^s_t \in \{L, H\} \), where the superscript \( s \) indicates that the private signal can be subjective. We assume that the signal is independent of the actual output \( y_t \) conditional on effort, and it is more likely to be high if the agent works:

\[
\Pr\{y^s_t = H | e_t = 1\} = p > \Pr\{y^s_t = H | e_t = 0\} = q.
\]
Upon observing the signal, the supervisor sends a public report $s_t \in S$, where $S$ is the set of possible reports. The supervisor’s report can depend on all the information available to him, and the supervisor can also randomize on his reports.

Following the supervisor’s report, the output $y_t$ is then realized and observed by all three parties. Denote $\phi_t = (s_t, y_t)$ as the publicly observable performance outcomes and $\Phi_t$ as the set of possible realizations of $\phi_t$. After observing $\phi_t$, the principal decides whether to pay out the end-of-period bonus $b_t$. Denote $W_t = w_t + b_t$ as the agent’s total compensation for the period.

Before describing the strategy and equilibrium concepts, we make a few remarks on the information structure and the form of compensation. In terms of the information structure, the key to our analysis is the supervisor’s reports. We emphasize that since the supervisor’s signals are his private information, he does not need to report them truthfully. He can, for example, delay reporting his signals and may never reveal them. For future discussions, the two examples below are useful.

**Example 1: T-period Truthful Reporting**

The supervisor reveals his private signals every $T$ periods. Formally, let $S = \{L, H\}^T \cup \{N\}$, where $N$ stands for no information. When the period $t = nT$ for some $n \in \mathbb{N}$, the supervisor reports his signals in the previous $T$ periods $s_t = (y_{(n-1)T+1}^s, \ldots, y_{nT}^s)$. Otherwise, $s_t = N$.

**Example 2: Two-Strike-Out Reporting**

The supervisor essentially sends a bad report when there are two consecutive low signals. Formally, let $S = \{G, B\}$, where $G$ stands for good performance and $B$ for bad performance. In period $t > 1$, the supervisor reports $s_t = G$ if his signal either this or last period ($y_t^s$ or $y_{t-1}^s$) is equal to $H$ and reports $s_t = B$ otherwise. When $t = 1$, the supervisor reports $s_1 = G$ if $y_1^s = H$. If $y_1^s = L$, the supervisor reports $G$ with some probability $\rho$ and reports $B$ with probability $1 - \rho$.

In addition to the reports, the outputs are also publicly observed each period. The outputs serve two roles. First, they play a productive role by providing benefits to the principal. Second, they have an informational role by allowing the principal to infer the effort of the agent. While the productive role is clearly necessary for the analysis—it determines the principal’s payoff—the informational role is not essential. As long as the principal eventually realizes his benefits, it is not important that he
learns about the output realization in every period. The main message of the paper is unchanged, for example, when the principal realizes his benefits with sufficient delay, so that essentially his only source of information is the supervisor’s reports.

In terms of the compensation form, we follow the standard setup (Levin, 2003) to include a base wage $w_t$, an end-of-period bonus $b_t$, and total compensation $W_t$. Since the end-of-period bonus $b_t$ is the difference between $w_t$ and $W_t$, the principal’s compensation package is determined by $w_t$ and $W_t$, so we do not include $b_t$ in our description of the principal’s strategy below. We include $b_t$ in our description because it sometimes helps with the exposition. Also notice that the agent’s base wage $w_t$ can be history-dependent. As a result, by adjusting the timing of payment—for example by moving the bonus in this period to the base wage in the next period—the principal can still reward the agent for receiving a good report from the supervisor even if $w_t = W_t$ for all $t$. We will revisit this point in Section 3.2.

2.3 Strategies and Equilibrium Concept

Since the supervisor is nonstrategic, we will only describe the strategies of the principal and the agent. We denote $h_t = \{w_t, d_t, \phi_t, W_t\}$ as the public events that occur in period $t$, and $h^t = \{h_n\}_{n=0}^{t-1}$ as the public history at the beginning of period $t$. We set $h^1 = \emptyset$. Let $H^t = \{h^t\}$ be the set of public history until time $t$. The principal observes only the public history. The agent observes his past actions $e^t = \{e_n\}_{n=1}^{t-1}$ in addition to the public history. Denote $H^t_A = H^t \cup \{e^t\}$ as the set of the agent’s private history at the beginning of period $t$.

We use $s^P$ and $s^A$, respectively, to denote the principal’s and the agent’s strategies. The principal’s strategy $s^P$ specifies, for each period $t$, the wage $w_t$ and the total compensation $W_t$. Notice that both $w_t$ and $W_t$ are functions that depend on the available public history. The agent’s strategy $s^A$ specifies, for each period $t$, his acceptance decision $d_t$ and his effort decision $e_t$. The agent’s decisions depend both on the public history and his private history of past efforts. In addition, denote the principal’s belief as $\mu^P$, which assigns to every information set of the principal, i.e., every element in the public history, a probability measure on the set of histories in the information set. Define the agent’s belief $\mu^A$ similarly.

Notice that we do not consider the mixed strategies by the principal or the agent. Allowing for mixed strategies does not improve the sustainability of the relational
contracts in part because the actions of the principal are publicly observable. As will be clear from our analysis below, public observability of the principal’s actions implies that the key to sustaining an efficient relational contract is to minimize the maximal bonus while maintaining enough expected bonus to induce effort. When the principal randomizes, she can only (weakly) increase the maximal bonus because randomization adds noise to the bonus payments and makes them more volatile. And when the agent randomizes, he makes the efficient relational contracts harder to sustain by lowering the expected discounted surplus of the relationship, making the principal more likely to renege on the bonus.

The same reasoning implies that the sustainability of relational contracts will not be affected if the model allows for a public randomization device. To the extent that the total expected bonus must be maintained to induce effort from the agent, public randomization adds fluctuation to the total expected bonus. In particular, the conditional total expected bonus following some realization of public randomization must be (weakly) higher than the total expected bonus prior to the public randomization. This makes it more difficult for the maximum bonus to be lower than the expected discounted surplus following this particular realization.

Our solution concept is Perfect Bayesian Equilibrium (PBE). To describe it formally, we introduce the following notations. Let the agent’s expected payoff following private history $h^t_A$ and $w_t$ be

$$\hat{U}(h^t_A, w_t, s^A, s^P) = E[\sum_{\tau=t}^{\infty} \delta^{\tau-t} \{ w + 1_{\{d_{\tau}=1\}}(c_{e_{\tau}} - W_{\tau} - w) \}|h^t_A, w_t, s^A, s^P].$$

Define $\tilde{U}(h^t_A, w_t, d_t, s^A, s^P)$ accordingly as the agent’s expected payoff following his acceptance decision in period $t$. Next, let the principal’s expected payoff following the agent’s private history $h^t_A$ be

$$\hat{\pi}(h^t_A, s^A, s^P) = E[\sum_{\tau=t}^{\infty} \delta^{\tau-t} \{ \pi + 1_{\{d_{\tau}=1\}}f(e_{\tau}) - W_{\tau} - \pi \}|h^t_A, s^A, s^P],$$

where recall that $f(e_{\tau})$ denotes the expected output given the agent’s effort choice. Since the principal does not observe the agent’s private history, we define

$$\hat{\Pi}(h^t, s^A, s^P) = E_{\mu'}[\hat{\pi}(h^t_A, s^A, s^P)|h^t].$$
as his expected payoff following public history $h^t$. Here, the expectation is taken over all of the agent’s possible private histories ($h^t_A$) according to the principal’s belief ($\mu^P$) conditional on observing public history $h^t$. Finally, we denote $\tilde{\pi}(h^t_A, w_t, d_t, \phi_t, s^A, s^P)$ as the principal’s expected payoff in period $t$ following the agent’s private history $h^t_A$, the principal’s wage offer $w_t$, the agent’s acceptance decision $d_t$, and the performance outcomes $\phi_t$. We define $\tilde{\Pi}(h^t, w_t, d_t, \phi_t, s^A, s^P)$ accordingly.

A PBE in this model consists of the principal’s strategy ($s^*P$), the agent’s strategy ($s^*_A$), the principal’s belief ($\mu^P$), and the agent’s belief ($\mu^A$), such that the following are satisfied. First, following any history $\{h^t_A, w_t\}$ and $\{h^t_A, w_t, d_t\}$, and for any $\tilde{s}^A$,

$$\hat{U}(h^t_A, w_t, s^*_A, s^*P) \geq \hat{U}(h^t_A, w_t, \tilde{s}^A, s^*P);$$
$$\tilde{U}(h^t_A, w_t, d_t, s^*_A, s^*P) \geq \tilde{U}(h^t_A, w_t, d_t, \tilde{s}^A, s^*P).$$

Second, following any history $h^t$ and $\{h^t, w_t, d_t, \phi_t\}$, and for any $\tilde{s}^P$,

$$\hat{\Pi}(h^t, s^*_A, s^*P) \geq \hat{\Pi}(h^t, s^*_A, \tilde{s}^P);$$
$$\tilde{\Pi}(h^t, w_t, d_t, s^*_A, s^*P) \geq \tilde{\Pi}(h^t, w_t, d_t, s^*_A, \tilde{s}^P).$$

Third, the beliefs are consistent with ($s^*P, s^*_A$) and are updated with the Bayes rule whenever possible. Note that the agent has private information about his effort. As a result, the agent’s belief about the past history depends on his actual effort levels. In contrast, the principal’s belief about the past history depends only on the agent’s equilibrium effort levels as long as the agent has not publicly deviated. When the agent publicly deviates, we assume the principal believes that the agent has never put in effort in periods with low public output.

When the signals are also publicly observed, a commonly used equilibrium concept is Perfect Public Equilibrium (PPE). PPE requires the strategies to depend only on the public history. This restriction is questionable when the supervisor’s reports (and thus the agent’s payoff) depend on the past history of signals. When his effort affects future reports, the agent’s private history contains payoff-relevant information and should be used to his advantage.
3 Analysis

In this section, we study how information structures affect the efficiency of the relational contract. We first review in Subsection 3.1 the condition to sustain the efficient relational contracts when the supervisor fully reveals his signals. Subsection 3.2 presents our main result that the supervisor can help sustain the efficient relational contracts by revealing less information.

Below, we restrict our analysis to the case that $q_0 = 0$ and $q = 0$. In other words, both the output and the supervisor’s signal are low when the agent shirks. The assumption that $q_0 = 0$ is made entirely for ease of exposition and can be relaxed. The assumption that $q = 0$, however, is made for analytical convenience, and we discuss the $q > 0$ case in Section 4.1.

3.1 Benchmark: Fully Revealing Reports

Suppose the reports fully reveal the signals, i.e., $s_t = y_t^s$ for all $t$. Levin (2003) implies that the optimal relational contract can take the following form. In each period, the principal offers the agent a base wage $w$ and promises him a bonus $b > 0$ if either the output is high ($y_t = y$) or the report is high ($s_t = H$).

Since the relational contract is stationary, the agent’s effort decision only affects his payoff within each period. Notice that if the agent shirks, the probability of a bonus is 0. If he works, the probability of bonus is given by $p_s = p_0 + p - pp_0$. To induce effort from the agent, the expected benefit must exceed the cost of effort:

$$p_s b \geq c. \quad (1)$$

When there is no restriction on payments, the principal can set the base wage to capture the entire surplus of the relationship. It follows that for the principal not to renege on the bonus, the following must be true:

$$\frac{\delta}{1-\delta} s (1) \geq b, \quad (2)$$

where recall that $s (1)$ is the per-period surplus if the agent works.

Combining equations (1) and (2) shows that a relational contract can induce effort
when
\[ \frac{\delta}{1 - \delta} s(1) \geq \frac{c}{p_s}. \]  

In other words, the incentive cost should be smaller than the discounted expected future surplus. Inequality (3) implies that the sustainability of the relational contract depends on the extremes. In other words, the set of discount factors (δ) that allow for efficiency is completely determined once the value of the maximal reneging temptation (c/p_s) and the expected per-period surplus in the relationship (s(1)) are given. When (3) is satisfied, the optimal relational contract can be achieved by setting
\[ b = \frac{\delta s(1)}{1 - \delta} \text{ and } w = u + c - p_s b. \]

3.2 Main Results

In this section, we show that the supervisor can help sustain the relational contract by sending out less-informative reports. We consider a class of efficient equilibrium where essentially the principal pays the agent a fixed base wage and a bonus on top of that if either the output is high or the supervisor’s report good. Our main result is that when the supervisor ties the reports to past signals, he can lower the discount factor necessary for supporting the efficient relational contract.

3.2.1 Spillover Reporting

The key to our result is the supervisor’s reporting rule. In particular, we consider the following class of reporting rules. The supervisor reports either G (good) or B (bad) in each period. These reports are partitioned into reporting cycles that end stochastically. The first reporting cycle starts in period 1, and each new reporting cycle starts if the supervisor reports B in the previous period.

Within each reporting cycle, the supervisor reports G if his signal is high. If the signal is low, he reports G with probability f (and B with probability 1 − f) if the signal in the previous period within the reporting cycle is high. If that signal is again low, the supervisor reports B. When the supervisor observes a low signal at the beginning of a reporting cycle (so that there is no within-cycle previous period), the supervisor reports G with probability \( \rho^*_f f \) and B with probability 1 − \( \rho^*_f f \), for some \( \rho^*_f \in (0, 1] \) to be explained below.

Figure 1a illustrates the reporting rule when the current period is not the first.
period of a reporting cycle, and Figure 1b illustrates the reporting rule when the current period is the first period of a reporting cycle.

![Figure 1a: Reporting rule (not in first period of reporting cycle)](image1a)

![Figure 1b: Reporting rule (in first period of reporting cycle)](image1b)

We choose $\rho_f^*$ as the unique value satisfying $\rho_f^* = p/ (p + (1 - p) \rho_f^* f)$ . This choice ensures that the probability of a good report in each period is constant along the equilibrium path. Notice that the probability of a good report is essentially determined by the conditional probability of a high signal in the past period. The choice of $\rho_f^*$ ensures that this conditional probability is constant within each reporting cycle. In particular, suppose the conditional probability of a high signal in period $t - 1$ is $\rho_{t-1}$. Conditional on a $G$ report in period $t$, the probability of a high signal in period $t$ is given by $\rho_t = p/ (p + (1 - p) \rho_{t-1} f)$ . The choice of $\rho_f^*$ then implies that when $\rho_{t-1}$ is equal to $\rho_f^*$, so is $\rho_t$.

We denote this class of reporting rules as $f$-spillover reporting because following a high signal, with probability $f$ the supervisor will send out a good report in the next period even if his signal is low. In other words, the benefit of a high signal spills over to the next period with probability $f$. Notice that when $f = 0$, there is no spillover: $G$ is reported if and only if the signal in the current period is high. This is the benchmark case in Subsection 3.1 in which the reports track the signals perfectly. When $f = 1$, a high signal means that the supervisor reports $G$ both for the current and the next periods. Essentially, the supervisor reports $G$ as long as there are no two consecutive low signals. This is the same as Example 2 above except that the reporting cycle restarts after each $B$-report.
3.2.2 Perfect Bayesian Equilibrium (PBE)

We now show that $f$-spillover reporting can help sustain efficient relational contracts. In particular, consider the following class of strategies.

The principal offers $w_{t} = w$ in period 1. If the agent has always accepted the contract, the principal offers for $t > 1$

$$w_{t} = \begin{cases} 
    w & \text{if } s_{t-1} = B \text{ and } y_{t-1} = 0, \\
    w + b/\delta & \text{otherwise,}
\end{cases}$$

where $w - c + \left( p_{s} + (1 - p_{s}) \rho_{f}^{*} f \right) b = u$. If the agent has ever rejected the offer, the principal offers $w_{t} = u - 1$. The agent’s total compensation at the end of period is given by $W_{t} = w_{t}$ for all $t$.

The agent accepts the principal’s contract offer if and only if $w_{t} > u$ or the principal has never deviated. The agent puts in effort if and only if there is no public deviation and the probability of a low signal in the previous period is smaller than $\rho_{f}^{*}$.

Before describing our main result, we comment on a few features of this class of strategies. First, notice that since $w_{t} = W_{t}$ for each $t$, no end-of-period bonus is paid out. However, this class of strategies is essentially equivalent to the following: the principal sets a base wage $w$ and gives a bonus $b$ whenever the report is good or the output is high. In the class of equilibria we focus on, the bonus, however, is not paid at the end of the period but is postponed to the beginning of next period through a higher $w_{t+1}$. We therefore denote this class of strategies as stationary strategies with a deferred bonus and refer to $b$ as the deferred bonus. The reason for the bonus postponement is that under $f$-spillover reporting, past signals, and therefore past effort choices, affect future reports. This implies that the agent may engage in multistage deviations under $f$-spillover reporting, and a deferred bonus can help prevent such deviations. We discuss this point again at the end of the section.

Second, the strategies specify that the principal offers $w_{t} = u - 1$ if the agent publicly deviates. The choice of $u - 1$ is made for convenience. One can also choose any $w_{t} < u$ since it will again lead the agent to choose his outside option. Third, we choose $w$ and $b$ so that the principal captures all of the surplus of the relationship. Notice, however, that the principal and the agent can divide the surplus arbitrarily by changing the base wage $w_{1}$ in period 1 and keep the rest unchanged.
Our main result shows that relative to full revelation of outputs, $f$-spillover reporting can reduce the discount factor necessary to support the efficient relational contract. Denote $\delta^\star(f)$ as the smallest discount factor such that there exists a PBE supported by a stationary strategy with a deferred bonus. Note that when $f = 0$, the supervisor reveals his signals fully, so $\delta^\star(0)$ is determined by Eq (3) in the benchmark case.

**Proposition 1:** Suppose $1 - p_0 - \delta \rho^\star > 0$, where $\rho^\star = p / (p + (1 - p) \rho^\star)$; then the following holds:

(i): $\delta^\star(1) < \delta^\star(0)$ if and only if $p_s / (1 - p_s) < \rho^\star \delta - p (1 - \rho^\star) \delta^2$.

(ii): If $\delta^\star(f) < \delta^\star(0)$ for some $f \in (0, 1)$, then $\delta^\star(1) < \delta^\star(f)$.

Part (i) provides the condition for 1-spillover reporting to lower the cutoff discount factor that sustains the efficient relational contract. Part (ii) shows that when full revelation of signals is not optimal, 1-spillover reporting has the lowest cutoff discount factor within the class. This allows us to focus below on 1-spillover reporting, which we refer to as spillover reporting for convenience.

To see why and when spillover reporting may help, we take a three-step approach. First, we introduce a value function for the agent, which is useful for describing when an efficient relational contract can be sustained under $f$-spillover reporting. Notice that the agent’s payoff depends on the supervisor’s signal in the previous period. Since the agent does not observe the signal, denote $\rho$ as the probability of a high signal in the previous period, and let $\rho = \rho^\star_f$ if the period is at the beginning of a reporting cycle. In this case, $\rho$ becomes a state variable that summarizes the agent’s payoff. Denote $V(\rho)$ as the agent’s value function, and let $V_e(\rho)$ ($V_s(\rho)$) be the agent’s maximum expected payoff if he works (shirks) this period. It follows that $V(\rho) = \max\{V_e(\rho), V_s(\rho)\}$.

For ease of exposition, let the agent’s outside option $u$ be 0, and define $p_f^\rho(\rho) = p + (1 - p) f \rho$ as the probability of a good report if the agent works. For convenience, we assume for now that the agent never takes his outside option. We will return to this assumption later. Under these assumptions,

$$V_e(\rho) = w - c + p_f^\rho(\rho) \left( b + \delta V(p/p_f^\rho(\rho)) \right) + (1 - p_f^\rho(\rho)) p_0 b;$$

$$V_s(\rho) = w + \rho f \left( b + \delta V(0) \right).$$

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To see the expression for $V_e(\rho)$, notice that if the agent works, a good report is sent with probability $p_f^\rho(\rho)$. In this case, he is rewarded with a deferred bonus $b$ and infers that the probability of a high signal is $p/p_f^\rho(\rho)$. With probability $1 - p_f^\rho(\rho)$, a bad report is sent. In this case, the agent still receives a deferred bonus (with probability $p_0$) if the output is high. But since a bad report is sent, a new reporting cycle starts next period and the agent’s continuation payoff is given by $\delta V(\rho^*_f) = 0$ (since the principal captures all of the surplus). Similarly, when the agent shirks, a good report is sent with probability $\rho f$. In this case, the agent is rewarded with a deferred bonus $b$, but he infers that the probability of a high signal is 0.

We now take our second step by deriving the condition for sustaining an efficient relational contract. As in the benchmark case, we start with the agent’s incentive constraint. Since $\rho = \rho^*_f$ along the equilibrium path, the agent puts in effort as long as $V_e(\rho^*_f) \geq V_s(\rho^*_f)$. From the expression for $V_e$ and $V_s$ above (and noting that $p/p_f^\rho(\rho) = \rho^*_f$ and $\delta V(\rho^*_f) = 0$), we can rewrite this inequality as

$$
(1 - \rho^*_f) p_s + \rho^*_f \delta (V(\rho^*_f) - V(0)) \geq c,
$$

where $p_s \equiv p_0 + p - pp_0$ is the probability that either the output or the signal is high.

Relative to the agent’s IC in the benchmark case, spillover reporting introduces two effects on the agent’s incentive constraints. The first one is an information-loss effect: when reports are less informative of the agent’s effort, they reduce the incentive of the agent. This is captured by the factor $1 - \rho^*_f$, where $\rho^*_f$ is the probability that the supervisor sends out a good report even if his signal is low. When $\rho^*_f$ is larger, the agent is more likely to have a good report even if the supervisor’s signal is low, making the bonus less sensitive to effort. The information-loss effect therefore makes the incentive constraint more difficult to satisfy. The information-loss effect is well known in the literature on the garbling of signals; see, for example, Kandori (1992).

The second is a smoothing effect, which is captured by $\rho^*_f \delta (V(\rho^*_f) - V(0))$. Under $f$-spillover reporting, part of the agent’s reward for working is paid in the future: the agent has a higher continuation payoff by working ($\delta V(\rho^*_f)$) than shirking ($\delta V(0)$). In other words, the rewards for working are smoothed across periods. This allows for the contemporaneous reward (the bonus) to be lowered while keeping the agent motivated. Notice that when the reward is lowered, it also occurs more
frequently. When $f = 0$ (so that the information is fully revealed), the supervisor sends out a good report with probability $p$ per period. When $f = 1$, the supervisor sends out a good report with probability $p + (1 - p) \rho^*$. We can interpret $\rho^*$ as the equilibrium amount of spillover in reporting since it is the probability that a good report is sent out even if the signal is low. The smoothing effect makes the incentive constraint easier to satisfy.

Before evaluating the overall effect from spillover reporting, we next list the principal’s non-reneging constraint. Since the principal captures all of the surplus, his non-reneging constraint is again

$$\frac{\delta}{1 - \delta} s (1) \geq b.$$  

(NR-g)

Combining this with the agent’s incentive constraint, we obtain the condition for sustaining an efficient relational contract:

$$\frac{\delta}{1 - \delta} s (1) \geq \frac{c - \rho^*_f f \delta (V(\rho^*_f) - V(0))}{p_s (1 - \rho^*_f f)},$$  

(NSC-g)

which corresponds to Eq (3) in the benchmark case. When $f = 0$, the right-hand side (RHS) is equal to $c/p_s$, so Eq (NSC-g) includes the benchmark as a special case. In general, $f$-spillover reporting can help sustain efficient relational contracts when the RHS is smaller than $c/p_s$. When $f > 0$, both the denominator and the numerator decrease. In the denominator, the $1 - \rho^*_f f$ factor reflects the information-loss effect described above. In the numerator, the $\rho^*_f f \delta (V(\rho^*_f) - V(0))$ reflects the smoothing effect. Note that the smoothing effect is absent in within-in period garbling and that explains why within-period garbling that leads to higher probability of a lower bonus does not help sustain relational contracts.

Given Eq (NSC-g), we take our third step by discussing when the overall effect of spillover reporting is positive. Using Eq (NSC-g), we see that spillover reporting helps if and only if

$$\delta \left( V(\rho^*_f) - V(0) \right) > c.$$  

(Gain)

This condition is easier to satisfy for a larger discount factor $\delta$. It follows because under spillover reporting, part of the reward is postponed, so the size of bonus has to increase to account for the “interest payment” associated with the postponement of bonus payment. When the agent is more patient, this increase is smaller, making
spillover reporting more effective. Next, notice that this condition is easier to satisfy if \( V(\rho^*_f) - V(0) \) is larger. When \( f = 0 \), \( V(\rho^*_f) = V(0) \) since a high signal last period gives no extra benefit. This suggests that the smoothing effect is dominated by the information effect when \( f \) is small. When \( f \) is larger, a high signal this period leads to a larger probability of a good report next period. In other words, the smoothing effect is likely to increase with \( f \), suggesting that \( f = 1 \) is optimal within this class of reporting rules.

To obtain the exact condition for when the smoothing effect dominates, one must calculate \( V(0) \) (since \( V(\rho^*_f) = 0 \)). This calculation, however, requires knowing the agent’s effort choice for \( \rho = 0 \), and moreover, his effort choice for all future realizations of \( \rho \)s that are associated with the continuation payoffs originating from \( V(0) \). Specifically, let \( \rho_t = 0 \) in period \( t \) and suppose the agent puts in effort. Now if the supervisor sends a good report, the agent then infers that the signal must be high, i.e., \( \rho_{t+1} = 1 \). This calculation then requires knowing the agent’s choice of \( e_{t+1} \) given \( \rho_{t+1} = 1 \) and so on.

When \( \delta = \delta^* (1) \), we show in the proof that the agent will put in effort if and only if \( \rho \leq \rho^*_f \). Essentially, the agent’s expected gain for effort is decreasing in \( \rho \), so that the agent will put in effort if and only if \( \rho \) is below some cutoff. Notice that when \( \rho \) is larger, there is a higher probability that the agent receives a good report even if the signal is low. This higher probability dampens the agent’s incentive to put in effort. Given the cutoff-effort rule, one can then compute the value function and therefore \( V(0) \). This leads to the condition in Proposition 1 for when spillover reporting can help, and the details are provided in the proof.

Figure 2 below illustrates the value function of the agent at the cutoff discount factor with \( f = 1 \), and define \( \rho^* \equiv \rho^*_1 \). The value function is piecewise linear in \( \rho \) and has a kink at \( \rho^* \). In particular, the agent prefers to work if and only if \( \rho < \rho^* \).
Given the value function (and the associated effort rule), we conclude the section by highlighting some properties of the relational contract. First, it is important that the agent does not know the supervisor’s exact signal. For example, if the agent knows that $\rho = 1$, then he prefers to shirk since $V(1) = V_s(1)$. In general, an important feature for a reporting rule to improve over fully revealing reporting is that the agent’s continuation payoffs cannot always be common knowledge. The lack of common knowledge is an essential feature for a reporting rule to improve efficiency.

Second, one must check for multistage deviation for the sustainability of the relational contract. To see this, consider, for example, that the agent has shirked and received a good report. He infers that $\rho = 0$. The value function then implies the agent should put in effort (since $V(0) = V_e(0)$). But once the agent works and the supervisor again sends a good report, the agent then infers that the signal must be high, and therefore $\rho = 1$. It follows that the agent’s optimal effort is then to shirk ($V(1) = V_s(1)$). In summary, for an agent who has just shirked, his subsequent optimal actions, given that the supervisor has been sending good reports, are to work and then to shirk. This implies that profitable multistage deviation might exist even if there are no profitable one-stage deviations.

The reason that multistage deviation can be more profitable is that under $f$-spillover reporting, the agent’s private effort choice affects his future expected payoff (conditional on the public observables). When the agent shirks, in particular, his belief about his future payoffs will be different from that of the principal. This implies that the players’ future payoffs cannot be summarized by a sufficient statistic.
based on the public history alone, so the one-stage-deviation principle does not apply. We formally check in the proof that there are no profitable multistage deviations, but finish the section by noting a particular type of multistage deviation, which is responsible for the postponement of the bonus.

Unlike in Levin (2003) where the timing of bonus is irrelevant so one can assume that the bonus paid out at the end of the period, here it is crucial that bonus is deferred to the beginning of the next period. To see why the deferral is necessary, suppose the agent shirks but receives a good report. If the bonus were paid immediately, the agent would strictly prefer to exit since he knows the signal is low ($\rho = 0$) and that $V(0)$ is less than his outside option (see Figure 2). In other words, this type of shirk-then-exit behavior can be profitable if the bonus payment is not deferred. With deferred rewards, the agent always stays.

4 Discussion

In the previous section, we show that spillover reporting can help sustain the efficient relational contract. In this section, we show that the mechanism behind spillover reporting is robust to a number of extensions in Subsection 4.1, and discuss general properties of reporting rules in Subsection 4.2. All formal descriptions of the setups, results, and proofs are relegated to the online appendix.

With the exception of Subsection 4.1.4, we assume in this section that outputs are observed only by the supervisor. This corresponds to the case in which the public signal is uninformative of the agent’s effort. This simplification allows us to focus on the main mechanism of the paper without providing superfluous details. All formal results can be adapted to allow for observable outputs.

4.1 Robustness

4.1.1 $q > 0$

In Section 3, we assume for tractability that $q = 0$, so the supervisor never receives a high signal when the agent shirks. When $q > 0$, the value function $V(\rho)$ no longer has an analytical solution, so checking multistage deviation becomes very difficult. In the online appendix, we compute $V(\rho)$ numerically for $q > 0$ and show that spillover
reporting can lower the bonus amount necessary for motivating the agent. We also prove formally that spillover reporting can help for a sufficiently small \( q \).

### 4.1.2 Multiple Effort Levels

The agent’s effort level is binary in Section 3. The online appendix considers a model with three levels of effort, \( e \in \{0, 1, 2\} \), and efficiency requires \( e = 2 \). We show that as long as the effort costs are sufficiently convex (\( (c(2) - c(1)) / (c(1) - c(0)) \) is large enough), spillover reporting can help sustain the efficient relational contract.

When the cost function is sufficiently convex, the gain of deviating from \( e = 1 \) to 0 is small relative to that from \( e = 2 \) to 1. If the agent does not gain from deviating to \( e = 1 \), he will not gain from deviating to \( e = 0 \). More generally, a sufficiently convex effort-cost structure implies that ruling out profitable local deviations is enough for ruling out global deviations. This suggests that spillover reporting can improve the sustainability of relational contracts when the cost structure is sufficiently convex.

### 4.1.3 Multiple Agents

Our next extension considers \( n > 1 \) identical agents. When the principal maintains \( n \) independent relationships with the agents, our result is unchanged since both the total surplus and the maximal reneging temptation increase \( n \)-fold and therefore cancel each other out. The optimal relational contract with \( n \) agents, however, is not independent. When the signals are fully revealed, the optimal relational contract specifies a fixed bonus pool that is paid out whenever at least one agent has a high signal, and it is shared equally among agents with high signals (Levin, 2002).

In the online appendix, we construct a corresponding relational contract with spillover reporting. There, the bonus pool is paid out at the beginning of the following period whenever at least one agent has a good report, where the report is obtained through spillover reporting as in Section 3. We provide conditions for when spillover reporting helps and establish a limit result for the gain of spillover reporting. Specifically, the minimal surplus for sustaining the efficient relational contract under full revelation of signals is \( \delta \) times more than that under spillover reporting as \( p \) goes to 0, and the limit is independent of \( n \).
4.1.4 Collusion

In the main model, we abstract away from incentive issues of the supervisor to focus on the gain from spillover reporting. Since the agent’s pay depends on the supervisor’s reports, one issue is that the supervisor may collude with the agent; see, for example, Tirole (1986) and the large literature that followed. In the online appendix, we study how the principal may design compensation for the supervisor to prevent collusion.

We show that by offering a fraction of the output to the supervisor, the principal can prevent the supervisor from always sending good reports about the agent. The key observation is that if the supervisor colludes with the agent by always sending a good report, the agent will shirk. This results in low outputs and therefore reduces the supervisor’s pay. In the online appendix, we provide the formal conditions under which the principal can prevent this type of collusion by giving a large enough share of the output to the supervisor.

Another possibility is that the principal may collude with the supervisor. This type of collusion may be prevented if the agent can catch the collusion with some probability. In this case, the agent takes his outside options forever if he discovers that the supervisor deviates from the reporting rule. When the supervisor has rent in the relationship, it can be shown that the fear of losing future rent in the relationship can deter the supervisor from colluding with the principal.

4.2 General Reporting Rules

In Subsection 3.2, we show that $f$-spillover reporting can help sustain efficient relational contracts. There are many other reporting rules the supervisor can use. What are the necessary conditions for a reporting rule to help? And if a reporting rule helps, how much gain can it achieve in terms of reducing the surplus necessary for an efficient relational contract? We explore these issues in the following subsection.

4.2.1 Necessary Conditions

A commonly studied reporting rule is T-period reviews, where the agent is evaluated and compensated every T periods. It has been shown that T-period reviews can help improve efficiency in a number of settings; see, for example, Radner (1985), Abreu, Milgrom, and Pearce (1991), and Fuchs (2007). However, T-period reviews cannot
help sustain efficient relational contracts in our setting, and it suffices to see this for two-period reviews.

**Proposition 2:** Let $s(1)$ be the expected future of the relationship per period. Suppose the supervisor reports his signals truthfully every other period. A necessary condition to sustain the efficient relational contract is

$$\frac{\delta}{1 - \delta} s(1) \geq \frac{c}{p}.$$

To see the intuition behind Proposition 2, denote $B_2(h)$ ($B_2(l)$) as the agent’s expected bonus if the second-period signal is high (low). The necessary condition for inducing effort from the agent in the second period is then given by $B_2(h) - B_2(l) \geq c/p$. Recall that the principal’s non-reneging condition implies that the discounted future surplus ($\delta s(1)/(1 - \delta)$) is greater than the maximal bonus, which is in turn bigger than the difference in the expected bonus ($B_2(h) - B_2(l)$). Combining this with the agent’s second-period incentive constraint gives the condition in Proposition 2.

The argument above also shows why T-period reviews cannot help and, more generally, why any reporting rule that restarts on predetermined dates cannot improve efficiency over the full revelation of signals. Therefore, for any reporting rule to help, the supervisor’s memory must be persistent in the sense that there cannot be a predetermined date after which all past histories become irrelevant. We discuss this property more formally in the online appendix. Notice that under $f$-spillover reporting, although the supervisor neglects all past histories following a $B$ report, the date of the $B$ report is not predetermined.

In a related paper, Fuchs (2007) shows that $T$-period-review contracts enhance the efficiency of the relationship when the outputs are privately observed by the principal. In this case, surplus must be destroyed when the agent is punished. By revealing information to the agent infrequently, $T$-period-review contracts reduce the expected punishment received, giving rare but harsh punishments. In contrast, no surplus is destroyed in our environment when the agent is punished (fails to receive a bonus). In fact, spillover reporting has the opposite effect of increasing the frequency of bonuses and reducing the bonus amounts.
Finally, notice that for a reporting rule to help, the persistence of memory is necessary but not sufficient. Recall from the discussion in Section 3.2 that an important feature for spillover reporting is that the agent’s continuation payoffs are not common knowledge. The lack of common knowledge is an essential feature for a reporting rule to improve efficiency, and this observation applies to other contexts as well. In a related paper, Ekmekci (2011) examines a product-choice game between a long-run seller and a sequence of short-run buyers. He defines a mapping from past outputs to signals as a rating system, which corresponds to a reporting rule in our context. Importantly, Ekmekci (2011) considers a Markovian rating system, i.e., the latest rating depends on the previous rating and the latest output. As a result, the seller’s continuation payoff is common knowledge, unless the seller has privately known types (such as commitment types). In contrast, the spillover reporting rule we propose is not Markovian, but rather hidden Markovian; the supervisor’s report depends on a hidden state variable, namely, the realization of the signals. This difference explains why the rating system cannot enhance efficiency in Ekmekci (2011) unless the seller has privately known types, while spillover reporting rule helps here even if the agent only has a single publicly known type.

4.2.2 Limits of Gain from Reporting

Under spillover reporting, a high signal in the current period guarantees a good report in the next period. Many other reporting rules can also defer the reward and may help sustain the efficient relational contract. The supervisor, for example, can send a good report as long as any of the past \( n \) signals is high. For a given level of surplus, one would like to know whether there are reporting rules that can sustain the efficient relational contract. This problem, however, is difficult because the set of reporting rules is large and checking multistage deviations is hard. Nevertheless, the next proposition partially addresses the problem by establishing a lower bound on the minimal surplus necessary to sustain efficiency.

**Proposition 3:** Let \( s(1) \) be the expected future surplus of the relationship per period. For any reporting rule to sustain the efficient relational contract, one must have

\[
\frac{\delta}{1 - \delta} s(1) \geq \sqrt{4p(1 - p)c}.
\]

In particular, the full revelation of signals is the optimal reporting rule when \( p = 1/2 \).
When the signals are fully revealed, recall that the smallest discounted future surplus to sustain efficiency must satisfy $S \geq c/p$, where $S = \delta s(1) / (1 - \delta)$. Proposition 3 shows that the smallest surplus to sustain efficiency under any reporting rule must satisfy $S \geq \sqrt{4p(1-p)c/p}$. In other words, the optimal reporting rule can lower the surplus necessary for sustaining efficiency by at most a factor of $1 - \sqrt{4p(1-p)}$. Notice that $\sqrt{4p(1-p)} = 1$ when $p = 1/2$. Proposition 3 therefore implies that the full revelation of signals is optimal when $p = 1/2$.

We now provide an intuition for the condition in Proposition 3, which arises through an argument on the variance of the agent’s payoff. Note that we can rewrite the condition as

$$\frac{S^2}{4} \geq p(1-p) \left( \frac{c}{p} \right)^2.$$

Here, the left-hand side can be viewed as an upper bound on the variance of the agent’s per-period pay that is allowed by the surplus, and the right-hand side is a lower bound necessary for effort. For the left-hand side, notice that the principal’s non-reneging constraint implies that the maximal bonus is bounded by $S$. Since any nonnegative random variable bounded by $S$ cannot have a variance above $S^2/4$, the left-hand side provides an upper bound on the variance of the agent’s realized pay per period, calculated from bonus payments.\(^4\)

To see that the right-hand side provides a lower bound, notice that the agent’s actual pay from period $t$ is a binary random variable whose value is $u_h$ (for a high signal) with probability $p$ and $u_l$ (for a low signal) with probability $1-p$. To induce effort from the agent, we have $u_h - u_l \geq c/p$. This implies that the variance of the agent’s actual pay per period must be greater than $p(1-p)(c/p)^2$.

Under a general reporting rule, the agent’s realized pay needs not to be the same as his actual pay each period since some of the actual pay can be deferred to the future. But for sufficiently long periods of time, the agent’s average realized pay per period will be close to his average actual pay. It follows that $S^2/4$ is the maximal feasible variance of average pay, and $p(1-p)(c/p)^2$ is the minimal variance necessary to induce effort. This leads to the condition in Proposition 3. Finally, when $p = 1/2$ (and setting the bonus equal to $S$), full revelation of signals is optimal because it

\(^4\)The maximal variance is obtained by a binary random variable that is equal to 0 half of the time and equal to $S$ the other half of the time.
generates the maximal variance of pay per period ($S^2/4$). No other reporting rule can generate enough variance to induce effort if full revelation fails to do so.

5 Conclusion

In this paper, we show that supervisors can improve the sustainability of relational contracts by revealing less information. We study $f$-spillover reporting rules in which the reward for an agent’s good performance is spread over periods. This reduces the required bonus size necessary for inducing effort and therefore reduces the principal’s gain from reneging on the bonus. We show that $1$-spillover reporting is optimal within this class of reporting rules.

More generally, we provide a necessary condition for an arbitrary reporting rule to lower the surplus necessary for sustaining the efficient relational contract. To improve over the full revelation of signals, a reporting rule must allow the memory to persist. Finally, we establish an upper bound for the gain from reporting in terms of how much it can lower the amount of surplus required to sustain efficient relational contracts. The upper bound implies that when the probability of a high signal is a half (under effort), full revelation of signals is optimal.

We also show that the mechanism is robust to a number of extensions, and, in particular, we briefly consider the collusion between the supervisor and the agent. We show that the principal can prevent stationary collusion by giving a share of the output to the supervisor. In general, there may be more-sophisticated types of collusion between the supervisor and the agent. For example, the supervisor’s collusive reports can depend on past realizations of outputs, and if too many past outputs are low, the supervisor can send out bad reports. This type of collusion may induce the worker to put in effort while extracting extra bonus from the principal. The principal, of course, may also offer more-complicated contracts to the supervisor or use other tools, such as turnover and rotation, to prevent these types of collusion. Formal study of how collusion affects relational contracts is an interesting line of future research.
References


Appendix A: Proof of Proposition 1

Proof. Part (i). To simplify the exposition, we set $u = 0$, and recall that $\rho^* = \rho_1^*$ (since $f = 1$). In an efficient relational contract, the agent always puts in effort and generates surplus $s(1) = p_0 y - c - w$ per period. Given the choice of $w$ and $b$ the surplus is captured entirely by the principal, so she will not deviate as long as $b \leq \delta s(1)/(1 - \delta)$. This implies that finding the cutoff discount factor $\delta^*(1)$ is equivalent to finding the smallest deferred bonus $b^*$ such that the agent always puts in effort. To do this, we take the following steps.

**Step 1: Recursive Formulation.** Given the supervisor’s reporting strategy, the agent’s payoff is completely determined by the probability that the signal is high in the last period $\rho$ (and $\rho = \rho^*$ when the report in the last period is bad). Let $V(\rho)$ be the agent’s value function at the beginning of a period assuming that he has accepted the principal’s offer. Let $V_e(\rho)$ ($V_s(\rho)$) be the agent’s value function if he works (shirks) this period, then $V(\rho) = \max\{V_e(\rho), V_s(\rho)\}$, and the following holds:

$$
\begin{align*}
V_e(\rho) &= w^* - c + (p + (1 - p)\rho) \max \left\{ 0, b^* + \delta V \left( \frac{p}{p + (1 - p)\rho} \right) \right\} \\
&\quad + (1 - p - (1 - p)\rho) (p_0 \max \{0, b^* + \delta V(\rho^*)\} + (1 - p_0) \max \{0, \delta V(\rho^*)\}) \\
V_s(\rho) &= w^* + \rho \max \{0, b^* + \delta V(0)\} + (1 - \rho) \max \{0, \delta V(\rho^*)\}.
\end{align*}
$$

The max operators capture the possibility that the agent can take his outside option at the beginning of next period. In addition, recall that if the agent starts at $\rho = \rho^*$ and works, the probability of a high signal last period is again $\rho^*$. This implies that to check the agent has the incentive to work every period, it suffices to check $V_e(\rho^*) \geq V_s(\rho^*)$. Moreover, at the smallest deferred bonus $b^*$ we must have $V_e(\rho^*) = V_s(\rho^*)$. Note that by substituting out $V_e$ and $V_s$, the resulting functional equation for $V$ can be written as $V = T(V)$, where $T$ is a monotone and nonexpansive operator that maps bounded functions on $[0, 1]$ to the same space, and thus, has a unique solution. Below, we construct the unique value function that this holds.

**Step 2: The value function.** Consider the following candidate value function:

$$
V(\rho) = \begin{cases} 
\alpha(\rho - \rho^*) & \text{for } \rho \leq \rho^* \\
\beta(\rho - \rho^*) & \text{for } \rho > \rho^* 
\end{cases},
$$

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where
\[
\alpha = \frac{(1 - p)(1 - p_0 - \delta \rho^*)}{1 - \delta^2 p (1 - \rho^*)} b^*, \quad \beta = \frac{1 - (1 - p_0)(1 - p) \delta \rho^*}{1 - \delta^2 p (1 - \rho^*)} b^*,
\]
and
\[
c = \left( p_s + \rho^* \left( (1 - p_s) - \frac{1 - (1 - p_s) \delta \rho^*}{1 - \delta^2 p (1 - \rho^*)} \right) \right) b^*,
\]
where recall \( p_s = p_0 + (1 - p_0) p \). In addition, \( V(\rho) = V_\epsilon(\rho) \) for \( \rho \leq \rho^* \) and \( V(\rho) = V_s(\rho) \) for \( \rho > \rho^* \) so that the agent puts in effort if and only if \( \rho \leq \rho^* \).

Before proceeding, we note the following. First, it can be checked that the choice of \( \alpha \) and \( \beta \) satisfy the following two equations that will be used later:
\[
(1 - p) ((1 - p_0) b^* - \delta \beta \rho^*) = \alpha; \quad \text{(alpha)}
\]
\[
b^* - \delta \alpha \rho^* = \beta. \quad \text{(beta)}
\]
Second, \( \beta > 0 \), and given \( 1 - p_0 - \delta \rho^* > 0 \), we then also have \( \alpha > 0 \). Third, the choice of \( b^* \) gives that
\[
w^* + \rho^* (b^* - \delta \alpha \rho^*) = 0, \quad \text{(b*)}
\]
where \( w^* = c - (p_s + (1 - p_s) \rho^*) b^* \) (given \( f = 1 \)).

For the candidate value function to be the value function, the following must hold:
\[
V(\rho) = V_\epsilon(\rho) \quad \text{for} \quad \rho \leq \rho^*; \quad \text{(PK-e)}
\]
\[
V(\rho) = V_s(\rho) \quad \text{for} \quad \rho > \rho^*; \quad \text{(PK-s)}
\]
\[
V_\epsilon(\rho) \geq V_\epsilon(\rho) \quad \text{for} \quad \rho \leq \rho^*; \quad \text{(IC-e)}
\]
\[
V_s(\rho) \geq V_s(\rho) \quad \text{for} \quad \rho > \rho^*. \quad \text{(IC-s)}
\]

To check these constraints, we first derive new expressions for \( V_\epsilon(\rho) \) and \( V_s(\rho) \) using the functional equations. The functional equations for \( V_\epsilon(\rho) \) and \( V_s(\rho) \) can be simplified by noting that \( V(\rho^*) = 0 \) and that for all \( \rho \), we have \( b^* + \delta V(\rho) > 0 \). This inequality follows because \( V(\rho) \) is increasing in \( \rho \) and \( b^* + \delta V(0) = b^* - \delta \alpha \rho^* = \beta \),
where we use Eq(beta) for the equality. Given the simplification, we have

\[
V_e(\rho) = w^* - c + (p + (1 - p)\rho)(b^* + \delta V\left(\frac{p}{p + (1 - p)\rho}\right)) + (1 - p - (1 - p)\rho)p_0b^*;
\]

\[
V_s(\rho) = w^* + \rho(b^* + \delta V(0)).
\]

To obtain the new expression for \(V_s(\rho)\), the functional equation implies \(\frac{dV_s(\rho)}{d\rho} = \beta\), where the second equality uses the expression of \(V(0)\) and the last equality uses Eq(beta). We also note that \(V_s(\rho^*) = 0\) because of Eq(b*). Together, this gives that

\[
V_s(\rho) = \beta(\rho - \rho^*). \quad (Vs-alt)
\]

To obtain the new expression for \(V_e(\rho)\), we have that \(\frac{p}{p + (1 - p)\rho} \geq \rho^*\) if and only if \(\rho \leq \rho^*\). Substituting for the expression of \(V\left(\frac{p}{p + (1 - p)\rho}\right)\) in the functional equation, we have

\[
\frac{dV_e(\rho)}{d\rho} = \begin{cases} 
(1 - p)\left((1 - p_0)b^* - \delta\beta\rho^*(\rho - \rho^*) \right) & \text{for } \rho \leq \rho^*, \\
(1 - p)\left((1 - p_0)b^* - \delta\alpha\rho^*(\rho - \rho^*) \right) & \text{for } \rho > \rho^*.
\end{cases}
\]

Noting that \(V_e(\rho^*) = 0\) (by using the expression of \(w^*)\), we get

\[
V_e(\rho) = \begin{cases} 
(1 - p)\left((1 - p_0)b^* - \delta\beta\rho^*(\rho - \rho^*) \right) & \text{for } \rho \leq \rho^*, \\
(1 - p)\left((1 - p_0)b^* - \delta\alpha\rho^*(\rho - \rho^*) \right) & \text{for } \rho > \rho^*.
\end{cases} \quad (Ve-alt)
\]

From the expressions of \(V_s(\rho)\) and \(V_e(\rho)\), we see immediately that \(V(\rho) = V_s(\rho)\) for \(\rho > \rho^*\). Also note that \((1 - p)\left((1 - p_0)b^* - \delta\beta\rho^*\right) = \alpha\) by Eq(beta), so \(V(\rho) = V_e(\rho)\) for \(\rho \leq \rho^*\). As a result, the promise-keeping constraints are satisfied.

Finally, we check the IC constraints. First, we check IC-e. Given Eq(Ve-alt), checking \(V_e(\rho) = V(\rho) > V_s(\rho)\) for \(\rho < \rho^*\) is equivalent to checking that \(\alpha < \beta\). Notice that

\[
\beta = \frac{1 - (1 - p_0)(1 - p)\delta\rho^*}{(1 - p)(1 - p_0 - \delta\rho^*)} > \alpha,
\]

where the inequality follows because

\[
1 - (1 - p_0)(1 - p)\delta\rho^* - (1 - p)(1 - p_0 - \delta\rho^*) \\
= 1 - (1 - p)(1 - p_0 - p\delta\rho^*) \\
< 0.
\]
Next, we check IC-s. From Eq(Ve-alt), IC-s \((V(\rho) > V_e(\rho)\) for \(\rho > \rho^*\) is equivalent to \((1 - p) ((1 - p_0)b^* - \delta \alpha \rho^*) < \beta\). This inequality is implied by \(\alpha < \beta\) because \((1 - p) ((1 - p_0)b^* - \delta \alpha \rho^*) - \alpha = (1 - p) \delta \rho^* (\beta - \alpha) < \beta - \alpha\). This finishes checking IC-s, and therefore, the IC constraints. The candidate value function is therefore the value function.

**Step 3: Condition for Improvement.** Using the expression of \(b^*\) in the value function, we obtain that the condition for sustaining the relational contract under spillover reporting is given by

\[
\frac{\delta}{1 - \delta} s(1) \geq \frac{c}{p_s + \rho^* \left(1 - p_s - \frac{1 - (1 - p_0) \delta \rho^*}{1 - \delta \rho^* (1 - \rho^*)}\right)}.
\]

(NSC-1)

It follows that spillover reporting improves the condition for sustaining the relational contract if and only if

\[
(p_s + (1 - p_s) \rho^*) - \rho^* \frac{1 - (1 - p_0) \delta \rho^*}{1 - \delta^2 \rho (1 - \rho^*)} > p_s,
\]

or equivalently,

\[
-\frac{p_s}{1 - p_s} + \rho^* \delta - p (1 - \rho^*) \delta^2 > 0.
\]

This finishes the proof of Part (i).

**Part (ii).** To find \(\delta^* (f)\), it is equivalent to find the minimal deferred bonus \(b^* (f)\) necessary for sustaining an efficient relational contract. Instead of finding \(b^* (f)\) directly, we provide a lower bound \(b(f) \leq b^* (f)\) for each \(f\). We show that if the lower bound is smaller than \(c/p_s\) for some \(f\), then the lower bounds \(b(f)\) are minimized at \(f = 1\). In this case, this lower bound \(b(1) = b^* (1)\) found in Part (i). This proves Part (ii).

**Step 1: A Necessary Condition.** We use \(V(\rho)\) again to denote the value of the agent when the probability of a high signal is \(\rho\) under \(f\)-spillover reporting, and define \(V_e(\rho)\) and \(V_s(\rho)\) accordingly. Suppressing the dependence of \(w^*\) and \(b^*\) on \(f\), we have

\[
V(\rho) = \max\{V_e(\rho), V_s(\rho)\};
\]

\[
V_e(\rho) = w^* - c + (p + (1 - p) \rho \rho f) \max\left\{0, b^* + \delta V\left(\frac{p}{p + (1 - p) \rho \rho f}\right)\right\} + (1 - p - (1 - p) \rho \rho f) (p_0 \max\{0, b^* + \delta V(\rho_f^*)\} + (1 - p_0) \max\{0, \delta V(\rho_f^*)\}) ;
\]

\[
V_s(\rho) = w^* + \rho \rho f \max\{0, b^* + \delta V(0)\} + (1 - \rho \rho f) \max\{0, \delta V(\rho_f^*)\} .
\]
Under the efficient relational contract, \( V_e(\rho_f^*) \geq V_s(\rho_f^*) \). Since \( V(\rho_f^*) = 0 \) and 
\[
\max\{0, b^* + \delta V(0)\} \geq b^* + \delta V(0), \quad V_e(\rho_f^*) \geq V_s(\rho_f^*) 
\]
implies that
\[
(1 - \rho_f^* f) p_s b^* \geq c + \rho_f^* f \delta (V(0) - V(\rho_f^*)). \tag{IC-f}
\]
Next, since \( V(0) \geq V_e(0) \), we have
\[
V(0) - V(\rho_f^*) \geq \delta p \left( V(1) - V(\rho_f^*) \right) - (1 - p_s) f \rho_f^* b^*. \tag{L1}
\]
Moreover, since \( V(1) \geq V_s(1) \), it follows that
\[
V(1) - V(\rho_f^*) \geq \left( f - p_s - (1 - p_s) \rho_f^* f \right) b^* + \delta f \left( V(0) - V(\rho_f^*) \right). \tag{L2}
\]
Combining Eq(L1) and Eq(L2), we obtain a lower bound \( V(0) - V(\rho_f^*) \), and using this lower bound in Eq(IC-f), we get, after some algebra,
\[
\frac{c}{b^*} \leq p_s + \rho_f^* f \left( 1 - p_s - \frac{1 - \delta \rho_f^* f (1 - p_s)}{1 - \delta^2 f (1 - \rho_f^*)} \right) = A(f).
\]
It follows that a necessary condition to sustain the relationship is
\[
\frac{\delta}{1 - \delta} s(1) \geq b^* \geq \frac{c}{A(f)}. \tag{NSC-f}
\]

**Step 2.** *The optimality of \( f = 1 \).* Notice that when \( f = 1 \), Eq(NSC-f) is exactly the same as the necessary and sufficient condition in Part (i) (Eq(NSC-1)). Since \( c/A(f) \) provides a lower bound for the surplus for sustaining the relational contract, it follows that choosing \( f = 1 \) is optimal as long as \( c/A(f) \) is minimized at \( f = 1 \), or alternatively, \( A(f) \) is maximized at \( f = 1 \).

To show this, let \( \rho_f^* f = x \) and note the following. First, \( x \) is increasing in \( f \). This follows because \( \rho_f^* = p/(p + (1 - p) x) \) and if \( dx/df \leq 0 \) for some \( f \), we must have \( d\rho_f^*/df \geq 0 \), which then contradicts \( dx/df \leq 0 \) since \( dx/df = \rho_f^* + f d\rho_f^*/df > 0 \).

Second, \( 2x (1 - p) \delta < 1 \) for all \( x \). To see this, notice that given \( x \) is increasing in \( f \), and at \( f = 1 \) we have \( x = \rho^* \), it suffices to show that \( 2 (1 - p) \rho^* \delta < 1 \). Recall
\( \rho^* = p/(p + (1 - p)\rho^*), \) and using \( \rho^* \) to substitute for \( p, \) we have

\[
\rho^* (1 - p) = \frac{\rho^* - (\rho^*)^2}{1 - \rho^* + (\rho^*)^2} < \frac{1}{2},
\]

where the inequality follows because it is equivalent to \( 1 - 3\rho^* + 3(\rho^*)^2 > 0, \) which holds for all value of \( \rho^*. \) This shows that \( 2x (1 - p) \delta < 1 \) for all \( x. \)

Now notice that

\[
A(f) = p_s + x \left( 1 - p_s - \frac{1 - \delta (1 - p_s)x}{\delta^2 (1 - p) x^2} \right) \equiv p_s + xB(x),
\]

where the equality uses \( pf = \rho^*_f f (p + (1 - p) \rho^*_f f) = x (p + (1 - p) x). \) If \( f\)-spillover reporting improves on full revelation of signals for some \( f, \) then \( A(f) > p_s \), and the corresponding \( B(x) > 0. \) In this case,

\[
\frac{d}{dx}(xB(x)) = B(x) + x\delta \frac{(1 - p_s) (1 - \delta^2 (1 - p) x^2) - 2x (1 - p) \delta (1 - \delta (1 - p_s) x)}{(1 - \delta^2 (1 - p) x^2)^2}
\]

\[
> B(x) + x\delta \frac{(1 - p_s) (1 - \delta^2 (1 - p) x^2) - (1 - \delta (1 - p_s) x)}{(1 - \delta^2 (1 - p) x^2)^2}
\]

\[
= B(x) + \frac{x\delta B(x)}{1 - \delta^2 (1 - p) x^2}
\]

\[
> 0,
\]

where the first inequality follows because \( 2x (1 - p) \delta < 0, \) which we showed above. This implies that if \( B(x) > 0, \) then \( xB(x) \) is increasing in \( x, \) and therefore, \( xB(x) \) is maximized at \( x_{\max} \equiv \rho^*, \) which corresponds to \( f = 1. \) \( \blacksquare \)
6 Online Appendix (Not for Publication)

6.1 q > 0

We show that spillover reporting helps the sustainability of efficient relational contracts when \( q > 0 \) by presenting two results. The first one is a continuity-type result that shows once spillover reporting helps for \( q = 0 \), it also helps for \( q \) close to 0. For given \( c, p, \) and \( \delta \), let \( S_1 \) be the smallest expected discounted surplus for sustaining the efficient relational contract under spillover reporting when \( q = 0 \).

**Corollary 1**: Consider \( p \) and \( \delta \) such that \(-p/(1-p)+\rho^*\delta - p(1-\rho^*)\delta^2 > 0\), where \( \rho^* = p/(p+(1-p)\rho^*) \). For each expected discounted surplus level \( S > S_1 \), there exists an associated \( q^* > 0 \) such that efficient relational contracts can be sustained under spillover reporting for all \( q \in [0,q^*] \).

**Proof.** The proof follows from standard continuity arguments. For given \( c, p, \) and \( \delta \), let \( b^* \) be the minimum deferred bonus necessary to induce effort under credit reporting. Proposition 1 implies that \( b^* < c/p \) when \(-p/(1-p)+\rho^*\delta - p(1-\rho^*)\delta^2 > 0\), where \( \rho^* = p/(p+(1-p)\rho^*) \). Now again normalize the agent’s outside option \( u \) to be 0, and for each \( w \) and \( b \), define the agent’s value functions \( V(\rho), V_s(\rho), \) and \( V_e(\rho) \) as in Proposition 1. Recall that at \( w^* \) and \( b^* \), we have \( V_e(\rho^*) = V_s(\rho^*), V_e(\rho^*) = 0, \) and \( b^* + \delta V(0) > 0 \). Since \( V_e(\rho^*) - V_s(\rho^*), V_e(\rho^*), \) and \( b + \delta V(0) \) all increase in \( b \), it follows that for each \( S > b^* = S_0 \), there exists a small enough \( \varepsilon(S) \) such that one can find \( w \) and \( b \in (b^*, S) \) such that

\[
V_e(\rho^*) > V_s(\rho^*) + 3\varepsilon;
V_e(\rho^*) > 2\varepsilon;
b + \delta V(0) > 2\varepsilon.
\]

Next, or each \( q > 0 \), let \( V^q(\rho) \) be the agent’s value function at the beginning of a period assuming that he has accepted the principal’s offer. Let \( V_e^q(\rho) \) be the agent’s value function if he works this period and \( V_s^q(\rho) \) be the agent’s value function if he
shirks. The value functions then satisfy the following:

\[ V^q(\rho) = \max\{V^q_e(\rho), V^q_s(\rho)\}; \]
\[ V^q_e(\rho) = w - c + (p + (1 - p)\rho) \max\{0, b + \delta V^q(p)\frac{p}{\rho + (1 - p)\rho}\} \]
\[ + (1 - p - (1 - p)\rho) \max\{0, \delta V^q(\rho^*)\}; \]
\[ V^q_s(\rho) = w + (q + (1 - q)\rho) \max\{0, b + \delta V^q\frac{q}{\rho + (1 - p)\rho}\} \]
\[ + (1 - \rho)\delta V^q(\rho^*) \max\{0, \delta V^q(\rho^*)\}. \]

It is clear that there exists a \( q^* \) such that for all \( q \in [0, q^*] \),

\[ \max\{|V^q(\rho) - V(\rho)|, |V^q_e(\rho) - V_e(\rho)|, |V^q_s(\rho) - V_s(\rho)|\} < \varepsilon \quad \text{for all } \rho \in [0, 1]. \]

As a result,

\[ V^q_e(\rho^*) - V^q_s(\rho^*) \]
\[ = V^q_e(\rho^*) - V^q_e(\rho^*) + V^q_e(\rho^*) - V^q_s(\rho^*) + V^q_s(\rho^*) - V^q_s(\rho^*) \]
\[ > -\varepsilon + 3\varepsilon - \varepsilon \]
\[ = \varepsilon. \]

Similarly,

\[ V^q_e(\rho^*) = V^q_e(\rho^*) + V^q_e(\rho^*) - V^q_e(\rho^*) > 2\varepsilon - \varepsilon = \varepsilon, \]

and

\[ b + \delta V^q(0) = b + \delta V(0) + \delta V^q(0) - \delta V(0) > 2\varepsilon - \varepsilon = \varepsilon. \]

These three inequalities imply that for the chosen \( w \) and \( b \), it is incentive compatible for the agent to accept the contract and exerts effort. □

Corollary 1 states that if spillover reporting improves the sustainability of efficient relational contracts when \( q = 0 \), it also does so for small \( q \). To see this, note that the minimum deferred bonus necessary for effort under spillover reporting when \( q = 0 \) is equal to \( S_1 \). When the expected discounted surplus \( S \) is larger than \( S_1 \), the principal can set a deferred bonus slightly larger than \( S_1 \) without reneging. With the new deferred bonus, the agent strictly prefers working over shirking when \( q = 0 \). It follows that for small enough \( q \) the agent also prefers working since his payoff from shirking is continuous in \( q \). This implies that spillover reporting continues to improve
the sustainability of efficient relational contracts when $q$ is small.

Beyond the continuity result above, however, it is difficult to provide general conditions for spillover reporting to improve over the full revelation of signals. The reason is that when $q > 0$, the value function $V(\rho)$ no longer has an analytical solution and it becomes difficult to check multistage deviation. As a result, our second result uses numerical methods instead. We numerically compute the minimum deferred bonus required for effort both under spillover reporting and under full revelation of signals. In particular, we compute $V(\rho)$ for each deferred-bonus level $b$ and then find the smallest deferred bonus level that sustains efficient effort.

Figure 3: Numerical Simulation

Figure 3 reports our findings for discount factor equal to 0.9 and the cost of effort equal to 2. The two panels on the left (upper left and bottom left) depict the minimum deferred bonus necessary for sustaining effort under full revelation ($b^f$). The bottom-left panel is a 3D plot that illustrates the value of $b^f$ for each $0 < q < p < 3$.
0.5. The top-left panel is the corresponding thermograph that projects the 3D plot into a 2D graph by using colors to represent values. The colder colors reflect smaller values and the warmer colors reflect larger ones. Since \( b^f \) is equal to \( c/(p - q) \), the colors become colder as \( p \) increases and become warmer as \( q \) increases. Moreover, all \((p, q)\) pairs on the same negative 45-degree line have the same color.

Next, the two middle panels report the minimum deferred bonus for sustaining effort under spillover reporting \( (b^g) \), where \( g \) means that the reports are garbled from the signals. The colors again become colder as \( p \) increases and warmer as \( q \) increases, indicating that the minimum deferred bonus under spillover reporting also decreases with \( p \) and increases with \( q \). Different from the two panels on the left, however, not all \((p, q)\) pairs on the same negative 45-degree line have the same color. The \((p, q)\) pairs with the same color are no longer straight lines and appear to have a slope larger than \(-1\).

The two panels on the right report the differences in the minimum deferred bonuses required to sustain efficiency between the cases of full revelation and spillover reporting, namely, \( b^f - b^g \). The thermograph in the upper-right panel makes it clear that there are values of \((p, q)\) such that spillover reporting lowers the minimum deferred bonus to sustain effort \( (b^f - b^g > 0) \). The gains from spillover reporting concentrate on the bottom left region, where the values of \( p \) and \( q \) are smaller. Figure 4 marks the region of \((p, g)\) in which spillover reporting lowers the minimum deferred bonus.
required to induce effort.

6.2 Multiple Effort Levels

Let the agent’s effort choice in period $t$ be $e_t \in \{0, 1, 2\}$, and the associated effort cost be given by $c(0) = 0$, $c(1) = c_1 > 0$ and $c(2) = c_2 > c_1$. The output is binary: $Y_t \in \{0, y\}$, and

$$
\text{Pr}\{Y_t = y\} = \begin{cases} 
  p & e_t = 2 \\
  q & e_t = 1 \\
  0 & e_t = 0
\end{cases},
$$

where $1 > p > q > 0$.

We assume that

$$
p y - c_2 > q y - c_1 \geq u + v > 0,
$$

so the joint surplus is maximized at $e = 2$, followed by $e = 1$, the outside options, and $e = 0$. Notice that we allow the relationship to be profitable even when the agent chooses $e = 1$. This assumption is not important for the analysis but as we will see below, it allows spillover reporting to help the relationship both in the extensive and intensive margins. For expositional convenience, define the discounted expected surplus as

$$
S = \frac{\delta}{1 - \delta} (p y - c_2 - u - v).
$$
We assume that the supervisor, and no other parties, observe the outputs. In other words, the supervisor’s signal is exactly equal to the output.

Compared to the binary-effort case with $q > 0$, the model above essentially adds a lower level of effort ($e = 0$). As a result, if one can rule out that $e = 0$ is used in a relational contract, the result from the binary-effort model can be directly applied. The result below shows that spillover reporting can improve the efficiency of the relational contracts when effort costs are sufficiently convex.

**Corollary 2:** For given $c_1, c_2, p, q,$ and $\delta$, let $S_1$ be the minimum expected discounted surplus for effort under spillover reporting in the binary-effort model. For any $S \in (S_1, \frac{c_2 - c_1}{p - q})$, efficient relational contracts are sustainable under spillover reporting when $(c_2 - c_1)/c_1 > M$ for some $M > 0$.

**Proof.** Define $V(\rho)$ as the agent’s value function at the beginning of a period assuming that he has accepted the principal’s offer. Let $V_i(\rho)$ be the agent’s value function if he puts in effort level of $i \in \{0, 1, 2\}$ this period. Normalize the agent’s outside option $u$ to be 0. For each base wage $w$ and deferred bonus $b$, the value functions satisfy the following:

$$
V(\rho) = \max\{V_0(\rho), V_1(\rho), V_2(\rho)\}
$$

$$
V_2(\rho) = w - c_2 + (p + (1 - p)\rho) \max\{0, b + \delta V(\frac{p}{p + (1 - p)\rho})\}
+ (1 - p - (1 - p)\rho) \max\{0, \delta V(\rho^*)\};
$$

$$
V_1(\rho) = w - c_1 + (q + (1 - q)\rho) \max\{0, b + \delta V(\frac{q}{q + (1 - p)\rho})\}
+ (1 - q - (1 - q)\rho) \max\{0, \delta V(\rho^*)\}.
$$

$$
V_0(\rho) = w + \rho \max\{0, b + \delta V(0)\} + (1 - \rho) \max\{0, \delta V(\rho^*)\}.
$$

It is clear that for each $w$ and $b$, there is a unique set of value functions that satisfy the functional equations above.

Next, suppose $b^*$ is the minimum deferred bonus for effort in the binary effort case and $w^*$ is the associated base wage so that the agent’s participation binds. Let $V_e(\rho)$, $V_s(\rho)$, and $V_b(\rho)$ be the agent’s value functions associated with $b^*$ and $w^*$, where we use the subscript $b$ stands for the binary case. Now let $w = w^* + c_1$ and $b = b^*$. We show below that for sufficiently small $c_1$, we have $V_2(\rho) = V_e(\rho), V_1(\rho) = V_s(\rho), V(\rho) = V_b(\rho)$, and $V_0(\rho) = w + \rho(b + \delta V_b(0))$ for all $\rho$. 6
To do this, it suffices to show that the constructed value function satisfies the set of functional equations above. Given the properties of $V_e(\rho)$, $V_s(\rho)$, and $V_b(\rho)$, we only need to show that $V_1(\rho) \geq V_0(\rho)$ for all $\rho$. Notice that

\[
V_1(\rho) - V_0(\rho) = (q + (1-q)\rho)(b + \delta V_b(\frac{q}{q+(1-p)\rho})) - \rho(b + \delta V_b(0)) - c_1 > (1-q)\rho(b + \delta V_b(0)) - c_1,
\]

where the inequality uses the fact that $V$ is increasing in $\rho$. Since $b + \delta V_b(0) > 0$ (by Proposition 1) and $b = b^*$ is independent of $c_1$, it is clear that for small enough $c_1$, $V_1(\rho) - V_0(\rho) > 0$ for all $\rho$.

The above implies that for small enough $c_1$, the agent is willing to choose $e = 2$ when $b = b^* = S_0$ and $w = w^* + c_1$. Finally, since $S > S_0$, the principal will not renege on the deferred bonus. This establishes that the stationary strategies with deferred bonus with $w = w^* + c_1$ and $b = b^*$ supports the efficient relational contracts.

Notice that $(c_2 - c_1)/c_1$ measures the convexity of effort cost since $c_2 - c_1$ is the marginal cost of effort from increasing $e = 1$ to $e = 2$ and $c_1$ is the marginal cost of effort from increasing $e = 0$ to $e = 1$. Corollary 2 therefore shows that when effort costs are sufficiently convex, spillover reporting can support the efficient relational contracts even if it is impossible to do so with perfect revelation of outputs. Notice that if the signals were revealed fully when $S \in (S_1, \frac{2c_2-c_1}{p-q})$, the agent would either choose a lower level of effort ($e = 1$) or forgo the relationship by taking his outside option. In the former case, spillover reporting improves the relationship in the intensive margin by making it more efficient. In the later case, spillover reporting improves the relationship in the extensive margin by sustaining an efficient relationship that would fail to start.

The intuition for Corollary 2 is as follows. When the cost function is sufficiently convex, the gain of deviating from $e = 1$ to $e = 0$ is small relative to the gain of deviating from $e = 2$ to $e = 1$. It follows that if the agent does not gain from deviating to $e = 1$, he will not gain from deviating to $e = 0$. Therefore, if spillover reporting helps sustain efficient relational contracts with binary effort, it can also help here when the effort cost is sufficiently convex.
This intuition suggests that spillover reporting may have wider applicability. For example, when there are multiple effort levels, ruling out profitable local deviations is sometimes enough for ruling out global deviations. Since checking local deviations requires essentially comparing payoffs from two (adjacent) effort levels, our result on spillover reporting can be applied to relax constraints that prevent local deviation. This suggests that spillover reporting can improve the sustainability of relational contracts for more general effort cost structures.

6.3 Multiple Agents

Suppose there are \( n \) identical agents, and the production is independent across them. For each agent \( i \), his output \( Y_i \in \{0, y\} \). The probability that \( Y_i = y \) is \( p \) if the agent \( i \) puts in effort, and is 0 otherwise. We assume that \( py - c > u + v > 0 \), where \( c \) is the agent’s cost of effort. In addition, the supervisor, and no other agents, observe the outputs. In other words, this is essentially an \( n \)-fold model of that in Section 3 with \( p = p_0 \). Below, we study two types of relationships. The first is the commonly used, but suboptimal, independent relationships. In the second one, the relationships are interdependent.

6.3.1 Independent Relationships

In this case, the principal deals with each of the agent separately. In other words, the relationships are independent of each other. To derive the condition for sustaining an efficient \( n \)-agent relationship with full signal revelations, note that agent \( i \) will put in effort if

\[ b_i \geq c/p. \]

The maximal reneging temptation of the principal occurs in the case when all agents receive a bonus. The principal’s gain from reneging is given by \( \sum_{i=1}^{n} b_i \).

Next, the principal’s surplus is given by

\[
\frac{\delta n}{1 - \delta} (py - c - u - \pi) = \frac{\delta ns (1)}{1 - \delta}.
\]

It follows that for the principal not to renege on the bonuses, the non-reneging constraint is given by

\[
\frac{\delta ns (1)}{1 - \delta} \geq \sum_{i=1}^{n} b_i.
\]
Combining this with the agent’s incentive constraint, we obtain that the condition for sustaining the efficient relational contract with full signal revelation is given by

\[
\frac{\delta_{ns}(1)}{1 - \delta} \geq \sum_{i=1}^{n} b_i \geq nc/p,
\]

or equivalently,

\[
\frac{\delta_{ns}(1)}{1 - \delta} \geq c/p.
\]

This is the same condition as the single-agent case. Note that we showed above that this condition is necessary, but clearly this condition is also sufficient.

Next, to obtain the condition for sustaining an efficient \(n\)-agent relationship with spillover reporting, let \(b_g\) be the deferred bonus necessary to induce effort under spillover reporting (see the proof of Proposition 1, part (i) for the expression of \(b_g\)). Here, the subscript \(g\) reflects that the reports are garbled from the signals. This implies that each agent \(i\) puts in effort if

\[b_i \geq b_g.\]

As before, the non-reneging constraint of the principal is given by

\[
\frac{\delta_{ns}(1)}{1 - \delta} \geq \sum_{i=1}^{n} b_i.
\]

Combining this with the agent’s incentive constraint, we obtain that the condition for sustaining the efficient (independent) \(n\)-agent relational contract with spillover reporting is

\[
\frac{\delta_{ns}(1)}{1 - \delta} \geq \sum_{i=1}^{n} b_i \geq nb_g,
\]

or equivalently,

\[
\frac{\delta_{s}(1)}{1 - \delta} \geq b_g.
\]

Again, this is the same condition as the single-agent case, and in addition, it is clear that this condition is also a sufficient one. It follows that under the independent relationships, spillover reporting helps with the \(n\)-agent case if and only if it helps with the single-agent case.
6.3.2 Interdependent Relationships

When the signals are fully revealed, the independent relationships considered above are suboptimal. Levin (2002) implies that the optimal relational contract takes the following form. In each period, there’s a bonus pool of $B^p(n)$ to be shared by agents with good signals. With probability $(1-p)^n$, however, no agent receives a good signal, and in this case, the bonus pool is not paid out.

We now derive the agent’s incentive constraint under the optimal $n$-agent relational contract (with full revelation of signals). Since the agents are symmetric under the relational contract, we suppress the subscript and let $b^p(n)$ be the expected bonus of the agent if his signal is good. The agent will put in effort if

$$b^p(n) \geq \frac{c}{p}.$$ 

Notice that $b^p(n)$ and $B^p(n)$ are linked through the following equation:

$$pb^p(n) = \frac{1}{n} (1 - (1-p)^n) B^p(n).$$

The left-hand side is the agent’s expected bonus. And the right-hand side is another way to calculate it: the expected total bonus is $(1 - (1-p)^n) B^p(n)$, where $1 - (1-p)^n$ is the probability that the bonus pool $B^p(n)$ is paid out, and the agent gets $1/n$ of it in expectation. The equation implies that the agent’s incentive constraint can be written as

$$B^p(n) = \frac{npb^p(n)}{1 - (1-p)^n} \geq \frac{np}{1 - (1-p)^n} \frac{c}{p}.$$ 

Next, the principal’s non-reneging condition is given by

$$\frac{\delta ns(1)}{1 - \delta} \geq B^p(n).$$

And combining this with the agent’s incentive constraint, we obtain that the necessary condition for sustaining an efficient $n$-agent relational contract with full signal revelation is

$$\frac{\delta s(1)}{1 - \delta} \geq \frac{p}{1 - (1-p)^n} \frac{c}{p}. \quad \text{(n-perfect)}$$

Note that this condition is also a sufficient condition; see Levin (2002).

We now consider the following $n$-agent relational contract with spillover report-
ing. For each agent, the supervisor sends the report according to the spillover reporting described in Section 3. As in Section 3, the principal formally pays out no end-of-period bonus. Instead, "the deferred-bonus pool" is paid out at the beginning of the next period, and it is divided equally among the agents with good reports. Each agent observes his own report at the end of each period and also a public signal for whether at least one agent has a good report. In addition, each agent observes the contracts offered by the principal to all agents at the beginning of each period. All agents take their outside options forever if any parties ever publicly deviates.

Let $\delta_0(n)$ be the smallest discount factor for an efficient $n$-agent relational contract under full revelation of signals, and $\delta_g(n)$ the corresponding discount factor under spillover reporting. Corollary 3 below provides the condition for when spillover reporting improves over full revelation of signals.

**Corollary 3** \( \delta_g(n) < \delta_0(n) \) if and only if

\[
\frac{(p + (1-p)\rho^*) (1 - (1 - p)^n)}{(1 - (1 - (p + (1-p)\rho^*))^n)} \frac{1}{(p + (1 - p) \rho^*) - \rho^*} < 1.
\]

**Proof.** At $\delta = \delta_g(n)$, denote $w(n)$ and $b^g(n)$ as the agent’s base wage and expected bonus (conditional on a good report) under spillover reporting. Let $B^g(n)$ be the corresponding deferred-bonus pool. Notice that $b^g(n)$ and $B^g(n)$ are linked through

\[
(p + (1-p)\rho^*) b^g(n) = \frac{1}{n} (1 - (1 - (p + (1-p)\rho^*))^n) B^g(n),
\]

or alternatively,

\[
B^g(n) = \frac{n (p + (1-p)\rho^*) b^g(n)}{(1 - (1 - (p + (1-p)\rho^*))^n)}.
\]

This implies that the principal’s non-reneging constraint can be written as

\[
\frac{\delta ns(1)}{1 - \delta} \geq B^g(n) = \frac{n (p + (1-p)\rho^*) b^g(n)}{(1 - (1 - (p + (1-p)\rho^*))^n)}.
\]

Next, we determine the value of $b^g(n)$. As in Proposition 1, each agent’s expected future payoff is determined by his belief of a high signal in the previous period, and
the value functions satisfy

\[
V(\rho) = \max\{V_e(\rho), V_s(\rho)\};
\]

\[
V_e(\rho) = w(n) - c + (p + (1 - p)\rho) \max\left\{0, b^g(n) + \delta V\left(\frac{p}{p + (1 - p)\rho}\right)\right\}
+ (1 - p - (1 - p)\rho) \max\{0, \delta V(\rho^*)\};
\]

\[
V_s(\rho) = w(n) + \rho \max\left\{0, b^g(n) + \delta V(0)\right\} + (1 - \rho) \max\{0, \delta V(\rho^*)\}.
\]

Note that this is exactly the same set of functional equations as that in Proposition 1 (with \(p_0 = 0\)). As a result, we obtain that

\[
c = \left((p + (1 - p)\rho^*) - \rho^* \frac{1 - (1 - p)\delta \rho^*}{1 - \delta \rho^*}\right) b^g(n).
\]

Combining the above with the principal’s non-reneging constraint, we obtain that the condition for sustaining the efficient relational contract is given by

\[
\frac{\delta s(1)}{1 - \delta} \geq \frac{(p + (1 - p)\rho^*)}{(1 - (1 - (p + (1 - p)\rho^*))n)} \frac{c}{(p + (1 - p)\rho^*) - \rho^* \frac{1 - (1 - p)\delta \rho^*}{1 - \delta \rho^*}}.
\]

Comparing this condition to the condition for full revelation of signals, we get that spillover reporting is an improvement if and only if

\[
\frac{(p + (1 - p)\rho^*)}{(1 - (1 - (p + (1 - p)\rho^*))n)} \frac{c}{(p + (1 - p)\rho^*) - \rho^* \frac{1 - (1 - p)\delta \rho^*}{1 - \delta \rho^*}} \leq \frac{p}{1 - (1 - p)\rho^*} \frac{c}{p}.
\]

This finishes the proof.

Notice that when \(n = 1\), this condition in Corollary 3 is equivalent to that in part (i) of Proposition 1 (with \(p_0 = 0\)). As \(n\) increases, it can be checked that the left-hand side increases with \(n\), so this condition becomes more difficult to satisfy. It can be shown, however, that for all \(n\), there exists ranges of \(p\) such that spillover reporting helps to sustain relationship. Moreover, for all \(n\), the minimal surplus for an efficient relational contract under full revelation is about \(\delta\) times more as that under spillover reporting when as \(p \to 0\).

**Corollary 4** Let \(S_0(p, \delta, n)\) be the minimal surplus necessary sustain an efficient
relational contract with full revelation of signal. Let $S_g(p, \delta, n)$ be the corresponding surplus with spillover reporting. For all $n$,

$$\lim_{p \to 0} \frac{S_0(p, \delta, n)}{S_g(p, \delta, n)} = 1 + \delta.$$ 

**Proof.** From the condition on the sustainability of efficient relational contracts, we have

$$\frac{S_0(p, \delta, n)}{S_g(p, \delta, n)} = \frac{(p + (1 - p) \rho^*) - \rho^* \frac{1 - (1 - p) \delta \rho^*}{1 - \delta^* p (1 - \rho^*)}}{p} \frac{A(p, n)}{B(p, n)},$$

where $A(p, n) = \frac{p}{1 - (1 - p) \rho^*}$ and $B(p, n) = \frac{(p + (1 - p) \rho^*)}{(1 - (1 - p + (1 - p) \rho^*))^2}$, and recall that $\rho^* = \frac{p}{p + (1 - p) \rho^*}$.

Now notice that we have $p = \frac{(\rho^*)^2}{1 - \rho^* + (\rho^*)^2}$, so $\lim_{p \to 0} \rho^* = 0$, and $\lim_{p \to 0} \frac{(\rho^*)^2}{p} = 1$. Now

$$\lim_{p \to 0} \frac{S_0(p, \delta, n)}{S_g(p, \delta, n)} = \lim_{p \to 0} \frac{(p + (1 - p) \rho^*) - \rho^* \frac{1 - (1 - p) \delta \rho^*}{1 - \delta^* p (1 - \rho^*)}}{p} \lim_{p \to 0} A(p, n) \lim_{p \to 0} B(p, n).$$

And for each $n$, $\lim_{x \to 0} \frac{x}{1 - (1 - x)} = \frac{1}{n}$, so $\lim_{p \to 0} A(p, n) = \lim_{p \to 0} B(p, n) = \frac{1}{n}$. In addition,

$$\lim_{p \to 0} \frac{(p + (1 - p) \rho^*) - \rho^* \frac{1 - (1 - p) \delta \rho^*}{1 - \delta^* p (1 - \rho^*)}}{p} = \frac{1 + \lim_{p \to 0} \rho^* - p + (1 - p) \delta (\rho^* - \delta p (1 - \rho^*))}{1 - \delta^2 p (1 - \rho^*)} = 1 + \delta.$$

Together, we then have

$$\lim_{p \to 0} \frac{S_0(p, \delta, n)}{S_g(p, \delta, n)} = 1 + \delta.$$  

**6.4 Collusion**

Suppose the principal can offer the supervisor the following history-dependent contract at the beginning of each period $t$,

$$w_t^s + \alpha_t Y_0,$$
where $\alpha$ is the share of the public output that goes to the supervisor. To simplify the analysis, we assume that $w^t_s = -\alpha t p_0 y_0$ so that the supervisor receives 0 expected payoff in equilibrium. We also assume that $\alpha$ share of the output directly accrues to the supervisor, and this allows us to focus on the relational contract between the principal and the agent. Now consider the following type of stationary collusion. The agent offers the supervisor a payment of $t_b > 0$ in each period. In return, the supervisor always sends a good report about the agent.

Now define $p_g \equiv p_s + (1 - p_s) \rho^*$ be the probability that the agent receives a deferred bonus under spillover reporting (when he works and does not engage in stationary collusion). Our result below shows that when $\delta < 1/(2 - p_g)$, if spillover reporting can help sustain the efficient relational contract, then the principal can prevent stationary collusion by giving enough share of the output to the supervisor.

**Corollary 5:** Suppose spillover reporting helps sustain the relational contract, i.e., $1 - p_0 - \delta \rho^* > 0$ and $-p_s/(1 - p_s) + \delta (\rho^* - \delta p (1 - \rho^*)) > 0$. Now if $\delta < 1/(2 - p_g)$, then there exists $\alpha^* < 1$ such that for all $\alpha > \alpha^*$, the supervisor and the agent will not engage in stationary collusion.

**Proof.** Let $b_g$ be the lowest deferred bonus necessary to sustain effort under spillover reporting. From the proof of Proposition 1, we have

$$b_g = \frac{c}{p_0 + (1 - p_0) (p + (1 - p) \rho^*) - \rho^* \frac{1 - (1 - p_0)(1 - p) \rho^*}{1 - \delta^2 p (1 - \rho^*)}}.$$ 

To prevent stationary collusion, it suffices to show that the joint payoff between the supervisor and the agent without collusion is smaller than their joint payoff under collusion. The joint payoff without collusion is 0 by design.

Now if the agent colludes with the supervisor, he will choose to shirk. This is because if the agent always receives a good report, he always receives a deferred bonus, and therefore, does not have an incentive to put in effort. Given the agent shirks, the supervisor’s payoff is given by $t_b - \alpha p_0 y_0$ per period. For the agent, his payoff under collusion is given by

$$-t_b + c + (1 - p_g) b_g,$$

where recall that $p_g = p_s + (1 - p_s) \rho^*$ is the probability that a deferred bonus is paid out to the agent under spillover reporting when the agent works.
It follows that the joint payoff of the agent and the supervisor is given by 
\[-\alpha p_0 y_0 + c + (1 - p_g) b_g.\] 
As a result, they will not collude if 
\[\alpha p_0 y_0 - c \geq (1 - p_g) b_g.\] 
(No-collusion condition)

It follows that a large enough \(\alpha\) can be chosen as long as
\[s(1) = p_0 y_0 - c \geq (1 - p_g) b_g.\]

To see that this condition is satisfied when \(\delta < 1/(2 - p_g)\), recall that for a relational contract to be sustainable under spillover reporting, the principal’s non-reneging condition is given by
\[\frac{\delta}{1 - \delta} s(1) \geq b_g.\]

It follows that
\[s(1) \geq \frac{1 - \delta}{\delta} b_g > (1 - p_g) b_g,\]
where the last inequality follows because \(\delta < 1/(2 - p_g)\).

The main idea for why collusion will not take place is as follows. If the collusion occurs so that the supervisor always sends out good reports, the agent will shirk. This will then hurt the supervisor’s payoff, and the damage is larger when the supervisor receives a bigger share of the output. For a large enough \(\alpha\), the cost from agent shirking is sufficiently large that the supervisor and the agent will not engage in collusion.

### 6.5 General Reporting Rules

Below, we show that any reporting rule with a predetermined restart date cannot help sustain efficient relational contract. Proposition 2 in the main text follows directly from it. We also provide a proof for Proposition 3.

**Proposition 2’**: Let \((U_t, \Pi_t)\) be the expected discounted payoffs of the agent and the principal evaluated at time \(t\). Suppose (3) fails. For all \(t\), if there exists a predetermined \(t' \geq t\) such that \((U_{t'}, \Pi_{t'})\) are independent of \(h_{t'}\), then \(e_t = 0\).

**Proof.** Note that in our setting, the feasible surplus of the game \(S\) cannot be raised by any reporting rule. Suppose \((U_{t'}, \Pi_{t'})\) are independent of \(h_{t'}\). Then \(e_{t' - 1}\) is solely
motivated by \( b_{t'} - 1 \), and since \( b_{t'} - 1 \leq S < c/ (p - q) \), it must hold that \( e_{t-1} = 0 \). Given that \( e_{t-1} = 0 \) regardless of \( h_{t'} - 1 \) and that (3) fails, the expected sum of bonuses \( E( b_{t-2} + \delta b_{t-1} ) \) should also be no larger than \( S \). This implies that \( e_{t-2} = 0 \) as well. By induction, \( e_\tau = 0 \) for all \( \tau \in \{1, 2, \ldots, t'\} \).

**Proposition 3:** Let \( s(1) \) be the expected future of the relationship per period. For any reporting rule to sustain the efficient relational contract, one must have

\[
\frac{\delta s(1)}{1 - \delta} \geq \sqrt{4p(1 - p)} \frac{c}{p}.
\]

In particular, full revelation of signals is the optimal reporting rule when \( p = 1/2 \).

**Proof.** Define \( S = \delta s(1) / (1 - \delta) \). When the signals are perfectly revealed, the necessary and sufficient condition for sustaining an efficient relational contract is given by

\[
\frac{c}{p} \leq S.
\]

We want to show that if \( S < \sqrt{4p(1 - p)} \frac{c}{p} \), it is impossible to construct an equilibrium in which the agent puts in effort. In particular, using standard argument as in Fuchs (2007), it suffices to show that there does not exist an equilibrium in which the agent always puts in effort (unless the relationship is terminated).

Consider an arbitrary information partition process. Pick one information set \( (h^i) \). Use \( x \) to denote the possible states within the information set. One interpretation of \( x \) is some output realizations \( y^t \) that falls into \( h^i \).

Let \( V(x) \) be the agent’s continuation payoff in state \( x \) after \( e_{t+1} \) is put in but before \( y_{t+1} \) is realized and \( W_{t+1} \) is paid out. Let \( V(x_i) \) be the agent’s continuation payoff in state \( x \) after \( e_{t+1} \) is put in, \( y_{t+1} \) is realized but before \( W_{t+1} \) is paid out. Within each state \( x \), we have \( x_i \in \{x_y, x_0\} \), where \( x_y \) denotes that \( Y_t = y \) is realized following \( x \), and \( x_0 \) denotes that \( Y_t = 0 \) is realized.

Note that

\[
V(x) = V(x) + p(V(x_y) - V(x)) + (1 - p)(V(x_0) - V(x)).
\]

And since the output \( Y_t \) is independent of the past state, we have \( Cov(V(x_i) - V(x), V(x)) = 0 \).
To induce effort, we need

\[ E_x[V(x_y) - V(x)] = E_x[V(x_y) - V(x)] - E_x[V(x_0) - V(x)] \geq \frac{c}{p}. \]

This helps give a lower bound for \( \text{Var}(V(x_i)) \). In particular,

\[
\begin{align*}
\text{Var}(V(x_i)) &= \text{Var}(V(x)) + \text{Var}(V(x_i) - V(x)) \\
&= \text{Var}(V(x)) + E_y[\text{Var}(V(x_i) - V(x))|Y] \\
&\quad + E_y[E_x[V(x_i) - V(x)|Y]] \\
&\geq \text{Var}(V(x)) + E_y[\text{Var}(V(x_i) - V(x))|Y] \\
&\quad + p(1 - p)\left(\frac{c}{p}\right)^2 \\
&\geq \text{Var}(V(x)) + p(1 - p)\left(\frac{c}{p}\right)^2,
\end{align*}
\]

where the first line follows because \( \text{Cov}(V(x_i) - V(x), V(x)) = 0 \), the second line uses the variance decomposition formula, the third line follows because \( E_y[\text{Var}(V(x_i) - V(x))|Y] \) is a binary value \( (Y \in \{0, y\}) \) such that with probability \( p \) its value is \( E_x[V(x_y) - V(x)] \) and with probability \( 1 - p \) its value is \( E_x[V(x_0) - V(x)] \), and \( E_x[V(x_y) - V(x)] - E_x[V(x_0) - V(x)] \geq c/p \).

Now let’s provide an upper bound for \( \text{Var}(V(x_i)) \). Suppose a public report \( s(x_i) \) will be sent out after state \( x_i \). Let \( b(s) \) be the bonus paid out to the agent (at the end of the period) following report \( s \). This allows us to write

\[ V(x_i) = b(s(x_i)) + \delta V(s(x_i))(x_i), \]

where \( V(s(x_i))(x_i) \) is the continuation payoff of \( x_i \), which goes to the information set by report \( s(x_i) \).

Note that for the principal to be willing to pay the bonus, we must have

\[
\max_s \{b_s + \delta E_{x_i}[V_s(x_i)|s]\} - \min_s \{b_s + \delta E_{x_i}[V_s(x_i)|s]\} \leq S.
\]

Because otherwise the expected payoff of the principal following some report will be below his outside option.
Decomposing the variance on the reports, we have

\[
\text{Var}(V(x_i)) = \text{Var}(E[b_s + \delta V_s(x)|s]) + E[\text{Var}(b_s + \delta V_s(x)|s)] \\
\leq \frac{1}{4}S^2 + \delta^2 E[\text{Var}(V_s(x)|s)].
\]

Now combining the upper and lower bound for \( \text{Var}(V(x_i)) \), we get that

\[
\frac{1}{4}S^2 + \delta^2 E[\text{Var}(V_s(x)|s)] \geq \text{Var}(V(x)) + p(1-p)\left( \frac{c}{p} \right)^2,
\]

or equivalently,

\[
E[\text{Var}(V_s(x)|s)] \geq \frac{1}{\delta^2}(\text{Var}(V(x)) + p(1-p)\left( \frac{c}{p} \right)^2 - \frac{1}{4}S^2).
\]

Now if \( 4p(1-p)\left( \frac{c}{p} \right)^2 > S^2 \), the inequality above implies that

\[
E[\text{Var}(V_s(x)|s)] > \frac{1}{\delta^2}(\text{Var}(V(x))).
\]

In particular, there will be one information set (associated with a signal) whose variance exceeds \( \frac{1}{\delta^2}(\text{Var}(V(x))) \). Now we can perform the same argument on this new information set, and we can construct a sequence of information set whose variance approaches infinity. This leads to a contradiction.

Therefore, to sustain an efficient relational contract, one must have

\[
S^2 \geq 4p(1-p)\left( \frac{c}{p} \right)^2.
\]

It follows that when \( p = 1/2 \), this condition becomes \( S \geq \frac{c}{p} \), which is exactly the condition for sustaining the efficient relational contract under full revelation of signals. This shows that full revelation of signals is the optimal reporting strategy when \( p = 1/2 \). □