Information Revelation in Relational Contracts*

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Abstract
This paper shows that efficiency of relational contracting can be enhanced by a simple and intuitive supervisor-administered performance evaluation system which we term credit-rollover reporting. Under credit-rollover reporting, the supervisor’s report depends on both the current and the past performance of the agent, resulting in a higher frequency of good reports than good performance. This allows the principal to reduce the size of bonus for each good report while maintaining the effort incentive of the agent. The reduced bonus size under credit-rollover reporting lowers the principal’s short-term gain from breaking his promise and makes efficient relational contracts more readily sustainable.

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1 Introduction

Employment relationships are often modeled as a principal-agent relationship. One way for the boss (the principal) to motivate the worker (the agent) is to use formal employment contracts which tie the worker’s pay to his performance. In many situations, however, formal contracts are difficult to write because, for example, the agent’s performance is hard to measure. One substitute for a formal contract is a relational contract, where the principal motivates the agent using informal promises; see for example, Malcomson (2012) for a review. Even if there is no third party to enforce these promises, relational contracts can be self-enforcing as long as the principal’s long-term value in carrying out the promises exceeds the short-term gain in breaking them.

Relational contracts are important in both small and large organizations. When an organization is large, however, the principal may not directly observe the performance of each agent. Instead, information about an agent’s performance is typically transmitted to the top through a middle manager (the supervisor). The supervisor, upon observing the agent’s performance, evaluates him and reports his performance to the principal. In these situations, the principal’s promises are based not on the agent’s performance but on the supervisor’s reports.

Consider, for example, the incentive system at Lincoln Electric, a leading manufacturer of welding machines famous for its high-powered incentive pay. An important part of the worker’s incentive pay is his year-end bonus, which depends on hard-to-verify criteria such as dependability, quality, and cooperation (Berg and Fast 1983). With more than 3000 workers in 1990, the top management does not observe the performance of each worker. As a result, the bonus is determined according to the evaluation by the worker’s supervisor.

The purpose of this paper is to study the effect of the supervisor’s report when the agent is motivated by relational contracts. Surprisingly, we find that the supervisor can improve the sustainability of relational contracts by submitting less-informative reports. In particular, we study in detail a class of reporting rules in which the supervisor evaluates the agent based on both his current and past performance. These reporting rules facilitate the relational contract by making the principal more credible through lowering his short-term gain from breaking his promise.
Specifically, we consider the repeated relationship between a principal and an agent where the agent’s output is observed and reported by a supervisor. The agent’s effort is his private information and stochastically affects his output. The agent’s output is binary (high and low) and his effort makes the high output more likely. Only the supervisor observes the agent’s performance, and he reveals (partially or in full) his information publicly by sending a report about the agent’s performance. The supervisor’s report is not contractible, but the principal can promise to pay the agent based on the supervisor’s report.

Within this environment, we focus on a class of reporting rules denoted as $f$-credit-rollover reporting. Essentially, the supervisor reports either $G$ (good) or $B$ (bad). In each period, the supervisor reports $G$ if the output is high. If the output is low, the supervisor reports $B$ unless the previous period’s output is high, in which case the supervisor reports $G$ with probability $f$ (and $B$ with probability $1 - f$). One can interpret $f$ as the rollover credit the agent earned from last period’s high output. The more credit the agent receives, the higher is the probability that he is forgiven for this period’s low output.

Notice that when $f = 0$, the supervisor’s report coincides with the output since there is no credit rollover. Our main result identifies the condition under which the supervisor can lower the cutoff discount factor necessary to sustain relational contracts. Moreover, we show that $f = 1$ obtains the lowest cutoff discount factor whenever the full revelation of outputs ($f = 0$) is not optimal. When the probability of high output is close to 0 and the discount factor given by $\delta$, $f = 1$ reduces the expected surplus necessary to sustain the relational contract (under $f = 0$) by a fraction of roughly $\delta / (1 + \delta)$.

The reason for why $f$-credit-rollover reporting can help sustain relational contracts is as follows. When $f > 0$, a high output not only leads to a bonus in the current period but also gives the agent a higher continuation payoff. Because the agent’s reward for a high output is spread over time, this allows the principal to motivate the agent with a smaller bonus (relative to $f = 0$), which reduces the principal’s short-term gain from reneging. Moreover, $f$-credit-rollover reporting has the feature that the relationship is stationary along the equilibrium path, so it helps to sustain the relationship by lowering the principal’s short-term deviation gain without affecting his long-term value.
While it is easy to explain why $f$-credit-rollover reporting can help, checking this formally is difficult. The reason is that the relationship under $f$-credit-rollover reporting is no longer stationary if the agent shirks. In particular, the agent’s probability of receiving a bonus in any given period is higher if he worked rather than shirked in the previous period: working increases the probability of a high output, which in turn enhances the probability of receiving a bonus in this period. When the agent’s future payoff depends on his private action in the past, standard recursive techniques cannot be applied. As a result, checking that the agent has no profitable deviation becomes nontrivial because one needs to check multistage deviations.

To show that the agent has no profitable deviation, we notice that under $f$-credit-rollover reporting the agent’s expected future payoff is completely determined by the probability of high output in the past period. This allows us to represent the agent’s future payoffs using a value function that depends only on this probability. We solve for the value function explicitly and use it check that the agent has no incentive to deviate. A consequence of preventing the multistage deviation is that the timing of bonus is relevant: they should be paid at the beginning of next period (as a higher base wage) instead of at the end of the current period.

Our paper contributes to two strands of the literature. First, it contributes to the theoretical works that explore the relationship between the information structure and efficiency. Within this literature, Kandori (1992) shows that garbling signals within periods weakly decreases efficiency in repeated games with imperfect public monitoring. Abreu, Milgrom, and Pearce (1992) (AMP hereafter), in the context of repeated games, and Fuchs (2007), in relational contracting with subjective evaluation, however, show that efficiency can be increased by bundling signals across a fixed number of periods. The AMP-Fuchs type of reporting rules does not help in our setting, and we discuss features of reporting rules necessary for improving efficiency in Subsection 4.3 in detail. Kandori and Obara (2006) show that when signals do not have full support, the use of private (mixed) strategies can give rise to equilibria that are more efficient. In our model, the principal’s action is publicly observed, so the use of mixed strategy does not help relax the incentive constraints of the players by better detecting deviations.

Second, this paper contributes to the literature that studies the use of external instruments to increase the efficiency of relational contracting. Baker, Gibbons, and
Murphy (1994) show that the use of an explicit contract can help increase the efficiency of the relationship by reducing the gain from reneging.\footnote{They also show, however, that explicit contracts can crowd out relational contracts by improving players’ outside options.} Rayo (2007) examines the role of ownership structure in sustaining relationships. He shows that when the actions of players are unobservable (and the First Order Approach is valid), the optimal ownership shares should be concentrated. When the actions are observable (so that the First Order Approach is invalid), the optimal ownership shares should be diffused. The external instrument explored in our paper is the use of information flows. Our result implies that the efficiency of the relationship can be enhanced with less information (in the sense that signals are intertemporally garbled). This suggests that strategically using intermediaries to manipulate information can increase the efficiency of the relationship.

The rest of the paper is organized as follows: we set up the model in Section 2 and present our main results in Section 3. Section 4 discusses the robustness of our results and examines properties of general reporting rules. Section 5 concludes.

\section{Setup}

Time is discrete and indexed by $t \in \{1, 2, ..., \infty\}$.

\subsection{Players}

There is one principal, one agent, and one supervisor. The players are all risk neutral, infinitely lived, and have a common discount factor $\delta$. The agent’s per-period outside option is given by $u$; the principal’s per-period outside option is $\pi$. To focus on the effect of information revelation, we assume that the supervisor is a nonstrategic player. His payoff is the same whether he stays in or exits the relationship and is normalized to 0.

\subsection{Production}

If the principal and the agent engage in production together in period $t$, the agent chooses effort $e_t \in \{0, 1\}$. The cost of effort is given by $c(0) = 0$ and $c(1) = c$. The
output is binary: \( Y_t \in \{0, y\} \), and

\[
\begin{align*}
\Pr\{Y_t = y | e_t = 1\} &= p; \\
\Pr\{Y_t = y | e_t = 0\} &= q,
\end{align*}
\]

where \( 1 > p > q \geq 0 \).

The production function is commonly used in the literature. Notice that the restriction to binary outputs can be easily relaxed and does not affect the result of the paper. For a model with multiple or continuous outputs, one can use the relative likelihood ratio to divide the outputs into two groups and transforms it into a model with binary effort. In contrast, the restriction to binary effort levels significantly simplifies our analysis, and solving the model with general effort cost structure is difficult. In Subsection 4.1, we show that when the effort costs are sufficiently convex, the main result of the paper continues to hold for three effort levels and discuss how the model can be generalized. Finally, to make the analysis interesting, we assume that the relationship is valuable if and only if the agent puts in effort. In other words,

\[ py - c > \pi + u > qy. \]

### 2.3 Information Structure

In each period \( t \), the agent’s effort, \( e_t \), is his private information. The supervisor observes the output and makes a public report, which is summarized by a public signal, \( s_t \). Notice that the supervisor’s payoff is always zero, so any reporting rule is incentive compatible for him. Denote the supervisor’s reporting rule by \( S_t \), which maps the set of past outputs into the set of possible signals, \( S \):

\[
S_t : \prod_{j=1}^{t} Y_j \rightarrow S.
\]

We also allow for the supervisor to randomize on his reports and denote \( \Sigma_t \) as the supervisor’s mixed strategy. Depending on the supervisor’s reporting rule, our setup incorporates the following commonly studied information structures.

**Example 1: (Full Revelation of Outputs)**
In standard relational contracting models with imperfect public monitoring (see, for example, Levin (2003)), outputs are observed by both the principal and agent each period. This corresponds to the reporting rule such that \( S = \{0, y\} \), and the supervisor’s report in each period \( t \) is equal to \( y_t \), the output in period \( t \). More formally, the reporting is given by

\[
S_t(y_1, \ldots, y_t) = y_t, \text{ for all } \{y_1, \ldots, y_t\}.
\]

**Example 2: (\( T \)-period Revelation)**

Another commonly used information structure is \( T \)-period revelation, where outputs are fully revealed every \( T \) periods and no information is revealed in between (see, for example, Radner (1985), Abreu, Milgrom, Pearce (1991) and Fuchs (2007)). In this case, \( S = \{0, y\}^T \cup \{N\} \), where \( N \) stands for no information. When \( t \neq nT \) for some natural number \( n \), the signal \( s_t = N \). When \( t = nT \), \( s_t = (y_{(n-1)T+1}, \ldots, y_{nT}) \). More formally, the reporting is given by

\[
S_t(y_1, \ldots, y_t) = \begin{cases} 
N & \text{if } t \neq nT \\
(y_{(n-1)T+1}, \ldots, y_{nT}) & \text{if } t = nT
\end{cases}.
\]

Notice that the examples above share two properties. First, there is full revelation of information in the long run so that eventually the entire past history of outputs is known. Second, any output stops impacting all future reports after a predetermined date. In the first example, this happens after every period. In the second example, this happens after at most \( T \) periods. Next, we present two reporting rules that violate one or both properties.

**Example 3:**

Let the set of signals be \( S = \{G, B\} \), where \( G \) can be thought of as a good performance and \( B \) a bad one. Essentially, the supervisor reports good performance if one of the two most recent outputs is high. Specifically, in period \( t > 1 \), the supervisor reports \( s_t = G \) if the output this or last period is equal to \( y \), and \( s_t = B \) otherwise: for \( t > 1 \),

\[
S_t(y_1, \ldots, y_t) = \begin{cases} 
G & \text{if } y_t = y \text{ or } y_{t-1} = y \\
B & \text{otherwise}
\end{cases}.
\]

6
When $t = 1$, the supervisor reports $s_1 = G$ if $y_1 = y$. If $y_1 = 0$, the supervisor reports $G$ with some probability $\rho \in (0, 1)$ and reports $B$ with probability $1 - \rho$.

**Example 4:**

Again let the set of the signals be $S = \{G, B\}$. In period $t$, the supervisor reports $s_t = G$ if more than half of the previous outputs are equal to $y$, and $s_t = B$ otherwise. More formally, the reporting rule is given by

$$S_t(y_1, ..., y_t) = \begin{cases} G & \text{if } \sum_{j=1}^{t} y_j > ty/2 \\ B & \text{if } \sum_{j=1}^{t} y_j \leq ty/2 \end{cases}.$$  

Under this reporting rule, the report in any period depends on the entire past history of outputs. In particular, each report is affected by the outputs in the distant past. Relatedly, each output affects the report in the arbitrarily far future.

In both Example 3 and 4, the principal never observes the entire past history of outputs. Since the outputs directly affect the profits, this means that the principal does not know her payoffs fully. In general, even if the principal may not observe his payoffs accurately or quickly, she typically has some information on her past average profits. In other words, a more complete model should allow the principal to obtain an additional source of information about past outputs. Adding another source of information to the principal will make the model more complicated, but we can show that when there is a sufficient delay in when the principal learns about the outputs, the main results of the paper are preserved.

### 2.4 Timing

The timing is as follows. Before the first period begins, the supervisor determines his reporting rule. At the beginning of period $t$, the principal offers a contract to the agent that consists of a contractible base wage $w_t$. The agent chooses whether to accept it: $d_t \in \{0, 1\}$. If the agent rejects the contract ($d_t = 0$), all players receive their outside options for the period. If the agent accepts it, he chooses $e_t$. The supervisor observes $y_t$ and sends the public report $s_t$. After observing $s_t$, the principal decides whether to pay out the bonus $b_t \geq 0$. Denote $W_t = w_t + b_t$ as the agent’s total compensation for the period. Just as in the analysis of relational contracts with a full revelation of outputs (e.g., Levin (2002)), this restriction to nonnegative
bonuses helps simplify the exposition without affecting the set of equilibrium payoffs sustainable by relational contracts.

2.5 Strategies and Equilibrium Concept

Since the supervisor is nonstrategic, we will only describe the strategies of the principal and the agent.

2.5.1 History and Strategies

We denote \( h_t = \{ w_t, d_t, s_t, W_t \} \) as public events that occur in period \( t \) and \( h^t = \{ h_n \}_{n=0}^{t-1} \) as the public history at the beginning of period \( t \). We set \( h^1 = \Phi \). Let \( H^t = \{ h^t \} \) be the set of public history until time \( t \) and \( H = \bigcup_t H^t \) be the set of public histories. The principal only observes the public history. The agent, however, also observes his past actions \( e^t = \{ e_j \}_{j=1}^{t-1} \) at the beginning of period \( t \). Denote \( H^t_A = H^t \cup \{ e^t \} \) as the set of the agent’s private history at the beginning of period \( t \).

We use \( s^P \) and \( s^A \) to denote the principal’s and the agent’s strategies, respectively. The principal’s strategy \( s^P \) specifies, for each period \( t \), the contractible base wage \( w_t \) and the total compensation \( W_t \). The principal’s decisions depend only on the public history available. The agent’s strategy \( s^A \) specifies, for each period \( t \), his acceptance decision \( d_t \) and his effort decision \( e_t \). The agent’s decisions depend both on the public history and his private history of past efforts.

Notice that we do not consider mixed strategies in our analysis. Allowing for mixed strategies does not improve the sustainability of the relational contracts in part because the actions of the principal are publicly observable. As will be clear from our analysis below, the public observability of the principal implies that the key to sustain an efficient relational contract is to maintain the total expected bonus necessary for effort while ensuring that the maximal bonus is smaller than the expected discounted surplus of the relationship. When the principal randomizes, she can only (weakly) increase the maximal bonus because randomization adds noises to the bonus payments and make them more volatile. And when the agent randomizes, he makes the efficient relational contracts harder to sustain by lowering the expected discounted surplus of the relationship without affecting the maximal bonus or the total expected bonus.

The same reasoning implies that the sustainability of the relational contracts will not be affected if the model allows for a public randomization device. To the
extent that the total expected bonus must be maintained to induce effort from the agent, the public randomization adds fluctuation to the total expected bonus. In particular, the conditional total expected bonus following some realization of the public randomization must be (weakly) higher than the total expected bonus prior to the public randomization. This makes it more difficult for the maximum bonus to be lower than the expected discounted surplus following this particular realization.

2.5.2 Equilibrium Concept

We use Perfect Bayesian Equilibrium (PBE) as the solution concept here. To describe the PBE, it is convenient to introduce a few payoff functions. The expected payoff of the agent following a private history $h^t_A$ and $w_t$ is given by

$$U(h^t_A, w_t, s^A, s^P) = E[t \sum_{\tau=t}^{\infty} \delta^{\tau-t} \{ \mu + 1_{\{d_t=1\}} (-ce_\tau + W_\tau - \mu) \}] h^t_A, s^A, s^P].$$

We can define $U(h^t_A, w_t, d_t, s^A, s^P)$, the expected payoff of the agent following his acceptance decision in period $t$, in similar fashion. The principal’s expected payoff following the agent’s private history $h^t_A$ is given by

$$\pi(h^t_A, s^A, s^P) = E[t \sum_{\tau=t}^{\infty} \delta^{\tau-t} \{ \pi + 1_{\{d_t=1\}} (y(q + (p - q)e_\tau) - W_\tau - \pi) \}] h^t_A, s^A, s^P].$$

Since the principal does not observe the agent’s private history, we define

$$\Pi(h^t, s^A, s^P) = E[\mu^P(\pi(h^t_A, s^A, s^P)|h^t)]$$

as his expected payoff following public history $h^t$. Here, the expectation is taken over all of the agent’s possible private histories ($h^t_A$) according to the principal’s belief ($\mu^P$) conditional on observing public history $h^t$. Finally, we denote $\pi(h^t_A, w_t, d_t, s_t, s^A, s^P)$ as the principal’s expected payoff in period $t$ following the agent’s private history $h^t_A$, the principal’s wage offer $w_t$, the agent’s acceptance decision $d_t$, and the signal $s_t$. We define $\Pi(h^t, w_t, d_t, s_t, s^A, s^P)$ similarly.

A PBE in this model consists of the principal’s strategy ($s^P$), the agent’s strategy ($s^A$), the principal’s belief ($\mu^P$), and the agent’s belief ($\mu^A$), such that the following
are satisfied. First, following any history \( \{h_A^t, w_t\} \) and \( \{h_A^t, w_t, d_t\} \),

\[
U(h_A^t, w_t, s^{A*}, s^{P*}) \geq U(h_A^t, w_t, \tilde{s}^A, s^{P*});
\]
\[
U(h_A^t, w_t, d_t, s^{A*}, s^{P*}) \geq U(h_A^t, w_t, d_t, \tilde{s}^A, s^{P*}).
\]

Second, following any history \( h^t \) and \( \{h^t, w_t, d_t, s_t\} \),

\[
\Pi(h^t, s^{A*}, s^{P*}) \geq \Pi(h^t, s^{A*}, \tilde{s}^P);
\]
\[
\Pi(h^t, w_t, d_t, s_t, s^{A*}, s^{P*}) \geq \Pi(h^t, w_t, d_t, s_t, \tilde{s}^A, \tilde{s}^P).
\]

Third, the beliefs are consistent with \( s^* \) and are updated with the Bayes rule whenever possible. In this game, the agent has private information about his effort. As a result, the agent’s belief about the past history of outputs depends on his \textit{actual effort levels}. In contrast, the principal’s belief about the past history of outputs depends only on the agent’s \textit{equilibrium effort levels} as long as the agent has not publicly deviated. When the agent publicly deviates, we assume the principal believes that the agent has never put in effort in the past.

When outputs are publicly observed, a commonly used equilibrium concept is Perfect Public Equilibrium (PPE). PPE requires the strategies to be public in the sense that they only depend on the public history. The restriction to such public strategies is questionable when the supervisor’s reports (and thus the agent’s payoff) depend on the past history of outputs. The reason is that when his effort affects future reports, the agent’s private history contains payoff-relevant information and should be used to his advantage.

3 Analysis

In this section, we study how the information structure affects the efficiency of the relational contract. We first review in Subsection 3.1 the condition required to sustain the efficient relational contracts when the supervisor fully reveals the outputs. Subsection 3.2 presents our main result that the supervisor can help sustain the efficient relational contracts by revealing less information.
3.1 Benchmark: Fully Revealing Reports

Suppose the reports fully reveal the outputs, i.e., $s_t = y_t$ for all $t$. Levin (2003) shows that in this setting the optimal relational contract is stationary. In each period, the principal offers the agent a base wage $w$ and promises him a bonus $b > 0$ for high output ($s_t = y$).

Since the relational contract is stationary, the agent’s effort decision only affects his payoff within each period. To induce effort from the agent, the cost of effort ($c$) must be smaller than the expected increase in bonus ($(p-q)b$). Or equivalently,

$$b \geq \frac{c}{p-q}.$$  \hspace{1cm} (1)

When there is no restriction on payments, the principal can capture the entire surplus of the relationship by setting the base wage such that the agent’s payoff inside the relationship is equal to his outside option:

$$w - c + pb = u.$$  

Finally, for the principal not to renege on the bonus, the following must be true:

$$b \leq \frac{\delta(py - c - u - \pi)}{1 - \delta},$$  \hspace{1cm} (2)

where $\frac{\delta(py - c - u - \pi)}{1 - \delta}$ is the discounted expected future surplus that is completely captured by the principal.

Combining equations (1) and (2) shows that a relational contract can induce effort if and only if

$$\frac{c}{p-q} \leq \frac{\delta(py - c - u - \pi)}{1 - \delta}.$$  \hspace{1cm} (3)

In other words, the incentive cost should be smaller than the discounted expected future surplus. Inequality (3) implies that the sustainability of the relational contract depends on the extremes. In other words, the set of discount factors ($\delta$) that allow for efficiency is completely determined once the value of the maximal reneging temptation ($\frac{c}{p-q}$) and the expected per-period surplus in the relationship ($py - c - u - \pi$) are given. When (3) is satisfied, the optimal relational contract can be achieved.
by setting

\[ w = u + c - \rho \frac{\delta (py - c - u - \pi)}{1 - \delta}, \]
\[ b = \delta \frac{(py - c - u - \pi)}{1 - \delta}. \]

3.2 Main Results

In this section, we show that the supervisor can help sustain the relational contract by sending out less-informative reports. In particular, we consider a class of efficient equilibrium where in every period the agent puts in effort, the supervisor sends out reports that recommend either a bonus or no bonus, and the principal pays the agent a fixed base wage and a bonus on top of that if the report recommends it. Our main result is that when the supervisor ties the reports to past outputs, he can lower the discount factor necessary for supporting the efficient equilibrium.

3.2.1 Credit-Rollover-Reporting Rule

The key to our result is the supervisor’s reporting rule. In particular, we consider the following class of reporting rules. The supervisor reports either \( G \) (good) or \( B \) (bad) in each period. In addition, the reports can be partitioned into reporting cycles which end stochastically. The first reporting cycle starts in period 1. And each new reporting cycle starts if the supervisor reports \( B \) in the previous period.

Within each reporting cycle, the reporting rule can be summarized as follows. In each period, the supervisor reports \( G \) if the output is high. If the output is low, the supervisor reports \( G \) with probability \( f \) (and \( B \) with probability \( 1 - f \)) if the output in the previous period (within the reporting cycle) is high. If the output in the previous period is again low, the supervisor reports \( B \). The only exception happens when a low output occurs at the beginning of a reporting cycle (so that there is no within-cycle previous period), the supervisor reports \( G \) with probability \( \rho(f)f \) and \( B \) with probability \( 1 - \rho(f)f \), for some \( \rho(f) \in (0, 1] \) to be explained below. Figure 1a illustrates the reporting rule when the current period is not the first period of a reporting cycle and Figure 1b illustrates the reporting rule when the current period is the first period of a reporting cycle.
We denote this class of reporting rules as *f-credit-rollover reporting* for the following reason. One can think that the reports depend on the agent’s total credit, a hidden variable known only to the supervisor. In each period, the total credit of the agent is determined based on the output and the credit rollover from the previous period. The agent receives one credit for high output, and he receives one rollover credit with probability $f$ from the previous period if the previous period’s output is high. In each period, the supervisor reports $G$ when the agent has a positive total credit and $B$ otherwise. Once $B$ is reported, a new reporting cycle starts in the following period. At the beginning of a reporting cycle, the agent is given one credit with probability $\rho$.\(^2\)

We choose $\rho(f)$ as the unique $\rho$ satisfying $\rho = p/(p + (1 - p) \rho f)$. This choice of $\rho$ maintains stationarity of the reporting so that the probability of a bonus in each period is constant along the equilibrium path. Notice that the probability of bonus is essentially determined by the conditional probability that output is high in the past period. The choice of $\rho$ ensures that this conditional probability is constant within each reporting cycle. In particular, suppose the conditional probability that the output is high in period $t - 1$ is $\rho_{t-1}$. Conditional on a $G$ report in period $t$, the probability of high output in period $t$ is given by $\rho_t = p/(p + (1 - p) \rho_{t-1} f)$. The choice of $\rho$ then implies that when $\rho_{t-1}$ is equal to $\rho$, so is $\rho_t$.\(^2\)

\(^2\)Alternatively, one can interpret $f$ directly as the credit the agent earns based on last period’s high output. The credit here is the probability that the agent is forgiven for this period’s low output.
To connect $f$-credit-rollover reporting with our previous examples, notice that when $f = 0$, there is no credit rollover. In this case, $G$ is reported if and only if the output in the current period is high. This is the benchmark case in Subsection 3.1 in which the report tracks the output perfectly and the worker’s bonus does not depend on his past outputs. When $f = 1$, a high output means that 1 credit will be rollover to the next period. As a result, the supervisor reports $G$ both for the current and the next period. When $f = 1$, the reporting rule is very similar to that in Example 3. Essentially, the supervisor reports $G$ as long as one of the two most recent outputs is high. The only difference is that we assume that the reporting process restarts after each $B$ report.

In general, when $f > 0$, the report depends on past outputs and can be thought of as a garbled signal of the agent’s outputs in the current and the previous periods. This implies that one cannot infer past outputs from the reports alone. When the supervisor reports $G$, it is not clear whether the current-period output is high or low, so the reporting rule results in a loss in information.

### 3.2.2 Perfect Bayesian Equilibrium (PBE)

We now show that $f$-credit-rollover reporting can help sustain efficient relational contracts. To obtain more analytical results, we first assume $q = 0$ so that the output is always low if the agent puts in no effort. We then extend our results to $q > 0$ and also present some simulation results.

For each case of $f$-credit-rollover reporting, consider the following class of strategies.

**The principal offers**

$$w_1 = w$$

in period 1. If the agent has always accepted the contract, the principal offers for $t > 1$

$$w_t = \begin{cases} 
    w & \text{if } s_{t-1} = B, \\
    w + \frac{b}{s} & \text{if } s_{t-1} = G.
\end{cases}$$

If the agent has ever rejected the principal’s offer, the principal offers

$$w_t = u - 1.$$
The agent accepts the principal’s contract offer if and only if

\[ w_t > \underline{w} \]

or the principal has never deviated. The agent puts in effort if and only if there is no public deviation and the probability of a low output in the previous period is smaller than \( \rho(f) \), where

\[ \rho(f) = \frac{p}{p + (1 - p) \rho(f_f)}. \]

Note that this class of strategies is essentially stationary along the equilibrium path: the principal pays out a bonus if and only if the report is good. One feature of the bonus is that it is not paid at the end of the period but is rather delayed to the beginning of next period. We therefore denote this class of strategies as stationary strategies with delayed bonus. When the output is publicly known, the timing of the bonuses does not affect the condition for sustaining an efficient relational contract; see, for example, MacLeod and Malcomson (1989) and Levin (2003). Under \( f \)-credit-rollover reports, past output affects future reports, and the timing of the bonus matters. The reason is that the agent may engage in multistage deviations, and we discuss this point in detail below.

Our main result shows that relative to full revelation of outputs, \( f \)-credit-rollover reporting can reduce the discount factor necessary to support the efficient relational contract. Denote \( \delta^*(f) \) as the smallest discount factor such that there exists a PBE supported by a stationary strategy with a delayed bonus. Note that when \( f = 0 \), the reporting rule reveals the outputs fully, and \( \delta^*(0) \) is determined by Equation (3) in the benchmark case.

**Proposition 1:** The following holds.

(i): \( \delta^*(1) < \delta^*(0) \) if and only if \( \delta + \delta^2 p - (1 + \delta + \delta^2) \rho^* > 0 \), where \( \rho^* = \frac{p}{p + (1 - p) \rho^f} \).

(ii): If \( \delta^*(f) < \delta^*(0) \) for some \( f \in (0, 1) \), then \( \delta^*(1) < \delta^*(f) \).

Part (i) provides a necessary and sufficient condition for 1-credit-rollover reporting to lower the cutoff discount factor that sustains the efficient relational contract. Part (ii) shows that when full revelation of outputs is not optimal, 1-credit-rollover reporting has the lowest cutoff discount factor within the class of \( f \)-credit-rollover reporting. This allows us to focus on 1-credit-rollover reporting below. For expositional convenience, we refer to 1-credit-rollover reporting simply as credit-rollover
The basic intuition for why credit-rollover reporting can help is that, by making the reports less informative of the outputs, the supervisor reduces the principal’s maximal reneging temptation by smoothing the bonus payments across periods. Specifically, under credit-rollover reporting, a high output increases both the current and future payoffs of the agent (since he is guaranteed a bonus next period). This allows the principal to reduce the current bonus (compared to the required bonus under reports that fully reveal outputs) while maintaining the agent’s incentive to work. The reduction in the size of the bonus makes the principal’s nonreneging constraint easier to satisfy. It is worth noting that since the bonuses are smaller in size, they are paid out more frequently. The probability of a bonus is \( p \) under full revelation of outputs, and it increases to \( p + (1 - p) \rho^* \) under credit-rollover reporting.

The gain of credit-rollover reporting, however, is limited by two countervailing forces. First, when the output in the previous period is high, the agent receives a bonus even if the output is low. Paying a bonus for low output reduces the agent’s incentive to work, and to maintain his incentive the principal must pay a larger bonus. Second, credit-rollover reporting implies that part of the reward paid to the agent for high output is postponed. When the bonus is postponed and the agent is impatient, the total expected bonus has to be increased, again making the principal’s nonreneging constraint harder to satisfy.

The condition in Part (i) identifies the condition under which the net benefit of credit-rollover reporting is positive. In particular, the condition shows that the net benefit of credit-rollover reporting is more likely to be positive when \( p \) is smaller and \( \delta \) is larger. When \( p \) is smaller, the agent is less likely to receive a bonus for low output because the probability of a high output in the previous period is smaller. This means that the first countervailing force mentioned above is less important. When \( \delta \) is bigger, the extra bonus the principal needs to pay due to postponing the reward is smaller. This means that the second countervailing force is less important. In summary, the cost of credit-rollover reporting is smaller when the high output is less likely and the agent is more patient. In these cases, credit-rollover reporting lowers the cutoff discount factor relative to full revelation of output.

For credit-rollover reporting to help better sustain the efficient relational contracts, a number of conditions must be satisfied for the constructed equilibrium. First, the
principal cannot know the history of past outputs. If the principal knew that the agent’s output is high (and thus the agent’s continuation payoff is high), she would also know that she would be required to pay the bonuses for two consecutive periods regardless of the following period’s output. This lowers the principal’s future surplus in the relationship and makes her more likely to renege.

Second, the agent cannot know the history of past outputs. The reason is that the previous period’s output affects the agent’s incentive to exert effort. When the previous period’s output is low, the benefit of a high output includes both a bonus this period and a guaranteed bonus next period. When the previous period’s output is high, the agent has already been guaranteed a bonus, so the benefit of a high output is lower. At the cutoff discount factor, the agent’s expected benefit of effort is exactly equal to the cost. If the agent knew last period’s output is high, however, he would strictly prefer to shirk. In general, withholding information enables the supervisor to pool the agent’s incentive constraints together, making it easier to induce effort.

Third, the bonus needs to be paid out at the beginning of next period instead of at the end of the current period. This contrasts with most models of relational contracts in which the timing of the bonus is irrelevant for the sustainability of relational contracts; see, for example, MacLeod and Malcomson (1989), Levin (2003). The difference arises because in those models the future payoffs of the players are determined only by the public history. Under credit-rollover reporting, in contrast, the agent’s private effort choice also affects his future expected payoff. This implies that it is impossible to capture players’ future payoffs using a sufficient statistic based on the public history alone, and the one-stage-deviation principle does not apply. As a result, the agent might find multistage deviation profitable.

Paying the bonus in the next period helps to prevent a specific type of multistage deviation in the following situation. Notice that at the cutoff discount factor, the agent is indifferent both between working and shirking and between staying in the relationship and exiting. If the bonus were paid out at the end of a period, however, even if the agent would not gain by shirking alone or exiting alone; he would strictly prefer to shirk, and if the bonus were paid out, he would immediately exit the relationship. The reason is that if the agent shirks and then receives a bonus, he knows that the output must be low. This implies that the agent’s future expected payoff from staying in the relationship is lower than his outside option, and the agent prefers
to exit. To prevent such shirk-and-exit behavior, the bonus needs to be paid out at the beginning of next period. In this case, the agent no longer receives the bonus if he exits.

The need to check that there is no profitable multistage deviation significantly complicates our analysis because of the large number of possible deviations involved. Even if the agent never deviates by exiting, there still remain many types of deviations. For example, the agent may prefer to shirk at the beginning of a reporting cycle, and in the contingency of \( n \) consecutive bonuses, he shirks again. Ruling out such deviations is challenging because a past action (shirking) has an enduring effect on future deviation payoffs. For any \( n \), the agent’s expected future payoff following \( n \) consecutive bonuses is different than if he has never shirked.

Our analysis circumvents the difficulty of checking multistage deviation by making note of the following. Credit-rollover reporting implies that the agent’s expected payoff is completely determined by the output in the previous period. Let \( \rho \) be the probability of a high output in the previous period, and let \( \rho = \rho^* \) if the period is at the beginning of a reporting cycle. In this case, \( \rho \) becomes a state variable that summarizes the agent’s expected payoff. This allows us to characterize the agent’s action recursively by finding a value function that maps the probability of high output in the previous period to the agent’s expected payoff. In our proof, we solve for this value function directly and verify that the agent never wants to deviate.

Figure 2 below illustrates the value function of the agent at the cutoff discount factor. The value function is piecewise linear in \( \rho \) and has a kink at \( \rho^* \). In particular, the agent strictly prefers to work if \( \rho < \rho^* \) and he strictly prefers to shirk if \( \rho > \rho^* \). Also notice that \( V(0) \) is less than \( u \), the agent’s outside option. This underscores the importance of postponing the bonus payment to the beginning of next period.
Proposition 1 provides the condition under which credit-rollover reporting can improve the sustainability of relational contracts relative to full revelation of outputs. We next provide a sense of the degree to which credit-rollover reporting can help.

Define $S_0$ as the smallest expected discounted surplus $\left(\frac{\delta}{1-\delta}(pq - c - u - v)\right)$ necessary to support an efficient relational contract when the outputs are revealed fully by the supervisor. Notice that $S_0 = c/p$ by the necessary and sufficient condition in Subsection 3.1. Next, define $S_1$ as the smallest corresponding expected discounted surplus under credit-rollover reporting. The ratio between $S_0$ and $S_1$ provides a measure for the gain from credit-rollover reporting over full revelation of outputs.

**Corollary 1:** The ratio of surplus is given by

$$R(p, \delta) \equiv \frac{S_0}{S_1} = 1 + \frac{\delta - (1 + \delta + \delta^2) \rho^* + \delta^2 p}{1 - p\delta^2(1 - \rho^*)},$$

where $\rho^* = p/(p + (1 - p) \rho^*)$. In particular, there exists $p^*$ such that $R(p, \delta)$ is decreasing for all $p \in (0, p^*)$, and

$$\lim_{p \to 0} R(p, \delta) = 1 + \delta.$$

Corollary 1 provides an explicit expression of $S_0/S_1$. Since $S_0 = c/p$, Corollary 1 implies that $S_1$ is also linear in $c$. Unlike $S_0$, however, $S_1$ depends on the discount factor $\delta$. Notice that $S_0/S_1 > 1$ if and only if $\delta - (1 + \delta + \delta^2) \rho^* + \delta^2 p > 0$, which is
exactly the condition in Proposition 1 for credit-rollover reporting to improve over full revelation of outputs. When \( S_0/S_1 > 1 \), Corollary 1 implies that efficient relational contracts can be sustained under credit-rollover reporting for any expected discounted surplus of the relationship \( S \in (S_1, S_0) \) but not under full revelation of outputs.\(^3\) Corollary 1 also shows that the ratio \( S_0/S_1 \) is decreasing in \( p \) for small enough \( p \), which is consistent with our intuition that the credit-rollover reporting is more effective for smaller \( p \).

Corollary 1 also shows that, when \( p \) is close to 0, credit-rollover reporting reduces the surplus necessary for the efficient relational contract by a factor of \( 1 + \delta \). To see this, let \( b_0 \) be the bonus necessary for effort under full revelation of outputs and \( b_1 \) the bonus for a \( G \) report under creditor-rollover reporting. When \( p \) is close to 0, a high output is very unlikely, so the negative incentive effect on the agent from credit-rollover reporting (the first countervailing force above) is negligible. This implies that the total expected bonus for a high output under credit-rollover reporting is essentially \( (1 + \delta) b_1 \). In other words, credit-rollover reporting basically spreads the bonus for a high output into two periods, and this reduces the size of the bonus by a factor of \( 1 + \delta \). Since the bonus is bounded by the expected future surplus, this implies that credit-rollover reporting reduces the surplus necessary for an efficient relational contract by a factor of \( 1 + \delta \), which approaches 2 as \( \delta \) goes to 1.

When \( p \) is close to 0, the discussion above suggests that further gain can be realized if the bonus is split into more than two parts. For example, a reporting rule that guarantees the agent \( n \) consecutive periods of bonus for a high output can presumably reduce the surplus by a factor of \( 1 + \delta + \ldots + \delta^{n-1} \) when \( p \) is close to 0. While such conjectures are reasonable, proving them is challenging in part because of the possibility of multistage deviations. The further back the bonus is affected by past outputs, the more difficult it is to check that there are no profitable multistage deviations. This also makes it hard to find the optimal reporting rule. Even if we cannot characterize the optimal reporting rule in general, we provide in the next section an upper bound on the gains from supervisor reporting.

Finally, we show that credit-rollover reporting also helps the sustainability of

\(^3\)Notice that for fixed \( p, c, \delta \), and outside options, the expected discounted surplus of the relationship is completely determined by the value of the output \( y \). Corollary 1 then gives the range of output values such that credit-rollover reporting enhances the sustainability of efficient relational contracts.
efficient relational contracts when \( q > 0 \) by presenting two types of results. The first one is a continuity result that shows once credit-rollover reporting helps for \( q = 0 \), it also helps for \( q \) close to 0. The second type numerically computes the minimum bonus for effort under credit-rollover reporting and under full revelation of outputs. For the first result, recall that \( S_1 \) is the smallest expected discounted surplus for effort under credit-rollover reporting when \( q = 0 \).

**Corollary 2:** Consider \( p \) and \( \delta \) such that \( \delta + \delta^2 p - (1 + \delta + \delta^2) \rho^* > 0 \), where \( \rho^* = p/(p + (1 - p) \rho^*) \). For each expected discounted surplus level \( S > S_1 \), there exists an associated \( q^* > 0 \) such that efficient relational contracts can be sustained under credit-rollover reporting for all \( q \in [0, q^*] \).

Notice that \( \delta + \delta^2 p - (1 + \delta + \delta^2) \rho^* > 0 \) is the same condition in Proposition 1 for credit-rollover reporting to help. Therefore, Corollary 2 states that if credit-rollover reporting improves the sustainability of efficient relational contracts when \( q = 0 \), it also does so for small \( q \). The reason for this is straightforward. Notice that the minimum bonus necessary for effort under credit-rollover reporting when \( q = 0 \) is equal to \( S_1 \). When the expected discounted surplus \( S \) is larger than \( S \), the principal can set a bonus slightly larger than \( S_1 \) without reneging. With the new bonus, the agent strictly prefers working over shirking when \( q = 0 \). It follows that for small enough \( q \) the agent also prefers working since his payoff from shirking is continuous in \( q \). This implies that credit-rollover reporting continues to improve the sustainability of efficient relational contracts when \( q \) is small.

Beyond the continuity result above, however, it is difficult to provide the general condition for when credit-rollover reporting improves over the full revelation of outputs. The reason is that when \( q > 0 \), the value function \( V(\rho) \) no longer has an analytical solution. As a result, checking multistage deviation analytically becomes difficult and numerical methods are used instead. In particular, we compute \( V(\rho) \) for each bonus level \( b \) and then find the smallest bonus level that sustains efficient effort.
Figure 3 reports our findings for discount factor equal to 0.9 and the cost of effort equal to 2. The two panels on the left (upper left and bottom left) depict the minimum bonus necessary for sustaining effort under full revelation ($b^f$). The bottom-left panel is a 3D plot that illustrates the value of $b^f$ for each $0 < q < p < 0.5$. The top-left panel is the corresponding thermograph that projects the 3D plot into a 2D graph by using colors to represent values. In particular, the colder colors reflect smaller values and the warmer colors reflect larger ones. Since $b^f$ is equal to $c/(p-q)$, the colors become colder as $p$ increases and become warmer as $q$ increases. Moreover, all $(p,q)$ pairs on the same negative 45-degree line has the same color.

Next, the two panels in the middle report the minimum bonus for sustaining effort under credit-rollover reporting ($b^g$), where $g$ means that the reports are garbled signals of the true output. Notice that the colors again become colder as $p$ increases and become warmer as $q$ increases, indicating that the minimum bonus under credit-rollover reporting also decreases with $p$ and increases with $q$. Different from the two
panels on the left, however, not all \((p, q)\) pairs on the same negative 45 degree line has the same color. The \((p, q)\) pairs with the same color are no longer straight lines and appear to have a slope larger than \(-1\).

Constructed based upon the left and middle panels, the two panels on the right report the differences in the minimum bonuses required to sustain efficient effort between the cases of full revelation and credit-rollover reporting, namely, \(b^f - b^g\). The thermograph in the upper-right panel makes it clear that there are values of \((p, q)\) such that credit-rollover reporting lowers the minimum bonus to sustain effort \((b^f - b^g > 0)\). The gains from credit-rollover reporting concentrate on the bottom left region, where the values of \(p\) and \(q\) are smaller. For a figure that marks clearly the region of \((p, q)\) in which credit-rollover reporting lowers the minimum bonus required to induce effort, please refer to Figure 4 in Appendix B. Moreover, the colors become colder as \(p\) increases, showing that the gain from credit-rollover reporting diminishes as \(p\) becomes larger. This is in line with our discussion of the \(q = 0\) case that credit-rollover reporting is less effective when \(p\) is larger.

4 Discussion

In this section, we extend our model in several ways. First, we show in Subsection 4.1 that when the effort costs are sufficiently convex, credit-rollover reporting can also improve over full revelation of outputs when there are three effort levels. Subsections 4.2 and 4.3 then discuss general properties of reporting rules that enhance efficiency. Subsection 4.2 proves an upper bound for the gain from the optimal reporting rules. Subsection 4.3 provides a necessary condition for any reporting rules to improve the sustainability of efficient relational contracts over fully-revealing reporting.

4.1 Multiple Effort Levels

Suppose, for example, that the agent’s effort choice in period \(t\) is given by \(e_t \in \{0, 1, 2\}\). Let the associated effort cost be given by \(c(0) = 0\), \(c(1) = c_1 > 0\) and \(c(2) = c_2 > c_1\). The output is binary: \(Y_t \in \{0, y\}\), and

\[
\Pr\{Y_t = y\} = \begin{cases} 
p & e_t = 2 \\
q & e_t = 1 \\
0 & e_t = 0 \end{cases}
\]
where $1 > p > q > 0$.

We assume that
\[ py - c_2 > qy - c_1 \geq u + v > 0, \]
so the joint surplus is maximized at $e = 2$, followed by choosing $e = 1$, taking the outside options, and setting $e = 0$. Notice that we allow the relationship to be profitable even when the agent chooses $e = 1$. This assumption is not important for the analysis but as we will see below, it allows credit-rollover reporting to help the relationship both in the extensive and intensive margins. For expository convenience, define the discounted expected surplus as
\[
S = \frac{\delta}{1 - \delta} (py - c_2 - u - \pi).
\]

Compared to the binary-effort case with $q > 0$, the model above simply adds a lower level of effort ($e = 0$). As a result, if one can rule out that $e = 0$ is used in a relational contract, the results from the binary-effort model can be directly applied. In particular, define $c = c_2 - c_1$ and recall that $S_1(p,q,\delta,c)$ is the minimum amount of surplus for effort under credit-rollover reporting. The result below shows that for these parameters credit reporting also improves the efficiency of the relational contracts when the effort costs are sufficiently convex.

**Corollary 3:** Let $c = c_2 - c_1$ and let $S_1(p,q,\delta,c)$ be the minimum expected discounted surplus for effort under credit-rollover reporting in the binary-effort model. For any $S \in (S_1, \frac{c}{p-q})$, efficient relational contracts are sustainable under credit-rollover reporting when $\frac{c}{c_1} > M$ for some $M > 0$.

Notice that $c/c_1$ measures the convexity of effort cost since $c$ is the marginal cost of effort from increasing $e = 1$ to $e = 2$ and $c_1$ is the marginal cost of effort from increasing $e = 0$ to $e = 1$. Result 1 therefore shows that when effort costs are sufficiently convex, credit-rollover reporting can support the efficient relational contracts even if it is impossible to do so with perfect revelation of outputs. Notice that if the outputs were revealed fully when $S \in (S_1, \frac{c}{p-q})$, the agent would either choose a lower level of effort ($e = 1$) or forgo the relationship by taking his outside option. In the former case, credit-rollover reporting improves the relationship in the intensive margin by making it more efficient. In the later case, credit-rollover
reporting improves the relationship in the extensive margin by sustaining an efficient relationship that would fail to start.

The intuition for Result 1 is straightforward. When the cost function is sufficiently convex, the gain of deviating from $e = 1$ to $e = 0$ is small relative to the gain of deviating from $e = 2$ to $e = 1$. It follows that if the agent does not gain from deviating to $e = 1$, he will not gain from deviating to $e = 0$. Therefore, if credit-rollover reporting helps sustain efficient relational contracts with binary effort, it can also help here when the effort cost is sufficiently convex.

Notice that this intuition suggests that credit-rollover reporting may have wider applicability. For example, when there are more effort levels or continuous effort levels, it is sometimes the case that ruling out profitable local deviations is enough for ruling out global deviations. Since checking local deviations requires essentially comparing payoffs from two (adjacent) effort levels, our results on credit-rollover reporting can be applied to relax constraints that prevent local deviation. This suggests that credit-rollover reporting can improve the sustainability of relational contracts for more general effort cost structures.

4.2 Limits of Gain from Reporting

In Subsection 3.2, we show that the supervisor can use credit-rollover reporting to improve the sustainability of efficient relational contracts. In particular, Proposition 1 provides the condition under which creditor-rollover reporting helps and Corollary 1 indicates how much it can reduce the surplus necessary to sustain an efficient relational contract. In general, one would like to understand when and by how much the supervisor can help sustain efficient relational contracts if arbitrary types of reporting rules can be used. While a full answer is beyond the scope of this paper, we provide a partial answer in Proposition 2 by establishing an upper bound on the gain from supervisor reporting.

**Proposition 2:** Recall that $S \equiv \frac{e}{1-\delta}(py - c - u - \pi)$ is the discounted expected future surplus of the relationship. For any reporting rule to sustain the efficient relational contract, one must have

$$S \geq \sqrt{4p(1-p)} \frac{c}{p-q}.$$
In particular, full revelation of outputs is the optimal reporting rule when \( p = \frac{1}{2} \).

Recall that \( c/(p-q) \) is the smallest bonus necessary to induce effort when outputs are fully revealed. Proposition 2 implies that the smallest bonus necessary to induce effort under any reporting rule must be at least \( \sqrt{4p(1-p)c/(p-q)} \). Since the bonus is limited by the future surplus, the optimal reporting rule can lower the surplus necessary for the efficient relational contract by at most a factor of \( 1 - \sqrt{4p(1-p)} \). In particular, Proposition 2 shows that fully-revealing reporting is the optimal reporting rule when \( p = 1/2 \) since \( \sqrt{4p(1-p)} = 1 \).

Next, we provide an intuition for the factor \( \sqrt{4p(1-p)} \). To simplify our discussion, we assume for illustrative purposes that \( \delta = 1 \). Since the gain from supervisor reporting is likely to be larger when the players are more patient, \( \delta = 1 \) does not limit the gain from supervisor reporting. Now notice that there are two ways to calculate the agent’s total payoff. One way is to link the agent’s payoff to the outputs he produced. Let \( u_t \) be the agent’s earned payoff for the output in period \( t \). In particular, \( u_t \) is a random variable whose value is either \( u_h \) (for a high output) or \( u_l \) (for a low output). Notice that the agent does not necessarily receive \( u_h \) or \( u_l \) in period \( t \). Under credit-rollover reporting, for instance, part of \( u_h \) is paid out in the following period in the form a guaranteed bonus. To induce effort from the agent, one must have \( u_h - u_l \geq c/(p-q) \). This implies that the variance of \( u_t \) satisfies \( Var(u_t) \geq p(1-p)(c/(p-q))^2 \).

Now let the total payoff of the agent earned for outputs up to \( T \) be \( U_T = \sum_{t=1}^{T} u_t \). Again notice that \( U_T \) is not necessarily equal to the agent’s total received payoff up to time \( T \) since part of \( U_T \) can be paid out in the form of future payoffs. Since \( Var(u_t) \geq p(1-p)(c/(p-q))^2 \), it follows that \( Var(U_T) \geq Tp(1-p)(c/(p-q))^2 \). This is the minimum amount of variability in payoffs to induce the agent to work in each period up to \( T \).

The second way to calculate the agent’s total payoff is to link it directly to the agent’s received payoff in each period. Let \( v_t \) be the payoff the agent receives in period \( t \). In other words, \( v_t \) is equal to the agent’s compensation minus the effort cost in period \( t \). For illustrative purposes, we may assume that \( v_t \) is a binary variable taking values in \( v_G \) (for good performance) and \( v_B \) (for bad performance), and \( v_G = v_B + b \), where \( b \) is the bonus paid to the agent. The more general case is considered in the
proof. Also let \( \rho \) be the probability of good performance and \( 1 - \rho \) the probability of bad performance. It follows that \( \text{Var}(v_t) = \rho(1 - \rho)b^2 \).

Now we can also write the agent’s payoff as \( U_T = \sum_{t=1}^{T} v_t + R_T \), where \( R_T \) is the agent’s residual payoff to be paid in the future. In other words, \( R_T = \sum_{t=1}^{T} u_t - \sum_{t=1}^{T} v_t \) reflects the part of the agent’s payoff (associated with the outputs in the first \( T \) periods) that has not been settled by the amount he has received so far. Under credit-rollover reporting, for example, \( R_T \) reflects the rollover credit. For the player’s strategy to be an equilibrium, \( R_T \) cannot grow unbounded. This implies that \( \text{Var}(U_T) \) is roughly given by \( T\text{Var}(v_t) = T\rho(1 - \rho)b^2 \) for a large enough \( T \).

Since \( \text{Var}(U_T) \) is larger than \( Tp(1 - p)(\frac{c}{p-q})^2 \), it follows that

\[
\rho(1 - \rho)b^2 \geq p(1 - p)(\frac{c}{p-q})^2.
\]

Given that \( \rho(1 - \rho) \) is smaller than \( 1/4 \) and \( b \) is bounded by the expected future surplus \( S \), one gets that \( S^2/4 \geq \rho(1 - \rho)b^2 \), and therefore,

\[
S \geq \sqrt{4p(1 - p)} \frac{c}{p-q}.
\]

Notice that when \( p = 1/2 \), the above implies that the future surplus must be bigger than \( \frac{c}{p-q} \). This is exactly the condition in the benchmark case in Subsection 3.1, and it follows that revealing outputs fully is the optimal reporting rule when \( p = 1/2 \).

### 4.3 Importance of Persistence of Memory

In this subsection, we provide a necessary condition for any reporting rule to perform better than fully revealing reporting in sustaining effort in our setting. This condition allows us to better understand how our reporting rule differs from some well known reporting rules, namely, \( T\)-period review contracts by Radner (1985), AMP (1991) and Fuchs (2007), and within-period signal garbling studied by Kandori (1992). It also explains why these reporting rules do not improve efficiency in our setting. Proposition 3 states the necessary condition formally.

**Proposition 3:** Let \((U_t, \Pi_t)\) be the expected discounted payoffs of the agent and the principal evaluated at time \( t \). Suppose (3) fails. For all \( t \), if there exists a predetermined \( t' \geq t \) such that \((U_{t'}, \Pi_{t'})\) are independent of \( h_{t'} \), then \( e_t = 0 \).
Proposition 3 shows that if a reporting rule has the property that it essentially restarts on predetermined dates, then it cannot improve efficiency over fully revealing reporting of outputs. In other words, for any reporting rule to help, the supervisor’s memory must be persistent in the sense that there cannot be a predetermined date after which all past histories become irrelevant. Notice that under $f$-credit-rollover reporting, although the supervisor neglects all past histories following a $B$ report, the date of the $B$ report is not predetermined.

Proposition 3 follows directly from a backward induction argument. Suppose it is known that the reporting rule restarts in period $t + 1$. In this case, the agent’s action in period $t$ does not affect his payoff in the future. Given that the maximal bonus is smaller than $c/(p - q)$, it is easy to see that the agent then has no incentive to exert effort in period $t$. In other words, $y_t$ is uninformative of the agent’s effort in period $t$, so the agent’s pay will not be tied to $y_t$. Essentially, period $t$ becomes a redundant period that is irrelevant for the relationship. As a result, one can view period $t - 1$ as the last period prior to period $t + 1$. Repeating the same argument, one can again show that the agent does not put in effort in period $t - 1$, and therefore, the agent will not put in effort in all periods prior to period $t + 1$.

Proposition 3 implies that some commonly studied reporting rules cannot improve the sustainability of relational contracts in our setting. Consider, for example, the $T$-period review contracts, where the outputs are fully revealed every $T$ periods; so the condition for Proposition 3 is satisfied. Fuchs (2007) shows that the $T$-period-review contracts enhance the efficiency of the relationship when the outputs are privately observed by the principal. In this case, surplus must be destroyed when the agent is punished. By revealing information infrequently, $T$-period-review contracts reduce the expected punishment received, giving rare but harsh punishments. In contrast, no surplus is destroyed in our environment when the agent is punished (fails to receive bonus). In fact, credit-rollover reporting has the opposite effect of increasing the frequency of bonuses and reducing the bonus amounts.

Another example of reporting rule is the within-period signal garbling; Kandori (1992). In this case, the supervisor reports a noisy signal of the true output in each period. Since the reports in each period is not affected by past history of outputs, the condition for Proposition 3 is also satisfied. It is clear why within-period signal garbling cannot improve the sustainability of efficient relational contracts. When the
reports are noisy signals of the true outputs, they are less indicative of the agent’s effort, and as a result, a bigger bonus is required to induce effort. But the bigger bonus makes the principal more likely to renege, rendering the efficient relational contracts harder to sustain.

Finally, an important feature for a reporting rule to improve over fully revealing reporting is that the agent’s continuation payoffs cannot always be common knowledge. The lack of common knowledge is an essential feature for a reporting rule to improve efficiency, and this observation applies to other context as well. In a related paper, Ekmekci (2011) examines a product choice game between a long-run seller and a sequence of short-run buyers. He defines a mapping from past outputs to signals as a rating system, which corresponds to a reporting rule in our context. Importantly, Ekmekci (2011) considers Markovian rating system, i.e., the latest rating depends on the previous rating and the latest output. As a result, the seller’s continuation payoff is common knowledge, unless the seller has privately known types (such as the commitment types). In contrast, the credit-rollover reporting rule we propose is not Markovian, but rather hidden Markovian; the supervisor’s report depends on a hidden state variable, namely, the rollover credit and the current period’s output. This difference explains why rating system cannot enhance efficiency in Ekmekci (2011) unless seller has privately-known types while credit-rollover reporting rule helps in our model even if the agent has only a single publicly-known type.

5 Conclusion

In this paper, we show that supervisors can improve the sustainability of relational contracts by revealing less information. We study $f$-credit-rollover reporting rules in which the reward for the agent’s good performance is spread over periods. This reduces the bonus size necessary for effort and therefore reduces the principal’s gain from reneging on the bonus. We show that 1-credit-rollover reporting is optimal within this class of reporting rules.

More generally, we provide a necessary condition for an arbitrary reporting rule to lower the surplus necessary for sustaining the efficient relational contract. To improve over full revelation of outputs, a reporting rule must allow the memory to persist in the sense that there cannot be a predetermined dates after which the relationship restarts. Finally, we establish an upper bound for the gain from reporting in terms
of how much it can lower the amount of surplus to sustain efficient relational contracts. The upper bound implies that when the probability of high output is a half (under effort), full revelation of outputs is optimal.

In our paper, we abstract away from incentive issues of the supervisor to focus on the gain from credit-rollover reporting. In general, there are a number of issues that can arise when information of the agent’s performance is monitored and evaluated by the supervisor. For example, the supervisor may not readily observe the agent’s performance, and in this case the principal needs to motivate the supervisor to monitor the agent. We can extend the model to allow for the supervisor to exert effort in monitoring the agent. When the cost of monitoring is small, it can be shown that credit-rollover reporting continues to improve the sustainability of efficient relational contracts.

Another issue is that the supervisor may collude with the agent or with the principal; see for example, Tirole (1986) and the large literature that followed. Given the repeated nature of the relationship, one way to model collusions in our context is to view them as self-enforcing side relational contracts. To prevent such side relational contracts, one possibility is to introduce supervisor turnover or rotation: collusion is less sustainable when the supervisor’s job tenure is limited. Formal study of how collusion affects relational contracts is an interesting and promising line of investigation and is left to future research.
References


Appendix A

Proposition 1: The following hold.

(i): \( \delta^*(1) < \delta^*(0) \) if and only if \( \delta + \delta^2 p - (1 + \delta + \delta^2) \rho^* > 0 \), where \( \rho^* = \frac{p}{p + (1 - p) \rho^*} \).

(ii): If \( \delta^*(f) < \delta^*(0) \) for some \( f \in (0, 1) \), then \( \delta^*(1) < \delta^*(f) \).

Proof. Part (i). To simplify the exposition, we set \( u = 0 \). Recall that \( w \) denotes the base wage and \( b \) is the bonus. In an efficient relational contract, the agent always puts in effort and the principal never reneges. To find the cutoff discount factor \( \delta^*(1) \), we may assume without loss of generality that the principal captures the entire surplus. In this case, the principal will not deviate as long as

\[
b \leq \frac{\delta}{1 - \delta}(pw - c - v).
\]

This implies that the problem of finding the cutoff discount factor \( \delta^*(1) \) is equivalent to finding the smallest bonus \( b^* \) such that the agent always puts in effort. To do this, we take the following steps.

Step 1: Recursive Formulation. Given the supervisor’s reporting strategy of the principal, the agent’s payoff is completely determined by the probability that the output is high in the previous period. Notice that we may assume that this probability is equal to \( \rho^* \) if a period is at the beginning of a reporting cycle. Let \( V(\rho) \) be the agent’s value function at the beginning of a period assuming that he has accepted the principal’s offer. Let \( V_e(\rho) \) be the agent’s value function assuming that he puts in effort this period and \( V_s(\rho) \) be the agent’s value function assuming that he shirks this period. These value functions satisfy the following functional equations:

\[
V(\rho) = \max\{V_e(\rho), V_s(\rho)\};
\]
\[
V_e(\rho) = w - c + (p + (1 - p)\rho) \max\{0, b + \delta V\left(\frac{p}{p + (1 - p)\rho}\right)\} + (1 - p - (1 - p)\rho) \max\{0, \delta V(\rho^*)\};
\]
\[
V_s(\rho) = w + \rho \max\{0, b + \delta V(0)\} + (1 - \rho) \max\{0, \delta V(\rho^*)\}.
\]

Notice that \( \frac{p}{p + (1 - p)\rho} \) is the probability that the output is high in the previous period given effort is put in. In addition, notice that the max operators capture the possibility that the agent can take his outside option at the beginning of next period.
Step 2: Modified Problem. For our analysis, we make the following assumptions on the value functions:

\[
V(\rho) = \max\{V_e(\rho), V_s(\rho)\};
\]
\[
V_e(\rho) = w - c + (p + (1 - p)\rho)(b + \delta V(\frac{p}{p + (1 - p)\rho}))
+ (1 - p - (1 - p)\rho)\delta V(\rho^*) ;
\]
\[
V_s(\rho) = w + \rho (b + \delta V(0)) + (1 - \rho)\delta V(\rho^*).
\]

In other words, we assume that the agent will not take the outside option. And we check in the end that for the parameters of relevance this is indeed the case, i.e.,

\[
\begin{align*}
  b + V(0) & \geq 0; \\
  b + \delta V(\frac{p}{p + (1 - p)\rho}) & \geq 0; \\
  V(\rho^*) & \geq 0.
\end{align*}
\]

Now notice that if the agent starts at \( \rho = \rho^* \) and puts in effort, the probability of high output in the last period is again \( \rho^* \). This implies that to check the agent has incentive to put in effort in each period, it suffices to check that

\[
V_e(\rho^*) \geq V_s(\rho^*).
\]

This implies that our problem is to solve the following problem

\[
\min_{w, b} b
\]

such that

\[
\begin{align*}
  V_e(\rho^*) & \geq V_s(\rho^*); \\
  V_e(\rho^*) & = 0.
\end{align*}
\]

Notice that the \( V_e(\rho^*) = 0 \) because we assume that the principal captures all of the surplus.

Step 3: Solution to the Modified Problem. We solve the modified problem by construction. Notice that at the smallest bonus level \( b^* \) we must have \( V_e(\rho^*) = V_s(\rho^*) \).

We construct the unique value function \( V(\rho) \) associated with \( V_e(\rho^*) = V_s(\rho^*) = 0 \) and
check that it satisfies all desired properties.

Let
\[
V(\rho) = \begin{cases} 
(b^* + \delta V(0))(\rho - \rho^*) & \text{for } \rho > \rho^* \\
(1 - p)(b^* - \delta \rho^*(b + \delta V(0)))(\rho - \rho^*) & \text{for } \rho \leq \rho^*,
\end{cases}
\]
where
\[
V(0) = \frac{p(1 - \frac{1}{\rho^*} + \delta(1 - \rho^*))}{1 - p\delta^2(1 - \rho^*)}b^*,
\]
\[
w^* = c - \frac{p}{\rho^*}b,
\]
and
\[
b^* = \frac{c/p}{(\frac{1}{\rho^*} - \frac{p}{\rho^*} + \delta(1 - \rho^*)\delta(1 - \rho^*)\rho^*)}. \tag{5}
\]
In addition, the agent puts in effort if and only if \(\rho < \rho^*\).

Notice that the constructed value function is piecewise linear with a kink at \(\rho^*\). In addition, the expressions for \(V_e(\rho)\) and \(V_s(\rho)\) are both readily obtainable from \(V(\rho)\). Both are also piecewise linear functions with a kink at \(\rho^*\).

Given the construction above, it can be checked immediately that \(V(\rho^*) = V_e(\rho^*) = 0\). Next, we check that the effort decisions specified are optimal. To do that, we first need to make sure that for \(\rho \leq \rho^*\), \(V_s(\rho) \leq V_e(\rho)\), or equivalently,
\[
w + \rho(b^* + \delta V(0)) \\
\leq w - c + [(p + (1 - p)\rho)(b^* + \delta V(\frac{p}{p + (1 - p)\rho}))]
\]
\[
= (1 - p)(b^* - \delta \rho^*(b^* + \delta V(0)))(\rho - \rho^*).
\]
Note that the above is satisfied if
\[
b^* + \delta V(0) \geq (1 - p)(b^* - \delta \rho^*(b^* + \delta V(0))).
\]
Let \(x = b^* + \delta V(0)\), and define \(T(x) = (1 - p)(b^* - \delta \rho^*x)\), then the above can be rewritten as
\[
T(x) \leq x.
\]
In addition also want to make sure that for \(\rho > \rho^*\), we have \(V_s(\rho) \geq V_e(\rho)\), or
equivalently,

\[ w + \rho (b^* + \delta V(0)) \]
\[ \geq w - c + [(p + (1 - p)\rho)(b^* + \delta V(\frac{p}{p + (1 - p)\rho})) \]
\[ = w - c + [(p + (1 - p)\rho)(b^* + \delta V(0))(\frac{p}{p + (1 - p)\rho} - \rho^*)] \]
\[ = (1 - p)(b^* - \delta \rho^*(1 - p)(b^* - \delta \rho^*(b^* + \delta V(0)))(\rho - \rho^*). \]

If we again let \( x = b^* + \delta V(0) \) and \( T(x) = (1 - p)(b^* - \delta \rho^*x) \), then the slope of \( \rho \) in the expression above is given by \( T(T(x)) \), and we need

\[ T(T(x)) \leq x. \]

Now note that \( T(x) \) is an affine function of \( x \) with slope \( -\delta \rho^*(1 - p) > -1 \). Let \( x^* \) be such that \( T(x^*) = x^* \), then

\[ (1 - p)(b^* - \delta \rho^*x^*) = x^* \]
\[ x^* = \frac{(1 - p)b^*}{1 + \delta \rho^*(1 - p)}. \]

Now note that if \( x \geq x^* \), then

\[ T(x) \leq x. \]

Moreover, since the slope of \( T(x) \) is equal to \( -(1 - p)\delta \rho^* > -1 \), this implies that, for \( x > x^* \),

\[ \frac{T(x^*) - T(x)}{x - x^*} = \frac{x^* - T(x)}{x - x^*} < 1. \]

By the linearity of \( T \), it follows that,

\[ \frac{T(T(x)) - T(T(x^*))}{T(x^*) - T(x)} = \frac{T(x^*) - T(x)}{x - x^*} < 1 \]

so that

\[ T(T(x)) - T(T(x^*)) \leq x - x^*, \]

or

\[ T(T(x)) \leq x. \]
The discussion above implies that, as long as
\[ b^* + \delta V(0) = x \geq x^* = \frac{(1 - p)b^*}{1 + \delta \rho^*(1 - p)}, \]
the action profile is optimal. In other words, we need
\[ V(0) \geq -\frac{1}{\delta} \left( \frac{p + \delta \rho^*(1 - p)}{1 + \delta \rho^*(1 - p)} \right) b^*. \]

Recalling from (4) that
\[ V(0) = \frac{p(1 - \frac{1}{\rho^*} + \delta(1 - \rho^*))}{1 - p\delta^2(1 - \rho^*)} b^*. \]

So
\[
\begin{align*}
V(0) + \frac{1}{\delta} \left( \frac{p + \delta \rho^*(1 - p)}{1 + \delta \rho^*(1 - p)} \right) b^* &= \left( \frac{p(1 - \frac{1}{\rho^*} + \delta(1 - \rho^*))}{1 - p\delta^2(1 - \rho^*)} + \frac{1}{\delta} \frac{p + \delta \rho^*(1 - p)}{1 + \delta \rho^*(1 - p)} \right) b^* \\
&= \frac{\delta(1 - p\delta^2(1 - \rho^*)) + \delta \rho^*(1 - p)}{\delta(1 - p\delta^2(1 - \rho^*) + \delta \rho^*(1 - p))} \geq 0.
\end{align*}
\]

This shows that the effort decisions are optimal. This shows that the function constructed is the true value function if the agent does not take his outside option.

Finally, we need to check that the agent never takes his outside option under the constructed value function. Given that \( V \) is increasing in \( \rho \) and \( V(\rho^*) \geq 0 \), the only condition to check is that
\[ b^* + \delta V(0) > 0. \]

But from the above, we see that
\[ b^* + \delta V(0) \geq \frac{(1 - p)b^*}{1 + \delta \rho^*(1 - p)} > 0, \]
and this shows that the proposed value function is the true value function.

**Step 4:** Comparison to \( f = 0 \). Notice that the bonus necessary to induce effort is given by \( c/p \) when \( f = 0 \). This implies that \( \delta^*(1) < \delta^*(0) \) as long as \( b^* < c/p \).
From Step 3, we see that
\[
\frac{c}{p b^*} = \frac{1}{\rho^*} - \frac{\rho^*}{p} + \delta \frac{(1 - \rho^* - \delta(1 - \rho^*)\rho^*)}{1 - p\delta^2(1 - \rho^*)}.
\]
Repeatedly using \(p(1 - \rho^*) = (1 - p)\rho^2\) and some algebra, we have
\[
\frac{c}{p b^*} - 1 = \frac{\delta (1 - \rho^* - \delta(1 - \rho^*)\rho^*)}{1 - p\delta^2(1 - \rho^*)} + \frac{p(\rho^* - 1)}{(1 - p)\rho^*} - \frac{\rho^* + (1 - \rho^*)\delta - \delta^2(\rho^* - p)}{1 - p\delta^2(1 - \rho^*)} = \frac{\delta - (1 + \delta + \delta^2)\rho^* + \delta^2 p}{1 - p\delta^2(1 - \rho^*)}.
\]
Therefore, \(b^* < c/p\) if and only if \(\delta - (1 + \delta + \delta^2)\rho^* + \delta^2 p > 0\). This finishes proving Part (i).

**Part (ii).** Now consider the general \(f\)-credit-rollover reporting so that the bonus is paid out with probability \(f\) following a low output this period but the output in the previous period is high. In this case, denote \(s \equiv \rho(f)\) as the stationary probability that a bonus is paid out at the beginning of a reporting cycle when the output is low. In particular, we have
\[
s = \frac{p}{p + (1 - p)s f}.
\]
Following Part (i), we use \(V(\rho)\) to denote the value of the agent when the probability of high output is \(\rho\). As in Part (i), to find \(\delta(f)\), it is equivalent to find the minimal bonus \(b^*(f)\) necessary for sustaining an efficient relational contract, i.e.,
\[
\min_{b} \quad b
\]
such that
\[
V_e(s) \geq V_a(s); \quad V_e(s) = 0.
\]
Instead of solving this problem directly, we solve below a modified problem that gives a lower bound \(b(f) \leq b^*(f)\) for each \(f\). We show that if this lower bound results in a number smaller than \(c/p\) for some \(f\), then the lower bounds \(b(f)\) are minimized at \(f = 1\). In this case, this lower bound \(b(1) = b^*(1)\) found in Part (i), and this proves
Part (ii).

**Step 1:** *Modified problem.* Consider the following type of deviation strategy of the agent. First, the agent accepts the contract at the beginning of period 1. Second, following the first $B$ report, the agent takes his outside option forever. Third, while he stays in the relationship, the agent shirks if and only if $\rho \leq s$. Denote the agent’s value function under this deviation as $V_d(\rho)$. Below, we solve the problem

$$\min_{w,b} b$$

such that

$$V_e(s) \geq V_d(s);$$

$$V_e(s) = 0.$$ 

Notice that $V_e(s) \geq V_d(s)$, so the solution to this problem provides a lower bound $b(f)$ associated with $f$-credit-rollover reporting necessary to sustain the efficient relational contract.

Now $V_e(s) = 0$ gives

$$w - c + (p + (1 - p)s) b = 0.$$ 

In addition, under the deviation strategy, we have

$$V_d(s) = w + sf(b + \delta V_d(0));$$

$$V_d(0) = w - c + p(b + \delta V_d(1));$$

$$V_d(1) = w + f(b + \delta V_d(0)).$$

Solving the above, we obtain

$$V_d(0) = \frac{\delta pc + p(1 + \delta f)b - (1 + \delta p)(p + (1 - p)s)f b}{1 - \delta^2 pf}.$$ 

Given the expression for $V_d(0)$, we can rewrite $V_e(s) \geq V_d(s)$ as

$$(c - (p + (1 - p)s)f) b + sf(b + \delta \frac{\delta pc + p(1 + \delta f)b - (1 + \delta p)(p + (1 - p)s)f b}{1 - \delta^2 pf}) \leq 0.$$ 

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Using \((p + (1 - p)s f) = p/s\), we find that the above is equivalent to
\[
\frac{-s + \delta (1 - s) - \delta^2 (fs (1 - p) - p (1 - s))}{1 - \delta^2 pf (1 - s)} \geq \frac{c}{pb} - 1. \quad \text{ (garbling)}
\]

This implies that for \(f\)-credit reporting to improve on \(f = 0\), one must have
\[
\frac{-s + \delta (1 - s) - \delta^2 (fs (1 - p) - p (1 - s))}{1 - \delta^2 pf (1 - s)} \geq 0.
\]

**Step 2: A necessary condition.** We show that \(p < 1/4\) is a necessary condition for
\[
\frac{-s + \delta (1 - s) - \delta^2 (fs (1 - p) - p (1 - s))}{1 - \delta^2 pf (1 - s)} \geq 0.
\]

To see this, notice that
\[
f = \frac{p(1 - s)}{(1 - p)s^2}.
\]

This implies that \(s\) strictly decreases with \(f\). In addition, when \(f = 0\) we have \(s = 1\), and when \(f = 1\) we have that \(s\) is minimized at \(\rho^*\), where \(\rho^* = \frac{p}{p + (1 - p)\rho^*}\). As a result, choosing \(f \in [0, 1]\) is then equivalent to choosing \(s \in [\rho^*, 1]\). Substituting out of \(f\), one find that the expression above is equivalent to
\[
\frac{p (1 - s) \left( \delta - (1 + \delta) s - \delta^2 p \frac{(1 - s)^2}{s} \right)}{(1 - p) s^2 - \delta^2 p^2 (1 - s)^2} \geq 0.
\]

Notice that for the expression above to be satisfied, one a necessary and sufficient condition is to find \(s \in [\rho^*, 1]\) such that
\[
\delta s - (1 + \delta) s^2 - \delta^2 p (1 - s)^2 > 0.
\]

The unconstrained maximization problem of the left hand side gives
\[
s^* = \frac{\delta (1 + 2p\delta)}{2 \left( 1 + \delta + p\delta^2 \right)}.
\]
and

\[ \delta s^* - (1 + \delta) s^* - \delta^2 p (1 - s^*)^2 = \frac{\delta^2 (1 + 2p\delta)^2}{4 (1 + \delta + p\delta^2)} - \delta^2 p. \]

This expression is positive iff \( p < \frac{1}{4} \). As a result, \( p < \frac{1}{4} \) is a necessary condition for

\[ f \frac{s + \delta (1 - s) - \delta^2 (fs(1 - p) - p(1 - s))}{1 - \delta^2 pf(1 - s)} \geq 0. \]

**Step 3:** *Optimality of \( f = 1 \).* By Step 2, we now assume that \( p \in (0, 1/4) \). To minimize the lower bounds \( b(f) \) (over \( f \)), Step 2 implies that it is equivalent to find solve the following constrained maximization problem.

\[ \max_s \frac{p (1 - s) \left( \delta - (1 + \delta) s - \delta^2 p \frac{(1 - s)^2}{s} \right)}{(1 - p) s^2 - \delta^2 p^2 (1 - s)^2} \]

such that

\[ 1 \geq s \geq \rho^*, \]

where \( \rho^* = \frac{p}{p + (1 - p)\rho^*} \).

Below, we show that the objective function is maximized at \( s = \rho^* \). To see this, it is useful to rewrite the objective function as

\[ A(s, p, \delta) \equiv B(s, p, \delta) \frac{p (1 - s)}{(1 - p) s^2 - \delta^2 p^2 (1 - s)^2}, \]

where

\[ B(s, p, \delta) \equiv \delta - (1 + \delta) s - \delta^2 p \frac{(1 - s)^2}{s} = \delta + 2\delta^2 p - (1 + \delta + \delta^2 p) s - \frac{\delta^2 p}{s}. \]

Notice that \( \frac{p(1 - s)}{(1 - p)s^2 - \delta^2 p^2 (1 - s)^2} \) decreases with \( s \) because the numerator is decreasing in \( s \) and the denominator is increasing in \( s \). In particular, the derivative of the denominator with respect to \( s \) is given by \( 2(1 - p)s + 2\delta^2 p^2 (1 - s) > 0 \).

It follows that \( A(s, p, \delta) \) is maximized at \( s = \rho^* \) as long as we can show that \( B(s, p, \delta) \) is maximized at \( s = \rho^* \) for \( 1 \geq s \geq \rho^* \). To show this, we notice that \( B \)
is concave in \( s \) since \( \frac{dB^2(s,p,\delta)}{ds^2} = - \frac{2\delta^2 p}{s^3} < 0 \). As a result, it suffices to show that \( \frac{dB(s,p,\delta)}{ds} < 0 \) for \( s = \rho^* \).

Now at \( s = \rho^* \),

\[
\frac{dB(\rho^*, p, \delta)}{ds} = -(1 + \delta + \delta^2 p) + \frac{\delta^2 p}{\rho^{*2}} = -(1 + \delta + \delta^2 p) + \delta^2 \frac{1 - p}{1 - \rho^*}.
\]

To show that the term above is negative, notice that

\[
\rho^* = \frac{-p + \sqrt{p^2 + 4p(1-p)}}{2(1-p)} = \frac{2}{\sqrt{1 + \frac{4(1-p)}{p}} + 1}
\]

and it is easy to see that that \( \rho^* \) increases with \( p \). Moreover, when \( p < 1/4 \), we have that

\[
\rho^* \leq \frac{2}{\sqrt{1 + \frac{4(1-1/4)}{1/4}} + 1} = \frac{2}{\sqrt{13} + 1} < \frac{1}{2}.
\]

It follows that

\[
-(1 + \delta + \delta^2 p) + \delta^2 \frac{1 - p}{1 - \rho^*} < -(1 + \delta + \delta^2 p) + 2\delta^2 (1-p) = -3\delta^2 p - (1 + \delta - 2\delta^2) < 0.
\]

This shows that \( \frac{dB(\rho^*, p, \delta)}{ds} < 0 \), and as a result, \( B(s, p, \delta) \) and \( A(s, p, \delta) \) are both maximized at \( s = \rho^* \).

Recall that \( f = \frac{p(1-s)}{(1-p)s} \), and this implies that the lower bounds \( b(f) \) are minimized at \( f = 1 \). Finally, when \( f = 1 \), the lower bound above for \( b(1) \) satisfies

\[
\frac{c}{pb(1)} - 1 = f \frac{-s + \delta (1-s) - \delta^2 (f s (1-p) - p (1-s))}{1 - \delta^2 p f (1-s)}
\]

for \( f = 1 \) and \( s = \rho^* \). Substituting for \( f \) and \( s \), we get

\[
\frac{c}{pb(1)} - 1 = \frac{\delta - (1 + \delta + \delta^2) \rho^* + \delta^2 p}{1 - p\delta^2 (1 - \rho^*)}.
\]

Comparing the condition that determines \( b^*(1) \) in Part (i), we have that \( b(1) = b^*(1) \). This implies that the minimal bonus necessary for the efficient relational contract is the smaller than the lower bound of bonus \( b(f) \) associated with other \( f\)-
credit-rollover reporting. This establishes the optimality of \( f = 1 \) and completes the proof. ■

**Corollary 1:** The ratio of surplus is given by

\[
\frac{S_0}{S_1} = 1 + \frac{\delta - (1 + \delta + \delta^2) \rho^* + \delta^2 p}{1 - p \delta^2 (1 - \rho^*)},
\]

where \( \rho^* = p / (p + (1 - p) \rho^*) \). In particular, there exists \( p^* \) such that \( S_0(p, \delta) / S_1(p, \delta) \) is decreasing for all \( p \in (0, p^*) \), and

\[
\lim_{p \to 0} \frac{S_0(p, \delta)}{S_1(p, \delta)} = 1 + \delta.
\]

**Proof.** Recall from Section 3.1 that \( S_0 = c / p \). Recall from the end of the proof of Part (i) of Proposition 1, the minimum bonus required to sustain efficient relational contracts is given by

\[
\frac{c}{pb^*} - 1 = \frac{\delta - (1 + \delta + \delta^2) \rho^* + \delta^2 p}{1 - p \delta^2 (1 - \rho^*)}.
\]

Since \( S_1 = b^* \), it is then immediate that

\[
\frac{S_0}{S_1} = 1 + \frac{\delta - (1 + \delta + \delta^2) \rho^* + \delta^2 p}{1 - p \delta^2 (1 - \rho^*)}.
\]

Since \( \rho^* = p / (p + (1 - p) \rho^*) \), \( \rho^* \) goes to 0 as \( p \) goes to 0. The expression for \( S_0 / S_1 \) then implies that

\[
\lim_{p \to 0} \frac{S_0(p, \delta)}{S_1(p, \delta)} = 1 + \delta.
\]

Now using \( p = \frac{\rho^*}{1 - \rho^* + \rho^*} \), we obtain that

\[
\frac{\delta - (1 + \delta + \delta^2) \rho^* + \delta^2 p}{1 - p \delta^2 (1 - \rho^*)} = \delta - (1 + \delta + \delta^2) \rho^* + \frac{\delta^2 (1 + \delta - (1 + \delta + \delta^2) \rho^*)}{1 - \rho^* + (1 - \delta^2) \rho^*} \rho^*^2
\]

\( \equiv R(\delta, \rho^*). \)
It is clear that
\[
\lim_{\rho^* \to 0} \frac{\partial R(\delta, \rho^*)}{\partial \rho^*} = -(1 + \delta + \delta^2).
\]
Moreover, since \( \rho^* \) is increasing in \( p \) for small enough \( p \), this implies that there exists a \( p^* \) such that \( S_0(p, \delta) / S_1(p, \delta) \) is decreasing for all \( p \in (0, p^*) \). □

**Corollary 2:** Consider \( p \) and \( \delta \) such that \( \delta + \delta^2 p - (1 + \delta + \delta^2) \rho^* > 0 \), where \( \rho^* = p/ (p + (1 - p) \rho^*) \). For each expected discounted surplus level \( S > S_0 \), there exists an associated \( q^* > 0 \) such that efficient relational contracts can be sustained under credit-rollover reporting for all \( q \in [0, q^*] \).

**Proof.** The proof follows from standard continuity arguments. For given \( c, p \) and \( \delta \), let \( b^* \) be the minimum bonus necessary to induce effort under credit reporting. Proposition 1 implies that \( b^* < c/p \) when \( \delta + \delta^2 p - (1 + \delta + \delta^2) \rho^* > 0 \), where \( \rho^* = p/ (p + (1 - p) \rho^*) \). Now again normalize the agent’s outside option \( u \) to be 0, and for each \( w \) and \( b \), define the agent’s value functions \( V(\rho), V_s(\rho) \) and \( V_e(\rho) \) as in Proposition 1. Recall that at \( w^* \) and \( b^* \), we have \( V_e(\rho^*) = V_s(\rho^*) \), \( V_e(\rho^*) = 0 \), and \( b^* + \delta V(0) > 0 \). Since \( V_e(\rho^*) - V_s(\rho^*) \) and \( b + \delta V(0) \) all increase in \( b \), it follows that for each \( S > b^* \equiv S_0 \), there exists a small enough \( \varepsilon(S) \) such that one can find \( w \) and \( b \in (b^*, S) \) in which

\[
V_e(\rho^*) > V_s(\rho^*) + 3\varepsilon;
\]

\[
V_e(\rho^*) > 2\varepsilon;
\]

\[
b + \delta V(0) > 2\varepsilon.
\]

Next, for each \( q > 0 \), let \( V^q(\rho) \) be the agent’s value function at the beginning of a period assuming that he has accepted the principal’s offer. Let \( V^q_e(\rho) \) be the agent’s value function if he puts in effort this period and \( V^q_s(\rho) \) be the agent’s value function if he shirks. The value functions then satisfy the following:

\[
V^q(\rho) = \max\{V^q_e(\rho), V^q_s(\rho)\};
\]

\[
V^q_e(\rho) = w - c + (p + (1 - p)\rho) \max\{0, b + \delta V^q(\frac{p}{p + (1 - p)\rho})\} + (1 - p - (1 - p)\rho) \max\{0, \delta V^q(\rho^*)\};
\]

\[
V^q_s(\rho) = w + (q + (1 - q)\rho) \max\{0, b + \delta V^q(\frac{q}{q + (1 - p)\rho})\} + (1 - \rho)\delta V^q(\rho^*) \max\{0, \delta V^q(\rho^*)\}.
\]
It is clear that there exists a $q^*$ such that for all $q \in [0, q^*],$

$$\max\{|V^q(\rho) - V(\rho)|, |V_e^q(\rho) - V_e(\rho)|, |V_s^q(\rho) - V_s(\rho)|\} < \varepsilon$$

for all $\rho \in [0, 1].$

As a result,

$$V_e^q(\rho^*) - V_s^q(\rho^*)$$

$$= V_e^q(\rho^*) - V_e(\rho^*) + V_s(\rho^*) - V_s(\rho^*)$$

$$> -\varepsilon + 3\varepsilon - \varepsilon$$

$$= \varepsilon.$$

Similarly,

$$V_e^q(\rho^*) = V_e(\rho^*) + V_e^q(\rho^*) - V_e(\rho^*) > 2\varepsilon - \varepsilon = \varepsilon,$$

and

$$b + \delta V^q(0) = b + \delta V(0) + \delta V^q(0) - \delta V(0) > 2\varepsilon - \varepsilon = \varepsilon.$$

These three inequalities imply that for the chosen $w$ and $b$, it is incentive compatible for the agent to accept the contract and exerts effort. □

**Corollary 3:** Let $c = c_2 - c_1$ and let $S_1(p, q, \delta, c)$ be the minimum expected discounted surplus for effort under credit-rollover reporting in the binary-effort model. For any $S \in (S_1, \frac{c}{p-q}),$ efficient relational contracts are sustainable under credit-rollover reporting when $c/c_1 > M$ for some $M > 0.$

**Proof.** Define $V(\rho)$ as the agent’s value function at the beginning of a period assuming that he has accepted the principal’s offer. Let $V_i(\rho)$ be the agent’s value function if he puts in effort level of $i \in \{0, 1, 2\}$ this period. Normalize the agent’s outside option $u$ to be 0. then for each base wage $w$ and bonus $b,$ the value functions satisfy
the following.

\[
V(\rho) = \max\{V_0(\rho), V_1(\rho), V_2(\rho)\}
\]
\[
V_2(\rho) = w - c_2 + (p + (1 - p)\rho)\max\{0, b + \delta V(\frac{p}{p + (1 - p)\rho})\} + (1 - p - (1 - p)\rho)\max\{0, \delta V(\rho^*)\};
\]
\[
V_1(\rho) = w - c_1 + (q + (1 - q)\rho)\max\{0, b + \delta V(\frac{q}{q + (1 - p)\rho})\} + (1 - q - (1 - q)\rho)\max\{0, \delta V(\rho^*)\}.
\]
\[
V_0(\rho) = w + \rho\max\{0, b + \delta V(0)\} + (1 - \rho)\max\{0, \delta V(\rho^*)\}.
\]

It is clear that for each w and b, there is a unique set of value functions that satisfy the functional equations above.

Next, suppose \(b^*\) is the minimum bonus for effort in the binary effort case and \(w^*\) is the associated base wage so that the agent’s participation binds. Let \(V_e(\rho), V_s(\rho),\) and \(V_b(\rho)\) be the agent’s value functions associated with \(b^*\) and \(w^*\), where we use the subscript \(b\) stands for the binary case. Now let \(w = w^*+c_1\) and \(b = b^*\). We show below that for sufficiently small \(c_1\), we have \(V_2(\rho) = V_e(\rho), V_1(\rho) = V_s(\rho), V(\rho) = V_b(\rho),\) and \(V_0(\rho) = w + \rho(b + \delta V_b(0))\) for all \(\rho\).

To do this, it suffices to show that the constructed value function satisfies the set of functional equations above. Given the properties of \(V_e(\rho), V_s(\rho),\) and \(V_b(\rho)\), we only need to show that \(V_1(\rho) \geq V_0(\rho)\) for all \(\rho\). Notice that

\[
V_1(\rho) - V_0(\rho) = (q + (1 - q)\rho)(b + \delta V_b(\frac{q}{q + (1 - p)\rho})) - (b + \delta V_b(0)) - c_1
\]
\[
> (1 - q)\rho(b + \delta V_b(0)) - c_1
\]
\[
\geq \frac{(1 - p)b}{1 + \delta \rho^*(1 - p)} - c_1,
\]

where the first inequality uses the fact that \(V\) is increasing in \(\rho\) and the second inequality uses the lower bound for \(b + \delta V_b(0)\) established in Proposition 1, Part (i). Since \(b = b^*\) is independent of \(c_1\), it is clear that for small enough \(c_1\), \(V_1(\rho) - V_0(\rho) > 0\) for all \(\rho\).

The above implies that for small enough \(c_1\), the agent is willing to choose \(e = 2\) when \(b = b^* = S_0\) and \(w = w^* + c_1\). Finally, since \(S > S_0\), the principal will not
renge on the bonus. This establishes that the stationary strategies with delayed bonus with \( w = w^* + c_1 \) and \( b = b^* \) supports the efficient relational contracts.

**Proposition 2:** Recall that \( S \equiv \frac{1}{1-\delta}(py - c - u - \pi) \) as the discounted expected future surplus of the relationship. For any reporting strategy to sustain the efficient relational contract, one must have

\[
S \geq \sqrt{4p(1-p)} \frac{c}{p-q}.
\]

In particular, full revelation of outputs is the optimal reporting strategy when \( p = \frac{1}{2} \).

**Proof.** First recall that when \( s_t = Y_t \) for all \( t \), the necessary and sufficient condition for sustaining cooperation is given by equation (3):

\[
\frac{c}{p-q} \leq \frac{\delta}{1-\delta}(py - c - u - \pi) \equiv S,
\]

where without confusion in this proof, \( S \) denotes the surplus of the relationship (when the agent puts in effort each period.) We want to show that if the inequality above fails, it is impossible to construct an equilibrium in which the agent puts in effort. In particular, using standard argument as in Fuchs (2007), it suffices to show that there does not exist an equilibrium in which the agent always puts in effort (unless the relationship is terminated).

Consider an arbitrary information partition process. Pick one information set \( (h_t) \). Use \( x \) to denote the possible states within the information set. One interpretation of \( x \) is some output realizations \( y_t \) that falls into \( h_t \).

Let \( V(x) \) be the agent’s continuation payoff in state \( x \) after \( e_{t+1} \) is put in but before \( y_{t+1} \) is realized and \( W_{t+1} \) is paid out. Let \( V(x_i) \) be the agent’s continuation payoff in state \( x \) after \( e_{t+1} \) is put in, \( y_{t+1} \) is realized but before \( W_{t+1} \) is paid out. Within each state \( x \), we have \( x_i \in \{x_y, x_0\} \), where \( x_y \) denotes that \( Y_t = y \) is realized following \( x \), and \( x_0 \) denotes that \( Y_t = 0 \) is realized.

Note that

\[
V(x) = V(x) + p(V(x_y) - V(x)) + (1-p)(V(x_0) - V(x)).
\]

And since the output \( Y_t \) is independent of the past state, we have \( Cov(V(x_i) - V(x), V(x)) = 0 \).
To induce effort, we need
\[ E_x[V(x_y) - V(x_0)] = E_x[V(x_y) - V(x)] - E_x[V(x_0) - V(x)] \geq \frac{c}{p - q}. \]

This helps give a lower bound for \( Var(V(x_i)) \). In particular,
\[
\begin{align*}
Var(V(x_i)) &= Var(V(x)) + Var(V(x_i) - V(x)) \\
&= Var(V(x)) + E_y[Var(V(x_i) - V(x))|Y] \\
&\quad + Var(E_x[V(x_i) - V(x)|Y]) \\
&\geq Var(V(x)) + E_y[Var(V(x_i) - V(x))|Y] \\
&\quad + p(1 - p)(\frac{c}{p - q})^2 \\
&\geq Var(V(x)) + p(1 - p)(\frac{c}{p - q})^2,
\end{align*}
\]
where the first line follows because \( Cov(V(x_i) - V(x), V(x)) = 0 \), the second line uses the variance decomposition formula, the third line follows because \( E_x[V(x_i) - V(x)|Y] \) is a binary value \( (Y \in \{0, y\}) \) such that with probability \( p \) its value is \( E_x[V(x_y) - V(x)] \) and with probability \( 1 - p \) its value is \( E_x[V(x_0) - V(x)] \), and \( E_x[V(x_y) - V(x)] - E_x[V(x_0) - V(x)] \geq \frac{c}{p - q} \).

Now let’s provide an upper bound for \( Var(V(x_i)) \). Suppose a public signal \( s(x_i) \) will be sent out after state \( x_i \). Let \( b(s) \) be the bonus paid out to the agent (at the end of the period) following signal \( s \). This allows us to write
\[ V(x_i) = b(s(x_i)) + \delta V_{s(x_i)}(x_i), \]
where \( V_{s(x_i)}(x_i) \) is the continuation payoff of \( x_i \), which goes to the information set by signal \( s(x_i) \).

Note that for the principal to be willing to pay the bonus, we must have
\[
\max_s \{ b_s + \delta E_{x_i}[V_s(x_i)|s] \} - \min_s \{ b_s + \delta E_{x_i}[V_s(x_i)|s] \} \leq S.
\]
Because otherwise the expected payoff of the principal following some signal will be below his outside option.
Decomposing the variance on the signals, we have

\[
\text{Var}(V(x_i)) = \text{Var}(E[b_s + \delta V_s(x)|s]) + E[\text{Var}(b_s + \delta V_s(x_i)|s)] \\
\leq \frac{1}{4}S^2 + \delta^2 E[\text{Var}(V_s(x)|s)].
\]

Now combining the upper and lower bound for \(\text{Var}(V(x_i))\), we get that

\[
\frac{1}{4}S^2 + \delta^2 E[\text{Var}(V_s(x_i)|s)] \geq \text{Var}(V(x)) + p(1 - p)(\frac{c}{p - q})^2,
\]

or equivalently,

\[
E[\text{Var}(V_s(x_i)|s)] \geq \frac{1}{\delta^2}(\text{Var}(V(x)) + p(1 - p)(\frac{c}{p - q})^2 - \frac{1}{4}S^2).
\]

Now if \(4p(1 - p)(\frac{c}{p - q})^2 > S^2\), the inequality above implies that

\[
E[\text{Var}(V_s(x_i)|s)] > \frac{1}{\delta^2}(\text{Var}(V(x))).
\]

In particular, there will be one information set (associated with a signal) whose variance exceeds \(\frac{1}{\delta^2}(\text{Var}(V(x)))\). Now we can perform the same argument on this new information set, and we can construct a sequence of information set whose variance approaches infinity. This leads to a contradiction.

Therefore, to sustain an efficient relational contract, one must have

\[
S^2 \geq 4p(1 - p)(\frac{c}{p - q})^2.
\]

It follows that when \(p = 1/2\), this condition becomes \(S \geq \frac{c}{p - q}\), which is exactly the condition for sustaining the efficient relational contract under full revelation of outputs. This shows that full revelation of outputs is the optimal reporting strategy when \(p = 1/2\).

**Proposition 3:** Let \((U_t, \Pi_t)\) be the expected discounted payoffs of the agent and the principal evaluated at time \(t\). Suppose (3) fails. For all \(t\), if there exists a predetermined \(t' \geq t\) such that \((U_{t'}, \Pi_{t'})\) are independent of \(h_{t'}\), then \(e_{t} = 0\).

**Proof.** Note that in our setting, the feasible surplus of the game \(S\) cannot be raised by any reporting rule. Suppose \((U_{t'}, \Pi_{t'})\) are independent of \(h_{t'}\). Then \(e_{t'} = 0\) is solely
motivated by $b_{t-1}$, and since $b_{t-1} \leq S < c/(p-q)$, it must hold that $e_{t-1} = 0$. Given that $e_{t-1} = 0$ regardless of $b_{t-1}$ and that (3) fails, the expected sum of bonuses $E(b_{t-2} + \delta b_{t-1})$ should also be no larger than $S$. This implies that $e_{t-2} = 0$ as well. By induction, $e_\tau = 0$ for all $\tau \in \{1, 2, ..., t'\}$. ■

Appendix B

![Diagram](image)

Figure 4: Region of $(p,q)$ where credit-rollover reporting enhances efficiency