Job Mobility, Wage Dispersion, and Technological Change: An Asymmetric Information Perspective

Jin Li*
Kellogg School of Management
Northwestern University

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Abstract

This paper develops a model of job mobility and wage dispersion with asymmetric information. Contrary to existing models in which the superior information of current employers leads to market collapse, this model generates a unique equilibrium outcome in which a) positive turnover exists and b) identical workers may be paid differently. The model implies that, in the presence of technological change that is skill-biased and favors general skills over firm-specific skills, the wage distribution becomes more spread out (corresponding to greater inequality) and job mobility increases.

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1 Introduction

There are two well-documented empirical regularities concerning the labor market for young workers. First, young workers frequently change jobs. A typical U.S. worker holds seven jobs in the first ten years of his labor-market experience; see, for example, Topel and Ward (1992). Second, the wage distribution of young workers widens over time, both unconditionally and conditionally (on the observables); see, for example, Farber and Gibbons (1996). Moreover, wage dispersion has increased significantly over the past thirty-five years; see, for example, Autor, Katz, Kearney (2005).

Job mobility and wage dispersion have both been studied extensively and important insights have been gained on both.[1] Existing models of job mobility, however, rarely discuss their implications for wage dispersion, nor do models of wage dispersion typically consider job mobility. This is surprising because models of either mobility or dispersion often share two common sets of assumptions. First, information about the abilities of workers is imperfect and is revealed over time. Second, workers can receive multiple offers from different firms at the same time. These assumptions reflect a set of underlying economic forces–namely imperfect information, learning, and multiplicity of wage offers–that are common to both job mobility and wage dispersion. Therefore, one might expect that patterns in job mobility and in wage dispersion are connected.

In this paper, I develop a model that offers a framework for studying the joint determination of wages and job changes as a result of asymmetric learning. The model applies to settings where the current employer knows more about a worker's ability than prospective employers do.[2] The superior information of the current employer creates a standard lemons problem. In this model, however, the presence of the lemons problem does not lead to market collapse. Instead, there exists a unique equilibrium allocation in which the current employer offers a wage equal to the average outside output of all types below the worker's ability, and outside firms compete for the workers by using mixed strategies. The unique equilibrium allocation determines both the allocation of workers with heterogeneous abilities to different firms and how wages change when workers change jobs.

The setup of this model is similar to Greenwald's (1986) classic asymmetric-information model, but with one important difference (described below). There are two periods, a single worker, and finitely many firms. At the beginning of period 1, firms compete for the worker using wage competition a la Bertrand. The worker picks a period 1 employer. At the end of period 1, the incumbent (period 1 employer) learns about the worker's ability while outside firms do not. Moreover, the worker may accumulate firm-specific human capital, which makes him more productive at the incumbent firm in period 2. Fully aware of the informational and production advantage of the incumbent, all firms offer wage contracts simultaneously to the worker at the beginning of period 2.

[1] Important models of job mobility include Jovanovic (1979), Greenwald (1986), Gibbons and Katz (1991), and Acemoglu and Pischke (1998,1999a,b). Important models of wage dispersion include Burdett and Judd (1983) and Burdett and Mortensen (1998). Mortensen (2001) offers an excellent survey of wage-dispersion models.

[2] Several recent papers test the empirical implications of the asymmetric information in the labor market; see, for example, Kahn (2008), Pinkston (2009), and Schoenberg (2007).

The key difference between our model and Greenwald's involves the timing of offers at the beginning of period 2. Greenwald assumes (as do many subsequent models) that the incumbent knows all the offers made by the outside firms before making a counteroffer. This timing of offers makes the lemons problem in the second-hand labor market so severe that the market collapses when there are no exogenous movers or differences in match qualities. In this model, in contrast, I assume that the incumbent and outside firms make their offers simultaneously. The assumption about the simultaneity of offers has been used in the literature; see, for example, Waldman (1984). This difference in timing alleviates the lemons problem and produces endogenous turnover and a non-degenerate wage dispersion.

In the unique equilibrium allocation of this model, the current employer's wage offer is strictly increasing in the worker's ability. The outside firms randomize their wage offers between the minimal and the maximal wage offers by the current employer. This randomization implies that some workers will receive outside offers higher than the current employer's offer. Consequently, endogenous turnover can arise in this model without requiring exogenous movers or differences in match qualities. In addition, the randomization generates wage dispersion for workers. The wage dispersion occurs at two levels. The first level indicates that different workers are in general paid differently because the current employer's wage is increasing in the worker's ability. The second level provides that workers of identical abilities can also be paid differently because outside firms randomize their wage offers so, for two identical workers, one may receive a high outside offer and the other may not.

A particular feature of the model is that there are explicit formulas for turnover probabilities and for wage distribution. The formulas on turnovers imply that, in contrast to models with symmetric information, turnover probability depends on both firm-specific and general human capital levels. The formulas for wage distribution allow one to compare the wage distribution of the workers who stay with the incumbent firm with those who have left.

The main application of the model uses these formulas to study the effect of technological change on wage inequality and job mobility. To the extent that technological change favors workers with higher abilities over those with lower abilities and favors general skills over specific skills in production, the model generates the following predictions. First, the wage distribution becomes more spread out as described in Bickel and Lehmann (1979), which corresponds to greater inequality. Note that the model is one of asymmetric information; it applies to workers with the same observables. Therefore, the widening of the wage dispersion refers to the increase in residual inequality, i.e., the dispersion of wages in observationally equivalent groups. Second, the job mobility of all worker types increases. Third, the proportional increase in job mobility is larger for workers with higher levels of firm-specific human capital. These patterns are consistent with empirical evidence on recent changes in wage inequality and job mobility in the United States.

Literature Review This paper is related to three strands of the literature. First, in terms of topic, this paper belongs to the literature that explores the effect of asymmetric information on labor market outcomes. Greenwald (1986) shows that asymmetric information reduces turnover and leads to compression in wages. Waldman (1984) shows that asymmetric information leads to distortion in promotion decisions.[3] Gibbons and Katz (1991) explore the consequence of asymmetric information on layoff decisions, and Acemoglu and Pishke (1998, 1999a) and Autor (2002) study the effect of asymmetric information in general.

This paper adds to the literature by showing that a mixed strategy can occur with asymmetric information. The mixed strategy implies that the turnover probability of a worker decreases continuously in the worker's ability, whereas the turnover probability takes a zero-one form in most of the existing literature.[4] In addition, the mixed strategy implies that identical workers can be paid differently, whereas there is typically no wage dispersion for workers of identical abilities in the literature.

Second, in terms of its analytical structure, this model is most closely related to the first-price auction models with privately informed bidders; see, for example, Engelbrecht-Wiggans, Milgrom, and Weber (EMW) (1983). While this type of auction has been studied in the auction literature, the implications of EMW have not been explored in the labor market context. Moreover, the focus of the auction literature has been on common-value auctions. Here, I mainly examine the case where firm-specific human capital is present (corresponding to an auction in which the better-informed bidder also has a higher value). I focus on how the comparison between the production advantage and the informational advantage of the incumbent can affect job mobility and wage distribution.

Also closely related are the macro-labor models such as those of Burdett and Judd (1983) and Burdett and Mortensen (1998). In these models, there is also a mixed strategy equilibrium where firms randomize their wage offers and thus generate wage dispersion. In the Burdett-Judd-Mortensen models, workers are identical and the number of wage offers they receive follow an exogenous Poisson process. Firms randomize to trade off the probability of hiring a worker against the profit made from the worker. In this model, in contrast, workers are heterogeneous and receive offers from all firms. Firms randomize to trade off the quality of the workers hired against the wage paid to the workers, taking into account the productivity and selection effects that arise from worker heterogeneity and asymmetric information.

Third, in terms of its applications, the paper belongs to the vast literature that studies the reasons for the recent changes in the patterns of wage inequality and job mobility. There are obviously many factors, such as globalization (Wood 1994) and labor market institutions (DiNardo, Fortin, and Lemieux 1995), that can explain the rise in wage inequality. This paper falls into the sub-literature that explains the wage inequality with changes in technology. The leading hypothesis in this sub-literature, the skill-biased technological change (SBTC), directly implies an increase in wage inequality in a competitive labor market in which the workers are paid their marginal products. Acemoglu (2002) argues, however, that the simplest models of SBTC imply that residual inequality and between-group inequality

[3] Golan (2005) shows that promotion can be efficient if the incumbent firm is allowed to make counteroffers.

[4] One exception is Novos (1995). The continuity of turnover probability arises in his model because, in addition to their abilities, workers also differ in their tastes for the job.

must move in the same direction, contradicting the empirical evidence on wage inequality in the 1970s.

Several influential papers develop more sophisticated models to show that techological change can affect residual inequality without necessarily affecting between-group inequality. Acemoglu (1999) shows that in a search model, SBTC can induce firms to create jobs that are more sensitive to worker skills, and thus, exacerbate wage inequality. Violante (2002) shows that in a frictional labor market, if human capital is vintage-specific, ex-ante identical workers can have different wages over time, and the differences in wages are larger when the speed of the technological change is faster. Shi (2002) develops a directed search model with heterogeneous firms and workers and shows that SBTC can generate increasing residual inequality.

While the above papers focus mostly on the rising inequality, Kambourov and Manovskii (2009) argue that inequality and mobility are closely linked. They develop and calibrate an equilibrium search model with occupation-specific human capital and heterogeneous experience levels within occupations. They show that as productivity shocks to occupations become more variable, both residual wage inequality and occupational mobility increase. Compared to Kambourov and Manovskii (2009), the source of the changes in this paper does not come from the demand side but rather from the supply side. In this model, all firms are identical, but the changes in technology affect the productivity distribution of the workers.

The rest of the paper proceeds as follows. I set up the model in Section 2. Section 3 solves the mixed strategy equilibrium and shows that the allocation is unique. Section 4 derives the equilibrium turnover probabilities and wage distributions and uses them to explore the model's implications on job mobility and wage distribution when technology changes. Section 5 concludes.

2 Model Setup

I set up the model formally in this section. Subsection 2.1 describes the model basics, including the types of players and their respective objective functions. Subsection 2.2 specifies the timing and information structure of the model and introduces notations for the strategies of the players. The solution concept of the model is provided in Subsection 2.3.

2.1 Worker and Firms

There is a single worker who lives for two periods. The worker has ability a, unknown at the beginning of period 1, which is drawn uniformly from $[0, 1]$.[5] The worker is risk neutral, has no disutility of effort, and does not discount the future. His utility is given by

$$u = w_1 + w_2, \tag{1}$$

[5] The uniform assumption is without loss of generality because we can interpret the worker's ability as his relative rank in the distribution.

the sum of his wage incomes in the two periods. Note that in this setup, the same analysis can be carried out if there is a continuum of workers as long as the firm has constant return to scale in production.[6]

There are N ex ante identical firms, $2 < N < \infty$.[7] The payoff of each firm is the sum of its payoffs in the two periods. A firm's period 1 payoff is 0 if it does not hire the worker. If it hires the worker, its period 1 payoff is

$$\pi_1 = y(a,t) - w_1, \tag{2}$$

where $y(a,t)$, the period 1 output of the worker, depends on the worker's ability a and an index t that reflects the state of technology common to all firms. I assume that

$$y(a,t) \geq 0, \ \frac{\partial y(a,t)}{\partial a} > 0, \frac{\partial y(a,t)}{\partial t} > 0 \quad \text{for all } a \text{ and } t, \tag{3}$$

so $y(a,t)$ is (uniformly) *strictly* higher if the worker has greater ability or the technology index is larger. The technology index t plays no role in the basic model analyzed in Section 3, but is central to the comparative static results presented in Section 4.

A firm's period 2 payoff is 0 if it does not hire the worker. If it hires the worker, its period 2 payoff is

$$\pi_2 = y(a,t) + 1_{\{incumbent\}} s(a,t) - w_2, \tag{4}$$

where $1_{\{incumbent\}}$ is an indicator function that takes the value of 1 if the firm is an *incumbent* (the worker's period 1 employer) and 0 otherwise. In other words, a firm's output equals $y(a,t) + s(a,t)$ if it is an *incumbent*, and $y(a,t)$ if it is an *outside firm* (i.e., a firm that does not hire the worker in period 1). I assume that

$$s(a,t) \geq 0, \ \frac{\partial s(a,t)}{\partial a} \geq 0, \ \frac{\partial s(a,t)}{\partial t} \geq 0 \text{ for all } a \text{ and } t, \tag{5}$$

so $s(a,t)$ is *weakly* higher if the worker has greater ability or the technology index is larger. In this paper, I interpret $y(a,t)$ as the general output (from the worker's general-purpose human capital) and $s(a,t)$ as the firm-specific output (from the worker's firm-specific human capital).

I also assume that

$$y(0,t) + s(0,t) < \int_0^1 y(a,t)da \equiv E[y(a,t)], \tag{6}$$

so the lowest inside output is smaller than the average outside output. This is a standard assumption in the literature to rule out trivial cases; see, for example, Gibbons and Katz (1991).

[6] When the production function displays decreasing return to scale and the workers are risk averse, Laing (1994) shows that involuntary layoffs can arise and may exceed efficient levels.

[7] When $N = 2$, the existence of the equilibrium remains, but the uniqueness is lost.

2.2 Timing and Information Structure

At the beginning of period 1, all firms simultaneously offer contracts to the worker. The contracts are restricted to be nonnegative, non-contingent, single-period wage offers. I allow for the firms to play mixed strategies, so each firm can draw its wage offer according to its own choice of offer distribution.[8] After all wage offers are made, the worker picks one firm from the N wage offers it receives. Contrary to the existing literature, this setup does not restrict the worker's behavior when a tie (multiple highest offers) occurs. Once the worker picks a firm, period 1 production takes place and the wage is paid. Through production, the incumbent observes the exact ability level of the worker. On the other hand, outside firms receive no information about the worker's ability. These information assumptions are extreme and are made for simplicity. The analysis can be adapted to a more general setting in which the information set of the incumbent is finer than that of outside firms.

At the beginning of period 2, all firms again simultaneously offer contracts to the worker. Since the incumbent observes the worker's ability, its wage offer may depend on the worker's ability. On the other hand, such dependence is not possible for outside firms. I again allow both the incumbent and the outside firms to play mixed strategies. After all offers are made in period 2, the worker picks one offer from the N period 2 wage offers and works for the firm whose offer is chosen. The worker can randomize his choice. Finally, after the worker chooses a period 2 employer, period 2 production takes place, the wage is paid, and the game ends.

More formally, the sequence of events is as follows:

1. At the beginning of period 1, firms simultaneously offer contracts to the worker. Each firm j ($j \in \{1, ..., N\}$) draws its offer from $[0, \infty]$ according to its choice of offer distribution $G_j \in \Delta R^+$, where ΔR^+ is the set of probability distributions on nonnegative real numbers.

2. The worker makes a decision $D_1 : (R^+)^N \to \Delta\{1, ..., N\}$ (the set of probability distributions on $\{1, ..., N\}$) to choose a firm from the N wage offers. Production takes place and the wage is paid. The incumbent observes the ability of the worker while outside firms do not.

3. At the beginning of period 2, all firms simultaneously offer contracts to the worker. For each firm $j \in \{1, ..., N\}$, if firm j hires the worker in period 1, it chooses a wage offer $w_j : [0, 1] \to \Delta R^+$ (the set of probability distributions on nonnegative real numbers) based on the worker's ability. If firm j does not hire the worker in period 1, it draws its offer randomly from $[0, \infty]$ according to its choice of offer distribution F_j.

4. The worker makes a decision $D_2 : (R^+)^N \to \Delta\{1, ..., N\}$ to choose a firm from the wage offers. Production takes place and the wage is paid.

[8] The restrictions on the available contracts are standard in the literature; see, for example, Greenwald (1986) and Gibbons and Katz (1991). In particular, the non-negativity assumption is made to rule out equilibria in which the wages accepted by the worker are not bounded below. The non-contingent assumption is made to fit with the assumption that outside firms cannot observe the output of the worker (so contracts based on outputs cannot be verified by courts). The single-period assumption is made to reflect the lack of commitment power of the firms (and the worker) and the associated lack of enforceability of long-term contracts.

A key assumption of the paper is that the offers are made simultaneously so the incumbent does not make a counteroffer. Many papers in this literature assume that the incumbent can make a counteroffer (Greenwald (1986), Lazear (1986), Gibbons and Katz (1991), and Golan (2005)), although the simultaneity assumption is also used; see, for example, Waldman (1984). To the extent that the incumbent would like to make a counteroffer if it could, the simultaneity assumption could be thought of as a limitation of the model. There are two justifications for this assumption. First, the assumption is made to capture that employees typically cannot credibly communicate the total value of outside offers to the incumbent: even if the exact monetary values of the outside offers are known to the incumbent, it is unlikely that the worker's non-monetary preferences can be known exactly.[9] In other words, the simultaneity assumption, while formally about the model's timing, should actually be interpreted in terms of differences in information structure. For example, I obtain identical results even if the incumbent makes its offer after the outside firms, as long as the incumbent does not know the outside offers. The second justification is that in a more dynamic setting, the arrival of outside offers may be endogenous. Committing not to match the outside offer may help the firm by reducing the worker's incentive to shop for outside offers (Postel-Vinay and Robin 2004). In a paper that provides direct evidence on the prevalence of a counteroffer, Barron, Berger, and Black (2006) report that 344 of the employers they survey would consider counteroffers, 437 would not, 46 are unsure, and 5 refuse to answer the question regarding the counteroffer.

2.3 Perfect Bayesian Equilibrium

According to the timing and information structure, the strategy of the worker is a 2-tuple (D_1, D_2), and the strategy of firm $j \in \{1, ..N\}$ is a 3-tuple (G_j, F_j, w_j). Given the strategies, I solve the Perfect Bayesian Equilibrium (PBE) of the model. The PBE requires that the strategies of the worker and the firms be sequentially optimal given their beliefs and that their beliefs be determined from Bayes Rule wherever possible. In particular, the PBE must satisfy the following criteria:

1. The worker's equilibrium period 2 contract choice D_2^* is optimal given any period 1 strategy D_1 of the worker and any strategies of the firms $\prod_{j=1}^{N}(G_j, F_j, w_j)$.

2. For each firm j, its period 2 strategy (F_j^*, w_j^*) is optimal given any period 1 strategy D_1 of the worker, any period 1 strategy of the firms $\prod_{j=1}^{N}(G_j)$, the period 2 equilibrium strategy of the worker D_2^*, and the period 2 strategies of all other firms (F_{-j}^*, w_{-j}^*).

3. The worker's choices (D_1^*, D_2^*) are optimal given the strategies of the firms: $\prod_{j=1}^{N}(G_j, F_j^*, w_j^*)$.

4. For each firm j, its strategy (G_j^*, F_j^*, w_j^*) is optimal given the worker's equilibrium strategy (D_1^*, D_2^*) and the equilibrium strategies of all other firms: $(G_{-j}^*, F_{-j.}^*, w_{-j}^*)$.

[9] Note that I do not model the wage-bargaining process between the firm and the worker. Golan (2009) shows that wage dispersion can occur for workers with the same observables when there is asymmetric information. Her insight is that firms are willing to offer higher wages to workers of higher productivities since failing to reach an agreement with such workers is more costly.

5. For each firm j, at the beginning of period 1, its belief about the worker's ability is the prior distribution of the worker's ability. In period 2, if j is the incumbent, it knows the exact ability of the worker. Otherwise, j's belief is equal to the prior distribution of the worker's ability. The worker does not know his ability in period 1. The worker knows his ability in period 2.

3 Equilibrium of the Model

I solve the equilibrium of the model in this section. First, I show that the model does not have a pure-strategy PBE. Next, I show in Theorem 1 that there exists a mixed-strategy PBE. Finally, I show in Theorem 2 that the equilibrium is essentially unique: every equilibrium of the model leads to the same allocation in terms of job mobility and wage distribution. All of the proofs are in the appendix.

For ease of exposition, I drop the technology index t in this section to write $y(a)$ and $s(a)$ instead of $y(a,t)$ and $s(a,t)$ because t plays no role in establishing the equilibrium. In addition, since all firms are ex ante identical, I let firm 1 be the incumbent in period 2 and write its equilibrium wage offer as $w_{In}(a)$.

Before describing the mixed-strategy PBE, I show in Lemma 1 that the model does not have a pure-strategy PBE. The absence of a pure-strategy equilibrium here stands in contrast to earlier results; see, for example, Greenwald (1986) and Gibbons and Katz (1991). The difference arises because the incumbent in this model cannot make counteroffers. When counteroffers are allowed, the incumbent has the opportunity to match an outside offer when the worker is worth more than the outside offer. Such matching exacerbates the adverse selection problem and helps sustain a low-wage equilibrium by discouraging the outside firms from making offers. When the incumbent cannot use counteroffers to respond to outside offers, such a low-wage equilibrium is no longer sustainable because outside firms can deviate by raising the wage.

Lemma 1 *There is no pure-strategy PBE.*

Although there is no pure-strategy PBE, Theorem 1 shows that a mixed-strategy PBE exists. Theorem 1 describes a) the wage offer made by the incumbent and b) the distribution of wage offers made by outside firms, which are the two building blocks for our analysis of the turnovers and the wage distribution in Section 4. I will start by describing these two building blocks and providing intuitions for them. Since the equilibrium involves a mixed strategy, it is implied that 1): the offer by the incumbent in period 2 makes the outside firms willing to randomize their offers, and 2): the randomization of the outside firms in period 2 makes the incumbent's offer optimal. To simplify the description of the incumbent's offer, I introduce the following definition.

Definition 1: The average outside output of a worker of ability a is defined as

$$B(a) = \frac{\int_0^a y(x)dx}{a} = E[y(x)|x \leq a]. \tag{7}$$

The first building block from Theorem 1 states that in period 2 the incumbent offers a wage that equals the average outside output of the worker. More formally,

$$w_{In}(a) = B(a), \quad \text{for all } a > 0. \tag{8}$$

It is easy to see that the incumbent's offer $B(a)$ is strictly increasing in the worker's ability. More importantly, by offering $w_{In}(a) = B(a)$, the incumbent makes the outside firms indifferent in their wage offers because the expected payoff of an outside firm when offering any $w \in [0, E[y(a)])$ is always 0. To see this, suppose $y(a) = a$, so $B(a) = \frac{1}{2}a$ and $E[y(a)] = \frac{1}{2}$. Now suppose an outside firm offers a wage of $w \in [0, \frac{1}{2}]$. If the outside firm does not hire the worker, its profit is zero. If it manages to hire the worker, the worker's ability must satisfy $a \leq 2w$ (because $w_{In}(a) = B(a) = \frac{1}{2}a$). Recall that the ability is uniformly distributed, so the expected output of the worker is w. Therefore, the expected profit of the firm is also zero when it hires the worker. Theorem 1 generalizes the logic to the case in which the worker's output is not equal to his ability.

The second building block of Theorem 1 describes the pattern of randomization of outside firms. To simplify the description, I introduce the following definition.

Definition 2: The maximum outside offer distribution F is defined as

$$F(w) = \prod_{j=2}^{N} F_j^*(w), \tag{9}$$

where $F_j^*(w)$ is the probability of firm j choosing an offer less than or equal to w in equilibrium. For the maximum outside offer to be less than w, the offer from each outside firm must be less than w, so the maximum outside offer distribution is the multiplication of the equilibrium offer distribution of all outside firms. Also define the "boundary" of support of F as

$$\begin{aligned} \underline{w} &= \inf\{w : F(w) > 0, w \geq w_{In}(a) \text{ for some } a\}; \\ \overline{w} &= \sup\{w : F(w) < 1\}. \end{aligned} \tag{10}$$

The concept of the maximum outside offer plays an important role in the analysis below because its distribution summarizes all the relevant information on the outside offers from the incumbent's point of view: the incumbent keeps a worker if and only if the maximum outside offer is less than its offer.

The second building block of the equilibrium states that in period 2 outside firms randomize their offers in $[0, E[y(a)])$ so that the maximum outside offer distribution F satisfies

$$\frac{1}{y(a) + s(a) - B(a)} = \frac{f(B(a))}{F(B(a))}, \quad \text{for all } a > 0. \tag{11}$$

This pattern of randomization implies that when the distribution of the maximum outside offer satisfies (11), the incumbent finds it optimal to offer a wage that equals the average outside output.

To illustrate this, the incumbent's expected payoff in period 2 by offering w to a worker of ability a is given by

$$F(w)(y(a) + s(a) - w), \tag{12}$$

where $y(a)+s(a)-w$ is the incumbent's profit for keeping the worker and $F(w)$ is the probability that the incumbent will keep him. Profit maximization implies that the incumbent's optimal wage choice w^* must satisfy

$$\frac{1}{y(a) + s(a) - w^*} = \frac{f(w^*)}{F(w^*)}. \tag{13}$$

Now note that equation (11) is identical to the equation above, except that w^* is replaced with $B(a)$. This means that when (11) holds, it is optimal for the incumbent to offer $w^* = B(a)$, the wage policy described in the first building block.

In summary, when the incumbent offers $w_{In}(a) = B(a)$, any wage offer in $(0, E[y(a)])$ is an optimal response for each outside firm. When the maximum offer distribution satisfies the differential equation (11), the incumbent finds it optimal to offer $w_{In}(a) = B(a)$. Therefore, this is an equilibrium strategy profile in period 2. To fully describe the PBE, I specify in Theorem 1 below the period 1 strategies and beliefs.

Theorem 1: *The following strategies and beliefs form a PBE:*

(i) In period 2, the worker chooses the highest wage offer. If there are multiple highest offers, the worker a) stays with the incumbent if its offer is one of the highest offers; b) randomizes otherwise.

(ii) At the beginning of period 2, the incumbent firm offers

$$w_{In}(a) = B(a) \quad \text{for all } a, \tag{14}$$

and each outside firm j ($j \in \{2, ...N\}$) offers a wage drawn independently from the distribution

$$F_j^*(w) = F(w)^{\frac{1}{N-1}}, \tag{15}$$

with

$$F(w) = C \exp\left(\int_{\underline{w}}^{w} \frac{1}{y(B^{-1}(x))+s(B^{-1}(x))-x} dx\right) \quad \text{for } w \in [\underline{w}, \overline{w}], \tag{16}$$

where $[\underline{w}, \overline{w}] = [y(0), E[y(a)]]$ *and* $C = \exp\left(-\int_{\underline{w}}^{\overline{w}} \frac{1}{y(B^{-1}(x))+s(B^{-1}(x))-x} dx\right)$.

(iii) In period 1, the worker chooses the highest wage offer. If there are multiple highest offers, the worker randomizes among them.

(iv) At the beginning of period 1, all firms offer

$$E[y(a)] + \int_0^1 F(B(a))(y(a) + s(a) - B(a))da. \tag{17}$$

(v) Each firm's belief about the worker's ability is equal to the prior if it has not hired the worker. The incumbent knows the worker's ability at the end of period 1. The worker does not know his ability in period 1. The worker knows his ability in period 2.

One prominent feature of the equilibrium is the randomization of wage offers. The randomization of wage offers is fundamental to the results in this paper because job mobility and wage dispersion follow directly from it. However, it may appear that the randomization is unappealing because in real life firms do not randomize their wage offers. Nevertheless, it can be shown that the wage distribution from the randomization is equivalent to a modified game in which outside firms independently form estimates of the ability of the worker (say through job interviews) and make nonrandomized wage offers contingent on the estimates.

One feature of the equilibrium Theorem 1 is that as the number of firms N increases, the offer distribution of each outside firm decreases. This appears counterintuitive because when there are more outside firms, one might expect the labor market to become more competitive and each firm to offer a higher wage. One possible way to resolve this is to distinguish competition based on observable characteristics (perfect information) from competition based on unobservables (asymmetric information). When the competition is based on the observable characteristics, a larger number of firms typically leads to higher maximal wages and each firm may bid for the worker more aggressively as well. In contrast, when the competition is based on unobservable characteristics, each firm may bid less aggressive because the problem of the winner's curse becomes more severe, and the distribution of the maximal wages may not change. In general, when firms compete on both observed and unobserved characteristics, the overall effect may still lead the firms to bid more aggressively as the number of firms increases.

Another possible resolution is not to view the model as one incumbent firm competing against a large number of outside firms simultaneously, but rather as one incumbent firm competing against a sequence of outside firms that arrive randomly. In this case, a bigger N can be thought of as a higher arrival rate of outside offers. As the arrival rate increases, one might expect that the worker's wage would also increase at a faster rate, confirming one's intuition that competition leads to higher wages. A fully dynamic model that captures the random arrival of outside firms, however, is less tractable. As will be seen in the next section, the current model generates an explicit formula of the turnover probabilities and the wage distributions, allowing one to study the effect of technological change on job mobility and wage inequality.

Finally, notice that the PBE in Theorem 1 is just one of the infinitely many PBEs of the game. In general, any set of outside offer distributions that satisfy $\prod_{j=2}^{N} F_j^*(w) = F$ can be sustained as an equilibrium and there are infinitely many of them. However, all PBEs of the model, as shown by Theorem 2 below, share the same two properties. First, the incumbent's wage offer must equal the average outside output. Second, the maximum outside offer distribution is unique and must satisfy the differential equation in (11). Note that the worker's wage and his mobility decision is completely determined by the incumbent's wage offer and the highest outside wage offer. Therefore, the identical incumbent offer and maximum outside wage offer distributions across all equilibria imply that the worker's mobility and wage distribution are also identical across all equilibria. In other words, this model has a unique equilibrium allocation in terms of job mobility and wage distribution.

Theorem 2: *In every PBE, all outside firms have zero expected profits in period 2. The incumbent's offer must satisfy* $w_{In}(a) = B(a)$ *for all* $a \in (0, 1]$, *and the distribution of the maximum outside offer must satisfy*

$$F(w) = C \exp\left(\int_{\underline{w}}^{w} \frac{1}{y(B^{-1}(x))+s(B^{-1}(x))-x} dx\right) \quad \text{for } w \in [\underline{w}, \overline{w}] \ ,$$

where $[\underline{w}, \overline{w}] = [y(0), E[y(a)]]$ *and* $C = \exp(-\int_{\underline{w}}^{\overline{w}} \frac{1}{y(B^{-1}(x))+s(B^{-1}(x))-x} dx)$.

By providing an explicit formula for the incumbent's offer and the maximum outside offer distribution, Theorem 2 allows one to explore patterns of job mobility and wage distribution. Notice that when $s(a) > 0$, turnover leads to loss in productivity and is inefficient. The next section studies how changes in technology affect turnover and, thus, the efficiency of the equilibrium.

4 Job Mobility, Wage Dispersion, and Effects of Technological Change

In this section, I study job mobility and wage dispersion using the above results. Subsection 4.1 applies the unique equilibrium allocation to derive formulas for turnover probability and wage distribution. I show that both turnover probability and wage distribution depend on a profit ratio that compares the incumbent's production advantage due to the worker's firm-specific human capital with its informational advantage with respect to the worker's general human capital. Subsection 4.2 shows that if technological change is skill-biased and favors general skills, the wage distribution will become more spread out (corresponding to greater inequality) and job mobility will rise. Subsection 4.3 relates the testable implications of the model with the empirical findings and argues that the model sheds light on the joint evolution of wage inequality and job mobility in the U.S. in the past thirty-five years.

Before proceeding, I make two remarks here. First, I use the word "workers" in the discussion, implying the model has multiple workers. I interpret the turnover probability of the worker as the proportion of workers who move to outside firms and $F(w)$ as the proportion of workers whose maximum outside wage offer is less than or equal to w. Second, I reinsert in this section the technology index t into the expressions and write $y(a,t)$, $s(a,t)$, and $B(a,t)$ to reflect the role of technology in affecting turnover probability and wage distribution.

4.1 Turnover Probability and Wage Distribution

I begin by deriving a formula of turnover probability, which will be used to describe the wage distributions and the comparative statics results in the next subsection. To simplify the expressions, I state the results in terms of staying probability.

Definition 3: Let $P(a,t)$ be the equilibrium probability that a worker of ability a stays with the incumbent in period 2 when the technology level is t.

Proposition 1: *For all a and t,*

$$P(a,t) = \exp(-\int_{a}^{1} \frac{1}{x(1+r(x,t))} dx),$$

where

$$r(a,t) \equiv \frac{s(a,t)}{y(a,t) - B(a,t)}. \tag{18}$$

The turnover probability formula here differs qualitatively with typical results in the asymmetric information literature; see, for example, Greenwald (1986) and Gibbons and Katz (1991). In these models, the turnover probability of a worker typically takes a zero-one form: a worker leaves the firm with probability 1 if his ability is below a certain threshold and otherwise leaves the firm with probability 0. Here, the turnover probability decreases continuously with the worker's ability and only the worker with the highest ability has a zero turnover probability.[10]

Notice also that there are qualitative differences in turnover between this model and the models based on symmetric information such as those in Jovanovic (1979). Specifically, the absolute level of general ability does not affect job mobility in symmetric-information models: a worker's turnover decision is determined by the differences in his expected output (which are equal to the wage offers) at different firms. In contrast, the absolute level of general ability matters for job mobility in this model.

In particular, the formula implies that general ability affects the turnover probability through $r(a,t)$, which can be interpreted as a profit ratio. The numerator of $r(a,t)$ is equal to the incumbent's "profit" from keeping the worker had the ability of the worker been known. In other words, the numerator reflects the incumbent's "profit" from its production advantage because of the firm-specific human capital. The denominator of $r(a,t)$ is equal to the incumbent's "profit" from keeping the worker had there been only general human capital. In other words, the denominator reflects the incumbent's profit source from its informational advantage on the worker's general output. This profit ratio $r(a,t)$ of production over the information advantage is key to the comparative statics results on technological change.

Next, I derive the wage-distribution formulas. Notice that the worker in period 2 either leaves the incumbent and becomes a mover or continues to work for the firm and becomes a stayer. The next lemma gives the formulas of the wage distributions of the movers, the stayers, and the two types combined. To simplify the exposition, I write the formulas in terms of the average outside output ($B(a,t)$) and the staying probability ($P(a,t)$).

Definition 4: Let $G(w)$ be the wage distributions of the stayers and movers combined, $G_S(w)$ be the wage distribution of the stayers, and $G_M(w)$ be the distribution of the movers.

[10] A related model of adverse selection in which turnover varies continuously with the worker's ability is Novos (1995). Turnover probability is continuous in ability in that model because workers also differ in their tastes for the job, whereas in this model the workers differ only in their ability.

Proposition 2

$$
\begin{aligned}
G(B(a,t)) &= aP(a,t); & (19)\\
G_S(B(a,t)) &= \frac{\int_0^a P(x,t)dx}{\int_0^1 P(x,t)dx}; & (20)\\
G_M(B(a,t)) &= \frac{\int_0^a (P(a,t)-P(x,t))dx}{\int_0^1 (1-P(x,t))dx}. & (21)
\end{aligned}
$$

The expressions of wage distributions enable us to compare the wage distribution of the movers with that of the stayers. This comparison reflects two conflicting forces. On the one hand, stayers on average have higher abilities because the incumbent's offer is increasing in ability. On the other hand, movers on average are luckier in receiving outside offers because for two workers of the same ability (who thus receive the same offer from the incumbent), the mover must have received a better outside offer. Therefore, the comparison of the mover-stayer wage distribution sheds light on the source of wage growth from learning (by the incumbent) and luck (from receiving good outside offers).

The key to this comparison is again the profit ratio $r(a,t)$ in (18). Corollary 1 below shows that when this ratio is increasing in ability, the wage distribution of the stayers FOSD that of the movers, so the stayers have a higher average wage. If this ratio is decreasing, the wage distribution of the movers FOSD that of the stayers, so the movers have a higher average wage.

Corollary 1: *The wage distribution of the stayers FOSD that of the movers if* $r(a,t) = \frac{s(a,t)}{y(a,t)-B(a,t)}$ *is increasing in* a*; the wage distribution of the movers FOSD that of the stayers if* $r(a,t) = \frac{s(a,t)}{y(a,t)-B(a,t)}$ *is decreasing in* a.

The mover-stayer wage gap, defined as the average wage of the stayers minus that of the movers, can be thought of as a tenure effect: both the movers and stayers have the same level of experience and the stayers have longer tenure than the movers. Corollary 1 suggests that the tenure effect is positive if the ratio of firm-specific output to general output is increasing in ability. If a more-able worker can better leverage his ability in larger firms possibly because a larger firm size allows for better matching (of ability to specific positions), then Corollary 1 suggests that positive tenure effects are more likely to be observed in larger firms, a well-documented empirical regularity.

It would also be interesting to know how the dispersion of the wage distribution of the movers compares with that of the stayers. While I do not have general results for such comparisons, Corollary 2 below shows that when there is no firm-specific output ($s(a,t) \equiv 0$), the wage distributions of the stayers and the movers are identical and the worker moves to an outside firm with ex ante probability $\frac{1}{2}$.

Corollary 2: *If* $s(a,t) \equiv 0$, *then* $P(a,t) = a$ *for all* a, *and* $G_s(w) = G_m(w)$ *for all* w. *Therefore,*

$$\int_0^1 P(a,t)da = \frac{1}{2}. \tag{22}$$

4.2 Effects of Technological Change

This subsection studies the effect of technological change on turnover and wage dispersion. Theorem 3 below shows that if a technological change is *log-skill-biased* and *general-skill-biased*, then job mobility increases and wage distribution becomes more *spread out* in the sense of Bickel and Lehmann (1979). Theorem 4 shows that if general output is sufficiently important in production, and if a technological change is *skill-biased*, then the increase in job mobility is larger for workers with higher levels of firm-specific human capital. Before stating these theorems, I first define the different types of technological changes.

Definition 5: A technological change is *skill-biased* if the increase in technology raises the output of the higher-ability workers more than that of the lower-ability workers:

$$\frac{\partial^2 y(a,t)}{\partial a \partial t} > 0 \text{ for all } a \text{ and } t. \tag{23}$$

Definition 6: A technological change is *log-skill-biased* if the increase in technology raises the output of the higher-ability workers proportionately more than that of the lower-ability workers:

$$\frac{\partial^2 \log y(a,t)}{\partial a \partial t} > 0 \text{ for all } a \text{ and } t. \tag{24}$$

One can show that a log-skill-biased technological change is also skill-biased, so log-skill-biased is a stronger notion.

Definition 7: A technological change is *general-skill-biased* if an increase in technology raises the general component of output proportionately more than the firm-specific part:

$$\frac{\partial \log y(a,t)}{\partial t} > \frac{\partial \log s(a,t)}{\partial t} \text{ for all } a \text{ and } t. \tag{25}$$

The definitions above introduce different types of technological changes. To state the theorems of how technological changes affect the wage distributions, I next introduce the definition by Bickel and Lehmann (1979) of "spread out", which compares the dispersion of distributions.

Definition 8: A distribution H_1 is more *spread out* than distribution H_2 if

$$H_1^{-1}(q) - H_1^{-1}(q') \geq H_2^{-1}(q) - H_2^{-1}(q') \quad for\ all \quad 0 \leq q' \leq q \leq 1. \tag{26}$$

In other words, the distance between the values corresponding to the two quantiles is larger for any two quantiles in the more "spread out" distribution. Alternatively, suppose there is a class of distributions $H(x,t)$; let $x(q,t)$ be the value of the qth quantile under index t. The distributions are more spread out as t increases if

$$\frac{\partial^2 x(q,t)}{\partial q \partial t} > 0 \text{ for all } q \text{ and } t. \tag{27}$$

A unique feature of "spread out" is that its order is preserved under translation. In other words, if $H_1(x)$ is more spread out than $H_2(x)$, then $H_1(x+t)$ is also more spread out than $H_2(x)$ for all t. This stands in contrast with Second Order Stochastic Dominance (SOSD), a popular measure of dispersion, which only compares distributions with the same mean. When two symmetric distributions have the same mean, then "spread out" implies SOSD. Since technological change typically affects both the mean and the dispersion of the wage distribution, I use "spread out" as the measure of wage inequality.

The main result here is that when a technological change is both log-skill-biased and general-skill-biased, turnover probability increases for workers of all ability levels and the wage distribution becomes more spread out.

Theorem 3: *Let the period 2 outside output be $y(a,t) > 0$ and the inside output be $y(a,t) + s(a,t)$, where $s(a,t) > 0$. If a technological change is log-skill-biased ($\frac{\partial^2 \log y(a,t)}{\partial a \partial t} > 0$) and general-skill-biased ($\frac{\partial \log y(a,t)}{\partial t} > \frac{\partial \log s(a,t)}{\partial t}$), the turnover probability increases and the wage distribution (of the movers and stayers combined) becomes more spread out as t increases.*

To see the intuition for the increased turnover, recall from Subsection 4.1 that both the levels of general and firm-specific human capital matter for turnover. Specifically, Proposition 1 shows that the turnover probability decreases with the profit ratio $r(a,t) = s(a,t)/\left(y(a,t) - B(a,t)\right)$. When a technological change is both log-skill-biased and general-skill-biased, essentially the denominator increases at a faster rate than the numerator, and this leads to a decrease in the profit ratio and thus an increase in turnover.

The intuition for the increased wage dispersion follows because the technological change makes the output distribution more spread out. Notice, however, that the wage is not equal to the worker's marginal product when there is asymmetric information. Nevertheless, the formula in Proposition 2 provides a link between the output distribution and the wage distribution, and I use them to confirm the intuition by calculating the wage distributions following a technological change.

A special case of Theorem 3 is when the technological change has no effect on firm-specific human capital. In this case, a skill-biased technological change is automatically general-skill-biased and leads to an increase in turnover. Corollary 3 below states this result formally. Notice that turnover is inefficient in this model because it leads to loss in firm-specific human capital. Corollary 3 therefore indicates a negative welfare effect of skill-biased technological change.

Corollary 3: *If $\frac{\partial^2 y(a,t)}{\partial a \partial t} > 0$ and $s(a,t_1) = s(a,t_2) > 0$ for all a and t, then for $t_1 < t_2$, $P(a,t_1) > P(a,t_2)$ for all $a > 0$.*

To explore further the effect of technological change on turnover, the next theorem shows that if general ability is sufficiently important in production, when a skill-biased technological change occurs, the increase in turnover is larger for workers with higher levels of firm-specific human capital.

Theorem 4: *Let the output be $y(a,t) > 0$ for the outside firms and $y(a,t) + ks(a)$ for the incumbent, where $ks(a) > 0$. If the technological change is skill-biased ($\frac{\partial^2 y(a,t)}{\partial a \partial t} > 0$) and $y(a,t) - B(a,t) > ks(a)$ for all a, then the proportionate increase in turnover increases with k, i.e.,*

$$\frac{\partial^2 \log(1 - P(a,t,k))}{\partial t \partial k} > 0 \text{ for all } a.$$

The intuition of Theorem 4 may be best gained through looking at the benchmark case in Corollary 2 where there is no firm-specific human capital. In this case, the aggregate turnover probability is always $\frac{1}{2}$. Consequently, for workers with little firm-specific human capital, the average turnover probability is always close to $\frac{1}{2}$, and thus any technological change cannot have a large effect on the aggregate turnover probability of such workers. When workers have more firm-specific human capital, their aggregate turnover probability is lower, and technological change can have a larger impact on it.

4.3 Empirical Implications

In this subsection, I relate the predictions of the model with the empirical findings on wage distribution and turnover. There has been extensive empirical literature on both of these topics. I first discuss the plausibility of the assumptions on technological change and then review the empirical findings on the recent rise in inequality and mobility.

Theorem 3 describes technological changes that are log-skill-biased and general-skill-biased. The idea that recent technological change has favored high-ability workers has received wide support from economists. The computer and internet revolution, together with associated technological and organizational changes, lend direct support to this idea. Many economists view skill-biased technological change as the leading explanation for the recent rise in inequality (Berman, Bound, and Griliches (1994), Autor, Katz, and Krueger (1998)). The idea that recent technological change has favored general ability also has some support. Sennett (2003) argues that aptitude, the general capacity for learning, has become increasingly valuable. In addition, Gould (2002) finds that the correlation of abilities across different sectors has recently increased. To the extent that general ability refers to the skill that can be carried over across different sectors, this points to an increase in the importance of general skills.

Theorem 3 shows that when the technological change is log-skill-biased and general-skill-biased, the wage distribution becomes more spread out. Since the model applies to workers with the same observables, the widening of the wage distribution in Theorem 3 refers to the increase in residual inequality, i.e., the dispersion of wages in observationally equivalent groups. Within the U.S., wage inequality has increased substantially in the past thirty-five years (Bound and Johnson (1992), Katz and Murphy (1992), Murphy and Welch (1993)). At least half of the increase in wage inequality results from the rise in residual inequality (Juhn, Murphy, and Pierce 1993).

Moreover, much of the change in residual inequality appears to concentrate on the top end of the distribution. For example, Lemieux (2006) shows that residual inequality has increased for the college-educated but has changed little for other groups since the

1990s. Autor, Katz, and Kearney (2005) report that after the 1990s, residual inequality has continued to rise for the 90/50 wage ratio but has declined for the 50/10 ratio. To the extent that asymmetric information is more relevant for better-educated workers, possibly because they are more likely to engage in jobs in which skills are harder to assess, Theorem 3 gives an explanation for why residual inequality increases more for college-educated workers.

Theorem 3 also shows that technological change leads to higher job mobility. There is some empirical support for the rising job mobility, although the evidence here is weaker than for increased wage inequality. Job mobility can be measured in a number of ways and investigated through different data sources. One measure of job mobility is to examine the fraction of workers with more than 10 or 20 years of tenure. Using CPS data, Farber (1999) reports that the incidence of long-term employment has declined from 1979-1996. Using Displaced Workers Surveys (DWSs), Gardner (1995) founds an increase in job loss among white-collars and workers in non-manufacturing industries from 1982-1991. Another measure of job mobility is job retention rates, which are often calculated from cross-sectional job tenure, using a method developed by Hall (1982). Using CPS data, Swinnerton and Wial (1995) show that there is an increase in job mobility in the 1980s, although Diebold et al. (1996) find little change in job mobility. Yet another measure of mobility is the fraction of workers who report no changes in a given time period, typically 10 years. Using this measure, Rose (1995) argue that mobility has increased more for male workers in the 1980s than in the 1970s. Farber (1999) offers an extensive review of the empirical findings on job mobility.

To the extent that the workers work for new firms when they change jobs in this model, a closer measure of turnover in this model is job-to-job mobility. A paper that sheds light on this measure is Stewart (2002), who studies employment to employment (EE) job changes. Stewart (2002) finds that the EE transition rate in the U.S. has increased 45% for men and 58% for women between 1975-2000. While a weakness of Stewart's (2002) finding is that his data cannot distinguish genuine EE transitions from those with short (less than two weeks) intervening unemployment spells, the large magnitude of the increase seems to support the increase in EE transition rates. It appears that much of the increase in the EE transitions rates arises from the periods before 1994. Fallick and Fleischman (2004) find that EE transition rates remain somewhat stable between 1994 and 2000 and drop sharply between 2000 and 2003.

Finally, Theorem 4 shows that the effect of technological change on turnover is larger for workers with more firm-specific human capital. To the extent that the level of firm-specific human capital is positively correlated with the worker's age, technological change should have a larger effect on older workers. Using the Displaced Workers Surveys (DWSs), Farber (1993) finds that the rates of job loss have increased slightly for older workers from 1982 to 1991. Moreover, Stewart (2002) finds that the proportionate increase in the employment to employment (EE) transition rate is increasing with the worker's age (even if the EE transition rate decreases with age). From 1975 to 2000, the proportionate increase in the EE rate for men was 23% for the age group 25-35, 79% for the age group 35-45, and 144% for the age group 45-55.

5 Conclusion

This paper develops a framework for studying job mobility and wage dispersion under asymmetric information. Contrary to existing work, the lemons problem from the superior information of the current employer does not lead to market collapse. Instead, there exists a unique equilibrium allocation in which the current employer offers a wage equal to the average output of all types below the ability of the worker and outside firms compete for the worker using mixed strategies. These mixed strategies lead to a non-degenerate wage distribution for all types of workers. This unique equilibrium allocation determines both the allocation of workers with heterogeneous abilities to different firms and also how wages change when workers change jobs (due to both selection and productivity effects).

I use the framework to study how technological change has affected the joint evolution of wage inequality and job mobility in the United States over the past thirty-five years. The model implies that, in the presence of technological change that is both skill-biased and general-skill-biased, wage distribution will become more spread out and job mobility will increase. The model also suggests that mobility should increase more for older workers. These patterns are broadly consistent with recent empirical evidence on changes in job mobility and wage inequality in the United States.

While the paper focuses on job mobility and wage inequality, the model can be used to shed light on other empirical patterns. For example, the model implies that, conditional on ability, there is wage dispersion for movers but not for stayers. This suggests that movers may face larger uncertainty in their careers than stayers. A piece of supportive empirical evidence is found in Baker, Gibbs, and Holmstrom (1994), who show that career outcomes are more variable for new hires than for comparable incumbents using personnel data. To the extent that technological change amplifies the productivity difference across workers, this model suggests that career outcomes may become even more variable for new hires. Another related testable implication is that wage dispersion is decreasing in the worker's ability in this model so wage inequality would be smaller at a higher ability level. One way to test this is to relate wage residuals to measures of ability, such as the residual of the Armed Forces Qualification Test score.

Finally, the framework developed in this paper can be used to study many other questions and issues related to the labor market when information asymmetry matters. These topics include general training (Acemoglu and Pischke, 1998, 1999ab) and layoff decisions (Gibbons and Katz, 1991). A unique feature of the model is that wage dispersion depends on productivity dispersion. This suggests that productivity dispersion can matter for training and layoff decisions, and to the extent that technological change affects productivity dispersion, these decisions on training and layoffs may be affected as well.

References

[1] Abowd, John, Francis Kramarz, and David Margolis. "High Wage Workers and High Wage Firms." *Econometrica,* Vol. 67, No. 2 (Mar, 1999), pp. 251-333

[2] Acemoglu, Daron. "Technical Change, Inequality, and the Labor Market." *Journal of Economic Literature*, Vol. 40, No. 1 (Mar, 2002), pp. 7-72

[3] Acemoglu, Daron and Jorn-Steffen Pischke. "The Structure of Wages and Investment in General Training." *Journal of Political Economy,* Vol. 107, No.3 (Jun, 1999), pp. 539-372.

[4] Acemoglu, Daron and Jorn-Steffen Pischke. "Why Do Firms Train? Theory and Evidence." *Quarterly Journal of Economics,* Vol. 113, No.1 (Feb. 1998), pp. 79-119.

[5] Akerlof, George. "The Market for Lemons: Qualitative Uncertainty and the Market Mechanism." *Quarterly Journal of Economics* Vol. 84, No.3 (Aug, 1970) pp. 488-500.

[6] Autor, David, Lawrence Katz, and Alan Krueger. "Computing inequality: have computers changed the labor market?" *Quarterly Journal of Economics,* Vol. 113, No 4. (Nov, 1998) pp.1169-1213.

[7] Autor, David, Richard Murnane and Frank Levy. "The Skill Content of Recent Technological Change: An Empirical Exploration." *Quarterly Journal of Economics* Vol. 118, No.4 (Nov, 2003) pp. 1279-1334.

[8] Autor, David, Lawrence Katz and Melissa Kearney. "Trends in U.S. Wage Inequality: Re-Assessing the Revisionists." (Sep, 2005), Mimeo.

[9] Autor, David, Lawrence Katz and Melissa Kearney. "The Polarization of the U.S. Labor Market." (Jan., 2006), Mimeo

[10] Baron, James and David Kreps, Strategic Human Resources: Frameworks for General Managers, (1999), Wiley.

[11] Barron, John, Mark Berger, and Dan Black. "Selective Counteroffers," *Journal of Labor Economics*, Vol. 24, No. 3, (July, 2006) pp. 385-410.

[12] Berman, Eli, John Bound, and Zvi Griliches. "Change in the Demand for Skilled Labor within U.S. Manufacturing: Evidence from the Annual Survey of Manufacturers." *Quarterly Journal of Economics*, Vol. 109, No. 2 (May, 1994), pp. 367-397.

[13] Bickel, Peter, and Erich Lehmann, "Descriptive statistics for nonparametric models IV. Spread" In: Jureckova, J. Eds., *Contributions to Statistics, Hajek Memorial,* Vol. 33-40. Reidel, London.

[14] Bound, John and George Johnson. "Change in the Structure of Wages in the 1980's: An Explanation of Alternative Explanations." *American Economic Review*, Vol. 82, No.3 (Jun, 1992), pp. 371-392.

[15] Burdett, Kenneth, and Kenneth Judd, "Equilibrium Price Dispersion." *Econometrica,* Vol 51, No 4. (Jul, 1983), pp. 955-69.

[16] Butters, Gerard. "Equilibrium Distributions of Sales and Advertising Prices." *Review of Economic Studies*, Vol. 44, No.3 (Oct, 1977), pp. 465-491.

[17] Engelbrecht-Wiggans, Richard, Paul Milgrom, and Robert Iber. "Competitive Bidding with Proprietary Information." *Journal of Mathematical Economics*, Vol. 11, (1983), pp. 161-69.

[18] Fallick, Bruce, and Charles Fleischman. "Employer-to-Employer Flows in the U.S. Labor Market: The Complete Picture of Gross Worker Flows" (2004), Finance and Economics Discussion Series 2004-34, Board of Governors of the Federal Reserve System.

[19] Farber, Henry. "Mobility and Stability: the Dynamics of Job Change in Labor Markets." (1999), in Orley Ashenfelter and David Card eds., *Handbook of Labor Economics, Vol 3B*, Amsterdam: Elsevier-North Holland, pp. 2439-2483.

[20] Farber, Henry and Robert Gibbons. "Learning and Wage Dynamics." *Quarterly Journal of Economics*, Vol. 111, No. 4. (Nov, 1996), pp. 1007-1047.

[21] Gardner, Howard. *Frames of Mind: The Theory of Multiple Intelligences.* (1983), New York: Basic Books.

[22] Gibbons, Robert, and Lawrence Katz. "Layoffs and Lemons." *Journal of Labor Economics,* Vol. 9, No. 4. (Oct., 1991), pp. 351-380.

[23] Gibbons, Robert, Lawrence Katz, Thomas Lemieux, and Daniel Parent. " Comparative Advantage, Learning, and Sectoral Wage Determination." *Journal of Labor Economics,* Vol. 23, No. 4 (Oct, 2005), pp. 681-723.

[24] Golan, Limor. "Counteroffers and Efficiency in Labor Markets with Asymmetric Information." *Journal of Labor Economics,* Vol. 23, No. 2 (April 2005), pp. 373-393.

[25] Golan, Limor. "Wage Signaling: A Dynmic Modle of Intrafirm Bargaining and Asymmetric Learning." *International Economic Review*, Vol. 50 (Aug 2009).

[26] Gottschalk,Peter and Robert Moffitt. "Changes in Job Instability and Insecurity Using Monthly Survey Data." *Journal of Labor Economics,* Vol. 17, No. 4, Part 2: Changes in Job Stability and Job Security. (Oct, 1999), pp. S91-S126.

[27] Gould, Eric. "Rising Wage Inequality, Comparative Advantage, and the Growing Importance of General Skills in the United States." *Journal of Labor Economics*, Vol. 20, No.1 (Jan, 2002), pp. 104-147.

[28] Greenwald, Bruce C. "Adverse Selection in the Labour Market." *The Review of Economic Studies*, Vol. 53, No. 3. (Jul, 1986), pp. 325-347.

[29] Hashimoto, Masanori. "Firm-specific Human Capital as a Shared Investment." *American Economic Review*, Vol 71, No.3 (Jun, 1981), pp. 475-482.

[30] Jaeger, David and Ann Stevens. "Is Job Stability in the United States Falling? Reconciling Trends in the Current Population Survey and Panel Study of Income Dynamics." *Journal of Labor Economics,* Vol. 17, No. 4, Part 2: Changes in Job Stability and Job Security. (Oct, 1999), pp. S1-S28.

[31] Johnson, William. "A Theory of Job Shopping." *Quarterly Journal of Economics*, Vol. 92, No. 2 (May, 1978), pp. 261-278.

[32] Jovanovic, Boyan. "Job Matching and the Theory of Turnover." *Journal of Political Economy*, Vol 87, No.5 (Oct, 1979), pp. 972-990.

[33] Jovanovic, Boyan. "Firm-specific capital and Turnover." *Journal of Political Economy,* Vol 87, No.6 (Dec, 1979), pp. 1246-1260.

[34] Jovanovic, Boyan and Robert Moffitt. "An Estimate of a Sectoral Model of Labor Mobility ." *Journal of Political Economy*, Vol 98, No.4 (Aug, 1990), pp. 827-852.

[35] Juhn Chinhui, Kevin M. Murphy and Brooks Pierce. "Wage Inequality and the Rise in Returns to Skill ." *Journal of Political Economy,* Vol. 101, No. 3 (Jun, 1993), pp. 410-442.

[36] Katz, Lawrence and Kevin Murphy. "Changes in Relative Wages, 1963-1987: Supply and Demand Factors." *Quarterly Journal of Economics,* Vol. 107, No. 1 (Feb, 1992), pp. 35-78.

[37] Kahn, Lisa. "Asymmetric Information betIen Employers." (2009), Mimeo.

[38] Kambourov, Gueorgui and Iourii Manovskii. "Occupational Mobility and Wage Inequality." *Review of Economic Studies* Vol 76, No.2 (April, 2009), pp. 721-759.

[39] Kambourov, Gueorgui and Iourii Manovskii. " Rising Occupational and Industry Mobility in the United States: 1968-1997." (2008), *International Economic Review* Vol 49, No. 1 (Feb, 2008), pp. 41-79.

[40] Lazear, Edward. "Raids and Offer-Matching," Research in Labor Economics,ed. Ronald Ehrenberg, Vol. 8, (1986) JAI Press, Greenwich, CT. pp. 141-165.

[41] Laing, Derek. "Involuntary Layoffs in a Model with Asymmetric Information Concerning Worker Ability." *Review of Economic Studies* Vol 61, No.2 (April, 1994), pp. 375-92.

[42] Lemieux, Thomas. "Increasing Residual Wage Inequality: Composition Effects, Noisy Data, or Rising Demand for Skill?" *American Economic Review*, Vol 96, No.3 (Jun, 2006), pp. 461- 499.

[43] Lucas, Robert and Edward Prescott. "Equilibrium Search and Unemployment." *Journal of Economic Theory*, Vol. 7, No. 2 (Feb, 1974), pp. 188-209.

[44] Milgrom, Paul and Robert Weber. "The Value of Information in a Sealed Bid Auction." *Journal of Mathematical Economics,* Vol 10, (1982), pp. 105-14.

[45] Mortensen, Dale. *Wage Dispersion: Why are Similar Workers Paid Differently?* (2001) M.I.T. Press.

[46] Murphy, Kevin and Finis Ilch. "The Structure of Wages." *Quarterly Journal of Economics.* Vol. 107, No. 1 (Feb., 1992), pp. 285-326.

[47] Neal, Derek. "Industry-Specific Human Capital: Evidence from Displaced Workers." *Journal of Labor Economics,* Vol. 13, No. 4 (Oct 1995), pp. 653-667.

[48] Neumark, David. "Changes in Job Stability and Job Security: A Collective Effort to Untangle, Reconcile, and Interpret the Evidence." NBER Working Paper 7472. (Jan., 2000).

[49] Neumark, David, Daniel Polsky, and Daniel Hansen "Has Job Stability Declined Yet? New Evidence for the 1990s." *Journal of Labor Economics,* Vol. 17, No. 4, Part 2: Changes in Job Stability and Job Security. (Oct., 1999), pp. S29-S64.

[50] Novos, Ian. "Imperfections in Labor Markets and the Scope of the Firm." International Journal of Industrial Organization, Vol 13, (1995) pp. 387-410.

[51] Parent, Daniel. "Industry-Specific Capital and the Wage Profile: Evidence from the National Longitudinal Survey of Youth and the Panel Study of Income Dynamics." *Journal of Labor Economics*, Vol. 18, No. 2 (April, 1995), pp. 306-323.

[52] Pinkston, Joshua " A Model of Asymmetric Employer Learning with Testable Implication." *Review of Economic Studies*, Vol. 76, No. 1 (Jan, 2009), pp. 367-394.

[53] Schoenberg, Uta. "Testing for Asymmetric Employer Learning." *Journal of Labor Economics*, Vol 25 (2007) pp. 651-692.

[54] Sennett, Richard. *Respect in a World of Inequality.* (2003), New York: W.W. Norton.

[55] Stewart, Jay. "Recent Trends in Job Stability and Job Security." BLS Working Paper No. 356, (2002).

[56] Topel, Robert and Michael Ward. "Job Mobility and the Careers of Young Men." *Quarterly Journal of Economics*, Vol. 107, No. 2 (May, 1992), pp. 439-479.

[57] Virag, Gabor. "Repeated Common Value Auctions with Asymmetric Bidders." *Games and Economic Behavior*, Vol. 61, No.1 (Oct, 2007), pp. 156-177.

[58] Waldman, Michael. "Job Assignments, Signalling, and Efficiency " *RAND Journal of Economics,* Vol. 15, No. 2 (Summer, 1984), pp. 255-267.

6 Appendix

Lemma 1: There is no pure strategy PBE.

Proof. I prove by contradiction. Suppose instead there is a pure strategy PBE.

Let w be the highest outside offer in period 2 in this equilibrium, so the incumbent can keep the worker if it offers any wage above w. Because the incumbent earns zero (in period 2) if it does not keep the worker, the incumbent will keep the worker in equilibrium if his inside output $(y(a) + s(a))$ is greater than w. This implies that outside firms never get the worker when $y(a) + s(a) > w$, so the expected profit of any outside firm, conditional on hiring the worker, is at most

$$E[y(a)|y(a) + s(a) \leq w] - w < 0.$$

Therefore, if an outside firm hires the worker with positive probability, it must have a negative payoff. Since outside firms can always guarantee themselves nonnegative payoffs (by offering zero wages), this implies that the outside firms must hire the worker with zero probability, or equivalently, the incumbent must keep the worker with probability 1 in this pure strategy PBE.

This implies that the incumbent must offer a wage greater or equal to w with probability 1. But any wage offer strictly greater than w cannot be optimal for the incumbent (because it can be replaced by a smaller wage, say the average of w and itself, that also keeps the worker but is smaller in amount), the incumbent must offer w with probability 1 in this equilibrium. Now consider an outside firm that deviates by offering $w' = w + \varepsilon$ for some $\varepsilon > 0$. This deviation hires the worker with probability 1 and gives to the deviating firm an expected profit of

$$E[y(a)] - w - \varepsilon \geq E[y(a)] - y(0) - s(0) - \varepsilon.$$

The deviation is profitable for small enough ε because $E[y(a)] - y(0) - s(0) > 0$ by the production assumption (6). This leads to a contradiction. ■

Theorem 1: *The following strategies and beliefs form a PBE:*

(i) In period 2, the worker chooses the highest wage offer. If there are multiple highest offers, the worker a) stays with the incumbent if its offer is one of the highest offers; b) randomize otherwise.

(ii) At the beginning of period 2, the incumbent firm offers

$$w_{In}(a) = B(a) \quad \text{for all } a, \tag{28}$$

and each outside firm j ($j \in \{2, ...N\}$) offers a wage drawn independently from the distribution

$$F_j^*(w) = F(w)^{\frac{1}{N-1}}, \tag{29}$$

with

$$F(w) = C \exp\left(\int_{\underline{w}}^{w} \frac{1}{y(B^{-1}(x))+s(B^{-1}(x))-x} dx\right) \quad \text{for } w \in [\underline{w}, \overline{w}], \tag{30}$$

where $[\underline{w}, \overline{w}] = [y(0), E[y(a)]]$ and $C = \exp\left(-\int_{\underline{w}}^{\overline{w}} \frac{1}{y(B^{-1}(x))+s(B^{-1}(x))-x} dx\right)$.

(iii) In period 1, the worker chooses the highest wage offer. If there are multiple highest offers, the worker randomizes among them.

(iv) At the beginning of period 1, all firms offer

$$E[y(a)] + \int_0^1 F(B(a))(y(a) + s(a) - B(a))da. \tag{31}$$

(v) Each firm's belief about the worker's ability is equal to the prior if it has not hired the worker. The incumbent knows the worker's ability at the end of period 1. The worker does not know his ability in period 1. The worker knows his ability in period 2.

Proof. I start with the beliefs. Since the worker does not know his own ability in period 1, the beliefs described by (v) is the only beliefs consistent with the (degenerated) Bayes rule.

Next, I examine the strategies in period 2. In period 2, the worker's utility is maximized by choosing the maximum wage offer. Therefore, the strategy described in (i) is optimal for the worker.

Now given the equilibrium strategy of the worker and the maximum outside offer distribution, the incumbent's payoff by offering w to a worker of ability a is

$$(y(a) + s(a) - w)F(w).$$

Maximization of the incumbent's profit gives the following first order condition:

$$\frac{1}{y(a) + s(a) - w_{In}(a)} = \frac{f(w_{In}(a))}{F(w_{In}(a))}. \tag{32}$$

The first order condition is a necessary condition for optimality, and it is easy to check that if a solution satisfies it and is also increasing in a (which is the case for $B(a)$), then the second order condition is also satisfied and the solution is optimal. Let $w_{In}(a) = B(a)$, then by the definition of F in (ii) I can check that

$$\frac{1}{y(a) + s(a) - B(a)} = \frac{f(B(a))}{F(B(a))}.$$

Therefore, $w_{In}(a) = B(a)$ satisfies (32) and thus maximizes the incumbent's expected payoff.

Given the incumbent's wage offer and the worker's strategy, the expected profit of an outside firm by offering w is

$$P(\text{Not Hiring}) \cdot 0 + P(\text{Hiring})(E[y(a)|B(a) < w] - w). \tag{33}$$

This expression is zero for all $w \le E[y(a)]$ because $E[y(a)|B(a) < w] = B(B^{-1}(w)) = w$. When $w > E[y(a)]$, this expression is negative because $E[y(a)|B(a) < w] - w = E[y(a)] - w < 0$. Therefore, the optimal response of each outside firm is to randomize over $(0, E[y(a)])$. This completes proving the optimality of the strategies in period 2.

Moving back to period 1, it is clear that the worker maximizes his utility by choosing the maximum wage offer because all the firms are ex ante identical. Therefore, the worker's strategy described in (iii) is optimal.

For the firms, wage competition implies that the equilibrium wage offer will be bid up to the expected output of the worker in period 1 plus the expected profit the firm makes if it is the incumbent in period 2. Therefore, it is an equilibrium that each firm offers in period 1

$$E[y(a)] + \int_0^1 F(B(a))(y(a) + s(a) - B(a))da.$$

This finishes the proof. ■

To prove Theorem 2, I first show that the incumbent's wage offer is *strongly increasing* in the worker's ability. Strongly increasing is an order on sets.[11] I use this order here because I have not shown that the incumbent's wage offer is single-valued yet.

[11] Let X and Y be two sets. $X \ge Y$ if $x \ge y$ for all $x \in X$ and $y \in Y$.

Lemma 2 *If $w_{In}(a_2) > \underline{w}$, I have $w_{In}(a_1) \geq w_{In}(a_2)$ for all $a_1 > a_2$.*

Proof. I prove by contradiction. Take two arbitrary ability levels $a_1 > a_2$ such that $w_{In}(a_2) > \underline{w}$. Let w_1 and w_2 be two equilibrium wage offers of the incumbent when the worker is of ability a_1 and a_2 respectively; i.e. $w_1 \in w_{In}(a_1)$ and $w_2 \in w_{In}(a_2)$. Suppose instead I have $w_1 < w_2$.

Define $\widetilde{F}(w)$ as the probability that the incumbent keeps the worker if it offers w.Clearly I have $\widetilde{F}(w_1) \leq \widetilde{F}(w_2)$, where recall $\widetilde{F}(w)$ is the probability that the incumbent keeps the worker if it offers w. Now if $\widetilde{F}(w_1) = \widetilde{F}(w_2)$, the incumbent's payoff by offering w_1 is

$$\widetilde{F}(w_1)(y(a_2) + s(a_2) - w_1) > \widetilde{F}(w_2)(y(a_2) + s(a_2) - w_2) \tag{34}$$

since $\widetilde{F}(w_1) = \widetilde{F}(w_2) > 0$. In other words, the incumbent strictly prefers offering w_1 to w_2 at a_2 if $\widetilde{F}(w_1) = \widetilde{F}(w_2)$, violating the assumption that $w_2 \in w_{In}(a_2)$. Therefore, I must have $\widetilde{F}(w_1) < \widetilde{F}(w_2)$

Now let $y_{In}(a) = y(a) + s(a)$. The expected profit of the incumbent by offering w_2 when the worker has ability a_1 is

$$\begin{aligned} & (y_{In}(a_1) - w_2)\widetilde{F}(w_2) \\ = \;& (y_{In}(a_1) - y_{In}(a_2))\widetilde{F}(w_2) + (y_{In}(a_2) - w_2)\widetilde{F}(w_2) \\ \geq \;& (y_{In}(a_1) - y_{In}(a_2))\widetilde{F}(w_2) + (y_{In}(a_2) - w_1)\widetilde{F}(w_1) \\ > \;& (y_{In}(a_1) - y_{In}(a_2))\widetilde{F}(w_1) + (y_{In}(a_2) - w_1)\widetilde{F}(w_1) \\ = \;& (y_{In}(a_1) - w_1)\widetilde{F}(w_1). \end{aligned} \tag{35}$$

The first inequality follows from $w_2 \in w_{In}(a_2)$, and the strict inequality follows because $y_{In}(a_1) - y_{In}(a_2) > 0$ and $\widetilde{F}(w_2) > \widetilde{F}(w_1)$.

This chain of inequality implies that the incumbent would strictly prefer offering w_2 to w_1 when the worker has ability a_1, thus contradicting the assumption that $w_1 \in w_{In}(a_1)$. Therefore, I must have $w_1 \geq w_2$. ∎

Using the monotonicity property in Lemma 2, I show that each outside firm must have zero expected profits. This is the content of Lemma 3.

Lemma 3 *The expected profit of each outside firm is zero.*

Proof. It suffices to show that the expected profit of one outside firm is zero. Because if one outside firm earns zero expected profit and another one earns positive profit, the one with zero expected profit can always mimic the one with positive profit to earn positive profit as Ill. I consider two cases.

Case 1: $F(\underline{w}) = 0$, where recall that F is the CDF of the maximal outside offers.

Since $F(\underline{w}) = 0$, the definition of $\underline{w}$ together with right-continuity of a CDF implies that for each $n > 0$, there exists $w_n > \underline{w}$ such that a): w_n is offered by an outside firm and b): $F(w_n) < \frac{1}{n}$.

By offering w_n, the expected profit of the outside firm is less than

$$F(w_n)(E[y(a)|w_{In}(a) \leq w_n)] - w_n) \tag{36}$$

because the probability that the firm hires the worker is no greater than $F(w_n)$ and the expected profit from hiring the worker is no greater than $E[y(a)|w_{In}(a) \leq w_n] - w_n$ because the incumbent's offer is monotone in the worker's ability (Lemma 2).

Equation (36) goes to zero as n goes to infinity because $F(w_n) < \frac{1}{n}$ and $E[y(a)|w_{In}(a) \leq w_n] - w_n$ is bounded. Because I have only finite many outside firms, this implies that there must be one outside firm whose profit is less than $\frac{1}{n}$ for all n and its profit must thus be zero. This finishes the proof for the case with $F(\underline{w}) = 0$.

Case 2: $F(\underline{w}) > 0$.

First, if an outside firm offers a wage less than $\underline{w}$, this wage offer hires the worker with probability zero (by the definition of $\underline{w}$) and gives the firm a profit of zero. Since all offers in the support of its equilibrium offers must yield the same profit for the firm, this implies the outside firm must have an expected profit zero (and I are done). Therefore, I may assume all outside firms offer wages greater than or equal to $\underline{w}$.

When all outside firms offer wages greater than or equal to $\underline{w}$, our assumption $F(\underline{w}) > 0$ implies that each outside firm must offer $\underline{w}$ with positive probability. In other words, $F_j^*(\underline{w}) > 0$ for all $j \in \{2, ..., N\}$ (recall the firm 1 is the incumbent). When all outside firms offer $\underline{w}$ (and the incumbent offers no greater than $\underline{w}$), without loss of generality, I may assume that the worker chooses firm 2 with probability $q \leq \frac{1}{2}$.

Now consider the payoff of firm 2 when it offers $\underline{w}$. Its expected output is less than or equal to

$$q \prod_{j=3}^{N} F_j^*(\underline{w}) \Pr(w_{In}(a) \leq \underline{w})(E[y(a)|w_{In}(a) \leq \underline{w}] - \underline{w}). \tag{37}$$

This expression follows because firm 2 can hire the worker only if all other outside firms offer $\underline{w}$ (with probability $\prod_{j=3}^{N} F_j^*(\underline{w})$), the incumbent offers less than or equal to $\underline{w}$ (with probability $\Pr(w_{In}(a) \leq \underline{w})$), and the worker chooses firm 2 in face of this tie (with probability q). Hence $q \prod_{j=3}^{N} F_j^*(\underline{w}) \Pr(w_{In}(a) \leq \underline{w})$ is the probability that firm 2 hires the worker with wage $\underline{w}$. And the firm 2's expected profit conditional on hiring the worker is at most $E[y(a)|w_{In}(a) \leq \underline{w}] - \underline{w}$.

Suppose firm 2 offers $\underline{w} + \varepsilon$ instead. Firm 2's expected profit with this offer is at least

$$\prod_{j=3}^{N} F_j^*(\underline{w}) \Pr(w_{In}(a) \leq \underline{w})(E[y(a)|w_{In}(a) \leq \underline{w}] - \underline{w} - \varepsilon).$$

If $E[y(a)|w_{In}(a) \leq \underline{w}] - \underline{w} > 0$, this is a profitable deviation for ε small enough because there is a discrete increase in the probability of hiring the worker. Therefore, I must have $E[y(a)|w_{In}(a) \leq \underline{w}] - \underline{w} = 0$, so firm 2 must have zero expected profit and I are done. ■

The zero expected profits result in Lemma 3 plays two important roles in our proof of Theorem 2. First, it is the key property used in Lemma 7 below that determines the equilibrium wage offer of the incumbent. Second, it helps prove Lemma 5, which states that the maximum outside offer distribution does not have an atom except possibly at the bottom.

Before proving Lemma 5, I first prove a technical lemma that states that the incumbent's offer does not have an atom at any $w > \underline{w}$. This is the content of Lemma 4.

Lemma 4 *If $w > \underline{w}$, then* $\Pr(w_{In}(a) = w) = 0$.

Proof. I prove by contradiction. If instead there exists a wage $w > \underline{w}$ such that $\Pr(w_{In}(a) = w) > 0$, then Lemma 2 implies that there exists $a_1 < a_2$, such that $w_{In}(a_1) = w_{In}(a_2) = w$.

Now take any $\varepsilon > 0$ and compare the expected profit of an outside firm betIen offering $w + \varepsilon$ and $w - \varepsilon$. Since by offering $w + \varepsilon$ ($w - \varepsilon$) the incumbent keeps the worker with a probability of at least(most) $F(w)$, the extra expected profit in offering $w + \varepsilon$ is at least

$$\begin{aligned} & F(w)(B(a_2) - w - \varepsilon) - F(w)(B(a_1) - w + \varepsilon) \\ = \; & F(w)(B(a_2) - B(a_1) - 2\varepsilon). \end{aligned} \tag{38}$$

Since $F(w) > 0$ and $B(a_2) - B(a_1) > 0$, for small enough ε the term above is positive. In other words, for small enough ε, offering $w - \varepsilon$ is strictly dominated by offering $w + \varepsilon$. Therefore, no outside firm would offer a wage in $(w - \varepsilon, w)$ for small enough ε.

Since no outside firms offer wage in $(w - \varepsilon, w)$, the incumbent that offers w has a profitable deviation by offering $w - \frac{\varepsilon}{2}$ for small enough ε, as long as the maximum outside offer distribution does not have a mass in w (so deviating to $w - \frac{\varepsilon}{2}$ does not affect the incumbent's probability of retaining the worker).

When the maximum outside offer distribution has a mass at w, let q be the probability that the worker leaves the incumbent if both the incumbent offer and the maximum outside offer is w. If $q = 1$, the incumbent never keeps the worker when the worker randomizes over w. In this case, the incumbent strictly prefers offering $w - \frac{\varepsilon}{2}$ to w, because it is cheaper and keeps the worker with the same probability.

When $q < 1$, the outside firm that offers w (there is such a firm because the maximum outside offer distribution has a mass at w) can profitably deviate by offering $w + \delta$, which increase the probability of hiring the worker and discretely increase the expected ability of the workers hired.

These cases combined show that there is always a profitable deviation when $\Pr(a|w_{In}(a) = w) > 0$ for $w > \underline{w}$. Therefore, I must have $\Pr(w_{In}(a) = w) = 0$ if $w > \underline{w}$. ∎

Lemma 5 *$F(w) = F(w_-)$ for all $w > \underline{w}$, where $F(w_-)$ is the left limit of F at w.*

Proof. I prove by contradiction. Suppose instead F has an atom at $w > \underline{w}$. To get a contradiction, it suffices to show that the incumbent will not offer a wage in $(w - \varepsilon, w)$. Because in this case, an outside firm that offers $w - \frac{\varepsilon}{2}$ has an expected profit (conditional on hiring the worker) of at least

$$\begin{aligned} & E[B(a)|w_{In}(a) \le w - \varepsilon] - w + \frac{\varepsilon}{2} \\ = \; & E[B(a)|w_{In}(a) \le w] - w + \frac{\varepsilon}{2} \\ = \; & \frac{\varepsilon}{2}, \end{aligned} \tag{39}$$

where the first equality follows from that the incumbent will not offer a wage in $(w - \varepsilon, w)$ and that incumbent's offer does not have an atom (Lemma 3), and the second equality follows from that the expected payoff of all outside firms must be zero (so the conditional profit of an outside firm offering w must be zero). This leads to a contradiction because it implies that an outside firm can have positive expected profit by offering $w-\frac{\varepsilon}{2}$, violating Lemma 3.

Now I show that the incumbent will not offer a wage in $(w-\varepsilon,w)$ for small enough ε and thus complete the proof. Since the incumbent's expected profit is increasing in the worker's ability, there exists an M such that if $w_{In}(a)\in(w-\frac{1}{M},w)$, I must have $y_{In}(a)>w+\frac{1}{M}$. Now take $\varepsilon=\frac{1}{2M}(F(w)-F(w_-))$. Note that $\varepsilon<\frac{1}{M}$, so if the incumbent offers a worker of ability a a wage in $(w-\varepsilon,w)$, I must have $y_{In}(a)>w+\frac{1}{M}$.

Suppose the incumbent offers a worker of ability a a wage in $(w-\varepsilon,w)$, its expected profit is at most $F(w_-)(y_{In}(a)-w+\varepsilon)$. If the incumbent deviates and offers $w+\varepsilon$ instead, its expected profit is at least $F(w)(y_{In}(a)-w-\varepsilon)$. Therefore, the difference in the expected profit after the deviation is at least

$$\begin{aligned} & F(w)(y_{In}(a)-w-\varepsilon)-F(w_-)(y_{In}(a)-w+\varepsilon) \\ = \ & (F(w)-F(w_-))(y_{In}(a)-w)-(F(w)+F(w_-))\varepsilon \\ > \ & (F(w)-F(w_-))\frac{1}{M}-(F(w)+F(w_-))\varepsilon \\ > \ & 0, \end{aligned} \tag{40}$$

where the first inequality follows because $y_{In}(a)>w+\frac{1}{M}$ and the second inequality follows because $\varepsilon=\frac{1}{2M}(F(w)-F(w_-))<\frac{F(w)-F(w_-)}{M(F(w)+F(w_-))}$. This shows that the incumbent would not offer a wage in $(w-\varepsilon,w)$ and finishes the proof. ■

Lemma 6 *For $w_1<w_2\in[\underline{w},\overline{w}]$, I have $F(w_1)<F(w_2)$.*

Proof. I prove by contradiction. Suppose instead I have $F(w_1)=F(w_2)$ for some $w_1<w_2\in[\underline{w},\overline{w}]$. Without loss of generality, I may assume that w_2 is the largest wage such that no outside firms makes offer betIen (w_1,w_2) (with positive probability), i.e. $w_2=\sup\{w: F(w)=F(w_1)\}$. This implies that, for any $\varepsilon>0$, I can find a wage $w\in[w_2,w_2+\varepsilon)$ that is offered in equilibrium by an outside firm.

Now take an outside firm that offers wage $w\in[w_2,w_2+\varepsilon)$, its expected profit is at most

$$F(w_2+\varepsilon)\Pr(w_{In}(a)\le w_2+\varepsilon)(E[B(a)|w_{In}(a)\le w_2+\varepsilon]-w_2), \tag{41}$$

where $F(w_2+\varepsilon)\Pr(w_{In}(a)\le w_2+\varepsilon)$ is an upper bound of the probability of hiring the worker and $E[B(a)|w_{In}(a)\le w_2+\varepsilon]$ is an upper bound of the expected output of the worker hired.

Suppose the firm deviates and offers $\frac{w_1+w_2}{2}$ instead. Its expected profit is at least

$$F(w_1)\Pr(w_{In}(a)\le w_1)(E[B(a)|w_{In}(a)\le w_1]-\frac{w_1+w_2}{2}), \tag{42}$$

where $F(w_1)\Pr(w_{In}(a)\le w_1)$ is a loIr bound of hiring the worker and $E[B(a)|w_{In}(a)\le w_1]$ is a loIr bound of the expected output of the worker hired.

Since $F(w_1)=F(w_2)$, it is clear that the incumbent would not offer a wage in (w_1,w_2) in equilibrium. Furthermore, Lemma 4 implies that the incumbent offers w_2 with probability zero. Therefore, I have

$$\Pr\left(w_{In}(a)\le w_1\right) = \Pr(w_{In}(a)\le w_2); \tag{43}$$

$$E\left(B(a)|w_{In}(a)\le w_1\right) = E(B(a)|w_{In}(a)\le w_2). \tag{44}$$

Substituting these expressions in (41) and (42), I see that the expected profit from offering $\frac{w_1+w_2}{2}$ is

$$\begin{aligned} & F(w_2)\Pr(w_{In}(a) \leq w_2)(E[B(a)|w_{In}(a) \leq w_2] - \frac{w_1+w_2}{2}] \\ > \ & F(w_2+\varepsilon)\Pr(w_{In}(a) \leq w_2+\varepsilon)(E[B(a)|w_{In}(a) \leq w_2+\varepsilon] - w_2] \end{aligned}$$

for small enough ε. This implies that the outside firm has a profitable deviation, and I have a contradiction. ■

Lemma 6, together with the zero expected profit lemma (Lemma 3), implies that the incumbent's wage offer must equal the average output for workers for all ability levels a such that $B(a) \in (\underline{w}, \overline{w}]$. This is stated formally in Lemma 7.

Lemma 7 *If $B(a) \in (\underline{w}, \overline{w}]$, then $w_{In}(a) = B(a)$.*

Proof. Suppose an outside firm offers a wage $w \in (\underline{w}, \overline{w}]$. It hires the worker with positive probability. This implies that the firm's conditional expected profit of hiring the worker is zero (Lemma 3). Since the incumbent's wage offer is strictly increasing by Lemma 2 and Lemma 4, the conditional expected profit of the firm is

$$\begin{aligned} & E[y(a)|w_{In}(a) \leq w] - w && (45) \\ = \ & E[a|a \leq w_{In}^{-1}(w)] - w \\ = \ & B(w_{In}^{-1}(w)) - w \\ = \ & 0, && (46) \end{aligned}$$

where the last equality follows from Lemma 3.

By Lemma 6, the support of maximum outside offer is dense in $(\underline{w}, \overline{w}]$. Therefore, $B(w_{In}^{-1}(w)) = w$ for all $w \in (\underline{w}, \overline{w}]$. Since B is strictly increasing, this implies that $w_{In}(a) = B(a)$ if $B(a) \in (\underline{w}, \overline{w}]$. ■

Lemma 8 *The equilibrium maximum outside offer distribution satisfies*

$$F(w) = C\exp\left(\int_{\underline{w}}^{w} \frac{1}{y(B^{-1}(x)) + s(B^{-1}(x)) - x}dx\right) \quad for\ w \in [\underline{w}, \overline{w}], \tag{47}$$

where $C = \exp\left(-\int_{\underline{w}}^{\overline{w}} \frac{1}{y(B^{-1}(x))+s(B^{-1}(x))-x}dx\right)$.

Proof. Since F does not have an atom above $\underline{w}$ (Lemma 5), the incumbent's expected profit by offering w to a worker of ability a is

$$(y(a) + s(a) - w)F(w). \tag{48}$$

Since F is strictly increasing, it is differentiable almost everywhere so the incumbent's profit maximization condition leads to the following first order condition:

$$\frac{1}{y(a) + s(a) - w_{In}(a)} = \frac{f(w_{In}(a))}{F(w_{In}(a))}. \tag{49}$$

Since I have $w_{In}(a) = B(a)$ for all $B(a) \in (\underline{w}, \overline{w}]$, the expression above can be written as

$$\frac{1}{y(B^{-1}(w)) + s(B^{-1}(w)) - w} = \frac{f(w)}{F(w)}.$$

Integrating this equation, I obtain (47) by using that $F(\overline{w}) = 1$ and F is right-continuous at $\underline{w}$. ∎

Lemma 8 pins down the distribution of the maximum outside offer in its support. To completely determine the maximum outside distribution, I need to specify the end points of F. This is done in Lemma 9 and 10.

Lemma 9 $\overline{w} = E[y(a)]$.

Proof. Outside firms that offer $w > E[y(a)]$ clearly have negative profits, so I must have $\overline{w} \leq E[y(a)]$.

Suppose instead $\overline{w} < E[y(a)]$. Let $a^* = B^{-1}(\overline{w})$. Since the incumbent's offer is strongly increasing in worker's ability (Lemma 2), all workers with ability greater than a^* must be offered at least $\overline{w}$. Moreover, identical reasoning as in Lemma 1 shows that the incumbent will choose $w_{In}(a) = \overline{w}$ for all $a > a^*$.

Now if an outside firm offers $w = \overline{w} + \varepsilon$, it hires the worker with probability 1 and its expected profit is

$$E[y(a)] - \overline{w} - \varepsilon > 0. \tag{50}$$

for small enough ε.This is a contradiction to the zero expected profit condition (Lemma 3) of outside firms. ∎

Lemma 10 $\underline{w} = y(0)$.

Proof. I prove by contradiction. If $\underline{w} < y(0)$, then an outside firm can deviate by offering $\underline{w} + \varepsilon < y(0)$ for some small ε. This outside firm hires the worker with positive probability (by the definition of $\underline{w}$) and earns positive conditional profit because the wage is loIr than the loIst possible output. This violates the zero expected profit of outside firms (Lemma 3).

If $\underline{w} > y(0)$, let $a_1 = B^{-1}(\underline{w}) > 0$, so $w_{In}(a_1) = \underline{w}$. Also define a_2 as the unique ability level such that: $y(a_2) + s(a_2) = \underline{w}$. Because $B(a) < y(a)$ for all $a > 0$, it follows that $a_1 > a_2$.

Now I must have $w_{In}(a) \geq \underline{w}$ for $a > a_2$ because otherwise the incumbent keeps such worker with probability zero. On the other hand, I also have $w_{In}(a_1) = \underline{w}$. Therefore, by the monotonicity lemma (Lemma 2), I must have $w_{In}(a) = \underline{w}$ for all $a \in (a_2, a_1]$.

Furthermore, I must have $F(\underline{w}) > 0$ because otherwise when it offer $\underline{w}$ its profit is zero.This implies that when an outside firm offers $\underline{w}$ (which happens with positive probability because $F(\underline{w}) > 0$), it cannot hire any worker with ability greater than a_2. Therefore, its conditional expected profit is

$$\begin{aligned} & E[y(a)|a \leq a_2] - \underline{w} \\ < \; & E[y(a)|a \leq a_1] - \underline{w} \\ = \; & B(a_1) - \underline{w} \\ = \; & 0. \end{aligned} \tag{51}$$

Furthermore, the outside firm hires the worker with positive probability because $\underline{w} > y(0)$. This leads to a contradiction because all outside firms must have expected profit of zero by Lemma 3. ■

Lemma 8-10 completely determine the distribution of the maximum outside offer by specifying the differential equation that governs it (Lemma 8) and its two end points (Lemma 9 and 10). Therefore, the distribution of the maximum outside offer is unique. This finishes the proof of Theorem 2.

Proposition 1: *For all a and t,*

$$P(a,t) = \exp\left(-\int_a^1 \frac{1}{x(1+r(x,t))}dx\right),$$

where

$$r(a,t) \equiv \frac{s(a,t)}{y(a,t) - B(a,t)}.$$

Proof. For a worker with ability $a > 0$, his offer from the incumbent is $B(a,t)$ (Lemma 7). This worker stays with the incumbent if and only if the maximum outside offer he receives is less than $B(a,t)$, which occurs with probability $F(B(a,t))$. In other words, I have

$$P(a,t) = F(B(a,t)). \tag{52}$$

Let $p(a,t) = \frac{\partial P(a,t)}{\partial a}$. Then (42) implies that $p(a,t) = f(B(a,t))\frac{\partial B(a,t)}{\partial a}$.

By Lemma 8,

$$\begin{aligned} \frac{p(a,t)}{P(a,t)} &= \frac{f(B(a,t))}{F(B(a,t))}\frac{\partial B(a,t)}{\partial a} \\ &= \frac{\frac{\partial B(a,t)}{\partial a}}{y(a,t)+s(a,t)-B(a,t)} \\ &= \frac{\frac{\partial B(a,t)}{\partial a}}{s(a,t)+a\frac{\partial B(a,t)}{\partial a}}, \end{aligned} \tag{53}$$

where the last equality uses $\frac{\partial B(a,t)}{\partial a} = \frac{y(a,t)-B(a,t)}{a}$.

Integrating equation (53), I obtain that

$$\ln P(1,t) - \ln P(a,t) = \int_a^1 \frac{\frac{\partial B(x,t)}{\partial x}}{s(x,t)+x\frac{\partial B(x,t)}{\partial x}}dx \quad \text{for all } a > 0. \tag{54}$$

Since $P(1,t) = F(B(1,t)) = 1$, the equation above gives

$$P(a,t) = \exp\left(-\int_a^1 \frac{\frac{\partial B(x,t)}{\partial x}}{s(x,t)+x\frac{\partial B(x,t)}{\partial x}}dx\right) \text{ for all } a > 0. \tag{55}$$

Finally, for a worker with ability $a = 0$, the comment following Lemma 10 implies that $P(0,t) = P(0_+,t) = F(B(0_+,t))$, so the formula remains correct (but the integral maybe improper and should be interpreted as a limit). ■

Proposition 2:

$$G(B(a,t)) = aP(a,t); \tag{56}$$

$$G_S(B(a,t)) = \frac{\int_0^a P(x,t)dx}{\int_0^1 P(x,t)dx}; \tag{57}$$

$$G_M(B(a,t)) = \frac{\int_0^a (P(a,t) - P(x,t))dx}{\int_0^1 (1 - P(x,t))dx}. \tag{58}$$

Proof. I first calculate $G(B(a,t))$. If a worker's wage is less than $B(y(a,t))$, it must be the case that both the incumbent offer and the maximum outside offer he has received are less than $B(a,t)$. The probability that the incumbent offer is less than $B(a,t)$ is a. The probability that the maximum outside offer is less than $B(a,t)$ is $F(B(a,t)) = P(a,t)$. Since the outside offers are independent of the incumbent's offer, this gives that $G(B(a,t)) = aP(a,t)$.

Next, I calculate the wage distribution of the stayers. Since a worker of ability a (who is offered $B(a,t)$ by the incumbent) stays with the incumbent with probability $P(a,t)$, the total number of stayers who receive less than $B(a,t)$ is $\int_0^a P(x,t)dx$. The wage distribution of the stayers in (20) follows immediately.

Finally, G is a Iighted average of G_S and G_M, so that I have

$$G_S(B(a,t)) \int_0^1 P(x,t)dx + G_M(B(a,t)) \int_0^1 (1 - P(x,t))dx = G(B(a,t)).$$

This gives the wage distribution of the movers. ■

Corollary 1: *The wage distribution of the stayers FOSD that of the movers if* $r(a,t) = \frac{s(a,t)}{y(a,t)-B(a,t)}$ *is increasing in* a*; the wage distribution of the movers FOSD that of the stayers if* $r(a,t) = \frac{s(a,t)}{y(a,t)-B(a,t)}$ *is decreasing in* a*.*

Proof. The likelihood ratio of movers over stayers is given by

$$\frac{\partial G_M(B(a,t))/\partial a}{\partial G_S(B(a,t))/\partial a} \tag{59}$$

$$= \left(\frac{\int_0^1 P(x,t)dx}{\int_0^1 (1 - P(x,t))dx} \right) \left(\frac{ap(a,t)}{P(a,t)} \right) \tag{60}$$

$$= \left(\frac{\int_0^1 P(x,t)dx}{\int_0^1 (1 - P(x,t))dx} \right) \frac{1}{\frac{s(a,t)}{y(a,t)-B(a,t)} + 1}, \tag{61}$$

where the first inequality uses the expressions obtained in Proposition 2. Notice that $\int_0^1 P(x,t)dx / \int_0^1 (1 - P(x,t))dx$ is a constant. It follows that if $\frac{s(a,t)}{y(a,t)-B(a,t)}$ is increasing, the likelihood ratio is decreasing in a. As a result, the decreasing likelihood ratio implies that the wage distribution of the stayers FOSD that of the movers. Similarly, a decreasing $\frac{s(a,t)}{y(a,t)-B(a,t)}$ implies that wage distribution of the movers FOSC that of the stayers. ■

Corollary 2: *If* $s(a,t) \equiv 0$*, then* $P(a,t) = a$ *for all* a*, and* $G_s(w) = G_m(w)$ *for all* w*. Therefore,*

$$\int_0^1 P(a,t)da = \frac{1}{2}. \tag{62}$$

Proof. By Proposition 1, $P(a,t) = \exp(-\int_a^1 \frac{\frac{\partial B(x,t)}{\partial x}}{s(x,t)+x\frac{\partial B(x,t)}{\partial x}}dx)$. When $s(a,t) \equiv 0$,

$$\frac{\frac{\partial B(x,t)}{\partial x}}{s(x,t)+x\frac{\partial B(x,t)}{\partial x}} \equiv \frac{1}{x}.$$

This implies that $P(a,t) = a$ for all a. Now by Proposition 2, the wage distribution of the stayers and movers combined is

$$G(B(a,t)) = aP(a,t) = a^2 = \frac{\int_0^a P(x,t)dx}{\int_0^1 P(x,t)dx} = G_s(B(a,t)).$$

This implies that the movers and stayers have the same wage distribution. Equation (22) follows immediately. ■

Theorem 3: *Let the period 2 outside output be* $y(a,t) > 0$ *and the inside output be* $y(a,t)+s(a,t)$*, where* $s(a,t) > 0$*. If a technological change is log-skill-biased (*$\frac{\partial^2 \log y(a,t)}{\partial a \partial t} > 0$*) and general-skill-biased (*$\frac{\partial \log y(a,t)}{\partial t} > \frac{\partial \log s(a,t)}{\partial t}$*), then the turnover probability increases and the wage distribution (of the movers and stayers combined) becomes more spread out as* t *increases.*

Proof. I first show that the turnover probability increases with t. First note that $a^2\frac{\partial B(a,t)}{\partial a} = ay(a,t) - \int_0^a y(x,t)dx$. Therefore,

$$\begin{aligned} & \frac{\partial \log(a\frac{\partial B(a,t)}{\partial a})}{\partial t} - \frac{\partial \log y(a,t)}{\partial t} \\ = \; & \frac{a(\partial y(a,t)/\partial t) - \int_0^a(\partial y(x,t)/\partial t)dx}{ay(a,t) - \int_0^a y(x,t)dx} - \frac{\partial y(a,t)/\partial t}{y(a,t)} \\ = \; & (\frac{\int_0^a y(x,t)dx}{ay(a,t) - \int_0^a y(x,t)dx})(\frac{\partial y(a,t)/\partial t}{y(a,t)} - \frac{\int_0^a(\partial y(x,t)/\partial t)dx}{\int_0^a y(x,t)dx}) \\ > \; & 0, \end{aligned} \tag{63}$$

where the last inequality follows from the log-skilled biased assumption, which implies that $\frac{\partial y(a,t)/\partial t}{y(a,t)} = \frac{\partial \log y(a,t)}{\partial t}$ is increasing in a, so $\frac{\partial y(a,t)/\partial t}{y(a,t)}$ is greater than an average of smaller ratios. The inequality above together with the general-skill biased assumption $(\frac{\partial \log y(a,t)}{\partial t} > \frac{\partial \log s(a,t)}{\partial t})$ implies that

$$\frac{\partial \log a\frac{\partial B(a,t)}{\partial a}}{\partial t} > \frac{\partial \log s(a,t)}{\partial t}. \tag{64}$$

With the inequality above, I see that

$$\begin{aligned}
& \partial\left(\frac{a\frac{\partial B(a,t)}{\partial a}}{s(a,t)+a\frac{\partial B(a,t)}{\partial a}}\right)/\partial t && (65)\\
= & \frac{1}{(s(a,t)+a\frac{\partial B(a,t)}{\partial a})^2}\left(\frac{\partial(a\frac{\partial B(a,t)}{\partial a})}{\partial t}s(a,t)-\frac{\partial s(a,t)}{\partial t}a\frac{\partial B(a,t)}{\partial a}\right)\\
= & \frac{a\frac{\partial B(a,t)}{\partial a}s(a,t)}{(s(a,t)+a\frac{\partial B(a,t)}{\partial a})^2}\left(\frac{\partial \log a\frac{\partial B(a,t)}{\partial a}}{\partial t}-\frac{\partial \log s(a,t)}{\partial t}\right)\\
> & \ 0.
\end{aligned}$$

This immediately implies that turnover probability increases with t (by Proposition 1).

Next, I show that the wage distributions become more spread out as t increases. Let $w(q,t)$ be the value of the qth quantile in the wage distribution with technology index t. I need to show that $\frac{\partial^2 w(q,t)}{\partial q \partial t} > 0$. Take $w = B(a,t)$, then Proposition 2 implies that the proportion of workers receiving wage less than w is $q(a,t) = a\exp(-\int_a^1 \frac{r(z,t)}{z}dz)$, where $r(a,t) = \frac{a\frac{\partial B(a,t)}{\partial a}}{s(a,t)+a\frac{\partial B(a,t)}{\partial a}}$. In other words, I have

$$w(q(a,t),t) = B(a,t), \tag{66}$$

where $q(a,t) = a\exp(-\int_a^1 \frac{r(z,t)}{z}dz)$. This implies that

$$\begin{aligned}
\frac{\partial w}{\partial q} &= \frac{\partial B/\partial a}{\partial q/\partial a}\\
&= \frac{a\frac{\partial B(a,t)}{\partial a}}{a\exp(-\int_a^1 \frac{r(z,t)}{z}dz)(1+r(a,t))}.
\end{aligned} \tag{67}$$

Take derivative with respect to t to the above expression, I have

$$\frac{\partial^2 w}{\partial q \partial t} = \frac{\exp(-\int_a^1 \frac{r(z,t)}{z}dz)\{\frac{\partial(a\frac{\partial B(a,t)}{\partial a})}{\partial t}(1+r(a,t)) - a\frac{\partial B(a,t)}{\partial a}[\frac{\partial r(a,t)}{\partial t} - \int_a^1 \frac{\partial r(z,t)/\partial t}{z}dz]\}}{a\exp(-2\int_a^1 \frac{r(z,t)}{z}dz)(1+r(a,t))^2}. \tag{68}$$

The denominator of is clearly positive. In the numerator, $\exp(-\int_a^1 \frac{r(z,t)}{z}dz) > 0$, and the term in the bracket is

$$\begin{aligned}
& \frac{\partial(a\frac{\partial B(a,t)}{\partial a})}{\partial t}(1+r(a,t)) - a\frac{\partial B(a,t)}{\partial a}[\frac{\partial r(a,t)}{\partial t} - \int_a^1 \frac{\partial r(z,t)/\partial t}{z}dz] && (69)\\
= & \frac{\partial(a\frac{\partial B(a,t)}{\partial a})}{\partial t} + a\frac{\partial B(a,t)}{\partial a}\int_a^1 \frac{\partial r(z,t)/\partial t}{z}dz + \left(\frac{\partial(a\frac{\partial B(a,t)}{\partial a})}{\partial t}r(a,t) - a\frac{\partial B(a,t)}{\partial a}\frac{\partial r(a,t)}{\partial t}\right)\\
> & \ 0,
\end{aligned}$$

where the inequality holds because $\frac{\partial(a\frac{\partial B(a,t)}{\partial a})}{\partial t} > \frac{\partial \log s(a,t)}{\partial t} \geq 0, a\frac{\partial B(a,t)}{\partial a}\int_a^1 \frac{\partial r(z,t)/\partial t}{z}dz > 0$ (

$\partial r(z,t)/\partial t > 0$ follows from the mobility part of the proof), and

$$
\begin{aligned}
& \left(\frac{\partial(a\frac{\partial B(a,t)}{\partial a})}{\partial t} r(a,t) - a\frac{\partial B(a,t)}{\partial a}\frac{\partial r(a,t)}{\partial t}\right) \qquad (70)\\
= & \left(\partial\left(\frac{a\frac{\partial B(a,t)}{\partial a}}{r(a,t)}\right)/\partial t\right) r(a,t)^2 \\
= & \left(\frac{\partial(a\frac{\partial B(a,t)}{\partial a})}{\partial t} + \frac{\partial s(a,t)}{\partial t}\right) r(a,t)^2 \\
> & \ 0
\end{aligned}
$$

This shows that the numerator is also positive, so I have $\frac{\partial^2 w}{\partial q \partial t} > 0$. Thus the wage distribution becomes more spread out when t increases. ■

Corollary 3: *If* $\frac{\partial^2 y(a,t)}{\partial a \partial t} > 0$ *and* $s(a,t_1) = s(a,t_2)$ *for all* a *and* t, *then for* $t_1 < t_2$, $P(a,t_1) > P(a,t_2)$ *for all* $a > 0$.

Proof. Note that $y(a,t) - B(a,t) = a\frac{\partial B(a,t)}{\partial a}$. Moreover

$$
\begin{aligned}
\frac{\partial(a\frac{\partial B(a,t)}{\partial a})}{\partial t} &= \frac{\partial(ay(a,t) - \int_0^a y(x,t)dx)}{\partial t} \qquad (71)\\
&= \int_0^a \left(\frac{\partial y(a,t)}{\partial t} - \frac{\partial y(x,t)}{\partial t}\right)dx \qquad (72)\\
&= \int_0^a \int_x^a \frac{\partial^2 y(z,t)}{\partial z \partial t} dz dx \\
&> 0.
\end{aligned}
$$

Therefore, when $s(a,t_1) = s(a,t_2)$, I have $\frac{B'(a,t_1)}{s(a)+aB'(a,t_1)} < \frac{B'(a,t_2)}{s(a)+aB'(a,t_2)}$ for all a. Proposition 1 immediately implies that $P(a,t_1) > P(a,t_2)$ for all $a > 0$. ■

Theorem 4: *Let the output be* $y(a,t) > 0$ *for the outside firms and* $y(a,t) + ks(a)$ *for the incumbent, where* $ks(a) > 0$. *If the technological change is skill-biased* ($\frac{\partial^2 y(a,t)}{\partial a \partial t} > 0$) *and* $y(a,t) - B(a,t) > ks(a)$ *for all* a, *then the proportionate increase in turnover increases with* k, *i.e,*

$$\frac{\partial^2 \log(1 - P(a,t,k))}{\partial t \partial k} > 0 \quad \text{for all } a.$$

Proof. It can be checked that

$$\frac{\partial^2 \log(1 - P(a,t,k))}{\partial t \partial k} = \frac{-\frac{\partial^2 P(a,t,k)}{\partial t \partial k}(1 - P(a,t,k)) - \frac{\partial P(a,t,k)}{\partial t}\frac{\partial P(a,t,k)}{\partial k}}{(1 - P(a,t,k))^2}.$$

By Corollary 3, one obtains that $\frac{\partial P(a,t,k)}{\partial t} < 0$. Moreover, notice that an increase in k increases $r(a,t,k)$ for all a and t. Proposition 1 then implies that $\frac{\partial P(a,t,k)}{\partial k} > 0$. It follows that $\frac{\partial P(a,t,k)}{\partial t}\frac{\partial P(a,t,k)}{\partial k} < 0$, and, therefore, it suffices to show that $\frac{\partial^2 P(a,t,k)}{\partial t \partial k} < 0$ to prove the theorem.

Notice that $P(a,t) = \exp(-\int_a^1 \frac{\frac{\partial B(x,t)}{\partial x}}{s(x,t)+x\frac{\partial B(x,t)}{\partial x}} dx)$, it can be checked that

$$\frac{\partial^2 P(a,t,k)}{\partial t \partial k} = \frac{1}{P(a,t,k)} \frac{\partial P(a,t,k)}{\partial k} \frac{\partial P(a,t,k)}{\partial t} + P(a,t,k) \frac{\partial^2 \log(P(a,t,k))}{\partial t \partial k},$$

where $\frac{\partial P(a,t,k)}{\partial k} \frac{\partial P(a,t,k)}{\partial t} < 0$, so it suffices to show that $\frac{\partial^2 \log P(a,t,k)}{\partial t \partial k} < 0$. To do this, I check that

$$\frac{\partial^2 \log P(a,t,k)}{\partial t \partial k} = -\int_a^1 \frac{s(x) \frac{\partial^2 B(x,t)}{\partial x \partial t} (x \frac{\partial B(x,t)}{\partial x} - ks(x))}{(ks(x) + x\frac{\partial B(x,t)}{\partial x})^3} dx < 0,$$

where the inequality follows because $\frac{\partial^2 B(x,t)}{\partial x \partial t} > 0$ (by Theorem 3) and $x\frac{\partial B(x,t)}{\partial x} = y(x,t) - B(x,t) > ks(x)$ (by assumption). This finishes the proof. ■