RELATIONAL CONTRACTS WITH SUBJECTIVE PEER EVALUATIONS*

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Abstract. We study optimal contracting in a setting where a firm repeatedly interacts with multiple workers, and can compensate them based on publicly available performance signals as well as privately reported peer evaluations. If the evaluation and the effort provision are done by different workers (as in a supervisor/agent hierarchy), we show that, using both the private and public signals, the first best can be achieved even in a static setting. However, if each worker is required to both exert effort and report on his co-worker’s performance (as in a team setting), the worker’s effort incentives cannot be decoupled from his truth-telling incentives. This makes the optimal static contract inefficient and relational contracts based on the public signals increase efficiency. In the optimal contract, it may be optimal to ignore signals that are informative of the worker’s effort.

1. Introduction

In modern labor markets, most workers perform jobs where objective performance measures are hard to obtain (Prendargast, 1999). Consequently, firms often rely on subjective performance measures to provide work incentives. For instance, a firm may use measures such as leadership skills, entrepreneurial drive or client satisfaction scores. Some of these subjective measures may be publicly observable. It is well-known that such publicly observable measures, even if they are subjective and imperfect, can be used as an input into compensation structure to sharpen worker incentives.

However, there may be other valuable performance-related information that is not publicly observable, but rather diffused within the organization. This is becoming increasingly the case with the prevalence of team-based organizations† coupled with increased complexity of tasks and decentralization of authority. In such settings, co-workers may have the most information about an individual’s performance and contribution towards the overall team.

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†See, Che and Yoo, 2001, and the references therein.
outcome (Fedor et. al, 1999; May and Gueldenzoph, 2006). However, it may be difficult for
the organization to elicit and use this information as it is privately observed by workers and
inherently subjective.

Indeed, firms often institute systematic processes to elicit subjective evaluation of a
worker’s performance from his co-workers. For example, in a typical organizational hier-
archy, the supervisor is often in charge of evaluating her subordinates’ performance and
providing this information to the owners of the firm (see Tirole, 1986 and the references
therein). In a team-production environment, the use of 360-evaluations is commonplace.
Under such evaluations the firm seeks a worker’s performance evaluation from several peo-
ple who worked closely with him during the evaluation period irrespective of their relative
position in the organizational hierarchy. As May and Gueldenzoph (2006) note, increasingly
“companies are turning to 360-degree multi-rater feedback and intragroup peer evaluation
systems for the purpose of managing performance and determining compensation rewards”
(p. 5). An estimated 90 percent of Fortune 1000 firms have implemented some form of
multi-source assessment that includes peer evaluations (Edwards and Ewen, 1996). Such
feedback is typically privately observed by the firm to ensure anonymity of the evaluator
and to encourage candid reporting.

It is well-documented that firms often combine both publicly observed subjective measures
and subjective private performance evaluation in their compensation policies. For example,
Field (2010) presents a Harvard Business Review (HBR) case study of a mutual fund com-
pany in which the performance evaluation system for portfolio managers involves paying
bonuses where 60 percent of the bonus is determined by the financial performance of the
fund they direct and 40 percent determined by the quality of teamwork, which is assessed
through structured feedback (gathered from team members and analyzed by top managers).
Another HBR case study by Rose and Sesia (2010) on the investment bank Credit Suisse
highlights a similar compensation structure.2,3

The central objective of this article is to explain how a firm should optimally use pub-
lic subjective measures along with dispersed private subjective measures to offer sharper
work incentives. When the performance measure is public (but nevertheless, subjective), a
common channel for incentive provision is a relational contract: The firm offers the work-
ers a discretionary bonus based on their publicly observed performance, and lives up to its
promised payments as reneging may result in future retaliation by the workers.4 But offer-
ing performance pay based on private evaluation is fraught with a unique set of challenges.
First, the firm must provide the workers the right incentives to report evaluations of their
co-workers truthfully. Moreover, as the subjective evaluations are reported privately to the

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2In addition to the financial service sector, similar compensation policy is also documented in other indus-
tries, such as consumer goods. Simon and Kindred (2012) reports a case study on Henkel, a manufacturer of
personal care products, where workers’ compensation includes a significant variable pay (bonus) component,
that depends on overall organization performance, team results, and the team leaders’ subjective evaluations
on the workers’ individual contributions.

3Note that some aspects of the firm’s performance (e.g., sales volume), are verifiable, and can, in principle,
affect a worker’s compensation through explicit contracts. But the firms often refrain from using such explicit
contracts as these measurable measures could be misaligned with the firm’s long-term goals. Instead, relational
contracts are often used to link pay to performance measures that may be non-verifiable but better reflect
the firm’s business objectives (see, Baker et. al, 1994).

4See, for instance, Bull (1987), Levin (2003), and Malcomson (2010) for a survey.
firm, the firm itself must have the incentive to report the true evaluations back to the workers. It turns out that truthful revelation of private subjective information may impose an additional agency cost as the workers’ incentive for exerting effort is interlinked with their incentive for truthful reporting.

We study a setting in which a firm can use both publicly observable and privately reported performance measures and analyze how incentives based on private information can be combined with relational contracts that rely on public information. An important finding is that the nature of the optimal contract depends crucially on the role played by agents in the organization: In particular, it depends on whether there is a separation of the informative and the productive roles. Accordingly, we distinguish between two settings.

First, we consider a simple model of organizational hierarchy where the informative and productive roles are played by different workers. We consider a supervisor-agent pair in which the agent is solely responsible for production and the supervisor’s only role is to (privately) observe and report the agent’s performance. Thus, the effort incentives are relevant only for the agent and the incentives for truthful reporting relevant only for the supervisor. The firm may offer different contracts to the supervisor and the agent that depend both on the agent’s publicly observed output and the private evaluation sent by the supervisor.

Second, we consider a more general model where the firm repeatedly interacts with two workers, but now each worker performs both an informative role and a productive role. The two agents work as a team in every period; each agent must exert effort towards the team output, and also (privately) evaluate his co-worker. (Note that such an environment is similar in spirit to ones where 360 evaluation is commonly used). The firm now has potentially two different channels for incentive provision: a “public performance bonus” based on the publicly observed team output and a “subjective performance bonus” based on the agents’ peer-evaluations that are privately reported to the firm.

The features of the optimal contract are substantively different in these two settings.

First, when the effort and the information roles are separated, we show that the first-best outcome can be achieved even in the static setting (thus, making relational contracts irrelevant). Here, the optimal contract penalizes the agent only if both the public output and the supervisor’s evaluation are at their respective worst levels. The supervisor is penalized only if the output is the best feasible one in spite of his evaluation of the agent being the worst. Finally, in order to penalize a worker (i.e., either the agent or the supervisor), the firm simply transfers a part of his wage to the other worker. Now, by choosing the penalty amounts appropriately, the firm can ensure that neither the agent wants to shirk on his effort nor the supervisor lies on his report (in anticipation of receiving a penalty transfer from the agent). Essentially, the optimal contract induces a lottery between the supervisor and the agent that elicits the first-best effort and truthful reporting.

Next, we consider the case when workers must play both informative and productive roles. Here we ask three questions: First, what is the optimal contract in the static setting? We find that in contrast to the setting with separation in roles, here the optimal static contract is necessarily inefficient. But similar to the previous setting, the peer evaluation is still sparingly used in the optimal contract; i.e., peer evaluation affects a worker’s pay only in the unlikely event that both the public output is at its lowest level and his co-worker sends the worst possible report about him. In this case the worker’s pay is lowered to punish him for poor performance; otherwise the worker earns a fixed compensation.
To see why the first best cannot be achieved, note that each worker now has two incentive constraints: to exert effort and to tell the truth about his peers. More importantly, these two constraints cannot be decoupled as a worker may resort to a “double deviation”: after he shirks, he may also want to lie about his co-worker’s performance. (Trivially, this issue does not show up in our earlier setting as we assume a separation between the information and the production role of a worker.) Due to such potential deviations, contracts in which a worker’s penalty amount is transferred to his co-worker are no longer feasible. This implies that incentive pay based on such private peer reports must call for a “joint punishment” in the form of surplus destruction. That is, whenever the firm punishes the worker by lowering his wage due to poor evaluation, the firm must also incur a cost in the form of lost surplus that it could have accrued otherwise. Indeed, if there is no joint punishment, the firm may manipulate the peer evaluations that it receives: As the reports are private, the firm has an incentive to understate a worker’s evaluation in order to lower his wage payment. Hence, if subjective peer evaluations are to be used to provide work incentives, surplus destruction is necessary in equilibrium and first best cannot be achieved (the literature on subjective performance measure with single-worker also makes this observation; see, for example, Levin, 2003; MacLeod, 2003; and Fuchs, 2007). The optimal contract is the one that minimizes the surplus destruction in equilibrium given the incentive constraints on the workers.

The inefficiency of the static optimal contract naturally leads us to our second question: How can efficiency be improved in a dynamic setting where incentives through the subjective performance bonus may be supplemented by a public performance bonus sustained through relational contracts? The main result of our article characterizes how the firms should optimally use these two instruments. Clearly, the firm would prefer to use a public bonus alone, and thus avoid surplus destruction, if possible. It turns out that when the firm is patient enough, a public performance bonus sustained through a relational contract suffices to provide effort incentives. In contrast, when the firm is impatient, the public bonus may be used along with subjective bonus pay (that depends on the private subjective evaluations) but only if the level of surplus destruction associated with the latter is small. If a large share of the surplus must be eroded in order to provide subjective bonus incentives, then relational contracts are not sustainable—the firm may actually prefer to renege on its public bonus promise since it has little to lose in terms of future surplus.

An important feature of our optimal contract is that the worker’s pay is independent of what he reports about his peer. To better understand this feature, we study a variation of our model in which the workers’ private signals about each other are correlated, and ask whether a worker’s pay is still independent of his reports. In a setting with correlation, a worker’s report about his peer contains information about the worker’s own effort, and we might expect that this information would be used in the optimal contract. However, we find that even in this setting, in the optimal contract, a worker’s pay may be independent of the peer report he submits. The firm may disregard some information, as the benefit of using such information might outweigh by the cost of eliciting it. This finding highlights that the nature of optimal contracting with subjective private peer evaluations is fundamentally different from optimal contracting with publicly observable signals.

However, we show that if we allow the agent’s effort to affect the degree of correlation between the agents’ private signals, then an agent’s pay may depend on his submitted peer report. To see the intuition, suppose that the signals are perfectly correlated when both agents work, but uncorrelated when at least one of them shirks. Then, the firm can detect
shirking by simply checking whether there is a mismatch between the two agents’ reports. By rewarding the agents only when their reported evaluations match (rather than solely based on their peer evaluations), the firm may be able to offer incentives more efficiently.

It is also interesting to note that our result on the sparing use of peer evaluations speaks to the debate in the management literature on the use of peer evaluation in organizations. Several scholars have observed that peer evaluation, albeit commonplace, is not always used to determine pay; rather, it is used more commonly for development and training purposes (see, Pieperl, 1999, and the references therein). The main reason cited is that peer evaluations are often plagued with various forms of rater biases, and, therefore, firms often find it unsuitable for use in determining compensation (Pieperl, 1999; May and Gueldenzoph, 2006). Our finding is consistent with the empirical evidence on the sparing use of peer evaluations in determining compensation, but we provide a different rationale that is derived from the firm’s optimal contracting problem. This result is also reminiscent of the findings in the subjective evaluation literature on individual-worker compensation that justify wage compression (MacLeod, 2003; Fuchs, 2007). Our finding indicates that such wage compression is a feature of the optimal contract even in a more general environment with multiple workers and peer evaluation.

Related literature: Starting with Telser (1980), a vast literature on relational contracts has flourished over the last few decades. The early contributions in this literature typically assume a public but non-verifiable performance signal and highlight how repeated interaction may alleviate moral hazard even in the absence of court-enforceable contracts (see Malcolmson, 2010, for a survey). More recently, some authors have focused on private or subjective evaluations (Levin, 2003; MacLeod, 2003; Fuchs, 2007; Chan and Zheng, 2011; Maestri, 2012), though most consider a single-worker framework and eschew the question of eliciting the truthful evaluation from the co-workers.

The articles that are most closely related to our analysis are Levin (2003), MacLeod (2003) and Fuchs (2007). Levin (2003) shows that the optimal contract involves just two pay levels, and prescribes termination following poor performance. MacLeod (2003) analyzes a static setting with a single risk-averse agent, and derives the optimal contract when there is a private (subjective) performance measure. He shows that the optimal contract results in a more compressed pay relative to the case with verifiable (objective) performance measures, and entails the use of bonus pay rather than the threat of dismissal. Fuchs (2007) considers a dynamic version of MacLeod’s setting. He characterizes the optimal contract in a finite horizon, and shows (similar to our setting) that surplus destruction on equilibrium path is necessary for incentive provision. However, resources are burnt only after the worst possible realizations. He also provides a partial characterization of the optimal contract in the infinite horizon where surplus destruction is endogenized through possible termination.

Three salient features distinguish our setting from these articles: First, we consider a multiple agent setting. Second, we allow both public and private performance measures. Third, and perhaps most substantively, the private information on performance must be collected from the workers by giving them adequate incentive for truthful reporting. This feature gives rise to an interaction between the agent’s incentives for effort exertion and his incentives for truthful reporting—an interaction that is absent in the environments of Levin, MacLeod, or Fuchs. A key contribution of our article is to illustrate how the interplay between these two incentives affects contracting. In particular, we characterize the optimal contract to show how relational contracts based on publicly observed performance measures...
may be used to supplement incentives based on the private peer evaluations.\textsuperscript{5} Although the nature of contracting in our setting is substantively different, it has some familiar features—the firm uses the private signal sparingly as such incentive payments must involve surplus destruction, and, consequently, the optimal contract leads to wage compression.

There is a sizeable literature on relational contracting with multiple agents that studies how relational contracting affects rent allocation among agents (Calzolari and Spagnolo, 2009; Board, 2011; Andrews and Barron, 2014; Barron and Powell, 2014), the structure of ownership (Rayo 2007), and task assignment (Mukherjee and Vasconcelos, 2011; Ishihara, 2013). Our article is more closely related to articles that focus on how relational contracting affects the form of compensation (Che and Yoo, 2001; Levin, 2002; Kvalev and Olsen, 2006; Baldenius and Glover, 2010; and Kim and Vikander, 2013). We contribute to this literature by allowing the principal to elicit private information from the agents through peer evaluation. We show that the principal can use the peer evaluation to its advantage even if some information is ignored so as to induce truthful reporting from the agents.

Some aspects of our environment are reminiscent of a few other articles in the incentive theory literature. The double-deviation issue discussed above also crops up in some models of delegated expertise, e.g., Gromb and Martimort (2007) (also see Malcomson, 2009). They consider a setting where a decision-maker hires experts to exert effort in order to obtain (private) signals about the optimal decision. The key question is how to provide optimal incentive for signal acquisition. As signal acquisition is private, an expert may deviate both in his effort choice and in his report of the signal. In contrast to our analysis, Gromb and Martimort assume verifiable reports (and decision outcomes) and analyze the issue of collusion between the players.

Our analysis also bears resemblance to Baker, et. al (1994) who study the interaction between explicit contracts based on verifiable measures and relational contracts based on non-verifiable measures.\textsuperscript{6} In contrast to Baker, et al. where performance measures are publicly observable, agents in our setting have private information about each other’s performance. Therefore, the optimal contract must also induce truthful revelation of information.

Finally, this article is linked, though somewhat tangentially, to the literature on community enforcement in an anonymous random matching setting. While the economic question is different, a key issue in community enforcement is truthful communication of information. The seminal papers by Kandori (1992) and Ellison (1994) highlight the difficulty with enabling transmission of information about past play. In their setting, players do not want to punish deviators (and thereby communicate information about off-path play) in order to prevent contagion and breakdown of cooperation. Recent work by Deb (2012) uses belief-free ideas and block strategies to address the issue of truthful revelation of information in the random matching setting, when players can send cheap talk messages before play. Also related is the article by Lippert and Spagnolo (2011) who study word-of-mouth communication in games played over a network, and consider the incentives of agents to truthfully report soft information about past play.

The rest of the paper is structured as follows. In section 2, we present a baseline model of subjective performance evaluation in which informative and productive roles are played by

\textsuperscript{5}Marx and Squintani (2009) consider a similar environment with peer monitoring. However, they are interested in a different question: They ask how agents can be motivated to undertake costly monitoring activity in order to implement the first-best action.

\textsuperscript{6}Also see MacLeod and Malcomson (1989).
separate workers. Section 3 discusses the optimal contract in this setting. A general model with no separation of roles is presented in section 4. Section 5 characterizes the optimal contract in this setting. Section 6 discusses some of our modeling assumptions. Section 7 concludes. Proofs are provided in the Appendix.

2. A baseline model

In this section we present a simple model of hierarchy in the spirit of Tirole (1986) where a firm hires two workers—a manager and an agent. The agent exerts effort to produce an output while the manager’s only role is to collect information on the agent’s performance and convey it to the firm. It turns out that, in this setting the first-best can be achieved even in the static one-period game. Hence, in what follows, we only focus on a stage game. We will present a repeated game later when considering a more general model.

The game is described in terms of three components: technology, contracts, and payoffs.

Technology: A principal, or a firm, $F$, hires a manager $M$ and an agent $A$ (or “subordinate”) to work in a given project. The project outcome, or the output, $Y \in \{y_1, ..., y_N\}$ depends only on the agent’s effort $e_A \in \{0, 1\}$. Effort is costly and privately observed by the agent, giving rise to a moral hazard problem. The cost of effort is $c$ if $e_A = 1$ and 0 otherwise. The outcome $Y$ may be conceived as a measure of the project’s success, where $y_1 < y_2 < ... < y_N$, and

$$\Pr(Y = y_j | e_A) = \begin{cases} \alpha_j & \text{if } e_A = 1 \\ \beta_j & \text{otherwise.} \end{cases}$$

We assume that $Y$ is publicly observable but not verifiable and is realized only at the end of the game.

Before the output is realized (but after the agent makes his effort choice), the manager obtains a noisy signal $s_M \in S = \{1, ..., n\}$ about the agent’s effort. Upon receiving the signal, the manager privately reports her signal to the firm. Let

$$\Pr(s_M = s | e_A) = \begin{cases} p_s & \text{if } e_A = 1 \\ q_s & \text{otherwise.} \end{cases}$$

The signal $s_M$ is a private signal of the manager: It is unverifiable, and also not observed by the other players (or any third party). As the signal $s_M$ is private and noisy, one can interpret $M$’s report on $A$ as her subjective evaluation of her subordinate’s performance. Without loss of generality, we assume that the set of reports sent by the manager coincide with the set of signals.

Assumption 1. (i) Let $\alpha_j \geq 0$, $\beta_j \geq 0$, $\sum_{j=1}^N \alpha_j = \sum_{j=1}^N \beta_j = 1$, and $\alpha_j/\beta_j > \alpha_k/\beta_k$ for any $j > k$. Similarly, (ii) let $p_s \geq 0$, $q_s \geq 0$, $\sum_{s \in S} p_s = \sum_{s \in S} q_s = 1$, and $p_s/q_s > p_r/q_r$ for any $s > r$. That is, both $Y$ and $s_M$ satisfy the monotone likelihood ratio property (MLRP). (iii) We assume that the set of signals $\{Y, s_M\}$ are mutually independent conditional on effort. Finally, (iv) let $\sum_{j=1}^N \alpha_j y_j - c > \sum_{j=1}^N \beta_j y_j$ so that it is efficient to implement $e_A = 1$. 
Contracts. The firm promises individual contracts to the manager and the agent. Both contracts specify payments based on the private report sent by the manager, $s$, and the public output, $Y$. Let $w_s^M(Y)$ and $w_s^A(Y)$ be the payments to the manager and the agent respectively. In addition, the manager’s contract specifies a reporting strategy ($m : S \rightarrow S$) and the agent’s contract specifies the effort levels ($e$) required of him.

As the manager’s report is private, note that the firm’s total payout to the agent cannot depend on the manager’s report. Otherwise, the firm would always claim to have received a report that minimizes its payout. In addition, public output is non-verifiable and hence, the firm’s payout cannot vary with it either. The total payroll expense of the firm must therefore be a constant. We assume that the firm commits to a total payroll expense $w.$ As the combined payment received by the manager and the agent cannot exceed the committed payroll expense, we have $w_s^M(Y) + w_s^A(Y) \leq w$ for all $Y$ and $s$. When $w_s^M(Y) + w_s^A(Y) < w$, we assume that the firm gives the remaining sum (i.e., $w - w_s^M - w_s^A$) to a disinterested third party. The use of this type of “money-burning” contracts dates back to at least Eswaran and Kotwal (1984) and is commonly used in similar environments; e.g., See MacLeod (2003).

A contract in the stage game is completely characterized by the tuple $\phi = \{e, m; w, w_s^A(Y), w_s^M(Y)\}$. Let $\Phi$ be the set of all such contracts.

Payoffs. We assume that all players are risk-neutral. Thus, the expected payoff of the firm is $\pi := \mathbb{E}[Y \mid e_A] - w$. The expected payoff of the manager and the agent are $u_M := \mathbb{E}[w_s^M(Y) \mid e_A, m]$ and $u_A := \mathbb{E}[w_s^A(Y) \mid e_A, m] - ce_A$, respectively. Let the outside options for all players be 0.

Time line. The game proceeds as follows:
- **Beginning of stage 1**. Firm offers contract $\phi \in \Phi$.
- **Stage 1.1**. Manager and the agent accept or reject. If both players accept, the game continues to the next stage.
- **Stage 1.2**. The agent exerts effort $e_A$.
- **Stage 1.3**. The manager privately obtain signal $s_M$ and privately sends her evaluation $s$ to the firm.
- **Stage 1.4**. Output $Y$ is realized.
- **End of Stage 1**. Transfers paid to the agents and the game ends.

Strategies and Equilibrium. The strategy of the agent $A$ has two components: Given the contract offered, he must choose (i) whether to accept or reject the contract and (ii) an effort level $e_A$. The strategy of the manager $M$ involves a choice to (i) accept or reject the contract, and (ii) a reporting strategy $m_M : S \rightarrow S$. Finally, the firm’s strategy is to choose a contract $\phi \in \Phi$. We use perfect Bayesian equilibrium (PBE) as our solution concept.

To characterize the optimal contract, by virtue of Revelation Principle, we focus on equilibria in which the manager always reports her signal truthfully, i.e., $m(s) = s$ for all $s \in S$.

3. The optimal contract under baseline model

We begin our analysis by first delineating the firm’s optimal contracting problem. We focus on the case where the first-best allocation requires the agent to induce effort, as the case of no effort is trivial. So, the firm’s problem is to maximize its payoff when $e_A = 1$, subject to a set of participation and incentive compatibility constraints.
First, as mentioned earlier, any contract based on peer evaluation involves a commitment from the firm to pay out a fixed sum $w$ in payroll expenses irrespective of the output and the manager’s reports. So, the total payroll expense must satisfy the following budget constraint:

\[(B) \quad w \geq w^M_s (Y) + w^A_s (Y) \quad \forall \ s \in S \text{ and } Y \in \{y_1, \ldots, y_N\}.\]

Second, as we are limiting attention to direct mechanisms (by the Revelation Principle), the contract should induce the manager to report her signals truthfully. Thus, we must have the following truth-telling constraint on the equilibrium path:

\[(T) \quad \mathbb{E}_Y [w^M_s (Y) \mid e_A = 1, s_M = s] \geq \mathbb{E}_Y [w^M_s (Y) \mid e_A = 1, s_M = s'] \quad \forall s \text{ and } s' \in S.\]

Third, the contract should make it optimal for the agent to exert effort (rather than shirk). So, we must have:

\[(IC) \quad \mathbb{E}_{\{Y, s\}} [w^A_s (Y) \mid e_A = 1] - c \geq \mathbb{E}_{\{Y, s\}} [w^A_s (Y) \mid e_A = 0].\]

Finally, the contract offered must ensure participation by each player; i.e., we have:

\[(IR_A) \quad u_A = \mathbb{E}_{\{Y, s\}} [w^A_s (Y) \mid e_A = 1] - c \geq 0,\]

and

\[(IR_M) \quad u_M = \mathbb{E}_{\{Y, s\}} [w^M_s (Y) \mid e_A = 1] \geq 0.\]

Denote $v = \mathbb{E} [Y \mid e_A = 1]$. Thus, the firm’s optimal contracting problem is given as follows:

\[
\max_{\phi \in \Phi} \quad \pi = v - w
\]

\[s.t. \quad (B), \quad (T), \quad (IC), \quad (IR_A), \quad \text{and} \quad (IR_M).\]

**Proposition 1. (Optimal contract under baseline model)** The optimal contract induces first best. Under the optimal contract, (i) the firm commits to a payroll expense of $c$, (ii) the manager reports truthfully and receives the transfer:

\[
\hat{w}^M_s (Y) = \begin{cases} 
-\Delta_M & \text{if } Y = y_N \text{ and } s = 1 \\
\Delta_A & \text{if } Y = y_1 \text{ and } s = 1 \\
0 & \text{otherwise}
\end{cases}
\]

and (iii) the agent exerts effort and receives the transfer:

\[
\hat{w}^A_s (Y) = \begin{cases} 
c + \Delta_M & \text{if } Y = y_N \text{ and } s = 1 \\
c - \Delta_A & \text{if } Y = y_1 \text{ and } s = 1 \\
c & \text{otherwise}
\end{cases}
\]

where $\Delta_M = \alpha_1 c / q_1 (\alpha_N \beta_1 - \alpha_1 \beta_N)$ and $\Delta_A = \alpha_N c / q_1 (\alpha_N \beta_1 - \alpha_1 \beta_N)$. 

Proposition 1 shows that under the optimal contract, both the manager and the agent earn a fixed compensation except in two cases: (i) when both the output and the manager’s evaluation are at their lowest, and (ii) the manager sends the worst evaluation but the output is at its highest level.\(^7\)

When both the output and the manager’s report are the worst possible ones, the agent is punished and his “penalty”, \(A\), is transferred to the manager. While such an incentive scheme does not require money burning, it distorts the manager’s truth-telling incentive—the manager is now always tempted to report the worst evaluation for the agent. To ensure truth-telling from the manager, the contract punishes her if the output is the best possible one in spite of her report being the worst, and, as before, her penalty, \(M\), is transferred to the agent. Now, one can choose values of these penalties such that the manager, in expectation, earns the same expected payoff for all evaluations that she may send and the agent’s expected payoff from exerting effort is at least as much as his payoff from shirking.

There are two important features of this contract: first, the contract achieves first best as punishment does not require money burning; whenever a player is punished, a transfer is made from the punished player to the other one. This observation has an immediate implication in a repeated game setting: even if the firm can offer a relational contracts based on the publicly observed output, such an incentive mechanism would be redundant. Second, contrary to the commonly observed compensation policies, the manager’s compensation depends on her evaluation of the agent.

To see why the contract is effective in inducing effort, notice that it essentially induces a lottery between the manager and the agent. The lottery has zero expected payoff for both the manager and the agent if he puts in effort. When the agent shirks, however, he has negative expected payoff from the lottery. So he is willing to put in effort because it improves his expected payoff.

Next, to see why this contract may not be adopted in practice, notice that a key requirement for the contract to work is that the manager cannot affect the distribution of the output. If the manager’s effort also matters for output, however, this contract provides a perverse incentive to the manager: The manager can increase her expected payoff from the contract if the worst output becomes more likely. The manager therefore prefers to sabotage the agent’s effort under this contract, and this may be part of the reason why the contract is rarely, if ever, observed. In practice, the manager’s role typically goes beyond evaluating the agent. From making plans to assigning tasks, from learning about the client’s demand to monitoring the worker’s behavior, the manager takes a number of actions that matter for the success of the project. When the effort of the manager matters, what form does the optimal contract take? We analyze this question in the next section.

### 4. A General Model

We now assume that both workers exert productive effort and play an informative role by evaluating each other’s performance. We will see that the optimal contract substantially differs from the baseline discussed above. For analytical tractability and a streamlined exposition, we do not explicitly model the organizational hierarchy and treat the manager and the agent in a symmetric fashion. So, we can also interpret the workers’ reports as subjective peer evaluation. To simplify notation, in what follows, we will denote the workers as agents

\(^7\)Such cross-checking between the two signals is somewhat similar in spirit to Cremer and McLean (1988).
A1 and A2. All players are assumed to be long-lived, and in each period \( t \in \{1, 2, \ldots \} \), they play the following stage game that is a slightly general version of our baseline model.

**Stage Game:** At the start of every period, the stage game begins with the firm \( F \) offering individual contracts to the two agents. For sake of brevity, we only elaborate on the aspects of the game that are different from our baseline model. Also, for expository clarity, we suppress the time index \( t \) for all variables unless mentioned otherwise.

**Technology.** We allow for both agents to exert effort and assume that each player receives and reports information about the other. Each agent (privately) exerts effort \( e_i \in \{0, 1\} \) in the project where the cost of effort is \( c \) if \( e_i = 1 \) and 0 otherwise. The project outcome \( Y \in \mathcal{Y} = \{y_1, \ldots, y_N\} \) is publicly realized at the end of the period and depends on the effort profile \( e = (e_1, e_2) \) where

\[
\Pr (Y = y_j | e) = \begin{cases} 
\alpha_j & \text{if } e = (1, 1) \\
\beta_j & \text{if } e = (0, 1) \text{ or } (1, 0) \\
\gamma_j & \text{if } e = (0, 0)
\end{cases}.
\]

The above technology specification captures a wide range of production environments. For example, when \( \beta_j > \gamma_j \) our setting allows the efforts to be substitutes, and if \( \beta_j = \gamma_j \), the efforts are purely complementary in the spirit of team production. Also note that as we are interested in implementing the effort profile \( e = (1, 1) \), \( \gamma_j \) plays no role in our analysis.

The signal on peer-evaluation is assumed to have the same structure as in our initial model. Before the team output is realized (and after the agents make their effort choices), each agent \( A_i \) privately obtains a noisy signal \( s_i \), about his co-worker \( A_i \)’s performance or contribution to the project, and privately reports his signal to the firm. Let

\[
\Pr (s_i = s | e_{-i}) = \begin{cases} 
p & \text{if } e_{-i} = 1 \\
q & \text{if } e_{-i} = 0
\end{cases}.
\]

We maintain Assumption 1 (i), (ii), and (iv) (i.e, \( Y \) and \( s_i \) satisfy MLRP and it is indeed efficient to implement \( e = (1, 1) \)), and unless mentioned otherwise, the set of signals \( \{Y, s_1, s_2\} \) are assumed to be mutually independent conditional on effort.

**Contracts.** In each period, the firm offers a contract to each of the two agents. The contract specifies the effort levels \( (e) \) required of the agents, the reporting strategies \( (m = (m_1, m_2)) \), and the transfer payments. In contrast to the baseline model, the transfer payment to an agent \( A_i \) not only depends on the peer evaluation \( s \) sent by \( A_i \) about his co-worker \( A_{-i} \)’s performance or contribution to the project, and \( \text{privately} \) reports his signal to the firm. Let

As before, the firm commits to a payroll expense \( w \) and offers a payment \( w^1_{rs}(Y) \) such that \( w^1_{rs}(Y) + w^2_{sr}(Y) \leq w \) for all \( r, s \) and \( Y \). Money is “burnt” whenever \( w^1_{rs}(Y) + w^2_{sr}(Y) < w \). We call the payments \( w^1_{rs}(Y) \) subjective performance bonus in order to emphasize their dependence on the subjective peer evaluations.

But in contrast to the baseline model, in addition to the subjective performance bonus, the firm also promises a public performance bonus \( B(Y) \geq 0 \) for the team that depends only on the publicly observable team output \( Y \). As the team output \((Y) \) is non-verifiable, the
bonus payment is offered as a relational contract (à la Bull, 1987) that is sustained through the threat of future retaliation of the agents should the firm renege on its promise. Let \( b^i(Y) \) be the bonus promised to agent \( A_i \) and, without loss of generality, we assume that \( b^i(Y) + b^2(Y) = B(Y) \) (i.e., the sum of the bonus of each agent is equal to the firm’s total promised bonus).

For tractability, we limit attention to symmetric contracts. We assume that \( b_1(Y) = b_2(Y) = b(Y) \) and \( w^1_{rs}(Y) = w^2_{rs}(Y) = w_{rs}(Y) \) for all \( Y \), \( r \) and \( s \). A contract in the stage game is completely characterized by the tuple \( \phi = \{e, m, w, b(Y), w_{rs}(Y)\} \). Let \( \Phi^* \) be the set of all such contracts.

Notice that our setup is equivalent to a setting in which the agent’s pay includes both a fixed contractible “salary” and a nonnegative discretionary pay that depends on the subjective reports and the publicly observable output. To see the equivalence, consider the general setting where \( W^i_{rs}(Y) \) is the total payment received by the agent \( A_i \). Define \( \omega^i = \min_{r,s,Y} W^i_{rs}(Y) \). In other words, \( \omega^i \) is the minimal payment that the firm commits to agent \( A_i \), and it can be interpreted as his base wage, or “salary.” The total bonus of \( A_i \) is then \( B^i_{rs}(Y) = W^i_{rs}(Y) - \omega^i \geq 0 \). Note that we decompose \( B^i_{rs}(Y) \) into two parts: (i) a “public signal”-based component \( b^i(Y) = \min_r B^i_{rs}(Y) \) that depends only on \( Y \) that is equivalent to the public performance bonus defined earlier, and (ii) a subjective component \( B^i_{rs}(Y) - b^i(Y) \) that also depends on the subjective reports. This subjective component of bonus along with the fixed salary is defined above as the subjective performance bonus, i.e., \( w^1_{rs}(Y) = \omega^i + [B^i_{rs}(Y) - b^i(Y)] \). We explicitly consider these two components of the discretionary pay to clearly highlight the interplay between the incentives based on public signal and the incentives based on subjective peer evaluation. Finally, we can define \( w = \max_{r,s,Y} \{w^1_{rs}(Y) + w^2_{rs}(Y)\} \), and this is the payroll expense the firm commits to.

**Payoffs.** We continue to assume that all players are risk-neutral and their outside option is 0. Given the contract, the expected payoff of the firm is \( \pi := E [Y \mid e] - (w + 2E [b(Y) \mid e, m]) \), and the expected payoff of agent \( A_i \) is \( u_i := E [b(Y) + w_{rs}(Y) \mid e, m] - ce_i \).

**Repeated game:** The stage game above is repeated in every period. All players have a common discount factor \( \delta \in (0,1) \). The public history of period \( t \) is given by the tuple \( h_t = \{\phi, Y, \hat{b}\} \) where \( \hat{b} \) is the bonus paid in period \( t \). Let the (public) history in period \( t \) be \( h^t = \{\{\phi, Y, \hat{b}\}_\tau \mid \tau = 1, ..., t - 1\} \) and let \( H^t \) be the set of all (public) histories in period \( t \).

**Strategies and Equilibrium.** As the peer evaluations are private, our model presents an infinitely repeated game with private monitoring. A technical challenge in analyzing such games is that they lack a recursive structure and the characterization of the equilibrium payoff set appears intractable (Kandori, 2002). Hence, in order to analyze the optimal contract while maintaining tractability we restrict the class of strategies.

Specifically, we allow strategies to depend on the full public history (all past team outputs and bonus payments as well as the current period’s team output), and the most recent private history (private reports or private signals in the current period). Therefore, the strategy of an agent \( A_i \) has three components: in each period, given the public history and the current contract, (i) a contract acceptance decision: \( H^t \times \Phi^* \rightarrow \{\text{accept}, \text{reject}\} \), (ii) an effort choice \( e_i : H^t \times \Phi^* \rightarrow \{0, 1\} \) and (iii) a reporting strategy that also depends on effort level: \( H^t \times \Phi^* \times \{0, 1\} \rightarrow M_i \) where \( M_i \) is the set of all functions \( m_i : S \rightarrow S \) that maps \( A_i \)’s private signal on \( A_{-i} \)’s performance in the current period into the set of reports. The firm’s strategy
is to choose in each period (i) a contract given the public history of the game: $H^t \to \Phi^*$, (ii) a public performance bonus $b$ given the public history of the game and the publicly observed output in the current period: $H^t \times Y \to \mathbb{R}_+$, and (iii) a subjective performance bonus $\hat{w}_{rs}$, which may depend on the public history, current period’s (public) output, and the private reports received from the agents in the current period: $H^t \times Y \times S \times S \to \mathbb{R}$.

Our class of equilibria is extremely similar in spirit to semi-public equilibria used in earlier literature by Compte (1998) and Harrington and Skrzypacz (2011). Semi-public equilibria are defined for repeated games with communication, and require that equilibrium actions depend only on the public history, and messages depend on the public history and the most recent private history. In our setting, the agents’ effort decision and the principal’s bonus payout depend only on the public history, but the agents’ reports and principal’s subjective bonus payments can depend on the most recent private information.

We consider this article as a first step in understanding the use of private peer evaluations in incentive provision, by allowing subjective performance bonus to depend only on the most recent private reports. But it is worthwhile to point out that restricting attention to such equilibria is not without loss of generality. For instance, this restriction implies that players cannot condition their actions on their complete private history. In particular, this rules out the possibility where the principal reviews the agents’ performance over a pre-specified length of time rather than in every period (e.g., “$T$-period review” strategies as in Fuchs, 2007), or cases where the agent may withhold effort due to past disagreements on subjective performance bonus.

To characterize the optimal equilibria, by virtue of Revelation Principle, we continue to focus on equilibria in which the agents always report their signals truthfully, i.e., $m_i(s) = s$ for all $i$ and $s$. We also assume that following a publicly observable deviation (i.e., if $\hat{b} \neq b(Y)$ for some agent for some period $t$ and some public signal $Y$), all players take their outside options forever. This assumption is without loss of generality because the outside options give all players their minmax payoffs (Abreu, 1988). Notice that there is another static Nash equilibrium that gives all players 0: The firm pays out a wage of 0 under all contingencies and neither agents puts in effort. Since both options yield the same payoffs to all players and do not affect the structure of the optimal contract, either option can be chosen.\(^5\)

5. THE OPTIMAL CONTRACT UNDER THE GENERAL MODEL

Our analysis proceeds in three steps. We first revisit the firm’s contracting problem and highlight how it differs from its baseline model. Next, we discuss the stage game equilibrium and illustrate why first best need not be feasible in this setting. Finally, we characterize the optimal contract in the repeated game.

5.1. The firm’s problem. We focus on the case in which it is optimal for the firm to induce effort. It is straightforward to establish that, within our class of equilibria, there is no loss of generality in restricting attention to stationary relational contracts where the contract

\(^5\)An alternative modeling choice is to assume that the parties switch to the optimal subjective contracts following a publicly observed deviation (see, e.g., Baker, et. al 1994). The main results of the model remain unaltered under this alternative assumption.
\{e, w, b(Y), w_{rs}(Y)\} is invariant over time.\(^9\) Therefore, the optimal contract maximizes the firm’s per-period payoff, \(\pi\), subject to a set of participation and incentive constraints.

As in our baseline model, the optimal contract must satisfy the budget constraint \((B')\) (for the firm), truth-telling \((T')\) and participation constraints \((IR')\) (for both agents) given as follows:

\[
(B') \quad w \geq w_{rs}(Y) + w_{sr}(Y) \quad \forall Y \in \mathcal{Y}, \text{ and } r, s \in S.
\]

\[
(T') \quad \mathbb{E}_{\{Y,r\}}[w_{rs}(Y) | e = 1, s_i = s] \geq \mathbb{E}_{\{Y,r\}}[w_{rs'}(Y) | e = 1, s_i = s] \quad \forall s \text{ and } s' \in S.
\]

and

\[
(IR') \quad u_i = \mathbb{E}_{\{Y,r,s\}}[b(Y) + w_{rs}(Y) | e = 1] - c \geq 0, \quad \forall i.
\]

While these three constraints closely parallel their counterparts in the baseline model, there are two other constraints that sharply differ from our earlier analysis.

First, the incentive-compatibility constraint significantly differs from that of the canonical moral hazard problem. We must now take into account the fact that, after shirking, the agent may find it optimal to not report his signal truthfully. In other words, a deviation in effort provision may be coupled with a deviation in the peer evaluation reporting strategy. The incentive constraint below reflects the fact that, conditional on shirking, the agent will always report the signal that maximizes his expected payoff. As we will see later, this potential “double deviation” plays a critical role in the characterization of the optimal contract. The incentive constraint can be written as:

\[
(IC') \quad \mathbb{E}_{\{Y,r,s\}}[b(Y) + w_{rs}(Y) | e = 1] - c \geq \max_{s'} \mathbb{E}_{\{Y,r\}}[b(Y) + w_{rs'}(Y) | e_i = 0, e_{-i} = 1].
\]

Second, the use the public performance bonus introduces an additional constraint on the firm. As the bonus payment is sustained through the threat of future punishment, it must be the case that the firm’s payoff from honoring its bonus promise exceeds its payoff from reneging on it. Therefore, we must have the following dynamic enforcement constraint:\(^{10}\)

\[
(DE) \quad \frac{\delta}{1 - \delta} \pi \geq \max_{j} 2b(y_j).
\]

The firm’s payoff from reneging is obtained from two observations: First, since both agents trigger punishment if the firm “cheats” either of the two agents, if the firm decided to renege on its promise, it would renege with both agents. Second, we have assumed that if the firm reneges, off the equilibrium path, the agents do not exert any effort and the firm and the agents earn their outside options of 0.

Thus, the firm’s optimal contracting problem is:

\(^9\)The formal proof is available from the authors. It uses a line of argument very similar to that used by Levin (2003) to prove optimality of stationary contracts within the class of public perfect equilibria. The restriction to semi-public strategies is important in proving stationarity in our setting.

\(^{10}\)Recall that the contract is assumed to be symmetric. So, for any \(y_j\) the aggregate bonus promise \(B(y_j) = 2b(y_j)\).
\[ P : \begin{cases} \max_{\phi \in \Phi^*} & \pi = v - \{ w + 2\mathbb{E}[b(Y) \mid e = 1] \} \\ \text{s.t.} & (B^\prime), (T^\prime), (DE), (IC^\prime), \text{ and } (IR^\prime). \end{cases} \]

(With a slight abuse of notation, we continue to denote \( v = \mathbb{E}[Y \mid e = 1] \).) Note that at the optimum, the participation constraints of both agents must bind. Otherwise, the firm can reduce \( b(Y) \) by a small positive amount for all \( Y \) and increase its payoff without violating any constraints. Thus, we can use \((IR^\prime)\) to eliminate \( \mathbb{E}[b(Y) \mid e] \) in the firm’s objective function. Consequently, the firm’s objective function reduces to the total surplus generated by the relationship, i.e.,

\[ \pi = \{ v - 2c \} - \{ w - 2\mathbb{E}_{(Y,r,s)}[w_{rs}(Y) \mid e = 1] \}. \]

Denote \( z := w - 2\mathbb{E}_{(r,s,Y)}[w_{rs}(Y) \mid e = 1] \), i.e., the surplus that is “destroyed” (in expectation) on equilibrium path when the firm relies on the peer evaluation. The firm’s problem can be conceived as one of minimizing \( z \) subject to the set of constraints given in program \( P \); i.e., we can reformulate the problem as:

\[ P' : \begin{cases} \min_{\phi \in \Phi^*} & z = w - 2\mathbb{E}_{(Y,r,s)}[w_{rs}(Y) \mid e = 1] \\ \text{s.t.} & (B^\prime), (T^\prime), (DE), (IC^\prime), \text{ and } (IR^\prime). \end{cases} \]

The optimal contract is the solution to the above program.

5.2. The stage game equilibria. In order to characterize the optimal contract in the repeated game, we first characterize the equilibrium in the stage game in which effort is induced through subjective performance bonus. This characterization turns out to be informative about the repeated game as well. The optimal contract in the stage game must solve the firm’s program that is obtained by setting \( b(Y) = 0 \ \forall Y \) in the program \( P' \). The following proposition characterizes the optimal contract in the stage game.

**Proposition 2.** *(Optimal contract in stage game)* In the optimal contract that induces effort in the stage game, the firm commits to a payroll expense

\[ \hat{w} = \frac{2q_1\beta_1}{\beta_1q_1 - \alpha_1p_1}c, \]

and the agent’s compensation is given by:

\[ \hat{w}_{rs}(Y) = \begin{cases} \frac{1}{2} \hat{w} - \frac{c}{\beta_1q_1 - \alpha_1p_1} & \text{if } Y = y_1 \text{ and } r = 1 \\ \frac{1}{2} \hat{w} & \text{otherwise}. \end{cases} \]
Observe two key features of the optimal contract above. First, the compensation of an agent, $A_i$, depends only on his co-worker $A_{-i}$’s report—what $A_i$ reports on his co-worker’s performance does not affect his own compensations. This, in particular, implies that it is incentive-compatible for each agent to report the signal about his co-worker’s performance truthfully. Notice that in the optimal contract, the firm gives up the opportunity to sharpen incentives using relative and/or joint performance evaluation (see, for instance, Che and Yoo, 2001). We will return to this issue later.

Second, the agents receive the entire committed payroll expense, except in the case where both the public and private signals are the worst possible ones—i.e., the team output is at its lowest and the agent gets the worst possible peer evaluation. To see the intuition, note that the optimal contract minimizes the amount of surplus destruction needed to induce effort. This is achieved by punishing the agent only when both public and private signals indicate the worst performance. As this signal is least likely to be realized when the agent is exerting effort, the above contract minimizes surplus destruction in equilibrium while still giving the agent sufficient incentive to exert effort.

This finding is consistent with the observation in the management literature that firms often do not use peer evaluations to determine pay but, rather, use them as inputs for staff training and developmental initiatives (Budman and Rice, 1994; Pieperl, 1999). While this literature cites inherent biases in peer evaluations as the key reason behind this observation (May and Gueldenzoph, 2006), our finding highlights that in some contracting environments, such sparing use of peer evaluation (in determining workers’ pay) may indeed be the optimal compensation policy.

Our result also echoes the findings in the literature on the single-worker compensation with subjective evaluation. In these models, the worker is “punished” only when the firm obtains the signal that is most informative of shirking (i.e., the signal least likely to arise when agents exert effort). In other words, the worker’s pay remains constant under all other signal realizations. This type of wage compression arises both in a static setting à la MacLeod (2003) and in a dynamic setting à la Fuchs (2007).

It is important to note an important difference between our setting and those discussed above: In our setting the contract must simultaneously address the agents’ moral hazard in signal reporting and effort provision. Indeed, less surplus could be destroyed in equilibrium if an agent was paid based on both the report he receives and the report he sends. For example, when an agent receives a bad report but sends a good one, he could be punished, and the penalty is paid to the other agent. However, such a contract fails to generate an adequate incentive for effort provision because an agent can shirk and then avoid punishment by lying on his report (i.e., sending a bad report on his co-worker). As mentioned earlier, an optimal contract in our setting must account for such “double deviations” where a deviation in effort provision may be coupled with a deviation in the peer evaluation reporting strategy.

This observation highlights the critical difference between the subjective (i.e., private) and objective (i.e., public) peer-evaluation. If the signal on peer-evaluation, $s_i$, is public, the double deviation problem does not arise and the firm can attain the first-best by using a contract that parallels the optimal contract in our baseline model. Thus, the subjectivity of the peer-evaluation necessarily compromises the efficiency of the optimal contract. And as we will see later, these characteristics of the optimal contract in the stage game continue to hold in the repeated game as well.
Observe that the expected surplus destruction under the optimal contract (in Proposition 2) is given by

\[ \hat{\eta} := \hat{w} - \frac{2\alpha_1 p_1}{\beta_1 q_1 - \alpha_1 p_1} \mathbb{E}_{\{Y, r, s\}}[\hat{w}_{rs}(Y) | e = 1]. \]

Thus, the firm’s payoff under the optimal contract is \( \check{\pi} = v - 2c - \hat{\eta} \). It follows that \( \check{\pi} \geq 0 \) if and only if

\[ \frac{\beta_1 q_1}{\alpha_1 p_1} \geq \frac{v}{v - 2c}. \]

Specifically, if the above condition fails, then the unique stage game equilibrium is the one with no effort exertion. Otherwise, there exist multiple equilibria: one in which the agents do not exert any effort, and another where the firm induces effort using subjective performance bonus as described in Proposition 2. We state this formally in the proposition below (we omit the proof as it is already discussed above).

**Proposition 3. (Stage-game Nash equilibria)** Suppose that \( \frac{\beta_1 q_1}{\alpha_1 p_1} < \frac{v}{v - 2c} \). Then there exists a unique Nash equilibrium in the stage game in which agents do not exert any effort. Otherwise, there exist multiple Nash equilibria in the stage game:

(i) In the worst equilibrium for the firm, the agents do not exert effort and all players get their outside option of 0.

(ii) In the best equilibrium for the firm, agents exert effort and the firm earns \( v - \frac{2\beta_1 q_1}{\beta_1 q_1 - \alpha_1 p_1} c \).

The intuition behind this result is as follows. Condition (4) simply states that it is worthwhile for the firm to induce effort through surplus destruction if and only if the performance signals are sufficiently precise. That is, the likelihood ratio \( \beta_1 q_1/\alpha_1 p_1 \) for the signal that triggers money burning (i.e., \( r = 1, Y = y_1 \)) needs to be sufficiently large. The larger is this likelihood ratio the more likely it is that the signal \( r = 1 \) and \( Y = y_1 \) is obtained when the agent shirks rather than exerts effort. Thus, when \( \beta_1 q_1/\alpha_1 p_1 \) is large, it is easier for the firm to induce effort—a relatively small spread between the reward (for good performance) and punishment (for poor performance) would induce the agent to exert effort. Consequently, only a small amount of surplus needs to be destroyed in equilibrium. Thus, the firm may find it profitable to induce effort through subjective performance bonus. How large does the likelihood ratio need to be? Condition (4) specifies the required threshold which is decreasing in \( v \) and increasing in \( c \). This is intuitive as the larger is the value that the workers create and smaller is the cost of effort, the more likely it is that the surplus generated through effort outweighs the surplus burnt in inducing effort.

On the other hand, if \( \beta_1 q_1/\alpha_1 p_1 \) is small, then effort exertion has little effect on the relative likelihood of the worst signals being realized, and the firm must make the spread in the agent’s compensation sufficiently large in order to induce effort. This implies that a considerable amount of surplus must be destroyed and, consequently, the firm’s payoff may become negative—a scenario in which it is optimal for the firm not to induce effort and earn its outside option of 0.
5.3. Characterizing the optimal contract in the repeated game. We now use our understanding of the stage game to solve for the optimal relational contract. Proposition 3 shows that subjective performance bonus is used in a one-shot game if and only if condition (4) holds. It turns out that the same condition continues to determine whether subjective performance bonus is used in the repeated setting.

Proposition 4 below characterizes the optimal relational contract. But before we present the formal statement, it is instructive to summarize three salient implications of this proposition. First, when the firm is patient enough ($\delta$ is large), then the optimal incentive scheme uses only the relational contract: The firm pays a fixed bonus for all public signals above a certain threshold, and subjective performance bonus is a lump sum that is independent of the agent’s performance (consequently, there is no surplus destruction on the equilibrium path). Second, when the firm is relatively impatient ($\delta$ is small), then it may use subjective performance bonus to sharpen effort incentives. But its decision to use subjective performance bonus depends on whether condition (4) holds—if condition (4) is violated, (i.e., in the stage game equilibrium, effort cannot be induced with subjective performance bonus), then the firm never uses subjective performance bonus to generate effort incentives. Finally, whenever subjective performance bonus is used to provide incentives, the optimal contract has features very similar to those in the stage game: An agent is “punished” through low subjective performance bonus only if both the lowest public output and the lowest private signal are realized, and an agent’s subjective performance bonus does not depend on the report he submits about his co-worker.

**Proposition 4. (Optimal contract)** Let $y_{k^*}$ denote the lowest value of the team output $y_j$ such that $\alpha_j > \beta_j$. There exists a threshold discount factor $\delta^* \in (0, 1)$ such that the following holds:

(A) If $\frac{\beta_{y_1}}{\alpha_1 p_1} < \frac{v}{v - 2c}$, then (i) subjective performance bonus is not used: $w = w^*$ and $w_{rs}(Y) = w^*/2$ for all $r, s$, and $Y$; (ii) Public performance bonus is feasible if and only if \( \delta \geq \delta^* \), and in this case, the public bonus associated with the optimal contract is of the form

$$b^*(Y) = \begin{cases} b^* & \text{if } Y \geq y_{k^*} \\
0 & \text{otherwise} \end{cases},$$

where $b^* = \frac{c}{\sum_{j=k^*}^N (\alpha_j - \beta_j)}$; and (iii) agents exert effort if and only if $\delta \geq \delta^*$.

(B) If $\frac{\beta_{y_1}}{\alpha_1 p_1} \geq \frac{v}{v - 2c}$, then (i) for $\delta \geq \delta^*$, the optimal contract is as described above and both agents exert effort; (ii) for $\delta < \delta^*$, the subjective performance bonus takes the form: $w = w^{**}$ and

$$w_{rs}(Y) = \begin{cases} \frac{1}{2}w^{**} - \Delta & \text{if } Y = y_1 \text{ and } r = 1 \\
\frac{1}{2}w^{**} & \text{otherwise} \end{cases},$$

where $\Delta > 0$. The public bonus takes the form

$$b^*(Y) = \begin{cases} b^{**}(\delta) & \text{if } Y \geq y_{k^*} \\
0 & \text{otherwise} \end{cases},$$

where $b^{**}(\delta)$ is increasing in $\delta$. Also, both agents exert effort.
Proposition 4 indicates that it is optimal to combine incentives only when the firm is impatient (δ is low) and the signals (private, public or both) are a sharp measure of the agents’ performance. And as equation (4) suggests, the smaller is the value (v) and the larger is the cost (c) of effort, the more precise the signals have to be for subjective performance bonus to be a part of the optimal contract.

Moreover, the subjective performance bonus is used (to punish the agents) only when the lowest team output is realized and at least one of the agents receives the worst report from his co-worker. That is, in equilibrium, subjective performance bonus is “compressed” across both public and private performance measures. This is similar to the results on wage compression discussed earlier for the static game. Our result shows that wage compression extends not only to a team-production environment, but also to environments with a richer set of performance signals that allows for both public and private measures.

To see the intuition behind Proposition 4, notice that providing incentives through subjective performance bonus involves surplus destruction, but no surplus needs to be destroyed while providing incentives through public bonus payments. However, a public bonus requires the promise to be credible, which, in turn, depends on the future benefits from the relationship (this is the essence of the dynamic enforcement constraint (DE)). If the firm is sufficiently patient (i.e., when δ is high), the future gains from the relationship between the agents and the firm is high, and the range of credible bonus payments is considerably large. Specifically, the firm can use public bonus alone to generate sufficiently strong incentives to achieve the first best.

However, when the firm is not very patient (i.e., when δ is low), then its ability to offer large bonus payments is limited. Consequently, the firm will not be able to induce effort from the worker if it relies only on the bonus payments. In this case, subjective performance bonus may be used to help induce effort. Notice that in Proposition 4 (part (B)), b∗∗(δ) is increasing in δ. This implies that the more impatient the firm is, the more it will rely on the subjective performance bonus payments to induce effort.

But the subjective performance bonus affects both sides of the (DE) constraint. On the one hand, subjective performance bonus reduces the public performance bonus amount necessary to induce effort. This makes (DE) easier to satisfy. On the other hand, subjective performance bonus leads to surplus destruction, making (DE) harder to satisfy. When condition (4) holds, the subjective signals are more informative of the peer’s effort. As a result, the benefit from reducing the public bonus outweighs the cost of surplus destruction, and, thus, subjective performance bonus can be used to relax (DE).\footnote{Note that the optimal bonus payment when condition (4) is violated (i.e., under the contract described in Proposition 4 (A)) is reminiscent of the optimal relational contract under moral hazard in Levin (2003).}

Notice that the double deviation problem discussed in the context of Proposition 2 (for the stage-game) continues to be relevant in the repeated setting. In order to minimize surplus destruction, we might expect that the optimal contract destroys surplus only when the team output is the lowest and the peer reports are the worst. Further, surplus destruction might even be avoided by punishing an agent for poor performance while transferring the penalty amount to his co-worker. But as discussed earlier, this is not incentive compatible, and consequently, even in the repeated setting, an agent’s report on his co-worker does not affect his own pay.

We conclude this section with the following observation: As the agents get the same payoff irrespective of the reports they send, it may appear that the agents may collude in
their reporting strategy by always sending good reports for each other. While collusion remains an important issue, notice that such a reporting strategy may be counterproductive for an agent as it would encourage his partner to shirk and therefore reduces the expected public performance bonus for the agent himself. To see this, note that under the optimal contract, both agents are indifferent between working and shirking. If agent $A_1$ always sends a good report about $A_2$, this will change $A_2$'s payoff, both when he works and when he shirks. When $A_2$ works, his expected payoff increases by $\alpha_1 p_1 \Delta$ as his payoff changes only when $Y = y_1$ and $r = 1$, which happens with probability $\alpha_1 p_1$. But, when $A_2$ shirks, such mis-reporting increases his expected payoff by $\beta_1 q_1 \Delta > \alpha_1 p_1 \Delta$. In other words, if $A_1$ always covers for $A_2$, $A_2$'s payoff may be higher when he shirks; hence, $A_2$ might shirk, and, in turn, lower $A_1$'s expected public performance bonus as the team output is likely to fall. We will briefly revisit this issue in the conclusion and also in Appendix B.

5.4. Optimal contracting with correlated signals. A key feature of the optimal contract is that an agent’s subjective performance bonus does not depend on his report about his co-worker. This observation is perhaps not surprising when the private signals are statistically independent (conditional on effort)—an agent’s report on his co-worker is not informative about his own effort level and making the agent’s pay independent of his own report trivially ensures truth-telling. But when signals are correlated, one might expect that an agent’s subjective performance bonus will depend on his report, as this signal contains information about his action. Interestingly, we find that this is not necessarily the case.

To be concrete, consider a variation of our setting in which the public signal is still uncorrelated with the private signals, but the two private signals are correlated (conditional on effort). In particular, let the level of correlation to vary with the effort level; e.g., suppose that private signals are correlated only if both workers exert effort. Note that under such correlated private signals, there are two ways of detecting shirking. On the one hand, a worker is more likely to have shirked if the co-worker sends a bad report about him. On the other hand, the worker is more likely to have shirked if his report does not match the report of the co-worker. Intuitively, the optimal contract should use one or the other of two channels depending on which one is more effective in detecting shirking.

The interested reader may refer to the working paper version of this article (Deb et al. 2015) for a complete analysis of this setting. It is straightforward to characterize the conditions under which the optimal contract compares reports to detect shirking (and punishes mismatch of reports). It turns out that, an agent’s compensation remains independent of his report on his co-worker’s performance unless the signal correlation in equilibrium is significantly strong. The intuition is that, when using subjective signals the contract must induce the agents to truthfully report their signals and address the double-deviation problem. If compensation depends on relative performance, an agent always has an incentive to manipulate his report: It may be too costly to mitigate this effect unless the correlation between signals is sufficiently high. An interesting implication of this observation is that even when the private signals are correlated, if the correlation is not sufficiently strong, the firm finds it optimal to disregard some (potentially valuable) information as it ignores the report sent by the worker while determining his pay.

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12One can think of the public signal as being generated by an independent source outside the team, such as a client satisfaction survey while the private signals are correlated due to some common shock.
6. Discussion

This section presents a few remarks about our modeling assumptions.

Information Structure: Below, we discuss three assumptions on our information structure. First, in our general model we assume that the agents exert effort towards a joint output. It is important for our results that the public performance signal reflects the agents’ joint effort choice. If, instead, there are multiple public signals, each reflecting the individual effort choices of the two agents, then the first best could be achieved. Specifically, consider a setting in which the agents work on independent projects each generating a public signal about the output but the agents still get private signals about each other’s effort choices. In this case, the firm’s problem is separable in two independent parts. Essentially, the firm can offer two independent contracts as described in our baseline model, where one contract induces effort from the first agent using the second agent in the manager’s role and the other contract does the opposite. Our baseline analysis implies that the first best can be achieved.

Second, we assume that the peer evaluations are reported privately. This is consistent with what is commonly observed. In most peer evaluation systems in practice, feedback is submitted privately, often even anonymously, in order to encourage candid reporting (Edwards and Ewen, 1996). An alternative possibility is to assume that peer reports are public. In this case, the double deviation problem remains but as there is no truth-telling constraint on the firm, the contract performance weakly improves. However, a complete characterization of the optimal contract is difficult, and beyond the scope of this article. As the public bonus is a function of the output and the reports, the truth-telling and the dynamic enforcement constraints becomes closely intertwined and renders the optimal contracting problem loses analytical tractability.

Finally, note that we do not consider self-evaluation in our model. While the agents observe and report a signal on their co-worker’s performance, they do not observe or report any additional signal on their own performance. When an agent’s self-evaluation is merely a report on his own effort level, such evaluations do not help the contract performance as the evaluations do not vary on the equilibrium path. But, if the agents receive additional signals on their own performances, self-evaluation may improve the contract performance, albeit such additional reporting may aggravate the double-deviation problem discussed earlier. A formal analysis of self-evaluation, however, is beyond the scope of our model. Our modeling choice is also consistent with what we observe in practice: Though self-appraisals are common in many organizations, much of the literature suggests that self-appraisal is used more as a developmental tool than as a performance evaluation tool (see, for instance, Mabe and West, 1982; and Wexley and Klimoski, 1984).13

Contract Space: We adopt a standard contract specification used in the literature on subjective evaluation. Our setup parallels that of MacLeod (2003) who considers a single agent setting with subjective evaluation in which the firm commits to a payroll expense. In our multiple agent setting we could assume that the firm commits to a payroll expense for each agent. Instead, we assume that the firm commits to a total payroll expense. This is a weaker

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13In the management literature, the usefulness of self-appraisals (except as a vehicle for personal development) is highly debated, mainly because of the lack of convergence between self-appraisals and supervisors’ ratings (see Campbell and Lee, 1998). Inconsistencies in feedback are often attributed to the “leniency bias” in self-evaluation (Xie et al., 2006; Nilsen and Campbell, 1993; Yammarino and Atwater, 1997).
assumption and also economizes on notation. In addition, commitment to a payroll expense is a standard assumption in the literature on team incentives that dates back to Malcomson (1986).

In the literature, a common alternative to explicit money-burning is to implement money-burning via termination; see for example, Levin (2003) and Fuchs (2007). Our main findings are unaffected if we use termination (instead of money-burning) as a disciplining device. The difference is that if termination is used, the amount of money-burning is limited by the size of the surplus in the relationship. As a result, the relational contract becomes less effective when the relationship has a smaller surplus.

While money-burning is necessary in our setting, it can be mitigated in a more general environment. For example, suppose that the firm interacts with several teams independently. The money that is burnt in one team is given evenly to all agents in the other teams; so, in the aggregate, money is never burnt. Similarly, consider the case in which the team has at least three workers. In this case, the money that is burnt for one worker can be given equally to all other workers who are not the evaluators, and again there is no money-burning in the aggregate. However, if monetary transfer among agents is not frictionless, (e.g., if agents are liquidity constrained) surplus destruction may again be necessary.

Strategy Space: We have already mentioned that it is necessary to restrict the class of strategies in our model in order to ensure tractability. And in this class of strategies, we induce truth-telling from the firm by using money-burning. It is worth noting that the literature on mechanism design and repeated games has suggested alternative monitoring devices (see, e.g., Crémér and McLean, 1988; and Ben-Porath and Kahneman, 1996). A commonly used strategy in this literature is to cross-check the reports from interested parties and punish them if there is a mismatch. In a recent article, Rahman (2012) adopts a different approach of “monitoring the monitor.” In his setting, a principal hires a monitor to supervise a worker, but the monitor privately observes the worker’s deviation. To overcome the private monitoring problem, on occasions, the principal secretly asks the agent to shirk and rewards the monitor if he is able to detect these “prompted deviations.” To apply Rahman’s strategy in our setting (to induce truth-telling by the firm), we would have to allow the agents to occasionally prompt each other on what peer evaluation to report, and then check if the firm reports back truthfully. We would have to allow agents to observe each other’s peer reports or credibly communicate their reports to each other, as well as allow side payments between agents. We do not adopt this alternate setting as this is not consistent with peer evaluation mechanisms that are observed in practice.

7. Conclusion

This article studies the optimal contracting problem of a firm in environments where the performance measures are non-verifiable and the workers have superior knowledge about each other’s performance. The firm can use relational contracts to tie pay to publicly observable performance measures, but it can also attempt to elicit private information from the workers and use this information for compensation design. As the workers’ information is inherently private and subjective, the agents and the firm must have the right incentives for truthful revelation of information.

\[^{14}\text{We thank Andy Skrzypacz for suggesting this example. See Malcomson (1986) for a formal analysis of such a contract.}\]
Our analysis suggests that when the informative and productive roles of a worker can be separated, contracts can achieve the first-best outcome, as the private information can be obtained without incurring any additional agency cost. However, if workers perform both informative and productive roles, then the use of private information is a more delicate matter. We show that the worker’s effort incentives cannot be decoupled from his truth-telling incentives. In particular, when the worker’s peer evaluation affects his own pay, contracts that are robust to single deviations may fall victim to a double-deviation in which the worker shirks and then lies about his peer’s performance. The interconnectedness of effort and truth-telling incentives implies that the firm may neglect some information about the worker’s effort to elicit truthful peer evaluations. The optimal contract fails to achieve first-best and must involve surplus destruction on the equilibrium path. Peer evaluations are used in the optimal relational contract only when the associated level of surplus destruction is small and the firm is relatively impatient.

We conclude with brief remarks on two issues. First, we have assumed that subjective peer evaluation is performed in each period. Frequent feedback about performance is not always optimal: For example, Fuchs (2007) shows that when subjective evaluation is used, reducing the frequency of evaluation can mitigate the amount of surplus destruction. In addition, Fong and Li (2013) show that less information can sometimes help in sustaining the relational contract by reducing the principal’s temptation to renge on the bonus. A natural next step is to study how different information-collection processes affect organizational performance. In addition to questions about the frequency of peer evaluations, one can also ask how peer evaluations interact with other information sources, such as self-evaluations. These issues are crucial in practice, and further research in this direction is needed.

Second, an important issue related to information collection is the potential for collusion among the workers. Collusion is known to be a general problem in contracting environments with multiple agents (see, for example, Laffont and Martimort, 2000). However, the extent to which collusion poses a problem depends on the nature of the team. Following the classification of teams suggested by Scott and Einstein (2001), collusion is more of an issue in the “work and service” teams, where a group of workers are assigned to a “routine manufacturing or service tasks.” In contrast, one would expect collusion to be less of an issue if agents do not anticipate repeated interaction. This may be the case in “project teams” that are formed for a specific task and usually reconfigured upon completion of the task. Calzolari and Spagnolo (2009) make a similar observation in the context of procurement contracting environments where small groups of frequently interacting agents help sustaining strong relational incentives but are also more conducive to collusive behavior.

How does collusion affect the optimal contract in our setting? Note that the optimal contract may not be “collusion-proof” in the sense that workers may be able to avoid the punishment associated with a bad peer evaluation by always submitting good reports about each other. But as noted earlier (in section 5) such a collusion scheme may not be profitable for the agents as it encourages both the agents to shirk. In fact, simple collusive schemes where the agents collude on both their effort decisions and reporting behavior may also hurt the agents for the same reason—when agents shirk in every period, they exert a negative externality on each other through lowering the expected bonus associated with the public output (see Appendix B for a formal example). This observation leads to the natural question of how the workers can collude more generally while maintaining their effort levels. How
collusion affects the design of peer evaluations and, thus, how information is collected within organizations is an interesting topic for future research.

APPENDIX

Appendix A. This appendix contains the proofs omitted in the text.

Proof of Proposition 1. Trivially, the proposed contract yields the first-best payoff of \( v - c \). So, it remains to show that this contract is indeed feasible. Note that (B) is satisfied as \( \hat{w}_s^A(Y) + \hat{w}_s^M(Y) = c \) (that exactly equals the committed payroll expenditure) for all \( Y, s, \) and \( r \). To check (T), note that for all \( s, \) \( E_Y [w_s^M(Y) | e_A = 1, s_M = s] = 0 \). For \( s \neq s_1 \), this holds by construction and for \( s = s_1, E_Y [w_s^M(Y) | e_A = 1, s_M = s] = \alpha_1 \Delta_A + \alpha_N (-\Delta_M) = 0 \). Similarly, \( E_Y [w_s^M(Y) | e_A = 1, s_M = s] = 0 \) for all \( s \) and \( s' \). Next, consider (IC): \( E_{\{Y,s\}}[w_s^A(Y) | e_A = 1] - c = c + \alpha_1 p_1 (-\Delta_A) + \alpha_N p_1 (\Delta_M) - c = 0 \), and \( E_{\{Y,s\}}[w_s^A(Y) | e_A = 0] = c + \beta_1 q_1 (-\Delta_A) + \beta_N q_1 (\Delta_M) = 0 \). The above observations also imply that both (IR\(_A\)) and (IR\(_M\)) bind. Hence, the proposed contract is indeed feasible. \( \blacksquare \)

Proof of Proposition 2. Step 1. Note that (T') must bind. For any two \( s \) and \( s' \in S \), we must have 
\[
E_{\{Y,r\}}[w_{rs}(Y) | e = 1, s_i = s] \geq E_{\{Y,r\}}[w_{rs'}(Y) | e = 1, s_i = s],
\]
and 
\[
E_{\{Y,r\}}[w_{rs'}(Y) | e = 1, s_i = s'] \geq E_{\{Y,r\}}[w_{rs}(Y) | e = 1, s_i = s'].
\]
Combining these two inequalities and using the fact that the expectations are independent of \( s_i \) (as the distribution of \( \{r, Y\} \) conditional on \( e \) is independent of \( s \)), we obtain that for all \( s \) and \( s' \in S \),
\[
E_{\{Y,r\}}[w_{rs}(Y) | e = 1] = E_{\{Y,r\}}[w_{rs'}(Y) | e = 1]. \quad (T'')
\]

Step 2. Observe that (T'') implies that \( E_{\{Y,r\}}[w_{rs}(Y) | e = 1, s_i = s] \) must be independent of \( s \). Now, the (IR) can be rewritten as 
\[
E_s [E_{\{Y,r\}}[w_{rs}(Y) | e = 1, s_i = s]] \geq 0.
\]
Thus, it must be the case that 
\[
E_{\{Y,r\}}[w_{rs}(Y) | e = 1, s_i = s] = E_{\{Y,r\}}[w_{rs}(Y) | e = 1] = 0 \quad \forall s \in S. \quad (IR'')
\]
Otherwise, the firm could achieve a higher payoff by reducing its committed payroll expense \( w \) and all subjective performance bonus \( w_{rs}(Y) \) by a small amount \( \varepsilon > 0 \) while still not violating any constraints. Now, if (IR'') holds, (T'') must hold as well. Also, using (IR''), one can rewrite (IC') as 
\[
0 \geq E_{\{Y,r\}}[w_{rs'}(Y) | e_i = 0] \quad \forall s' \in S. \quad (IC'')
\]
Thus, the firm’s problem reduces to maximizing \( v - w \) subject to (B'), (IR'') and (IC'').

Step 3. Define the Lagrangian for the firm’s problem as 
\[
\mathcal{L} = (v - w) + \sum_{s \in S} \sum_{r \in S} \sum_{j=1}^N \eta_{jr} (w - w_{rs}(y_j) - w_{sr}(y_j)) + \sum_{s \in S} \lambda_s \sum_{j=1}^N \alpha_j (\sum_{r \in S} p_r w_{rs}(y_j) - c) + \sum_{s \in S} \mu_s \sum_{j=1}^N (-\beta_j \sum_{r \in S} q_r w_{rs}(y_j)).
\]
Now, the first-order conditions with respect to \( w \) and \( w_{rs}(y_j) \) \( \forall j, s, \) and \( r \) are:

\[
\sum_{r,s \in S} \sum_{j=1}^{N} \eta_{sr}^{j} = 1, \text{ and}
\]

\[
\lambda_s \alpha_{j} p_r - \mu_s \beta_{j} q_r = \eta_{sr}^j + \eta_{rs}^j.
\]

Also, the complementary slackness conditions are: \( \forall j, r, \) and \( s \),

\[
\eta_{sr}^j (w - w_{rs}(y_j) - w_{sr}(y_j)) = \lambda_s \sum_{j=1}^{N} \sum_{r \in S} \alpha_{j} p_r w_{rs}(y_j) - c = -\mu_s \sum_{j=1}^{N} \sum_{r \in S} \beta_{j} q_r w_{rs}(y_j) = 0.
\]

**Step 4.** We claim that there exists a set of non-negative Lagrange multipliers such that the proposed compensation schedule along with the set of multipliers satisfy the first-order conditions and the complementary slackness conditions.

Note that \( \sum_{j=1}^{N} \alpha_{j} \sum_{r \in S} p_r w_{rs}(y_j) - c = \sum_{j=1}^{N} \beta_{j} \sum_{r \in S} q_r w_{rs}(y_j) = 0 \forall s \in S \). Also note that \( \tilde{w} - \tilde{w}_{rs}(y_j) - \tilde{w}_{sr}(y_j) = 0 \forall j > 1 \) and \( \tilde{w} - \tilde{w}_{rs}(y_j) - \tilde{w}_{sr}(y_j) = 0 \forall r, s > 1 \). Thus, any set of \( \eta_{sr}^j \), \( \lambda_s \) and \( \mu_s \) values satisfies the complementary slackness conditions as long as

\[
\eta_{sr}^j = \eta_{sr}^1 = 0 \quad \forall \, r, s \in S.
\]

Now, consider the following values of the multipliers:

\[
\mu_s = \nu (q_1 p_s - q_s p_1) \quad \text{for some } \nu > 0, \quad \text{and } \lambda_s = \mu_s \beta_1 q_1 / \alpha_1 p_1.
\]

By MLRP, \( q_1 p_s - q_s p_1 \) = \( q_1 q_s (p_s / q_s - p_1 / q_1) > 0 \). Hence, \( \mu_s \geq 0 \), and therefore, \( \lambda_s \geq 0 \) \( \forall s \in S \). Also, given the proposed values of the multipliers in (7) and (8), it is routine to check that the first-order condition (6) is satisfied if \( s \) and/or \( r = 1 \).

Hence, it remains to show that one can find values of \( \eta_{sr}^j \) for \( s, \ r > 1 \), such that they satisfy the first-order conditions.

**Step 5.** For \( s, \ r > 1 \), let

\[
\eta_{sr}^j = \eta_{rs}^j = \frac{1}{2} \left( \lambda_s \alpha_{j} p_r - \mu_s \beta_{j} q_r \right) = \frac{\nu \beta_1 q_1 \beta_j q_r}{2 \alpha_1 p_1} \left( q_1 p_s - q_s p_1 \right) \left( \frac{\alpha_j p_r}{\beta_j q_r} - \frac{\alpha_1 p_1}{\beta_1 q_1} \right),
\]

and

\[
\nu = \left[ \sum_{j=1}^{N} \sum_{s, r \in S} \frac{\beta_1 q_1 \beta_j q_r}{2 \alpha_1 p_1} \left( q_1 p_s - q_s p_1 \right) \left( \frac{\alpha_j p_r}{\beta_j q_r} - \frac{\alpha_1 p_1}{\beta_1 q_1} \right) \right]^{-1}.
\]

As before, by MLRP, \( \eta_{sr}^j \geq 0 \). Also, by construction, the first-order conditions (6) are now satisfied. Thus, the proposed compensation scheme, along with the Lagrangian multipliers, satisfies all first-order conditions and the complementary slackness condition. Hence, the proposed compensation scheme is optimal.

**Proof of Proposition 4. Step 1.** We argue that without loss of generality, we can restrict attention to the class of contracts where

\[
w = w_{rs}(Y) + w_{sr}(Y) \quad \text{for all } Y > y_1.
\]
We show this by contradiction. Suppose that this is not true for some \( y_j > y_1, r, \) and \( s = s^* \in S \). Consider a new subjective performance bonus scheme where

\[
\begin{align*}
w_{r,s}^*(y_j) &= w_{r,s}^*(y_j) + \varepsilon, \\
w_{r,s}^*(y_1) &= w_{r,s}^*(y_1) - \frac{\alpha_1}{\alpha_1} \varepsilon,
\end{align*}
\]

and \( w_{r,s}'(Y) = w_{r,s}(Y) \) for all \((r,s,Y) \neq (r,s^*,y_1)\) or \((r,s^*,y_j)\). Note that under the new payment scheme, (a) the value in the objective function does not change; (b) constraint \((B')\) remains satisfied; and (c) \((DE)\) is not affected. Now to check \((T')\), note that if the agent reports \( s^* \), we have, by construction,

\[
\mathbb{E}_{(Y,r)} [w_{rs}^*(Y) | e = 1] = \mathbb{E}_{(Y,r)} [w_{rs}'(Y) | e = 1].
\]

And if the agent reports \( s' \neq s^* \), then

\[
\mathbb{E}_{(Y,r)} [w_{rs}'(Y) | e = 1] = \mathbb{E}_{(Y,r)} [w_{rs}'(Y) | e = 1],
\]

since \( w_{rs}'(Y) = w_{rs}(Y) \) for all \((Y,r,s')\) where \( s' \neq s^* \). Finally, to check \((IC')\), we need to make sure that

\[
\mathbb{E}_{(Y,r,s)} [b(Y) + w_{rs}'(Y) | e = 1] - c \geq \mathbb{E}_{(Y,r,s)} [b(Y) + w_{rs}(Y) | e = 1] \forall s'.
\]

Note that for all \( s \), by construction,

\[
\mathbb{E}_{(Y,r,s)} [b(Y) + w_{rs}'(Y) | e = 1] - c = \mathbb{E}_{(Y,r,s)} [b(Y) + w_{rs}(Y) | e = 1] - c.
\]

On the right-hand side of \((IC')\), if \( s' \neq s^* \),

\[
\mathbb{E}_{(Y,r)} [b(Y) + w_{rs}'(Y) | e_i = 0] = \mathbb{E}_{(Y,r)} [b(Y) + w_{rs}(Y) | e_i = 0]
\]

as none of the values are changed. And if \( s = s^* \), we have

\[
\mathbb{E}_{(Y,r)} [b(Y) + w_{rs}'(Y) | e_i = 0] = \mathbb{E}_{(Y,r)} [b(Y) + w_{rs}(Y) | e_i = 0] + \beta_j \varepsilon - \beta_j \frac{\alpha_1}{\alpha_1} \varepsilon < \mathbb{E}_{(Y,r)} [b(Y) + w_{rs}(Y) | e_i = 0].
\]

This shows that if under any contract, we have \( w > w_{rs}(Y) + w_{sr}(Y) \) some \( Y > y_1, r, \) and \( s \), there always exists another contract that makes the inequality bind but gives the same payoff to the firm.

**Step 2.** We claim that if the surplus destruction is positive, then the agent’s \((IC')\) constraint

\[
\mathbb{E}_{(Y,r,s)} [b(Y) + w_{rs}(Y) | e = 1] - c \geq \max_{s'} \mathbb{E}_{(Y,r)} [b(Y) + w_{rs}(Y) | e_i = 0] \forall s'
\]

must bind with equality. To see this, suppose that the above is slack. Now define

\[
w_{rs}'(y_1) = (1 - \varepsilon) w_{rs}(y_1) + \frac{\varepsilon}{2} w.
\]

In this case,

\[
\mathbb{E}_r [w_{rs}'(y_1) | e = 1] = \sum_{r \in S} p_r \left[ (1 - \varepsilon) w_{rs}(y_1) + \frac{\varepsilon}{2} w \right] = (1 - \varepsilon) \sum_{r \in S} p_r w_{rs}(y_1) + \frac{\varepsilon}{2} w.
\]

This implies that the truth-telling constraint \((T')\) remains satisfied. We can also check that \((B')\) and \((DE)\) are also satisfied. For small enough \( \varepsilon \), the \((IC')\) constraint remains satisfied. Note that this change reduces the surplus destruction by \( \varepsilon \) (in proportion). This implies that the \((IC')\) constraint must bind.
**Step 3.** Define $k^*$ to be the unique index such that $\alpha_{k^*} > \beta_{k^*}$ but $\alpha_{k^*-1} < \beta_{k^*-1}$. If the surplus destruction is positive, then there exists a $b$ such that

$$b(Y) = \begin{cases} b & \text{if } Y \geq y_{k^*} \\ 0 & \text{otherwise} \end{cases}.$$  

Note that for $j < k^*$, by decreasing $b(y_j)$, we (weakly) relax $(DE)$ and $(IC')$, and we do not affect other constraints. Similarly, for $j \geq k^*$, by increasing $b(y_j)$, we relax $(IC')$ and we do not affect other constraints. In this case, $(DE)$ remains satisfied as long as $b(y_j) = \max_i\{b(y_i)\}$. Also, note that the above value of $b(y_j)$ is unique. If there exists a $b(y_j) < b$ for some $j \geq k^*$, we can increase $b(y_j)$ by $\varepsilon_1$ and relax the $(IC')$ constraint. But from Step 2 above, we know that if $(IC')$ is relaxed, then the firm can set

$$w_{rs}'(y_1) = (1-\varepsilon_2)w_{rs}(y_1) + \frac{\varepsilon_2}{2}w,$$

for some small enough $\varepsilon_2$, and reduces the the level of surplus destruction.

**Step 4.** Next, consider the case in which effort can be induced using only the relational contract (i.e., no surplus is destroyed in equilibrium). From $(IC')$ and the formulation of the optimal bonus pay as given in Step 3, we obtain that $b = b^* := 2c/\sum_{j=k^*}^N(\alpha_j - \beta_j)$ if effort must be induced by using bonus pay alone. In this case, a necessary and sufficient condition for sustaining such a relational contract is

$$\frac{\delta}{1-\delta}[v - 2c] \geq \frac{2c}{\sum_{j=k^*}^N(\alpha_j - \beta_j)}. \tag{9}$$

The above condition is satisfied when $\delta$ is higher than a threshold, say $\delta^*$, at which the above inequality binds.

**Step 5.** If $\delta < \delta^*$, the optimal contract may use subjective performance bonus, and surplus will be destroyed. In this case, suppose that the maximum bonus is given by $b$; then, we can rewrite the $(DE)$ constraint in the firm’s program $\mathcal{P}'$ as

$$\frac{\delta}{1-\delta}\pi \geq 2b. \tag{DE}$$

(Recall that the punishment payoff of the firm, $\pi$, is 0). Now the program can be solved in two steps. First, for a fixed $b$, we choose $w_{rs}(y_1)$ to minimize the surplus destruction, and second, we choose the largest $b$ for which the above $(DE)$ holds. Note that in the first step, the problem is the same as the static one with the agent’s cost of effort equal to

$$c(b) = c - \sum_{j=k^*}^N(\alpha_j - \beta_j)b. \tag{10}$$

Therefore, we can apply Propositions 2 and 3 to conclude the following: If

$$\frac{\beta_1q_1}{\alpha_1p_1} \geq \frac{v}{v - 2c(b)}, \tag{11}$$

then the firm uses the following subjective contract in equilibrium: There exists some $w$ so that

$$w_{rs}(y_1) = \begin{cases} \frac{1}{2}w - \frac{\alpha_1p_1c(b)}{\beta_1q_1-\alpha_1p_1} & \text{if } r = 1 \\ \frac{1}{2}w & \text{otherwise}, \end{cases}$$
and the expected surplus destruction in this case is

$$z = \frac{2\alpha_1 p_1 c(b)}{\beta_1 q_1 - \alpha_1 p_1}.$$ 

Otherwise, no subjective performance bonus is used. This solves for the first step. In the second step, for \((DE)\) to be satisfied, we need

$$\frac{\delta}{1 - \delta} \left[ v - 2c - \frac{2\alpha_1 p_1 (c - \sum_{j=k}^N (\alpha_j - \beta_j) b)}{\beta_1 q_1 - \alpha_1 p_1} \right] \geq 2b. \tag{12}$$

That is, we need to find the largest \(b\) for which the above condition (12) holds. Note that at \(b = 0\), the left-hand side is

$$\frac{\delta}{1 - \delta} \left[ v - 2c - \frac{2\alpha_1 p_1 c}{\beta_1 q_1 - \alpha_1 p_1} \right] = \frac{\delta}{1 - \delta} [v - 2c - \hat{z}].$$

Here we have two cases.

**Case 1:** If \(\beta_1 q_1 / (\alpha_1 p_1) < v / (v - 2c)\) (i.e., effort cannot be induced in a Nash equilibrium of the stage game), \(v - 2c - \hat{z} < 0\). Also, at \(b = c / (\sum_{j=k}^N (\alpha_j - \beta_j))\), condition (12) is violated (recall that we are considering the case where (9) fails). Thus, there is no value of \(b\) that satisfies (12). So, \(b = 0\). As (11) is also violated in this case (note that \(c(0) = c\)), no subjective performance bonus is used either.

**Case 2:** If \(\beta_1 q_1 / (\alpha_1 p_1) \geq v / (v - 2c)\) (i.e., effort can be induced in a Nash equilibrium of the stage game), \(v - 2c - \hat{z} \geq 0\). Also, as noted above, at \(b = c / (\sum_{j=k}^N (\alpha_j - \beta_j))\), the condition (12) is violated. Since both sides of condition (12) are linear in \(b\), this implies that the highest \(b\) that satisfies (12), \(b^* (\delta)\) (say), is the one at which (12) holds with equality. That is,

$$b^* (\delta) = \frac{\delta [\beta_1 q_1 (v - 2c) - \alpha_1 p_1 v]}{2 \left[ \beta_1 q_1 (1 - \delta) - \alpha_1 p_1 \left( 1 - \delta \left( 1 - \sum_{j=k}^N (\alpha_j - \beta_j) \right) \right) \right]}.$$

This observation completes the proof. 

**Appendix B.** In this appendix we show that stationary collusion—i.e., in all periods, all agents shirk but always send good peer evaluations—may not be incentive compatible as it hurts the agents by exacerbating the free-riding problem associated with the effort provision. But to simplify the analysis, we study a special case of our model by considering a binary signal structure and a specific form of the production function.

Consider a team with two agents where in every period, each agent exerts effort \(e \in \{0, 1\}\) to produce a team output \(Y \in \{0, y\}\). Let \(\Pr(Y = y) = \alpha (e_1 + e_2)\) where \(\alpha < 1/2\). As before, assume that each agent \(A_i\) receives a signal \(s_i \in \{0, 1\}\) that is informative of his co-worker \(A_{-i}\)'s effort level. We assume that \(\Pr(s_i = 1 \mid e_{-i} = 1) = p\) and \(\Pr(s_i = 1 \mid e_{-i} = 0) = 0\). Let the cost of effort be \(c\) if \(e = 1\) and 0 otherwise. By Proposition 4, we know that the optimal contract is of the following form: for each agent \(A_i\), the firm commits to a payroll expense \(w\) and promises an additional bonus \(b\) (> 0) if \(Y = y\). Moreover, a part, \(\Delta\) (> 0),
of the payroll expense is burnt if \( Y = 0 \) and the report received by \( A_i, r_i \) (say), is 0. Thus, the agent \( A_i \)'s compensation \( \omega_i \) is:

\[
\omega_i = \begin{cases} 
  w + b & \text{if } Y = y \\
  w & \text{if } Y = 0 \text{ and } r_i = 1 \\
  w - \Delta & \text{if } Y = 0 \text{ and } r_i = 0
\end{cases}
\]

Now, we argue that there exists a \( p^* \) such that for all \( p > p^* \), the optimal contract uses subjective evaluation and the agent will not engage in stationary collusion (i.e., choose \( r_i = 1 \) and \( e_i = 0 \) in all periods) if the following condition holds:

\[
\frac{\delta}{1 - \frac{1}{\delta}} \frac{1}{2} (\alpha y - c) < \frac{c}{\alpha} < \frac{\delta}{1 - \frac{1}{\delta}} (\alpha y - c).
\]

The argument is as follows. Notice that the agent’s (IR) and (IC) constraints are given as:

\[
\begin{align*}
  w + 2ab - (1 - 2\alpha) (1 - p) \Delta - c & \geq 0, \quad \text{(IR)} \\
  \alpha b + (p + (1 - 2p) \alpha) \Delta & \geq c. \quad \text{(IC)}
\end{align*}
\]

Also, the firm’s non-reneging constraint (DE) is given by

\[
b \leq \frac{\delta}{1 - \frac{1}{\delta}} (\alpha y - c) - (1 - 2\alpha) (1 - p) \Delta. \quad \text{(DE)}
\]

In the optimal contract, it is clear that the agent’s (IR) constraint must be binding, i.e., \( w = (1 - 2\alpha) (1 - p) \Delta + c - 2ab \). Thus, the optimal contracting problem minimizes the surplus destruction and is given as:

\[
\min_{b, \Delta} \Delta \quad \text{s.t. (IC) and (DE)}
\]

Note that when the first-best is not achieved the (IC) must be binding, so \( b = \frac{1}{\alpha} (c - (p + (1 - 2p) \alpha) \Delta) \). Plugging the value of \( b \) back in (DE) (which must also bind at the optimum) one obtains that that the optimal \( \Delta \) is given by

\[
\Delta^* = \frac{c}{\alpha} - \frac{\delta}{1 - \frac{1}{\delta}} (\alpha y - c) - \frac{\delta}{1 - \frac{1}{\delta}} (1 - 2\alpha) (1 - p).
\]

Notice that for \( p \) sufficiently close to 1, we must have

\[
\frac{1}{\alpha} (p + (1 - 2p) \alpha) > \frac{\delta}{1 - \frac{1}{\delta}} (1 - 2\alpha) (1 - p),
\]

and, thus, \( \Delta^* > 0 \) for such a \( p \). Given \( \Delta^* \), it follows that the optimal contract sets:

\[
b^* = \frac{1}{\alpha} (c - (p + (1 - 2p) \alpha) \Delta^*), \quad \text{and } w^* = (1 - 2\alpha) (1 - p) \Delta^* + c - 2ab^*.
\]

Notice also that \( b^* < c/\alpha \), and therefore, if the agents engage in stationary collusion, each prefers to shirk. As a result, his payoff is given by \( w^* \). But when \( p = 1 \), we have

\[
\Delta^* = \frac{1}{1 - \alpha} \left( c - \frac{\alpha \delta}{1 - \frac{1}{\delta}} (\alpha y - c) \right), \quad b^* = \frac{\delta}{1 - \frac{1}{\delta}} (\alpha y - c),
\]

and

\[
w^* = c - \frac{2\alpha \delta}{1 - \frac{1}{\delta}} (\alpha y - c).
\]
Note that under condition (13), $w^* < 0$. Since $w^*$ is continuous in $p$, it follows that there exists a $p^*$ such that for all $p > p^*$, $w^* < 0$. Therefore, the agent will not engage in stationary collusion as it would yield a negative payoff and violate the agent’s participation constraint.

REFERENCES


