Are Reservations Recommended?

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We examine the role of reservations in capacity-constrained services with a focus on restaurants. Although customers value reservations, restaurants typically neither charge for them nor impose penalties for failing to keep them. However, reservations impose costs on firms offering them. We demonstrate that reservations can increase a firm's sales by altering customer behavior. When demand is uncertain, reservations induce more customers to patronize the restaurant on slow nights. The firm must then trade off higher sales in a soft market with sales lost to no shows on busy nights. We consequently evaluate various no-show mitigation strategies, all of which serve to make reservations more likely in equilibrium. Competition also makes reservations more attractive; when there are many small firms in the market, it is never an equilibrium for none to offer reservations.

Key words: Service management; reservations; restaurants; capacity management.

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1 Introduction

Restaurant reservations are a curious phenomenon. Customers value them, but restaurants give them away. Indeed, firms such as TableXchange and Today's Epicure have tried to profit from the resulting arbitrage opportunity by creating markets to sell reservations. What makes offering reservations even more remarkable is that they are costly to provide. Fischer (2005) identifies three costs to offering reservations. These include additional staff needed to take reservations and added complexity from having to balance the needs of walk-in customers with commitments made to reservation holders. Even if technology is substituted for personnel, the restaurant must pay licensing and commission fees to services such as OpenTable. One New York restaurateur estimates his annual costs of offering reservations at $125,000 (Collins, 2010).

Fischer's final consideration is no shows. Customers can generally fail to keep reservations without penalty, but restaurants suffer if they hold capacity for customers that never come. No shows represent a real problem. Bertsimas and Shioda (2003) report a no-show rate of 3% to 15% for the restaurant they studied. More generally, rates of 20% are not unusual (Webb Pressler, 2003) and special occasions such as New Year's Eve can push rates to 40% (Martin, 2001).

Why then should restaurants offer reservations? One reason is the operational benefits they provide.

Reservations regulate the flow of work. By staggering seatings, a restaurateur can assure that waiters are not overwhelmed by a rush of customers followed by the bartender and kitchen being swamped with orders. Reservations thus allow fast service without excessive capacity (Fischer, 2005). Reservations would then be appealing when either customers are delay sensitive or the firm's costs increase with arrival variability. Further, reservations may allow a restaurant to estimate demand and improve staffing and sourcing decision. This is particularly important when variable costs are high. Chicago-based Alinea is known for intricate, multicourse menus. It requires that customers make reservations. To the extent that Alinea incurs significant expense in preparing for its guests, knowing how many are coming on a given night is crucial.

We abstract from these issues and focus on reservations' ability to influence consumer behavior and thus increase sales. We consider a restaurant with limited capacity whose sole decision is whether or not to offer reservations. (Our model also applies to other firms that serve both reservations and walk ins customers and whose revenues depend on choices customers make after they arrive for service.) At the time reservations are made (assuming they are offered), customers are uncertain how they will value the service at the time of consumption. Customers learn their valuations before traveling to the service facility. Since there is a fixed cost to accessing the facility, those with low realized values fail to keep their reservations. If reservations are not offered, customers learn their value for the service and decide whether to walk in for service. Walking in incurs a travel cost and a risk of an additional penalty if one cannot get a seat.

We assume the market-size is uncertain and takes one of two values. The firm can serve all customers on a slow night, but it lacks the capacity to serve everyone on a busy night. When reservations are not offered, customers risk not getting a seat despite having incurred the cost to travel to the firm. This is costly to the firm on slow nights since some customer who would have had a positive net utility from dining out choose to stay home. Reservations counteract this by guaranteeing all customers in the market a seat on a slow night. However, reservations also impose a penalty on the firm because some sales will be lost to no shows on busy nights. The restaurant hence faces a trade off as reservations are valuable in a weak market but costly in a strong one. We derive conditions under which this comparison favors offering reservations.

That is, reservations may indeed be recommended. Further, we explore various strategies for limiting the impact of no shows. We also show that competition can make offering reservations even more attractive. In a market with many firms, it is unlikely that none of the firms will offer reservations.

Below, we review the literature. §3 presents the basic model while the following section analyzes the trade off between reservations and relying on walk-in traffic. §5 evaluates strategies of mitigating no shows. §6 examines the impact of competition. §7 concludes. Proofs are in the Appendix.

2 Literature review

The existing literature on reservations or advance sales emphasizes pricing or segmentation. In Png (1989), a firm with limited capacity sells to risk-averse customers who are uncertain of their need for a service at some future point. The firm offers reservations and overbooks; bumped customers receive a specified compensation. Reservations act as insurance and allow for higher prices than either selling outright in advance or selling in the spot market. Dana (1998) considers a competitive market with capacitated firms. Market segments differ in their willingness to pay and the certainty with which they need service. If those with low valuations are more certain to need the service, the firms sell some capacity early at a discount but raise the price closer to the time of service delivery. In DeGarba (1995), advance selling allows the firm to capture the same revenue as first-degree price discrimination. Xie and Shugan (2001) and Shugan and Xie (2005) develop similar ideas. (See Courty, 2000, and Shugan, 2002, for reviews.) None of this work addresses the restaurant industry. Restaurants do not charge in advance, and menu prices are independent of holding reservations. Also, they do not publicize compensation schemes for reservation holders denied service. Further most of these papers focus on monopoly settings (Dana, 1998, and Shugan and Xie, 2005, are exceptions) while we extend our analysis to competitive environments.

Dana and Petruzzi (2001) study a newsvendor model in which customers incur a cost to shop. The stocking level thus affects demand. They focus on determining the stocking level and do not consider altering the timing of sales. Deneckere and Peck (1995) examine similar issues in a competitive environment. We take the restaurant's capacity as exogenous but allow reservations. See Netessine and Tang (2009) for more on the role of strategic consumers in operations management.

Appointment systems have been widely studied, particularly in health care. See, for example, Robinson and Chen (2003) or Savin (2006). Gupta and Denton (2008) provides a survey. This work generally ignores strategic consumer actions. Lariviere and Van Mieghem (2004) consider delay-sensitive consumers choosing arrival times. An appointment system allows them to pick sequentially and results in an arrival pattern that minimizes both period-to-period variation in arrivals and total delay costs. Customers are not allowed to decline seeking service. We ignore delay costs but explore how reservations can increase sales by encouraging more customers to come in.

There have been some studies applying revenue management to restaurants. Thompson (2010) provides a review and notes that existing research does not clearly state under what conditions reservations should be offered. We show that reservations are advantageous when demand is uncertain if the no show rate is sufficiently low. Bertsimas and Shioda (2003) consider how many reservations a restaurant should accept and how to accommodate walk-in customers given reservation commitments. Both demand streams are exogenous. Here, offering reservations changes the number of customers patronizing the firm. Çil and Lariviere (2010) examine a model with two segments, one of which will only patronize the firm if reservations are offered while the other incurs a cost to walk in. They examine how capacity should be allocated between the segments and show that it may be optimal to save no capacity for late arriving walk-in customers even if they are more valuable to the firm. We focus on a single segment who can be served either with or without reservations and focus on the question of whether reservations should be offered at all.

3 Model fundamentals

We consider one restaurant serving customers in a single sales period. Customers are *a priori* homogeneous. The net utility from planning to stay home is normalized to zero. Each customer incurs a cost $T \geq 0$ to travel to the restaurant whether or not she holds a reservation. A walk-in customer who fails to get a seat incurs a denial cost $D \geq 0$, which represents the inconvenience of changing plans. A strictly positive denial cost implies that the opportunities open to a customer who has been turned away are not as attractive as the customer's outside option of not visiting the restaurant. Note that while both T and D are costs incurred by customers, they are not paid to the firm.

While *a priori* homogeneous, customers are *ex post* differentiated by their value for dining out. Each customer draws a valuation independently from a known, continuous distribution, $F(V)$ with density $f(V)$ and support $(0, \Omega)$ for $\Omega > T$. $\bar{F}(V) = 1 - F(V)$. Customers learn their valuations before incurring the travel cost. Hence, only those whose realized valuation V exceeds T will even consider patronizing the restaurant. We will term $V - T$ the customer's net utility.

The number of customers in the market is uncertain and takes one of two levels, θ or 1 with $\theta > 1$.[1] High demand occurs with probability ρ for $0 < \rho < 1$. Market demand uncertainty does not reflect variation over observable conditions (e.g., Fridays are busier than Tuesdays) but variation given specific circumstances. One should interpret the model as saying that given that it is Friday night, the restaurant will be busy with probability ρ. Customers and the firm cannot learn the market size until they interact. That is, the firm learns it is a busy night from the number of customers that request service while customers can update their belief on whether it is a busy night based on whether they are able to get a reservation or receive service.

Customers are atomistic; they are sufficiently small relative to the market that the only aggregate uncertainty is the total number of customers in the market. Thus, while it is uncertain whether any specific consumer will have a positive net utility from dining out, the number of customers interested in dining out is certain to be $\theta \bar{F}(T)$ on a busy night and $\bar{F}(T)$ on a slow night.

The firm's sole decision is whether to offer reservations. Its menu and pricing are fixed, and the expected spending is the same for all customers regardless of their realized value for dining out. The restaurant's objective is thus to maximize unit sales. The firm can serve K customers per evening where we assume that every customer requires the same amount of capacity. That is, we are not differentiating based on party size. One can interpret the model as assuming that all arrivals to the restaurant are parties of the same size.

We assume $\theta \bar{F}(T) > K > 1$. On a slow night, the restaurant can serve everyone who would have a

[1] More generally we could have demand as taking values θ_H and θ_L with $\theta_H > K > \theta_L$. Setting $\theta_L = 1$ just represents a normalization of demand and capacity to economize on notation.

positive net utility to dining out but cannot do so on a busy night.

Events proceed as follows. The firm announces its policy. The number of customers in the market is realized. If it offers reservations, customers simultaneously decide whether or not to ask for one. Requesting customers learn immediately if they are successful. Subsequently, each customer learns her valuation V. Reservation holders then decide whether to use them while non-holders simultaneously decide whether to walk in. As customers arrive, reservation holders are seated first. If the number of walk-in customers exceeds the available capacity, seats are rationed randomly. Thus, if M reservations have been given out and $\kappa > K - M$ customers walk in, the probability of any one walk-in customer being seated is $(K - M)/\kappa$. This implicitly assumes the restaurant does not realize a given reservation holder is a no show until it is too late to give her seat to a walk-in customer. (We will discuss allowing the firm to re-offer the seats of no shows in §5.) If reservations are not offered, customers wait to learn their valuations and then decide whether or not to walk in. If the number of walk ins exceeds capacity, seats are rationed randomly.

4 Analysis: Walk ins versus reservations

To determine whether reservations are recommended we need to contrast firm sales under two policies, offering reservations and relying solely on walk-in traffic. We examine the latter first.

4.1 The no reservation case

When reservations are not offered, a customer with realized valuation V faces a lottery. By spending the travel cost T, she either receives a benefit of V or incurs a penalty of D. The probability of getting a seat δ is thus critical to her decision. Let δ denote her anticipated probability of getting a seat. Her expected utility from walking in is $\delta V - (1 - \delta) D - T$, and she will walk in if $\delta \geq \frac{T+D}{V+D}$. Since all customers have the same chance of getting a seat, anyone with realized valuation $V' \geq V$ will also walk in if the customer with value V walks in. Thus, the equilibrium between customers will be in the form of a cutoff strategy in which all customers whose valuations exceed some value $V^* \geq T$ attempt to receive service. Following standard arguments (Deneckere and Peck, 1995; Dana and Petruzzi, 2001), the chance of getting a seat is given by the fill rate. If only those with valuations exceeding V^* walk in, the chance of getting a seat is:

$$\delta\left(V^*\right)=\frac{\bar{F}\left(V^*\right)(1-\rho)+K\rho}{\bar{F}\left(V^*\right)(1-\rho+\rho\theta)}. \tag{1}$$

Proposition 1 *Suppose the restaurant does not offer reservations.*

1. The unique equilibrium has all customer with valuations greater than V^* walking in to the restaurant, where V^* is found from
$$\bar{F}\left(V^*\right)=\frac{\rho K\left(V^*+D\right)}{\rho\theta\left(T+D\right)-\left(1-\rho\right)\left(V^*-T\right)}. \tag{2}$$
The equilibrium probability that an arriving customer gets a seat is $\frac{T+D}{V^*+D}$.
2. $\bar{F}\left(V^*\right)>K/\theta$, and V^* is increasing in ρ.
3. The firm's sales are
$$\left(1-\rho\right)\bar{F}\left(V^*\right)+\rho K. \tag{3}$$

In equilibrium, the marginal customer must be indifferent between walking in and staying home. Hence, it must be the case that there is a positive chance of not being seated, i.e., we must have $\theta\bar{F}\left(V^*\right)>K$ and the restaurant is oversubscribed on busy nights. At the same time, some customers with a positive net valuation stay home so the number of customers walking in on a slow night is just $\bar{F}\left(V^*\right)$. Thus, while the restaurant fully utilizes its capacity on busy night, it loses some potential sales on slow nights. Demand uncertainty is essential to this last point. As ρ goes to zero and demand is certain to be low, V^* goes to T and the only customers who stay home are those with a negative net utility for dining out.

It is straightforward to show that V^* increases with θ, T, and D, but falls in K. Further, consider two valuation distributions, F and G, and let V_F^* and V_G^* be the corresponding cutoffs. If $F\left(V\right)\geq G\left(V\right)$ for all V, $V_F^*\leq V_G^*$. A higher cutoff value does not translate into a smaller crowd. The chance a walk-in customer gets a seat falls as V^* increases (holding T constant and D constant). A better restaurant (in the sense of stochastically larger valuation distribution) is more crowded. Also, as less capacity is available, fewer customers walking in does not offset the loss of seats; the chance of getting a seat falls as K decreases.

4.2 Reservations and the restaurateur's problem

For simplicity, suppose the restaurant's entire capacity is offered via reservations. (We relax this assumption in §5.) It does not overbook, so reservation holders are guaranteed seats. (We discuss overbooking in

§7.) If demand exceeds K, reservations are rationed randomly. We do not *a priori* assume reservations eliminate walk ins; if fewer than K reservations are given out, the restaurant will accept walk-in customers. However, in equilibrium, a restaurant will not serve both reservation and walk ins. To see why, let $\mathbb{V} = E\left[\max\left\{0, V - T\right\}\right]$. $\mathbb{V}$ is a representative customer's expected value for holding a reservation. Because $\mathbb{V} \geq 0$, all customers request one, and the restaurant commits all of its capacity to reservations. Recall that the firm is unable to re-offer the seats of no-show reservation holders. A customer who was unable to secure a reservation consequently does not walk in because she knows that she will not get a seat.

If the restaurant offers reservations, it will give out 1 on a slow night and K on a busy nights. Its sales will then be

$$(1-\rho)\,\bar{F}\left(T\right) + \rho K \bar{F}\left(T\right). \tag{4}$$

Comparing (4) and (3) allows us to determine when reservations are recommended.

Proposition 2 *The restaurant offers reservations if*

$$(1-\rho)\left(\bar{F}\left(T\right) - \bar{F}\left(V^*\right)\right) \geq \rho K F\left(T\right). \tag{5}$$

Reservations are never offered if

$$F\left(T\right) \geq \frac{(1-\rho)\left(1-\frac{K}{\theta}\right)}{1-\rho+\rho K}. \tag{6}$$

Reservations alter the behavior of some customers whose realized valuations fall between T and V^*. When reservations are not offered, these customers are certain to stay home since their expected reward is too small relative to the cost of walking in. If, however, reservations are offered and they manage to secure one, they patronize the firm. On slow nights, everyone in the market receives a reservation and the firm's sales are thus certain to increase. The outcome on busy nights is less sanguine. When reservations are not offered, capacity is fully utilized so it is not possible to increase the firm's sales. Instead, reservations lower sales. The firm pre-commits capacity to customers who ultimately do not want service (i.e., have realized valuations below T) and thus become no shows.

If the firm opts to offer reservations, it must be that the gain on slow nights outweighs the loss on busy nights. Reservations are consequently important on evenings in which the firm has plenty of available seats but are costly when the place is hopping. That is, the restaurant offers reservations not to manage demand when business is good but to entice more customers to come in when demand would otherwise be slow.

Costly no shows are essential to our story: The trade off goes away if they are not an issue. If customers knew their valuations when making reservations, sales in the high state would be K, and the firm would always offer them. Alternatively, if the no show rate $F(T)$ is too high, reservations are never offered, as the bound (6) demonstrates. The bound is decreasing in K and ρ. Hence, for any given no show rate there exists a capacity level or chance of a large market sufficiently high that reservations would never be offered.

Proposition 3 *Suppose that for a given set of parameters K, θ, D, and T and valuation distribution F, (5) holds and the firm prefers offering reservations.*

1. The restaurant would offer reservations for any K' such that $1 < K' \leq K$.
2. The restaurant would offer reservations for any $\theta' \geq \theta$.
3. The restaurant would offer reservations for any $D' \geq D$.
4. The restaurant would offer reservations for any valuation distribution G larger than F in the reversed hazard rate order.[2]
5. The restaurant would offer reservations for any $T' \leq T_0$ if $F(\psi T)/F(T)$ is decreasing in T for $\psi > 1$.

Offering reservations becomes more attractive (in the sense that the gain from offering reservations relative to relying in walk ins increases) as shrinking capacity or an increasing denial penalty further limits walk-in demand. Similarly, when a larger valuation distribution raises the walk-in cutoff, reservations become more attractive. Together these give the empirical predictions that smaller restaurants or better quality restaurants (in the sense of higher valuation distributions) are more likely to offer reservations.

The roles of T and ρ are less clear. A higher travel cost lowers both reservation and walk-in sales. Reservation sales fall because no shows increase in all states. Walk-in sales fall because a higher T implies a higher V^*, lowering sales in the low demand state. The first effect dominates, and reservations become less attractive as T and the resulting no-show rate increases. The required regularity condition holds for many common distributions.

[2] For distributions F and G with respective densities f and g, F is smaller than G in the reversed hazard rate order if $\frac{f(V)}{F(V)} \leq \frac{g(V)}{G(V)}$ for all $V \geq 0$ (Shaked and Shanthikumar, 1994).

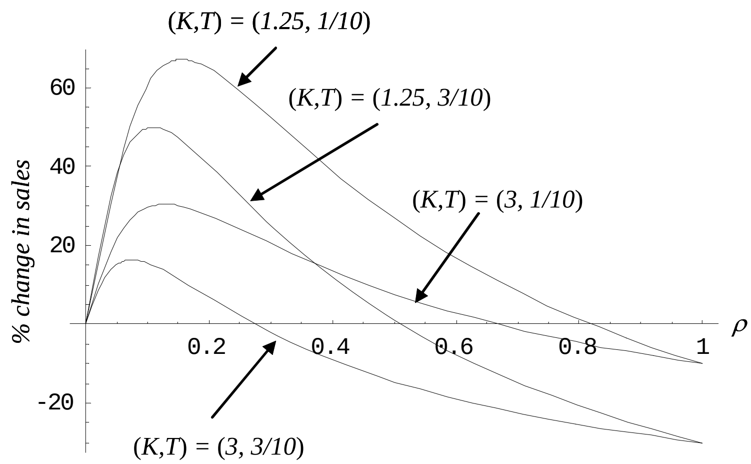

Figure 1: Percent change in sales when moving from not offering reservations to offering reservations. $F(v) = v$ for $0 \leq v \leq 1, D = 7/4$, and $\theta = 5$.

While reservation sales increase linearly with the probability of high demand, walk-in sales are not necessarily monotone in ρ. It is consequently difficult to generate comparative statics analytically. However, we know that sales with and without reservations are equal as ρ falls to zero but relying on walk ins is always better for ρ sufficiently high. This suggest that the gains from offering reservations are not monotone in ρ. Figure 1 show that this can, indeed, be the case. The percentage increase in sales peaks at intermediate values of ρ while the location of the peak tends to be at lower values of ρ for higher travel costs. Further, as discussed above, the gain from reservations is higher when capacity is tight.

5 Mitigating no shows

In the preceding analysis, no shows create a trade off. The restaurant wants to offer reservations in order to increase sales on slow nights but must weigh this gain against sales lost to no shows when demand is high. The firm then has an incentive to limit the impact of no shows. Here, we consider three such strategies: re-offering seats, implementing a partial reservation policy, and imposing a no-show penalty.

5.1 Re-offering seats

In our base model, no shows translate directly into lost sales. Alternatively, one might suppose that at least some reservation holders call to cancel or that the restaurant only briefly holds seats for late-arriving reservations customers. In either scenario, the firm would have some capacity that although initially committed to reservations holders can be offered to walk-in customers. Here we suppose that a fraction $\lambda \in [0,1]$ of the capacity held for no shows is made available to walk-in customers. If K reservations are given out, $\lambda F(T)K$ seats will be available for walk ins.

We also suppose customers rationally anticipate that capacity will be available. The availability of re-offered seats is only relevant when customers have been denied reservations (i.e., on busy nights). With a single segment, these customers are the only ones who might walk in. Thus, we are assuming that customers are willing to walk in even though they know that all capacity had been previously committed to reservations.

Given these assumptions, it is straightforward to show that customers who were denied reservations use a cutoff equilibrium. They know that it is a busy night and walk in if their valuation exceeds some value $V_\lambda > T$. The resulting number of walk ins $(\theta - K)\bar{F}(V_\lambda)$ must exceed the available number of seats $\lambda F(T)K$.

In such a setting, the firm's sales when offering reservations are

$$(1-\rho)\bar{F}(T) + \rho K\left(\bar{F}(T) + \lambda F(T)\right),$$

and the restaurateur prefers relying on reservations over walk ins if

$$(1-\rho)\left(\bar{F}(T) - \bar{F}(V^*)\right) \geq \rho K(1-\lambda)F(T). \tag{7}$$

Comparing (7) with (5), we see that re-offering seats lowers the costs of no shows and makes offering reservations attractive over a larger range of parameters. In particular, as the fraction of seats re-offered λ goes to one, offering reservations is always the best policy. To the extent that the restaurant can affect λ (e.g., limiting how long it will hold tables for late arrivals), it will seek to increase λ as much as possible.

5.2 Partial reservations

Re-offering limits the cost of no shows by increasing the salvage value of capacity. An alternative approach is to minimize the exposure to no shows by limiting the number of reservations given out. Thus far, we have assumed that the firm makes its entire capacity available via reservations. Clearly, this is suboptimal.

The firm would not offer reservations if it were sure that demand was going to be high. Once reservation requests exceed the low demand level of one, the restaurateur knows demand is high and could do better by refusing any more reservation requests. Giving out additional reservations increases the number of no shows but does nothing to increase demand on slow nights.

We now study allowing the firm to set a reservation level M such that $0 \leq M \leq 1$; $K - M$ seats are thus available for walk-ins customers. We suppose that capacity unclaimed by no shows cannot be re-offered (i.e., no shows result in lost sales). We again assume that customers are again willing to consider walking in even if they have been denied a reservation. These customer follow a cutoff equilibrium with those whose realized value for the service exceeds V_M walking in.

While the structure of the equilibrium is intuitive, the behavior of V_M is not immediately obvious. On the one hand, fewer walk-in customers are in the market (relative to the no reservation case) since some already hold reservations. This suggests that V_M should be decreasing in M. However, there are two countervailing forces. First, those customers are competing for less capacity since M seats have already been allocated to reservation holders. Second, being denied a reservation is informative since this is more likely to happen when demand is high. The following lemma shows that these latter factors dominate and that a smaller fraction of customers will walk in as more capacity is given out.

Lemma 1 *V_M is increasing in M.*

Given this equilibrium behavior, the restaurant's sales are

$$\begin{aligned}\Pi(M) &= (1-\rho)\left(M\bar{F}(T) + (1-M)\bar{F}(V_M)\right) + \rho\left(M\bar{F}(T) + K - M\right) \\ &= M\bar{F}(T) + (1-\rho)(1-M)\bar{F}(V_M) + \rho(K-M).\end{aligned}$$

Note that the first term is increasing in M. Since V_M is increasing in M, we then have that the other terms are decreasing.

Proposition 4 *Suppose that the firm makes $0 \leq M \leq 1$ seats available via reservations.*

1. If

$$(1-\rho)\left(\bar{F}(T) - \bar{F}(V^*)\right) \geq \rho F(T), \tag{8}$$

then $M = 1$ is optimal.

2. If (8) fails, then $M = 0$ is optimal.
3. Reservations are never offered if $F(T) \geq (1-\rho)\left(1-\frac{K}{\theta}\right)$.

Out of a continuum of possibilities, only two reservations levels need to be considered. M either maximizes sales on a slow night or is set to zero. For intuition on why intermediate reservations are never optimal, consider the limiting case as D goes to infinity. This would yield a customer equilibrium of $\bar{F}(V_M) = \frac{K-M}{\theta-M}$, and firm sales of

$$M\bar{F}(T) + (1-\rho)(1-M)\frac{K-M}{\theta-M} + \rho(K-M),$$

which is convex in M. The exact shape of the function can vary; it may be monotonically increasing or decreasing or may first decrease and then increase. All result in a corner solution. For finite D, $\bar{F}(V_M)$ generally mimics the behavior of $\frac{K-M}{\theta-M}$, and sales are maximized at either zero or one.

Comparing (8) to (5), we again have that limiting the impact of no shows increases the range of parameters over which reservations are preferred. However, unlike the setting in which all seats can be re-offered, it is never the case that reservations are preferred regardless of the no show rate. Part 3 of the proposition shows that for a given no show rate there exists a capacity level and probability of a large market sufficiently high that reservations are never offered.

5.3 No-show penalties

The previous options manipulate the impact of no shows or the firm's exposure to no shows but do not alter the root cause of the problem, the customer's propensity for failing to come in. Our last alternative directly lowers the no-show rate. We now suppose that the restaurant charges customers a fee $p > 0$ when they fail to keep their reservations. Such charges have existed in the US restaurant industry for well over a decade (Fabricant, 1996) but remain controversial. No-show penalties currently range from \$25 to \$175 (Frumkin, 2009). Such charges might seem exorbitant, but restaurateurs insist that the fees are secondary to shaping customer behavior. As one New York restaurateur put it "It's not really about being punitive. It's about trying to keep the dining room full." (Frumkin, 2009)

If a customer holds a reservation and has a realized valuation less than T, she now faces a choice. She can keep her reservation and receive a net utility of $V - T < 0$ or she can stay home, pay the penalty and have a net utility of $-p$. She thus keeps the reservation if $V > T - p$ and the restaurant experiences a no-show rate of $F(T-p)$.

The question then is how the firm should set the no-show fee. An obvious option is $p = T$ which would eliminate no shows completely. Since this possibility is open to the firm and will result in higher sales than relying on walk ins, we immediately have that the firm will *always* prefer reservations to walk ins if it can charge a no show fee. Remarkably, however, the firm may prefer not to set the no show fee to the travel cost.

To see this point, note that because no-show fees bring revenue to the restaurant, examining only unit sales is no longer sufficient. We must compare the margin on no shows with the margin on actually serving customers. We assume that there are no costs incurred for no shows so the margin on these customers is p and will denote the margin on serving customers as π. The firm's expected margin when giving out a reservation is then

$$\mu(p) = \pi \bar{F}(T-p) + pF(T-p) = \pi + (p-\pi) F(T-p) \text{ for } 0 \leq p \leq T.$$

There are two possibilities to consider. First, if $\pi \geq T$, then we must have $p \leq \pi$ and $\mu(p)$ is increasing in p. The optimal solution then is $p^* = T$, and all no shows are eliminated. Alternatively, if $\pi < T$, the firm can choose a no-show fee such that it makes more money on no shows than on customers who actually show up while still inducing some no shows. When $\pi < T$, this is, in fact, optimal.

Proposition 5 *Suppose that $\pi < T$ and $F(V)$ is log-concave.*

1. The optimal no-show fee p^* is less than T and satisfies
$$p^* = \pi + \frac{F(T-p^*)}{f(T-p^*)}. \tag{9}$$
2. Consider two valuations distributions $F(V)$ and $G(V)$ such that $G(V)$ is larger than $F(V)$ in the reversed hazard rate order. Let p_F^* and p_G^* be the corresponding optimal no-show fees. Then $p_F^* > p_G^*$.

The optimal no-show fee p^* is greater than π, and thus the restaurant is better off when customer *do not* keep their reservations. The proposition is illustrated by the power function distribution,

$$F(V) = \left(\frac{V}{\Omega}\right)^k \text{ for } k > 0 \text{ and } 0 \leq V \leq \Omega,$$

which yields $p^* = \frac{k}{k+1}\pi + \frac{1}{k+1}T$. The no-show fee decreases with k, which also corresponds with $F(V)$ increasing in the reverse hazard rate order.

No-show fees interact with our other mitigation strategies in interesting ways. If the firm eliminates no shows, it is indifferent between giving out all of its capacity or only part of its capacity via reservations. If it sets $p^* < T$, then it strictly prefers giving out as many reservations as possible. If the firm can re-offer a fraction λ of unused seats, it can make money twice on unkept reservations. The restaurant will not eliminate no shows as long as $T > \pi(1-\lambda)$, and the optimal no show fee is decreasing in λ. When there are greater opportunities to re-offer seats, the restaurant is more tolerant of no shows.

Finally, we should highlight an implicit assumption behind our analysis. We have throughout assumed that customers have a positive expected value to holding a reservation and thus all customers in the market would request one. If the firm imposes a no-show fee, this need not be the case since customers with low realized valuations end up with a negative net utility. We have thus implicitly assumed that the valuation distribution puts sufficient weight on large values that holding a reservation has a positive expected value even if $p^* = T$. This holds if $E[V] \geq T$.

6 Competition

We now suppose there are two restaurants, A and B. Travel and denial costs are the same for both firms. Restaurant j has capacity K_j for $j = A, B$. Let $\tilde{K} = K_A + K_B$ and $\alpha = 1 - \beta = K_A/\tilde{K}$. We assume $1 \leq K_A \leq \alpha\theta\bar{F}(T)$ and $1 \leq K_B \leq \beta\theta\bar{F}(T)$. Thus either firm could serve the entire market by itself on a slow night, but together they cannot serve all customers on a busy night even if customers are allocated in proportion to the fraction of industry capacity they control. This structure keeps the competitive market comparable to our monopoly setting since the industry as a whole could serve all interested customers on a slow night but not on a busy night. We relax the assumption that either firm could serve the entire market on a slow night below.

The sequence of events is slightly more involved. Both firms announce their policies, and customers arrive to the market. If at least one offers reservations, customers wanting one make their requests simultaneously before learning their valuations. If both offer reservations, customers seek only one reservation. Once valuations are learned, reservation holders determine whether to use their reservations or stay home. Customers without reservations wait to learn their realized valuations and determine whether to walk in and if so to which firm (*i.e.,* they can only patronize A or B). If no one offers reservations, customers choose whether to walk in (and to which firm) after learning their valuations.

It remains to specify the joint distribution of customer values. We focus on a simple case. Suppose each customer draws a valuation V from distribution F and her value for dining at A is the same as her value for B, *i.e.*, $V_A = V_B$.

We begin with the walk-in equilibrium. Consider a customer with realized value $V > T$. Since she values dining at the two restaurants the same, she goes to the firm offering a better chance of getting a seat. In equilibrium, the firms must offer the same probability of successfully securing a seat. Suppose customers with valuations greater than some $\tilde{V}$ attempt to dine out and randomize between the firms in proportion to their capacities (i.e., a given customer goes to firm A with probability α and to B with probability β). The chance of getting a seat at either firm would then be

$$\frac{\bar{F}\left(\tilde{V}\right)(1-\rho)+\tilde{K}\rho}{\bar{F}\left(\tilde{V}\right)(1-\rho+\rho\theta)}.$$

It remains to find $\tilde{V}$. Since the chance of a walk-in customer getting a seat is the same as when there is a monopolist restaurant with capacity $\tilde{K}$, the cutoff can be found from (2). We take this monopoly setting as the base case against which to compare the competitive outcome.

Now consider A's problem. Suppose B does not offer reservations. If A also forgoes reservations, its sales are $(1-\rho)\,\alpha\bar{F}\left(\tilde{V}\right)+\rho K_A$. If it offers reservations, A gets the entire market in the low demand state. Expected sales are $(1-\rho)\,\bar{F}(T)+\rho K_A\bar{F}(T)$, and reservations are preferable if

$$(1-\rho)\left(\bar{F}(T)-\alpha\bar{F}\left(\tilde{V}\right)\right)\geq\rho\alpha\tilde{K}F(T)\,. \tag{10}$$

Comparing (10) with (5), it is easy to see that if B does not offer reservations, A would find reservations attractive over a larger range of parameters than a monopolist with capacity $\tilde{K}$.

If B offers reservations and A does not, A's sales are ρK_A. If both offer reservations, they must offer customers the same chance of getting a reservation in equilibrium. A's sales are consequently $(1-\rho)\,\alpha\bar{F}\,(T)+\rho K_A\bar{F}\,(T)$, and A offers reservations if

$$(1-\rho)\,\alpha\bar{F}\,(T) \geq \rho\alpha\tilde{K}F\,(T)\,. \tag{11}$$

Proposition 6 *Suppose that firm A is the smaller firm so that $\alpha \leq 1/2$.*

1. The equilibrium has the following structure:
   (a) If $\rho\tilde{K}F\,(T) \geq \frac{1-\rho}{\alpha}\left(\bar{F}\,(T)-\alpha\bar{F}\left(\tilde{V}\right)\right)$, neither restaurant offers reservations.
   (b) If $\frac{1-\rho}{\alpha}\left(\bar{F}\,(T)-\alpha\bar{F}\left(\tilde{V}\right)\right) \geq \rho\tilde{K}F\,(T) > \max\left\{\frac{1-\rho}{\beta}\left(\bar{F}\,(T)-\beta\bar{F}\left(\tilde{V}\right)\right), (1-\rho)\,\bar{F}\,(T)\right\}$, A offers reservations and B does not.
   (c) If $\frac{1-\rho}{\beta}\left(\bar{F}\,(T)-\beta\bar{F}\left(\tilde{V}\right)\right) \geq \rho\tilde{K}F\,(T) > (1-\rho)\,\bar{F}\,(T)$, only one restaurant offers reservations, and it can be A or B.
   (d) If $(1-\rho)\,\bar{F}\,(T) > \rho\tilde{K}F\,(T)$, both restaurants offer reservations.
2. If a monopolist with capacity $\tilde{K}$ would offer reservations, both restaurants offer reservations.
3. If $F\,(T) \geq \frac{(1-\rho)\left(1-\tilde{K}/\theta\right)}{1-\rho+\rho\alpha\tilde{K}}$, reservations are never offered.

Competition makes reservations viable over a larger range of market parameters. This is driven by the smaller firm. The first firm to offer reservations enjoys a windfall in a soft market if its competitor does not match its policy. This slow-night windfall is larger for the small firm because each firm's slow night sales are proportional to the fraction of capacity it controls. The amount of sales lost to no shows is also proportional to capacity implying that the smaller firm pays a smaller price in the high-demand state . Indeed, the amount of no shows the larger firm would incur may be sufficiently big that it would rather forego any low demand sales than lose sales in the high demand state.

Figure 2 provides an example, showing the prevailing equilibrium for combinations of industry capacity and A's share of capacity.[3] For the given parameters, a monopolist would only offer reservations if $\tilde{K} \leq$

[3] Note that we cannot guarantee for a specific set of parameters, that all of the equilibrium possibilities of the proposition exist. Also, the assumption $\alpha\tilde{K} > 1$ limits the range of parameters we can consider. In the region

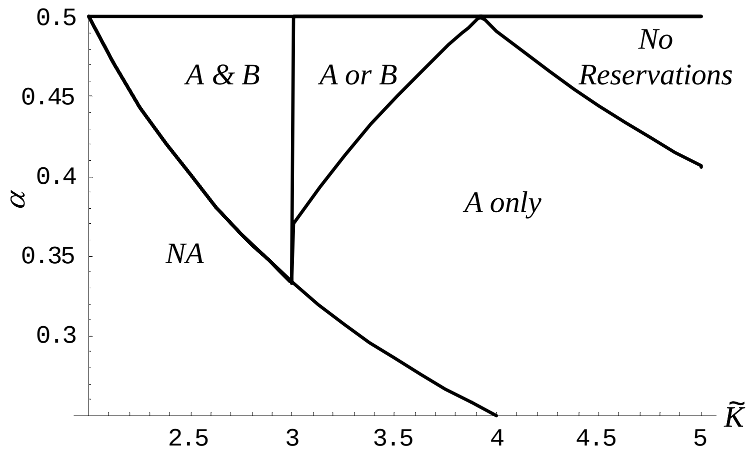

Figure 2: Equilibrium outcome with $F(v) = v$ for $0 \leq v \leq 1$ and parameters $\rho = 0.2$, $T = 1/3$, $D = 1/4$, and $\theta = 12$. Our analysis is not valid in region *NA* since $\alpha < 1/\tilde{K}$.

1.78 but both firms offers reservations for $\tilde{K} \leq 3$ (region "A & B" in the figure). As capacity expands, we move to the region marked "A *or* B"; here, only one firm will offer reservations and it can be A or B. This is the only region in which there are multiple equilibria. As the split of capacity becomes more skewed, only A offers reservations as the larger firm now finds it unprofitable to match offering reservations. If capacity expands sufficiently, walk-in business is robust on slow nights and reservations lead to many no shows on busy nights. Thus, no one offers reservations.

To generalize these results to $N \geq 2$ firms, we consider two cases. In the first, the restaurants are "lumpy." Each has capacity greater than one and thus can serve the entire market on a slow night. In the second case, the firms are arbitrarily small. Any one firm's capacity is inadequate for serving the entire market on a slow night, and the amount of industry capacity available via reservations $\tilde{K}_R$ can take on any value. In both settings, we assume that in any walk-in equilibrium customers first employ a common cutoff value and then split in proportion to firm capacities; the cutoff remains $\tilde{V}$ as discussed above.

Proposition 7 *Suppose that there are $N \geq 2$ firms in the market and that $\theta \bar{F}(T) > \tilde{K} > 1$.*

1. Suppose the firms each have capacity $\tilde{K}/N > 1$. The equilibrium has the following structure:
   (a) If $\tilde{K} > N\frac{1-\rho}{\rho}\left(\frac{\bar{F}(T)}{F(T)} - \frac{\bar{F}(\tilde{V})}{NF(T)}\right)$, no firm offers reservations.

marked NA in the figure, industry capacity is too small relative to the fraction held by firm A and our results do not apply.

(b) If $\frac{N}{R}\frac{1-\rho}{\rho}\frac{\bar{F}(T)}{F(T)} \geq \tilde{K} > \frac{N}{R+1}\frac{1-\rho}{\rho}\frac{\bar{F}(T)}{F(T)}$ for $R = 1, ..., N-1$, then R firms offer reservations.

(c) If $\frac{1-\rho}{\rho}\frac{\bar{F}(T)}{F(T)} \geq \tilde{K}$, all N firms offer reservations.

2. Suppose the firms are arbitrarily small and $F(T) \leq (1-\rho)\left(1 - 1/\tilde{K}\right)$. In equilibrium:

$$\tilde{K}_R = \frac{1-\rho}{\rho}\frac{\bar{F}(T)}{F(T)}. \tag{12}$$

When restaurants are sizeable, the range of industry capacities in which everyone offers reservations is independent of the number of firms in the market but the region in which no one offers reservations is decreasing in the number of restaurants. Thus, splitting market capacity more finely makes having at least one firm offer reservations more likely. The intuition is similar to the duopoly case. The less capacity each firm has, the less is lost to no shows in the high demand state. However, unless market capacity is severely limited, not every restaurant offers reservations. Indeed, the amount of capacity available via reservations $R\tilde{K}/N$ will be very close to $\frac{1-\rho}{\rho}\frac{\bar{F}(T)}{F(T)}$ when N is large.

When firms are small, this holds exactly. A market with many small restaurants will consequently never have all firms following the same policy. Instead, some will offer reservations and have sales levels that do not vary with the realized number of customers. Those that do not offer reservations will make do with boom-and-bust sales. On busy nights, they are turning customers away; on slow nights they are empty.

Before closing this section, we return to our strategies for mitigating no shows and consider their effectiveness in a duopoly. The first two strategies, re-offering seats and partial reservation policies, will remain attractive to the firm in a competitive market. Neither of these strategies affects the customer's decision of whether to ask for a reservation or from whom to ask for one. Thus either strategy serves to reduce the cost of no shows and will expand the range of parameters over which reservations are offered.

The story for no-show penalties is less positive. Suppose that problem parameters are such that without no-show penalties, restaurants in a duopoly would both offer reservations. If we now allow no-show penalties, the firms would still offer reservations but not impose no show-penalties. To see why consider the customer's problem. She is indifferent to holding a reservation from either firm *if* they impose the same no show penalty p^*. However, if one of the firms drops its penalty from p^* to $p^* - \varepsilon$, all customers would prefer this firm. It would get all of the market on a slow night while incurring only a trivial penalty on busy nights. Its competitor then has an incentive to cuts its no-show penalty as well. The firms are effectively pushed into undifferentiated Bertrand competition which ultimately drops the price to zero.

If instead only one firm offers reservations in equilibrium, it can demand a no-show penalty and enjoy higher profits. This would expand the range over which reservations are part of the market equilibrium. There is a caveat. Relative to the monopoly case, the firm imposing a no-show penalty faces an additional constraint in the duopoly setting. No-show penalties lower a customer's expected utility but now she has the option of patronizing the other firm. If the no-show fee is sufficiently high, some customers may opt to wait to learn their valuations and then walk in to the competitor.

7 Conclusion

We have examined whether a restaurant should offer reservations with an emphasis on the ability of reservations to shape customer behavior. Uncertain demand is central to our model. Customers cannot *a priori* verify whether demand is high or low. Consequently, slow nights are exceptionally slow when reservations are not offered because customers with low valuations stay home rather than risk being denied service. Reservations address this problem by guaranteeing potential diners seats, generating more demand in a slack market. The downside is sales lost to no shows in a large market. Thus, reservations are valuable to the restaurant when business is slow but costly when demand is high. In contrast, customers would not value reservations if they knew the market were small but would prize them highly on busy nights.

No shows are hence also crucial to our model, and the restaurateur has an incentive to reduce their impact. If the firm can re-offer the seats of no shows to those initially denied reservations, offering reservations becomes more attractive. The firm might also limit the number of reservations given out. We show that it is either optimal to not take reservations or offer just enough capacity via reservations to serve a small market completely. Alternatively, the restaurant may impose a no-show penalty and completely eliminate no shows. While it is always possible to eliminate no shows, the firm may prefer to allow some if its margin on serving customers is sufficiently small.

Our results carry over to competitive environments. Competition expands the range of parameters over which reservations are offered. This is particularly true for a smaller firm, and a market with many small firms almost certainly has some firms offering reservations.

While we have offered a unique perspective on reservations, it is certainly not the only reason a restaurant may offer free reservations. We have purposefully suppressed reservations' operational benefits to emphasize their impact on consumers. To the extent that the firm can use reservations to manage the flow of work or forecast demand, we have systematically underestimated their value. Further, firms may use reservations to segment customers, relying on them to attract customers with different characteristics. For example, one might suppose the existence of a segment that will only dine out if they have a reservation as in Çil and Lariviere (2010). More subtly, in our model, segments could differ in their travel or denial costs. If a restaurant does not offer reservations, high-cost customers are under-represented among its customers. Reservations move the firm's sales mix closer to the underlying market mix. Tweaking the sale mix would be worthwhile if the segments differ in both their costs and spending proclivities. If high-cost customers are more likely to run up large tabs, reservations would be warranted if the gain in the average bill is sufficient to compensate for the resulting no shows.

There also approaches to limiting the impact of no shows that we have not considered. Most important among these is overbooking. Anticipating that not all customers will keep their reservations, the restaurateur could simply give out reservations above the restaurant's capacity much as airlines overbook flights. There are several reasons why we have not analyzed this option. First, because we have assumed that customers are atomistic, the firm's overbooking problem is trivial. If it gives out $M = K/\bar{F}(T)$ reservations on a busy night, it will yield exactly K customers and never has excess capacity or delayed customers. A more insightful model would have to incorporate discrete customers so there is a possibility that the firm has more than K reservation customer arriving for service. Further such a model would also have to explicitly consider the sequence of arrivals over time. Overbooking in service systems with sequential arrivals and processing is challenging subject because high yields at some point in the planning horizon can impose delays that persist after customers who arrived in an overbooked period have left the system. See, for example, Muthuraman and Lawley (2008) or Liu, Ziya, and Kulkarni (2010). Finally, these potential delays introduce an additional difficulty in analyzing overbooking. A customer with, say, a 7:30 reservation presumably values be seating at 7:30 more than being seated at 8:00. The potential delays that overbooking introduces thus systematically lower the expected value of keeping one's reservation. Hence, the no show rate a firm sees will not be independent of its overbooking policy.

An alternative mitigation strategy is to simply charge for the restaurant's service at the time a reservation is requested. A restaurant would then be much more like an airline or Broadway theater, and the risk of later not wanting the service would be shifted to the customer. Chef Grant Achatz has proposed this scheme for his new Chicago project, Next Restaurant. This model has clear advantages. Besides making the firm's revenue independent of no shows, it allows for differential pricing across days of the week and times in the evening. It also alters the cash flow of the restaurant, allowing the firm to book its revenue before having to layout cash for labor and supplies (Wells, 2010).

Appendix: Proofs

Proof of Proposition 1: In equilibrium, a customer with valuation V^* must be indifferent between walking in and staying home, implying that $\delta(V^*) = \frac{T+D}{V^*+D}$. Equation (2) then follows. For uniqueness, note that the left-hand side of (2) is decreasing in V' while the right-hand side is increasing. $\bar{F}(V^*)$ must exceed K/θ because walks in would otherwise be guaranteed a seat regardless of the demand realization and any customer with a realized valuation V such that $T < V < V^*$ would have a positive expected value to walking in. Given that $\theta\bar{F}(V^*) > K$, the fill rate (1) is decreasing in ρ for a fixed V^*; V^* must increase to compensate. Finally, the expected number of walk-in customers is $\bar{F}(V^*)(1-\rho+\rho\theta)$ and (3) follows from noting that the restaurant can serve at most K customers on a busy night. $\square$

Proof of Proposition 2: Reservations are preferable if they lead to higher sales:

$$(1-\rho)\bar{F}(T) + \rho K\bar{F}(T) \geq (1-\rho)\bar{F}(V^*) + \rho K \Longrightarrow (1-\rho)\left[\bar{F}(T) - \bar{F}(V^*)\right] \geq \rho K\left[1-\bar{F}(T)\right].$$

For the bound on $F(T)$, $\bar{F}(V^*) \geq \frac{K}{\theta}$ so $(1-\rho)\frac{K}{\theta} + \rho K$ underestimates walk-in sales. Reservations are never offered if $\bar{F}(T)(1-\rho+\rho K) \leq (1-\rho)\frac{K}{\theta} + \rho K$, which give the bound. $\square$

Proof of Proposition 3: For the first part, as K falls, V^* increase. Hence, the left-hand side of (5) increases and the right-hand side falls as K decreases. Results for θ and D follow similarly. For the rest, note that (5) implies that $\frac{F(V^*)}{F(T)} \geq \frac{\rho K}{1-\rho} + 1$. Let V_F^* [V_G^*] be the equilibrium cutoff for F [G]. If G is larger than F in the reversed hazard rate order, we have that $G(t) \leq F(t)$ for all t and $F(t)/G(t)$ is decreasing in t (Shaked and Shanthikumar, 1994). The former implies that $V_G^* \geq V_F^*$; the latter give $F(T)/G(T) \geq F(V_F^*)/G(V_F^*)$. We then have

$$\frac{F(V_F^*)}{F(T)} \leq \frac{G(V_F^*)}{G(T)} \leq \frac{G(V_G^*)}{G(T)}.$$

For the travel cost, consider $T' < T$ and with respective equilibrium cutoffs V' and V^*. Define $\psi = V^*/T$ and $\psi = V'/T'$. We first show that $\psi' \geq \psi$. Since $T_0 > T_1$, it must be that $V^* > V'$ and $\frac{T+D}{V^*+D} > \frac{T'+D}{V'+D}$ because a higher cutoff value means fewer customers walking in which implies a higher probability of getting a seat. If $\psi' < \psi$, we would have to have

$$\frac{T'+D}{\psi' T'+D} > \frac{T'+D}{\psi T'+D} > \frac{T+D}{\psi T+D},$$

which implies $\frac{T+D}{V^{*}+D} < \frac{T'+D}{V'+D}$. Given that $\psi' \geq \psi$, we then have

$$F(V^{*})/F(T) = F(\psi T)/F(T) \leq F(\psi T')/F(T') \leq F(\psi' T')/F(T') = F(V')/F(T'). \ \square$$

Proof of Lemma 1: We have either $1-M$ or $\theta - M$ customers competing for $K-M$ seats. If all customers with realized valuations greater than $\tilde{V}$ walk in, the fill rate is:

$$\begin{aligned} \delta\left(\tilde{V}, M\right) &= \frac{\bar{F}\left(\tilde{V}\right)(1-M)(1-\rho) + (K-M)\rho}{\bar{F}\left(\tilde{V}\right)((1-\rho)(1-M) + \rho(\theta - M))} \\ &= 1 - \tilde{\rho}(M) + \tilde{\rho}(M)\frac{K-M}{(\theta - M)\bar{F}\left(\tilde{V}\right)}, \end{aligned}$$

where $\tilde{\rho}(M) = \frac{(\theta-M)\rho\theta}{(\theta-M)\rho\theta + \theta(1-M)(1-\rho)}$. Since $\theta > 1$, $\tilde{\rho}'(M) \geq 0$. We then have $\frac{\partial\delta\left(\tilde{V},M\right)}{\partial\tilde{V}} > 0$ and

$$\frac{\partial\delta\left(\tilde{V}, M\right)}{\partial M} = -\tilde{\rho}(M)\frac{\theta - K}{(\theta - M)^{2}\bar{F}\left(\tilde{V}\right)} + \rho'(M)\left(\frac{K-M}{(\theta-M)\bar{F}\left(\tilde{V}\right)} - 1\right),$$

which is negative if $\bar{F}\left(\tilde{V}\right) \geq \frac{K-M}{(\theta-M)}$. In equilibrium, we must have $\bar{F}(V_M) \geq \frac{K-M}{(\theta-M)}$ and $\delta(V_M, M) = \frac{T+D}{V_M+D}$. An increase in M must then be offset by an increase in V_M. $\square$

Proof of Proposition 4: $\Pi(1) > \Pi(0)$ if

$$\bar{F}(T) + \rho(K-1) \geq (1-\rho)\bar{F}(V_0) + \rho K,$$

which simplifies to (8). To show that $M \in (0,1)$ is never optimal, we have

$$\begin{aligned} \Pi(M) &= M\bar{F}(T) + (1-\rho)(1-M)\bar{F}(V_M) + \rho(K-M) \\ &< M\bar{F}(T) + (1-\rho)(1-M)\bar{F}(V_0) + \rho(K-M) \\ &= M(\bar{F}(T) + \rho(K-1)) + (1-M)((1-\rho)\bar{F}(V_0) + \rho K) \\ &= M\Pi(1) + (1-M)\Pi(0) \\ &\leq max\{\Pi(0), \Pi(1)\}. \end{aligned}$$

The sufficient condition for reservations follows from substituting $\bar{F}(V^{*}) = \frac{K}{\theta}$ into (8). $\square$

Proof of Proposition 5: Setting $\mu'(M)$ to zero yields (8). The log-concavity of $F(V)$ implies that the reversed hazard rate is decreasing and that the solution is unique. The second part follows from the definition of the reversed hazard rate order and the log-concavity of $F(V)$. $\square$

Proof of Proposition 6: First, because $\bar{F}(T) - \alpha \bar{F}\left(\tilde{V}\right) \geq \alpha \bar{F}(T)$, (11) implies (10). Next, the conditions for B to offer reservations are

$$\begin{aligned} (1-\rho)\left(\bar{F}(T) - \beta \bar{F}\left(\tilde{V}\right)\right) &\geq \rho \beta \tilde{K} F(T) && (13) \\ (1-\rho)\beta \bar{F}(T) &\geq \rho \beta \tilde{K} F(T). && (14) \end{aligned}$$

Comparing (11) and (13), it is evident that the former always holds if the latter does because $\beta \geq \alpha$. Comparing (10) and (14), one sees that they are scalings of each other. Hence, the firms would have same best responses to a competitor offering reservations.

Now suppose (10) fails (as in part 1a of the proposition), it must also be the case that B does not want to be the first to offer reservations. Also, A (and hence B) would not respond to a competitor offering reservations by also offering reservations. Thus, no one offering reservations is the only outcome. If, however, (10) holds while (11) and (13) fail (part 1b), A is willing to offer reservations if B does not, and B never is willing to offer reservations. If (13) holds but (11) does not, both firms are willing to offer reservations if the other does not do likewise. Since (11) fails, neither will match the others offering reservations. Hence, one firm will offer reservations and it can be either A or B. If (10) and (11) both hold, offering reservations is a dominant strategy for A and B's best response is to offer reservations (part 1d).

For part 2, (5) immediately implies (11) and both offer reservations. The proof of the final is similar to the corresponding part of Proposition 2. □

Proof of Proposition 7: With lumpy firms and no one offering reservations, a firm earns $\pi_0 = (1-\rho)\frac{\bar{F}(\tilde{V})}{N} + \rho\frac{\tilde{K}}{N}$. If one firm offers reservations, it makes $\pi_1 = (1-\rho)\bar{F}(T) + \rho\frac{\tilde{K}}{N}\bar{F}(T)$. $\pi_0 > \pi_1$ requires $\tilde{K} > N\frac{1-\rho}{\rho}\left(\frac{\bar{F}(T)}{F(T)} - \frac{\bar{F}(\tilde{V})}{NF(T)}\right)$. If $R \geq 1$ firms offer reservations, a firm offering reservations earns $\pi_R = \frac{(1-\rho)\bar{F}(T)}{R} + \rho\frac{\tilde{K}}{N}\bar{F}(T)$. A firm without reservations makes $\pi_{NoR} = \rho\frac{\tilde{K}}{N}$. π_R decreases monotonically with R while π_{NoR} is independent of R. For R firms to offer reservations in equilibrium, it must be the case that $\pi_R \geq \pi_{NoR} \geq \pi_{R+1}$, which leads to (1b). For (1c), one compares π_N and π_{NoR}.

For the case of small restaurants, we first show that given $F(T) \leq (1-\rho)\left(1 - 1/\tilde{K}\right)$ the equilibrium must involve reservations. Suppose that no firm offered reservations in equilibrium. Letting k be the capacity of a representative firm, this would require:

$$(1-\rho)\frac{k}{\tilde{K}}\bar{F}\left(\tilde{V}\right)+\rho k \geq k(1-\rho)\bar{F}(T)+\rho k\bar{F}(T)=k\bar{F}(T),$$

which implies $F(T) \geq (1-\rho)\left(1-\bar{F}\left(\tilde{V}\right)/\tilde{K}\right)$ but $(1-\rho)\left(1-\bar{F}\left(\tilde{V}\right)/\tilde{K}\right) > (1-\rho)\left(1-1/\tilde{K}\right)$. Next, we show that $\tilde{K}_R$ cannot be less than 1. For $1 > \tilde{K}_R > 0$, the profit of a firm offering reservations is $k\bar{F}(T)$ and is constant in $\tilde{K}_R$. The profit of a firm not offering reservations is $\pi\left(\tilde{K}_R\right) = (1-\rho)\left(1-\tilde{K}_R\right)\frac{k}{\tilde{K}-\tilde{K}_R}\bar{F}(V_R)+\rho k$, where V_R is the equilibrium appropriate cutoff value for those customers who were not able to get a reservation. One can easily show that $\pi'\left(\tilde{K}_R\right) < 0$. Hence, as more firms offer reservations but $\tilde{K}_R$ remains less than one, those not offering reservations have an increasing gain from offering reservations. Consequently, $\tilde{K}_R < 1$ cannot be an equilibrium. In equilibrium, the two policies must offer the same returns, which leads to (12). □

References

Bertsimas, D. and R. Shioda. 2003. Restaurant Revenue Management. *Operations Research*. **51** 472 - 486.

Çil, E. and M. A. Lariviere. 2010. Saving Seats for Strategic Customers. working paper, Northwestern University.

Collins, G. 2010. Table for 2? Get Ready to Wait in Line. *New York Times*. Jun. 8.

Courty, P. 2000. An Economic Guide to Ticket Pricing in the Entertainment Industry. *Louvain Economic Review*. **66** 167 - 192.

Dana, J. D. 1998. Advance-Purchase Discounts and Price Discrimination is Competitive Markets. *Journal of Political Economy*. **106** 395 - 422.

Dana, J. D. and N. C. Petruzzi. 2001. Note: The Newsvendor Model with Endogenous Demand. *Management Science*. **47** 1488-1497.

DeGarba, P. 1995. Buying Frenzies and Seller-Induced Excess Demand. *RAND Journal of Economics*. **26** 331 - 342.

Deneckere, R. and J. Peck. 1995. Competition Over Price and Service Rate When Demand is Stochastic: A Strategic Analysis. *The RAND Journal of Economics*. 26 148 - 162.

Fabricant F. 1996. American Express Plan Cuts Restaurant No-Shows. *New York Times*. Jan. 31.

Fischer, J. W. 2005. *At Service Your Service*, Hoboken, NJ: John Wiley & Sons.

Frumkin, P. 2009.Frustrated Restaurateurs Reserve Right to Charge Fees for No-shows. *Nation's Restaurant News*. Apr. 19.

Gupta, D., and B. Denton. 2008. Appointment Scheduling in Health Care: Challenges and Opportunities. *IIE Transactions* **40** 800-819.

Lariviere, M. A. and J. A. Van Mieghem. 2004. Strategically Seeking Service: How Competition Can Generate Poisson Arrivals. *Manufacturing & Service Operations Management*. **6** 23 - 40.

Liu, N, S. Ziya, V. G. Kulkarni, Dynamic Scheduling of Outpatient Appointments Under Patient No-Shows and Cancellations. *Manufacturing & Service Operations Management*. 12 347-364.

Martin, M. 2001. Side Dish. *Riverfront Time*. May 2.

Muthuraman, K. and M. Lawley. 2008. A Stochastic Overbooking Model for Outpatient Clinical Scheduling with No-shows. *IIE Transactions* **40** 820-837.

Netessine, S. and C. Tang. 2009. *Consumer-Driven Demand and Operations Management Models*, Springer.

Png, I. L. 1989. Reservations: Customer Insurance in the Marketing of Capacity. *Marketing Science*. **8** 248 - 264.

Robinson, L. W. and R. R. Chen. 2003. Scheduling Doctors' Appointments: Optimal and Empirically-Based Heuristic Policies. *IIE Transactions*, **35** 295 - 307.

Savin, S. 2006. Managing Patient Appointments in Primary Care in Randolph W. Hall, editor, *Patient Flow: Reducing Delay in Healthcare Delivery*, New York, NY: Springer.

Shaked, M. and J. G. Shanthikumar. 1994. *Stochastic Orders and Their Applications*, San Diego, CA: Academic Press.

Shugan, S. 2002. Service Marketing and Management: Capacity as a Strategic Marketing Variable in Barton A. Weitz and Robin Wensley, editors, *The Handbook of Marketing*, Thousand Oaks, CA: Sage Publications.

Shugan, S., and J. Xie. 2005. Advance-Selling as a Competitive Marketing Tool. *International Journal of Research in Marketing*. 22, 351-373.

Thompson, G. M. 2010. Restaurant Profitability Management The Evolution of Restaurant Revenue Management. *Cornell Hospitality Quarterly*. **51** 308-322

Webb Pressler, M. 2003. Wonder Why You're Simmering? Turnover, Atmosphere Shape Policies on Making Diners Wait. *The Washington Post*. July 27. F5.

Wells, P. In Chicago, the Chef Grant Achatz Is Selling Tickets to His New Restaurant. *New York Times*. May 4.

Xie J., and S. Shugan. 2001. Electronic Tickets, Smart Cards, and Online Payments: When and How to Advance Sell. *Marketing Science*. **20** 219 - 243.