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Monopoly Pricing under a Medicaid-Style Most-Favored-Customer Clause and Its Welfare Implication*

Peter Klibanoff and Tapas Kundu

Abstract

To control Medicaid's expenditure on prescription drugs, 1990 legislation established a rebate program guaranteeing Medicaid a rebate on each unit purchased by Medicaid participants. The rebate is the difference between the minimum price and the average manufacturer price (minimum price rule) or a proportion of the average manufacturer price (average price rule). We characterize the optimal pricing strategy of a third-degree price discriminating monopolist under these rules. Under the minimum price rule, the minimum price gross of rebate always increases whereas prices gross of rebate in at least some of the markets always decrease. In contrast, under the average price rule, these prices may move in the same direction in all markets, with all increasing in some circumstances and all decreasing in others. We also examine the effects of such provisions on social welfare. We analyze a modified version of our minimum price rule model suitable for applications beyond Medicaid.

KEYWORDS: monopoly pricing, Medicaid, MFC, welfare

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1 Introduction

Medicaid is a U.S. government program to pay for health-care services for some low-income families and individuals. It is funded jointly by the federal and state governments. Growing concern over the rapid increase in Medicaid's spending for outpatient prescription drugs led to the enactment of the Medicaid rebate program in 1990. This rebate program, established by the Omnibus Budget Reconciliation Act of 1990, requires drug manufacturers to offer rebates to Medicaid based on the discounts offered to other large purchasers. This is a form of "most favored customer" (MFC) clause. In particular, Medicaid collects a rebate on each unit purchased by Medicaid participants. The unit rebate is calculated as the difference between the minimum price and the quantity-weighted average price (minimum price rule), or a fraction[1] of the quantity-weighted average price (average price rule), whichever is higher.

As Medicaid participants constitute a significant fraction of the whole market,[2] the Medicaid rebate program provides drug manufacturers with a strategic incentive to alter their price distribution in the market. We analyze a model in which a monopolist optimally determines her pricing strategy, subject to minimum or average price rules. We examine these two types of MFC clauses separately. More specifically, we are interested in examining their effect on pricing when the monopolist practices third degree price discrimination across markets and a fraction of consumers in each market is covered by Medicaid. As Medicaid consumers do not pay for their drugs directly,[3] we allow for their demand to be less price sensitive than that of non-Medicaid purchasers. We also examine how these rebates affect social welfare.

Although the rebate program seems to have succeeded in lowering Medicaid's inflation-adjusted drug expenditures (Congressional Budget Office, 1996), its overall effects are complex. The savings to Medicaid, if any, would not generally be the same as those calculated without taking into account the change in optimal pricing strategy. Non-Medicaid purchasers are also affected by the rebate rule. For example, Duggan and Scott Morton (2006) estimate that for the top 200 drug treatments, the average price of a non-Medicaid prescription would have been 13.3 percent lower in 2002 if the Medicaid MFC clause had not been in effect. The rebate rule also affects drug manufacturers' profit ad-

[1] As of 2006, this fraction was 15.1% for branded drugs. See Hearne (2006).

[2] Estimates suggest a Medicaid market share around 15% as of 2006 (Jacobson et al., 2007). Moreover, the healthcare reform legislation passed in 2010 is expected to result in significant expansion of Medicaid enrollment beginning in 2014 (Holahan and Headen, 2010).

[3] In some states, however, they do have small co-payments (Hearne, 2006). Furthermore, their purchases may be influenced by physicians and others (including those running state drug formularies).

versely. Thus the aggregate welfare effects of this cost-saving mechanism are not obvious.

What do we find? A quick preview of some of our results follows. Our analysis of the minimum price rule is done with two markets. Under this rule, the minimum price charged always rises compared to the no regulation case. In contrast, the maximum price will (weakly) fall. The maximum price will remain unchanged if the demand function of a Medicaid participant in a market is the same as that of a non-Medicaid consumer. The welfare effect may be good or bad. A useful sufficient condition for the minimum price rule to be welfare improving is that it result in higher aggregate quantity.

Under the average price rule, all market prices move in the same direction if either Medicaid demand is sufficiently price insensitive or Medicaid demand is nearly the same as non-medicaid consumers' demand. In the former case, prices decrease, while in the latter, prices rise. As with the minimum price rule, the welfare effect of the average price rule is ambiguous in general. When prices in all markets fall, both welfare and aggregate quantity increase, while if all prices increase this is welfare and quantity decreasing.

Though the motivation for this paper mainly comes from the MFC clauses that are featured in the Medicaid reimbursement policy, a broad class of contractual problems features similar MFC clauses, especially in the form of minimum price rules.[4] Modeling applications of minimum price rules in a more general context requires a modified formulation. Though Medicaid collects a rebate from the sale price on each unit purchased by a Medicaid-covered consumer, the amount of rebate is not known at the time of purchase. The rebate is calculated only later, once the total Medicaid purchases, as well as the relevant minimum and quantity-weighted average prices are known. Furthermore, the rebate is paid by the manufacturer directly to Medicaid, and is essentially invisible to consumers. Therefore, the effective demand by Medicaid consumers is likely to be based on the pre-rebate market prices. In other applications of minimum price rules, MFCs are often aware of the minimum price at the time of purchase or more directly involved in the rebate process. To facilitate this wider application, we also analyze a rebate-responsive version of the minimum price rule in which MFC demand is directly affected by the price net of rebate (i.e., the minimum price). We find that our results on pricing and welfare in the context of Medicaid also hold true qualitatively with this alternative version of the minimum price rule.

[4] For example, external referencing policy in drug pricing in the context of Europe (see Heuer et al., 2007, Garcia Mariñoso et al., 2010), agreements between health care providers and health practitioners (see Martin, 2000), or long term trading contracts with price protection, such as natural gas contracts (see Crocker and Lyon, 1994).

Understanding the effects of these regulations is not simply of interest for evaluation of Medicaid policy, but is also important as a guide to future regulation. For example, recently there has been debate about the appropriate regulatory regime to govern drug purchases and reimbursement under Medicare, the US government program of health insurance for the elderly (Jacobson et al., 2007).

1.1 Related Literature

The literature related to the Medicaid rebate program and its rebate rules has been primarily empirical. The only theoretical models of monopoly behavior under these rules that we know of are in the brief theory sections of Scott Morton (1997a) and Congressional Budget Office (1996). The seminal Scott Morton (1997a) is closely related to and an important motivation for our analysis. We borrow the third degree price discrimination structure and the possibility that Medicaid consumers' demand may be less elastic than other consumers' demand from her model, but there are a number of key differences in our formulation and treatment of the problem.

First, we do not limit our analysis to the case of linear market demand curves – we allow general downward sloping, continuously differentiable demands. Nor do we limit ourselves to polar cases in terms of price sensitivity for Medicaid consumers' demand – we allow for all convex combinations of an inelastic portion with a portion identical to that of non-Medicaid consumers.

Second, we analyze how these MFC clauses could affect social welfare, an aspect not studied in Scott Morton (1997a, 1997b) or Congressional Budget Office (1996). Third, we find conditions under which non-MFC prices in all markets increase when an average price rule is imposed and also conditions under which these prices decrease in all markets as a result of an average price rule.

Finally, in our formulation of minimum and average price rules, we assume that the effective demand of Medicaid consumers is based on the pre-rebate prices, motivated by the fact that the amount of the rebate is neither determined at the time of purchase nor does the ultimate rebate involve any party except the manufacturer and Medicaid. In contrast, Scott Morton assumes that the demand of Medicaid consumers is a function of the post-rebate price. This is most related to our alternative minimum price rule formulation, in which we also assume MFC demand depends on post-rebate prices. In comparing this formulation to Scott Morton, in addition to the first two differences pointed out above, we note that we provide a full characterization of the solution, and, even in the special case of linear demand, this solution only coincides with that in Scott Morton (1997a) under additional and restrictive assumptions.

The welfare aspect of our work has close connections with the literature on the welfare effects of third degree price discrimination by a monopolist. The effect of price discrimination on social welfare was first studied by Robinson (1933). Schmalensee (1981) reexamined the problem, and provided a sufficient condition for welfare to decrease under uniform pricing as compared to third degree price discrimination. He shows that uniform pricing can lead to a decrease in welfare only if it leads to an decrease in aggregate demand. As stated above, we show a similar result for the minimum price rule – imposition of this rule can lead to a decrease in welfare only if it leads to a decrease in aggregate demand. Varian (1985) extends Schmalensee's results and proves additional results in a setting where demand in any market can depend on prices in other markets and marginal cost is constant or increasing.[5] Varian's (1985) techniques prove useful in our welfare analysis of the minimum price rule.

Concerning the welfare effects of an average price rule, the closest work is Armstrong and Vickers (1991), which analyzes (see their case 2) the welfare effect of a somewhat related price regulation. They use a convexity property of the consumer surplus function to establish that consumer surplus decreases when moving from a given uniform price across all markets to price discrimination with a constraint that quantity weighted average price is at most the given uniform price. Moreover, when the negative effect of price discrimination on consumer surplus is sufficiently small, they show the increase in producer's surplus dominates the loss in consumers' surplus, and therefore, aggregate welfare increases if the producer is allowed to price discriminate to a small extent. Unfortunately, the benefits of this convexity property of the consumer surplus function are largely limited to circumstances where one of the benchmark pricing schemes is uniform. As neither unconstrained prices nor prices under an average price rule are generally uniform, we are not able to benefit from their techniques.

The empirical work on the Medicaid rebate program includes two United States General Accounting Office (GAO) studies (1991, 1993), a Congressional Budget Office report (1996), Scott Morton (1997a, 1997b) and Duggan and Scott Morton (2006). All of these papers find some evidence of post-rebate rule

[5]Schwartz (1990) extends Varian's analysis to the case of decreasing marginal cost functions. Cowan (2007) and Cowan and Vickers (2007) obtain additional welfare results by imposing functional form restrictions on demand.

increases in drug prices for non-Medicaid buyers. Especially notable is Scott Morton (1997b)'s finding that products with higher ex-ante price dispersion show a greater increase in price when the rebate rule is in effect, consistent with the theory.

Rules like the minimum price rule have been studied in a number of other contexts. The impact of similar MFC clauses in oligopoly settings has been studied extensively in the theoretical literature. Most of the research explores the situation where the sellers strategically exploit the clause to soften price competition.[6] Spier (2003a, 2003b) studies uses of MFC clauses in settlement of litigation. The use of a minimum price rule with long term contracts has been studied by Butz (1990) in the context of durable goods monopoly. In his analysis, this rule is used as a strategic device by the monopolist in its intertemporal game with consumers to change consumer demand by changing beliefs about future prices. Thus even in the monopoly context, the emphasis has been on strategic effects. Our analysis differs substantially from those mentioned in this paragraph because our focus is on the unilateral/own-price effects of such clauses rather than the strategic effects operating through competitor or consumer reaction. In particular, none of our pricing or welfare results may be derived from this literature.

Our analysis of the rebate-responsive minimum price rule is related to the literature on the theory of pricing with external referencing and with parallel imports. Applications are common in the context of drug pricing in Europe and in North America (see Garcia Mariñoso et al., 2010, Pecorino, 2002, and Jelovac and Bordoy, 2005). In external referencing, a product's price in one market (call it the target price) is required to be below a function of the price of the same product in another market (call it the reference price). An example would be one country requiring that a drug be no more expensive than in a neighboring country. Parallel imports refers to allowing the importation of a product that may also be produced domestically. This leads to an indirect link between the target price and the reference price. If a country imports, then the domestic price is effectively bounded by the foreign price plus the cost of importing. Among the findings of this literature is that external referencing or parallel imports may lead to an increase in the reference price. In the context of a rebate-responsive minimum price rule, considering the minimum price as the reference price and the MFC price as the target price, we show a similar result.

[6] See, for example, Besanko and Lyon (1993), Cooper (1986), Cooper and Fries (1991), Neilson and Winter (1992, 1993, 1994), Png (1991), Png and Hershleifer (1987), and Salop (1986).

This paper is organized as follows. In section 2, we describe the general model and specify the monopolist's objective function under the two rules. In section 3, we examine the monopolist's problem without an MFC clause. In section 4, we solve the optimization problem under the minimum price rule and examine its welfare implications. Section 5 carries out a similar investigation for the average price rule. In section 6, we present the analysis of the rebate-responsive form of the minimum price rule and a discussion of the possibility that any of these rebate rules might lead the monopolist to drop a market. Section 7 concludes. Proofs not included in the main text are collected in an Appendix.

2 The Model

Consider a monopolist selling a single good in n different markets, indexed by i. We assume that the monopolist cannot discriminate between consumers within a market, but it can prevent arbitrage by consumers across markets. The presence of a MFC provision divides consumers in each market into two categories: MFCs and non-MFCs. If all consumers in market i were non-MFCs, the demand function in market i would be given by a downward sloping, non-negative, continuously differentiable demand curve, $q_i(p_i)$, for the product, where p_i is the price charged in market i.

In the context of Medicaid, MFCs' price sensitivity may be different from non-MFCs' price sensitivity, as Medicaid consumers do not pay for their drugs directly. Their purchases, however, may be influenced by physicians and others (including those running drug formularies in some states) as well as possible co-payments. Overall, the literature suggests viewing Medicaid demand as less (and, at worst, equally) price sensitive compared to non-Medicaid demand (see e.g., Congressional Budget Office, 1996, Scott Morton, 1997a, Danzon and Towse, 2003). To incorporate various possibilities, we describe MFC demand as follows: If all consumers in market i were MFCs, the demand function in market i would be given by $(1-\beta)\, q_i\,(p_i) + \beta z_i$, for constants $z_i > 0$ and $\beta \in [0, 1]$, where p_i is the price charged in market i.

The constant β measures how price insensitive MFC demand is, compared to non-MFC demand. Specifically, the price elasticity of non-MFC demand in market i is $\frac{p_i q_i'(p_i)}{q_i(p_i)}$, while for MFC demand in market i it is $\frac{(1-\beta)p_i q_i'(p_i)}{(1-\beta)q_i(p_i)+\beta z_i} = \frac{p_i q_i'(p_i)}{q_i(p_i)+\frac{\beta}{1-\beta}z_i}$ when $\beta < 1$. Thus, as β increases, the MFCs become less price elastic compared to non-MFC consumers. This functional form is a convenient and tractable way to capture the assumption that MFC demand is at most as elastic as non-MFC demand, while allowing the non-MFC demand to be quite general.

For simplicity, we assume that the fraction of MFCs in each market is the same and we denote this fraction by $\gamma \in [0,1]$.[7] Therefore, total demand in market i is given by

$$\begin{aligned} & (1-\gamma)\, q_i(p_i) + \gamma\left((1-\beta)\, q_i(p_i) + \beta z_i\right) \\ = \;& (1-\beta\gamma)\, q_i(p_i) + \beta\gamma z_i. \end{aligned} \tag{2.1}$$

Again for simplicity, we consider a linear cost function $C(q) = cq$. We also assume there are gains from trade in all markets, i.e., $q_i(c) > 0$. Without any MFC provision, the monopolist's total profit can be written as

$$\sum_{i=1}^{n} (p_i - c)\left((1-\beta\gamma)\, q_i(p_i) + \beta\gamma z_i\right). \tag{2.2}$$

Within this model of third-degree price discrimination, we analyze the consequences of MFC clauses. In particular, the MFC clauses related to Medicaid each involve a rebate, which we denote by r, on each unit purchased for a Medicaid-covered consumer. For practical reasons, the rebate amount is calculated only retrospectively, once the Medicaid purchases are known, and is paid directly from the manufacturer to Medicaid.[8] Thus, the rebate amount is essentially invisible to consumers at the time of purchase. We will assume, therefore, that demand from MFCs is unaffected by the rebate amount.[9] To avoid confusion between prices gross and net of rebate, we refer to the (gross of rebate) prices, p_i, as *market prices* while the (net of rebate) prices that Medicaid pays for each unit purchased by MFCs in market i, $p_i - r$, are referred to as *post-rebate prices*. With an MFC clause in effect, the monopolist will take

[7] If the monopolist were able to additionally price discriminate based on insurance plan, thus sub-dividing the markets as defined here, one might expect γ to vary across these new "markets" and would want to modify our analysis to allow a different γ_i for each market.

[8] See e.g., Congressional Budget Office report (1996) and Scott Morton (1997a, 1997b).

[9] While we think this is the most appropriate model for the Medicaid application, in a later section we also consider a general version of the minimum price provision rule where MFCs' demand is affected by the post-rebate price (i.e., the price ultimately paid). The latter model may be more relevant for other applications.

the rebate into account and chooses market prices to maximize

$$\begin{aligned} & \sum_{i=1}^{n}(p_i - c)\left((1-\beta\gamma)\, q_i\left(p_i\right) + \beta\gamma z_i\right) \\ & -r\gamma \sum_{i=1}^{n}\left((1-\beta)\, q_i\left(p_i\right) + \beta z_i\right) \\ = \ & \sum_{i=1}^{n}(p_i - r - c)\left((1-\beta\gamma)\, q_i\left(p_i\right) + \beta\gamma z_i\right) + r(1-\gamma)\sum_{i=1}^{n} q_i\left(p_i\right). \end{aligned} \tag{2.3}$$

Notice that γ and β affect the monopolist's problem under an MFC clause in distinct ways – although each results in demand $(1-\beta\gamma)\, q_i(p_i) + \beta\gamma z_i$ in each market becoming less price sensitive (increasing the fraction of consumers having less price sensitive demand has the same impact on market demand as making the demand of each of the less price sensitive consumers even less elastic), only γ determines the demand not subject to rebates, $(1-\gamma)q_i(p_i)$. This is why it is meaningful to distinguish the two, whereas it would not be in a setting without an MFC clause, where only total demand would matter.

There are two different rules that Medicaid uses to calculate the per unit rebate: the minimum price rule and the average price rule. We study them separately.

Under the minimum price rule, Medicaid claims the difference between p_q, the quantity weighted average market price, and $p_{\min} \equiv \min(p_1, \ldots, p_n)$, the minimum price charged in any market:

$$r = p_q - p_{\min}. \tag{2.4}$$

Under the average price rule, Medicaid claims a fraction $\alpha \in [0,1]$ of the quantity weighted average market price.[10] So,

$$r = \alpha p_q \text{ where } p_q = \frac{\sum_{i=1}^{n} p_i\left((1-\beta\gamma)\, q_i\left(p_i\right) + \beta\gamma z_i\right)}{\sum_{i=1}^{n}\left((1-\beta\gamma)\, q_i\left(p_i\right) + \beta\gamma z_i\right)}. \tag{2.5}$$

We will assume throughout our analysis that all n markets are served whether or not the MFC provisions are imposed. To this end we impose the following:

Assumption 1 *The demand functions q_i are positive for every market i at the monopolist's optimal market prices for the problem without an MFC provision,*

[10] As of 2006, when discounting average price, Medicaid uses $\alpha = 0.151$ for branded drugs and $\alpha = 0.11$ for generic drugs (Hearne, 2006).

the problem with the minimum price rule and the problem with the average price rule.

The above assumption simplifies our analysis by ruling out those situations where an MFC provision might lead the firm to drop some markets that it would serve in the absence of regulation. In general, for example, choosing not to serve a relatively small market that had the lowest price without the minimum price rule could be profitable under the minimum price rule because of its effect on the rebate. While a complete analysis of the possibility of dropping markets is beyond the scope of this paper, section 6.2 discusses some findings on pricing and welfare when Assumption 1 is relaxed.

We also assume that demand in each market is such that profit in that market (assuming no MFC clause) is a strictly concave function of price in that market whenever demand is positive. This assumption ensures that the unique solution of the monopolist's profit maximization problem without an MFC provision may be found by solving the first-order conditions. Formally, the following is assumed for the remainder of the paper:

Assumption 2 *For each market i, $(p_i - c)q_i(p_i)$ is strictly concave in p_i whenever $q_i(p_i) > 0$.*

Additional assumptions will be needed to support the first-order approach under the minimum and average price rules. We defer discussion of those to the sections on these respective rules.

In addition to the pricing implications of the MFC clauses, we are interested in the social welfare effects. To measure these, we use the classical Marshallian welfare criterion, consumers' surplus plus producers' surplus.[11] Since we allow for the possibility that MFC demand may have an inelastic component, z_i, consumer surplus for these consumers is technically infinite, rendering Marshallian welfare insensitive to changes in market prices. This is no more than a technicality for our purposes, however, as it can easily be remedied in a way that does not bias the results and simply removes the infinities from the analysis by assuming MFC demand is zero when market prices become high enough.[12] More specifically, assume there is a non-binding finite upper bound on market prices, $\overline{M}$, such that demand in all markets is zero at

[11] See Schmalensee (1981) and Varian (1985) for discussions on the legitimacy of this measure.

[12] This is just the simplest way of getting finite consumer surplus. Any MFC demand that doesn't decline below βz_i until price is above the price that chokes off non-MFC demand, $q_i^{-1}(0)$, and that has a finite area under the demand curve as price rises from $q_i^{-1}(0)$ to infinity leads to the same results.

prices above $\overline{M}$. This is equivalent to saying that the inelastic component of demand isn't really perfectly inelastic, but rather is inelastic until price hits $\overline{M}$, and zero thereafter. For any price vector $P = (p_1, \ldots, p_n)$ and associated demand $x(P) = (x_1(p_1), \ldots, x_n(p_n))$, the Marshallian welfare measure will thus be given by

$$W(P) \equiv \sum_{i=1}^{n} \left[(p_i - c)\, x_i(p_i) + \int_{p_i}^{\overline{M}} x_i(v)\, dv \right] \tag{2.6}$$

As we will be interested in the *changes* in welfare brought about by the various MFC clauses and not absolute welfare levels, *any* non-binding, finite $\overline{M}$ yields the same pricing and welfare results.

One consequence of adopting this welfare criterion is that the rebate funds collected do not enter into the measure of welfare directly because they are a pure transfer from the monopolist to consumers/Medicaid. Any welfare effect of such a policy will operate only through the change in prices it generates. If instead, as is sometimes assumed in the optimal regulation and public finance literatures, there were a greater weight placed on consumer surplus compared to producer surplus, then rebates would have a direct welfare value as well. Though we will not analyze this possibility at length, we note that the sufficient conditions derived below for welfare to increase remain sufficient when consumer surplus is overweighted. However, even in the extreme case when no weight is given to producer surplus so that the rebate amount is treated as pure consumer surplus gain, we can show through examples that the pricing reaction of the monopolist may be strong enough to turn consumer surplus negative. Thus, just as we will demonstrate is the case for the Marshallian welfare criterion, MFC clauses may be good or bad for consumer surplus.

We do not explicitly consider at least two characteristics of the actual Medicaid rebate policy. First, participation in the Medicaid rebate program on the part of drug manufacturers is voluntary in the following sense: a manufacturer could choose not to enroll drugs in the rebate program in exchange for giving up coverage for them under Medicaid, effectively eliminating sales to Medicaid-covered consumers. This could be modeled by including a participation constraint (i.e., that profits under the rebate program should be at least as high as profits without rebates when no Medicaid consumers are served). In practice, it appears that this constraint is not binding. Nearly all branded and generic drug manufacturers enrolled when the rebate program was introduced (Scott Morton, 1997a). Furthermore, in our model, it can be shown that this participation constraint is trivially satisfied when Medicaid demand has almost no inelastic component (i.e., β is close to 0). For higher values of

β, the constraint can be shown to be satisfied under a restriction on the range of relative values of the inelastic component of Medicaid demand as it varies across markets (i.e., the range of ratios of the z_i's).

Second, the actual Medicaid rebate (at least for branded drugs) is calculated as the rebate from the average price rule, or the rebate from the minimum price rule, whichever is higher. We analyze the two rebate forms separately. It is clear that these separate analyses can still give much insight into the combined problem. If at the optimal solution to the combined problem, only one of the two clauses, but not both, is binding, the solution will be exactly either the solution to the minimum price rule problem or the solution to the average price rule problem and our analysis may be directly applied. If at the optimal solution, however, both clauses are simultaneously binding, then the solution to the combined problem may differ from the optimal solutions obtained through our separate analyses. Ideally we would have liked to analyze this case as well, however it appears to us to be quite intractable. Furthermore, we have been unable to locate evidence that would suggest the dual-binding case occurs in practice.

As a final remark on the model, note that, in common with the literature on third-degree price discrimination and its welfare effects, we describe consumer behavior using demand functions as the primitive and imposing restrictions directly on these functions. Nonetheless, it is worthwhile to briefly investigate the utility foundations of the demand functions we assume. In particular, this may help in further understanding the role of the parameter β in our model. Since consumer surplus is an exact measure of aggregate consumer welfare only in the absence of income effects (see e.g., Varian, 1985), consider a quasi-linear utility function for a non-MFC in market i, $v_i(q) + y$, where y is the numeraire and, for $q > 0$, v_i is strictly increasing, concave and continuously differentiable. Given a price p, such an individual will demand $q_i(p)$ units where $q_i(p)$ is determined by the equation $v_i'(q_i(p)) = p$ whenever $q_i(p) > 0$. This is how a non-MFC's demand function in market i could be derived from utility maximizing behavior.

How about the behavior of an MFC in market i? Again, assume a quasi-linear utility function, $u_i(q) + y$ with u_i, for $q > 0$, strictly increasing, concave and continuously differentiable. If

$$u_i^{'}((1-\beta)q_i(p) + \beta z_i) = p \tag{2.7}$$

whenever $q_i(p) > 0$ then demand for such a consumer will be exactly $(1 - \beta)q_i(p) + \beta z_i$ as long as the price p is not so high that $q_i(p) = 0$. Under Assumption 1, it is never optimal for the monopolist to set prices so high, and thus equation (2.7) fully determines MFC demand. We have in mind that u_i is an induced utility, reflecting some combination of the inherent marginal benefit of consuming the product with the effective lowering of marginal cost due to Medicaid coverage. The parameter β functions as a rough index of the way this nets out – for all quantities $q < z_i$, the marginal utility at that quantity for MFCs is higher than the corresponding marginal utility for a non-MFC individual, and this difference is increasing in β. In other words, as the impact, as indexed by β, of Medicaid coverage increases, marginal utility of consuming the drug increases compared to the marginal utility of a non-MFC in a way that translates into decreased price sensitivity of the MFCs.

3 The Benchmark Case: No MFC Provision

As a point of comparison, it is useful to begin our analysis by looking at the profit maximization problem for the monopolist when there is no MFC clause. Without one, the monopolist receives revenue p_i for each unit sold in market i, irrespective of the split between MFCs and non-MFCs within the market. The monopolist therefore chooses prices to maximize profits as defined in (2.2). We call this the unconstrained problem.

Let p_i^m denote the optimal monopoly price in market i. Given our assumptions, p_i^m is the unique solution to the equation

$$(1-\beta\gamma)(p-c)q_i'(p) + (1-\beta\gamma)\,q_i(p) + \beta\gamma z_i = 0. \tag{3.1}$$

With no MFC clause, the social welfare is therefore given as

$$\sum_{i=1}^{n}\left[(p_i^m - c)\left((1-\beta\gamma)\,q_i(p_i^m) + \beta\gamma z_i\right) + \int_{p_i^m}^{\overline{M}}\left((1-\beta\gamma)\,q_i(v) + \beta\gamma z_i\right)dv\right].$$

As it is convenient, without loss of generality, we henceforth assume $p_1^m < p_2^m < \ldots < p_n^m$.[13] It will also be helpful to denote the uniform monopoly price (i.e., the profit maximizing price under the constraint that the same price must

[13] If this is strictly violated, simply reindex the markets so that their numbering agrees with the monopoly price induced order. In cases where there is equality in monopoly prices across markets, a similar analysis can be carried out by first combining these markets into one. To see this, let us consider a situation where $p_1^m < ... < p_k^m = p_{k+1}^m < ... < p_n^m$. If we define a market indexed by k' by combining market k and market $k+1$ such that $q_{k'} = q_k + q_{k+1}$ and $z_{k'} = z_k + z_{k+1}$, then $p_{k'}^m$ remains the same as p_k^m and p_{k+1}^m. This returns us to a situation where strict inequality is maintainted among the optimal individual monopoly prices in each of these markets.

be charged in each market) by p^u, the unique solution to

$$\sum_{i=1}^{n}\left[(p-c)(1-\beta\gamma)\,q_i'(p)+(1-\beta\gamma)\,q_i(p)+\beta\gamma z_i\right]=0.$$

4 Minimum Price Rule

We now examine the minimum price rule problem. Under the minimum price rule, combining (2.3) and (2.4), the monopolist chooses market prices to maximize

$$\sum_{i=1}^{n}(p_i-c)\left((1-\beta\gamma)\,q_i(p_i)+\beta\gamma z_i\right) \qquad (4.1)$$
$$-\gamma\left(p_q-p_{\min}\right)\sum_{i=1}^{n}\left((1-\beta)\,q_i(p_i)+\beta z_i\right).$$

Let $\widehat{p}_i$ denote the market price charged in market i in the solution to this problem. For simplicity and tractability of our results, we consider the two market case ($n=2$). We also assume the following strengthening of Assumption 2:

Assumption 3 *(4.1) is strictly concave in (p_1, p_2) whenever $(q_1(p_1), q_2(p_2)) \gg 0$.*

Just as Assumption 2 ensured that first-order conditions determined the unique solution to the monopolist's unconstrained problem, Assumption 3 does the same for the minimum price rule problem. To see that this strengthens Assumption 2, notice that when $p_1 = p_2$, (4.1) reduces to $\sum_{i=1}^{2}(p_i - c)\left((1-\beta\gamma)\,q_i(p_i)+\beta\gamma z_i\right)$, and thus Assumption 3 implies the strict concavity of $(p_i - c)q_i(p_i)$.

4.1 Pricing Analysis

Our next result shows that the optimal price in market 1 will remain (weakly) below the optimal price in market 2 after the minimum price rule is imposed. The key to this is showing that the monopolist would always prefer to charge a uniform price compared to a situation of charging a high price in the first market and a low price in the second market.

Lemma 1 *Suppose Assumptions 1 and 3 hold. Then $\widehat{p}_1 \leq \widehat{p}_2$.*

With the aid of this lemma, we can describe the effect of the minimum price rule on prices.

Proposition 1 *Suppose Assumptions 1 and 3 hold. If the minimum price rule is imposed, the minimum market price increases and the maximum market price decreases compared to the unconstrained case (i.e., $\widehat{p}_1 \geq p_1^m$ and $\widehat{p}_2 \leq p_2^m$). When some but not all of the consumers are MFCs ($\gamma \in (0,1)$), these changes are strict. Further, the monopolist will charge a uniform price if and only if*

$$\begin{aligned}(p^u - c)(1-\beta\gamma) q_2'(p^u) + ((1-\beta\gamma) q_2(p^u) + \beta\gamma z_2) \\ \times \left(1 - \gamma \frac{\sum_{i=1}^{2}((1-\beta) q_i(p^u) + \beta z_i)}{\sum_{i=1}^{2}((1-\beta\gamma) q_i(p^u) + \beta\gamma z_i)}\right) \leq 0.\end{aligned} \tag{4.2}$$

In such a scenario, the optimal uniform price will be the uniform monopoly price, p^u.

The first part of the above proposition shows that the minimum price rule raises the minimum market price but lowers the maximum market price. The basic intuition for this is that the monopolist pays a rebate based on the difference between the minimum market price and the quantity weighted average market price. The monopolist therefore, all else equal, prefers to set prices so that the minimum market price is close to the quantity weighted average market price. This force pushes the minimum market price up and the maximum market price down. At an extreme, the two prices coincide and the monopolist prefers to charge a uniform price. When this happens, the rebate equals zero. Therefore, the only equal market price that can be optimal for the monopolist to charge is the uniform monopoly price, p^u.

The second part of the above proposition provides a necessary and sufficient condition under which the monopolist prefers to charge a uniform price. The condition has a simple interpretation – it says that, starting from uniform monopoly prices (p^u, p^u), a marginal increase in p_2 reduces the monopolist's profit (net of rebate) from sales in market 2. Alternatively, one could write a similar condition examining a marginal decrease in p_1. Part of the proof of the proposition shows that it is enough to look at the change in only one of the markets. When (4.2) fails, market prices may be found by replacing $p_{\min}$ with p_1 in (4.1) and setting the partial derivatives with respect to p_1 and p_2 equal to zero. This ensures that $\widehat{p}_1$ and $\widehat{p}_2$ exactly balance the marginal gain in profit due to reduction in rebate with the marginal loss in profit because of deviation from the unconstrained monopoly prices.

4.2 Welfare Analysis

We start by describing a general result from Varian (1985) about change in welfare. We apply the result in our setting to derive bounds on the change in welfare resulting from the imposition of the minimum price rule.

Consider an m - good economy for any finite $m > 0$. Let $\mathbf{x}(\mathbf{P}) = (\mathbf{x}_1(p_1), \ldots, \mathbf{x}_m(p_m)) \in \mathbb{R}_+^m$ denote the vector of demands associated with price vector $\mathbf{P} = (p_1, \ldots, p_m) \in \mathbb{R}_+^m$. Assume that unit cost of production is constant and equal for each good and let $\mathbf{c} = (c, \ldots, c) \in \mathbb{R}_+^m$ denote the vector of production costs. The Marshallian welfare measure, as before, is defined as the sum of consumers' surplus and producers' surplus.[14]

When changing from a price vector $\mathbf{P}^0 \in \mathbb{R}_+^m$ to a price vector $\mathbf{P}^1 \in \mathbb{R}_+^m$, let $\triangle\mathbf{x} \in \mathbb{R}^m$ and $\triangle W \in \mathbb{R}$ denote the vector of changes in demand and the change in welfare respectively (i.e., $\triangle\mathbf{x} = \mathbf{x}(\mathbf{P}^1) - \mathbf{x}(\mathbf{P}^0)$ and $\triangle W = W(\mathbf{P}^1) - W(\mathbf{P}^0)$).

Fact 1 (Varian 1985) *The change in welfare, $\triangle W$, satisfies the following bounds:*

$$\left(\mathbf{P}^0 - \mathbf{c}\right) \cdot \triangle\mathbf{x} \geq \triangle W \geq \left(\mathbf{P}^1 - \mathbf{c}\right) \cdot \triangle\mathbf{x}$$

Proof. See the proof of Fact 2 in Varian (1985). ■

The next result uses Fact 1 and the solutions to the monopoly pricing problems with and without the minimum price rule to obtain bounds on the change in welfare resulting from this rule.

Proposition 2 *Suppose Assumptions 1 and 3 hold. The change in welfare, when moving from the unconstrained problem to a minimum price rule, satisfies the following lower bound:*

$$\triangle W \geq (\widehat{p}_1 - c)\,\triangle Q$$

where $\triangle Q$ denotes the corresponding change in aggregate demand. Furthermore, if $q_i(p)$ is concave in $p \geq 0$ for $i = 1, 2$, then the change in welfare satisfies the following upper bound:

$$(\widehat{p}_1 - c)\,\triangle Q - \triangle\pi + (1 - \beta\gamma)(\widehat{p}_2 - \widehat{p}_1)(q_2(\widehat{p}_2) - q_2(p_2^m)) \geq \triangle W$$

[14] In our formulation of social welfare, we consider a finite upper bound in prices, given by $\overline{M}$, such that demand becomes zero at prices above $\overline{M}$. In Varian (1985), there is no finite upper bound in prices ($\overline{M} = \infty$) as he did not explicitly consider demand with an inelastic portion. However, it can be shown easily that all the results on welfare bounds in Varian (1985) go through with finite $\overline{M}$.

where $\triangle\pi$ denotes the corresponding change in the monopolist's profit (excluding the rebate from the profit calculation).

The bounds in Proposition 2 use knowledge of only the realized change in aggregate demand, $\triangle Q$, the minimum price, $\widehat{p}_1$, the manufacturing cost, c, and the loss in profits to the monopolist due to the minimum price rule, $\triangle\pi$, to bound the change in welfare. As the monopolist can always do best when pricing is unrestricted, $\triangle\pi$ is never positive. Moreover, $(1-\beta\gamma)(\widehat{p}_2-\widehat{p}_1)(q_2(\widehat{p}_2)-q_2(p_2^m))$ is always positive as $\widehat{p}_1 \leq \widehat{p}_2 \leq p_2^m$. Thus the bounds are always possible to satisfy.

It is interesting to note that even if costs (and thus profits), for example, are unobserved, the lower bound implies that welfare (and consumer surplus) always increases when imposing the minimum price rule leads to an increase in aggregate demand. Similarly, if aggregate demand is decreased, welfare can decrease by no more than the decrease in aggregate demand valued at the minimum price. Under concavity of the demand functions, the upper bound implies that a large enough decrease in aggregate demand ($\triangle Q < [\triangle\pi-(1-\beta\gamma)(\widehat{p}_2-\widehat{p}_1)(q_2(\widehat{p}_2)-q_2(p_2^m))]/(\widehat{p}_1-c)$) generates a decrease in welfare.

5 Average Price Rule

We now analyze the average price rule. We allow for n markets here as, for the average price rule, this additional generality comes at no cost and may be helpful in applications. Under the average price rule, the monopolist chooses prices to maximize

$$\sum_{i=1}^{n}(p_i-c)\left((1-\beta\gamma)\,q_i(p_i)+\beta\gamma z_i\right)-\alpha\gamma p_q\sum_{i=1}^{n}\left((1-\beta)\,q_i(p_i)+\beta z_i\right). \quad (5.1)$$

Let $\widehat{p}_i$ denote the optimal market price in market i after the average price rule is imposed. Then $\widehat{p}=(\widehat{p}_1,\widehat{p}_2,\ldots,\widehat{p}_n)$ solves the first-order conditions:

$$\begin{aligned}&\frac{d}{dp_i}\left[\sum_{i=1}^{n}(p_i-c)\left((1-\beta\gamma)\,q_i(p_i)+\beta\gamma z_i\right)\right]\\&-\alpha\gamma\frac{d}{dp_i}\left[p_q\sum_{i=1}^{n}\left((1-\beta)\,q_i(p_i)+\beta z_i\right)\right]=0 \quad \text{, for } i=\;1,2,\ldots,n. \quad (5.2)\end{aligned}$$

This says that optimal market prices under the average price rule equate the marginal gain in profit due to reduction in total rebate paid with the marginal loss in profit due to deviation from the unconstrained monopoly prices. As in our analysis of the minimum price rule, we strengthen Assumption 2 to ensure that the first-order conditions determine a unique global optimum. To this end, assume the following:

Assumption 4 *(5.1) is globally concave in* $(p_1, p_2, \ldots, p_n)$ *whenever* $(q_1(p_1), \ldots, q_n(p_n)) \gg 0$.

5.1 Pricing Analysis

How do the market prices under the average price rule compare to those in the unconstrained problem? Observe that at the solution of the unconstrained problem, $(p_1^m, \ldots, p_n^m)$, the first term in (5.2) is equal to zero for all markets i. Hence, the left-hand side of (5.2), computed at $(p_1^m, \ldots, p_n^m)$, reduces to

$$-\alpha\gamma\frac{d}{dp_i}\left[p_q\sum_{i=1}^{n}((1-\beta)q_i(p_i)+\beta z_i)\right]|_{p=\left(p_1^m,\ldots,p_n^m\right)}\,. \tag{5.3}$$

Therefore, the sign of this expression is the key to understanding whether imposing the average price rule will raise or lower market prices.

The firm's only motive for moving prices away from the unconstrained monopoly level is to reduce the rebates it has to pay. If, at unconstrained market prices, raising prices increases the total rebate, (5.3) is negative, implying that the average price rule will result in lower market prices. Similarly, if raising prices decreases the total rebate, (5.3) is positive, implying that the average price rule will result in higher market prices.

In general, either case is possible. However, the parameter β, governing how much of MFC demand is inelastic, is very helpful in determining which case is relevant. We show that when β takes extreme values, one can unambiguously compare the market prices under the average price rule, $\widehat{p}$, to the unconstrained prices. When MFC demand is similar to that of other consumers (i.e., β low enough), the average price rule will increase market prices. If, instead, MFC demand is much closer to inelastic (i.e., β high enough), the average price rule will decrease market prices. The following proposition formalizes this claim:

Proposition 3 *Suppose Assumptions* 1 *and* 4 *hold. There exist* $\underline{\beta}$ *and* $\overline{\beta}$, $0 < \underline{\beta} \leq \overline{\beta} < 1$, *such that (i) for* $\beta < \underline{\beta}$, *all market prices strictly increase under the average price rule compared to the unconstrained case, and (ii) for* $\beta > \overline{\beta}$, *all market prices strictly decrease under the average price rule compared to the unconstrained case.*

What is the intuition for the role of β in determining these effects? Under the average price rule, the total rebate paid is a fraction of the quantity weighted average market price times the total MFC demand for the product. The monopolist, all else equal, prefers to reduce the rebate it pays. Starting at the solution of the unconstrained problem, an increase in market prices generates two effects on the total rebate. First, the quantity weighted average price increases. Second, total MFC demand for the product falls. When β takes values close to zero, i.e., when MFC demand is similar to that of other consumers, the demand reduction effect dominates and so by increasing prices from the unconstrained monopoly level, the firm can reduce the rebate it pays. On the other hand, when β takes values close to one, i.e., when MFC demand is almost inelastic, there is little demand reduction and the effect on quantity weighted average price dominates, leading the monopolist to reduce the rebate by decreasing prices. Note that the boundaries of these regions, $\underline{\beta}$ and $\overline{\beta}$, will vary with the fraction of MFCs, γ, the firm's cost, c, the demand functions q_i and the total of the inelastic demand terms, $\sum_{i=1}^{n} z_i$. The determination of these boundaries is described in the proof. For intermediate values of β, prices in different markets may move in different directions.

5.2 Welfare Analysis

The change in social welfare engendered by the average price rule depends on how it causes market prices to move. From Proposition 3, we know that for $\beta < \underline{\beta}$, all market prices increase, and move further away from the competitive price (which is $p_i = c$ for all i). As a result, aggregate quantity falls and social welfare decreases.[15] Conversely, for $\beta > \overline{\beta}$, all market prices decrease and move toward the competitive price. As a result, aggregate quantity rises and social welfare (and also consumer surplus) increases. This argument proves the following result:

Proposition 4 *Suppose Assumptions* 1 *and* 4 *hold. For* $\beta < \underline{\beta}$*, social welfare decreases when the average price rule is imposed and (ii) for* $\beta > \overline{\beta}$*, social welfare increases when the average price rule is imposed.*

Thus, at least in the cases where all market prices move in the same direction, whether the policy is welfare improving is easy to detect by looking to see if market prices fall.

[15] The effect on consumers alone may be positive or negative, as the rebate may or may not compensate for the price increase.

6 Extensions

6.1 Rebate-responsive Minimum Price Rule

In our analysis, we have assumed that demand from Medicaid consumers (to the extent that it is price-sensitive) is based on pre-rebate market prices. This appears reasonable in the Medicaid context, especially because the rebates are not transparent to consumers. However, as mentioned in the Introduction, clauses similar to the minimum price rule appear in other contexts as well. Often, the analogue to rebates in these applications are more immediate and visible than under Medicaid.

For example, consider long term trading contracts with price protection, such as natural gas contracts (see e.g. Crocker and Lyon, 1994). Sellers often sign agreements with large buyers (or buyers with large sellers) to provide the buyers (or sellers) with price protection over an extended time period. Consider this as an n period problem, where demand may change from period to period. A section of buyers, treated as most favored customers, will be paying the minimum price that prevails over the n periods. However, the seller is allowed to charge different prices in different periods to other customers. As long as it is not possible to substitute demand in one period for demand in another, we can treat these n different periods as n different markets with distinct demand curves. If the section of most favored customers remains a fixed fraction of the total consumers in every market, this formulation will directly fit our model.

As another example, consider a consumer electronics manufacturer who sells her product in different locations through retailers. Retailers differ in their bargaining power, depending on the size and elasticity of their individual markets. Assuming a high level of search cost, this would typically result in high dispersion in retail prices. Now consider an exogenous mechanism that can reduce the search cost for a section of consumers. For example, with the growth of web based transactions, almost every retailer now maintains a web site that allows online purchase of electronics. Not everybody can easily access or feels comfortable using that market, but for those who do, search cost is reduced to a large extent. Assuming that the fraction of consumers who may exercise the online purchasing option remains relatively constant across different markets, this implies that a section of consumers from every market now pay the minimum price (ignoring differences in retailer service provision and return policies).

What is important from a theoretical perspective is that the long term trading agreements or exogenous shifts in location of consumers create a crossmarket effect among the individual market prices in the monopolist's objective function. In each market, a fraction of the consumers is now paying a price that is the minimum of the prices charged in other markets. Thus, to expand the scope of application, and also as a robustness check on our Medicaid analysis, this section presents and analyzes the minimum price rule under the assumption that the prices relevant to MFC demand are the post-rebate (i.e., minimum) prices.

Under this rebate-responsive minimum price rule, MFCs in market i receive a rebate r, which is the difference between the market price, p_i, and the minimum price, $p_{\min}$. The post-rebate price for MFCs in market i, is therefore given by

$$p_i - r = p_i - (p_i - p_{\min}) = p_{\min}.$$

Unlike what we assumed earlier, MFC demand depends on the post-rebate price, $p_i - r$. The monopolist therefore chooses market prices, p_i, to maximize

$$\begin{aligned} & (1-\gamma)\sum_{i=1}^{n}(p_i - c)\,q_i(p_i) \\ & +\gamma\sum_{i=1}^{n}((p_i - r) - c)\,((1-\beta)\,q_i(p_i - r) + \beta z_i) \\ = \quad & (1-\gamma)\sum_{i=1}^{n}(p_i - c)\,q_i(p_i) \\ & +\gamma\sum_{i=1}^{n}(p_{\min} - c)\,((1-\beta)\,q_i(p_{\min}) + \beta z_i)\,. \end{aligned} \tag{6.1}$$

6.1.1 Pricing Analysis

We define $\widetilde{p}_i$ as the optimal monopoly price if facing only the non-MFCs in market i. Without loss of generality and because it will prove convenient, we order the markets so that $\widetilde{p}_1 < \widetilde{p}_2 < \ldots < \widetilde{p}_n$. Note that this ordering of the markets is on the basis of the optimal monopoly prices when facing the non-MFCs only, and that this is different from the way we ordered markets in the previous sections. Here, $\widetilde{p}_i$ solves

$$(p-c)q_i'(p) + q_i(p) = 0.$$

The following condition is useful in characterizing the optimal prices:

$$\beta\gamma\sum_{i=1}^{n} z_i + (1-\beta\gamma)\sum_{i=1}^{n}\left[(\widetilde{p}_n - c)q_i'(\widetilde{p}_n) + q_i(\widetilde{p}_n)\right] \geq 0. \tag{Condition U}$$

If the same price is being charged in all markets, the left-hand side of Condition U is the derivative of the profit function with respect to price evaluated at a price of $\widetilde{p}_n$. Therefore, given strict concavity, Condition U is necessary and sufficient for the optimal uniform price to be above $\widetilde{p}_n$. Note that, by definition, $(\widetilde{p}_n - c)q_n'(\widetilde{p}_n) + q_n(\widetilde{p}_n)$ is zero, whereas, by concavity, $(\widetilde{p}_n - c)q_i'(\widetilde{p}_n) + q_i(\widetilde{p}_n)$ is negative for any other i. The next result describes the optimal solution and shows that Condition U determines whether this solution involves uniform pricing.

Proposition 5 *Suppose Assumptions* 1 *and* 2 *hold. If Condition U is violated, the solution of the profit maximization problem under the rebate-responsive minimum price rule is of the form*

$$(\underbrace{\widehat{p}, \ldots, \widehat{p}}_{k\ times}, \widetilde{p}_{k+1}, \ldots, \widetilde{p}_n)$$

where $\widehat{p} \in [\widetilde{p}_k, \widetilde{p}_{k+1})$ *and* $k \in \{1, 2, \ldots, n-1\}$*. If Condition U holds, the solution will be of the form* $(\underbrace{\widehat{p}, \ldots, \widehat{p}}_{n\ times})$ *(i.e., uniform pricing) where* $\widehat{p} \geq \widetilde{p}_n$*.*

Several comments are in order. First, note that Assumption 2 is enough to guarantee strict concavity of the profit function in the minimum market price and the validity of the first-order approach, unlike in our earlier minimum price rule analysis. The reason for this is that, since the minimum price (post-rebate price) rather than the market price affects MFC demand, the expressions involving prices in the objective function (6.1) are either linear in price (the inelastic part) or in the form of a standard profit function addressed by Assumption 2.

Second, under rebate-responsiveness, the minimum price may be charged in more than one market, even though distinct prices would be charged in each market in the absence of the minimum price rule.[16] Furthermore, the analysis of n markets with the rebate-responsive minimum price rule turns out to be no more messy or difficult than the two market case, which was less true without rebate-responsiveness and led to our choice to present only the two market case in our earlier analysis.

[16]This point is germane to the relation with Scott Morton's (1997a) analysis of minimum price provision. Her model corresponds to the rebate-responsive minimum price rule assuming linear demand and $\beta = 0$ or $\beta = 1$. Comparing our result under those assumptions to her solution, we see that Scott Morton (1997a) must be implicitly assuming that the minimum price is charged in only one market (i.e., $k = 1$ in our proposition).

Finally, as was true without rebate-responsiveness, market prices may decrease in some of the markets under the minimum price rule compared to the unconstrained case. In those markets where the minimum price is not charged under the rule, the monopolist will optimally charge the monopoly price as if demand in that market came only from non-MFCs. In those markets, before the rule was imposed, the optimal market price was higher (with equality if $\beta = 0$) than the optimal monopoly price based on only the non-MFC section. Therefore, these prices decrease under the rebate-responsive minimum price rule. However, as was true without rebate-responsiveness, prices cannot fall in all markets. In particular, the minimum market price charged under the rule will always be higher than the minimum market price under unconstrained price discrimination. Formally:

Proposition 6 *Suppose Assumptions* 1 *and* 2 *hold. The minimum market price increases under the rebate-responsive minimum price rule, compared to the unconstrained case. In those markets where the minimum price is not charged, market prices decrease under the rebate-responsive minimum price rule.*

6.1.2 Welfare Analysis

As before, we apply Fact 1 to derive bounds on the change in welfare resulting from the imposition of the rebate-responsive minimum price rule. The following proposition uses Fact 1 and the solutions to the monopoly pricing problems with and without the minimum price rule to obtain bounds on the change in welfare.

Proposition 7 *Suppose Assumptions* 1 *and* 2 *hold. The change in welfare, when moving from no minimum price rule to a rebate-responsive minimum price rule, satisfies the following lower bound:*

$$\triangle W \geq (\widehat{p} - c) \triangle Q$$

where $\triangle Q$ *denotes the corresponding change in aggregate demand. Furthermore, if* $q_i(p)$ *is concave in* $p \geq 0$ *for all* $i = 1, 2, \ldots, n$*, then the change in welfare satisfies the following upper bound:*

$$(\widehat{p} - c) \triangle Q - \triangle \pi + B \geq \triangle W$$

where $\triangle\pi$ denotes the change in the monopolist's profit and

$$B = \begin{cases} 0 & \textit{if Condition U holds,} \\ \begin{array}{l} (1-\gamma)\sum\limits_{i=k+1}^{n}\left(\left(\widetilde{p}_i - \widehat{p}\right)\left(q_i\left(\widetilde{p}_i\right) - q_i\left(p_i^m\right)\right)\right) \\ \quad + \frac{\beta\gamma(1-\gamma)}{1-\beta\gamma}\sum\limits_{i=k+1}^{n}\left(\widehat{p} - \widetilde{p}_i\right) z_i \end{array} & \textit{otherwise,} \end{cases}$$

where k denotes the number of markets in which the minimum price is charged.

The lower bound in Proposition 7 uses knowledge of only the realized change in aggregate demand, the minimum price and the cost to bound the change in welfare. As was true without rebate-responsiveness, it is important to note that even if cost, for example, is unobserved, the lower bound implies that welfare (and consumer surplus) always increases when imposing the minimum price rule results in an increase in aggregate demand.

6.2 Dropping Markets

Throughout the paper, we have used Assumption 1 to ensure that an MFC clause will not lead the firm to drop any markets. For the analysis of the minimum price rule with two markets as in section 4, we can extend our analysis to the case where dropping markets is permitted. In particular, a necessary condition for the monopolist to serve only one market is that $q_1(p_2^m) = 0$. If this holds, then the first market will be dropped if and only if the profit from serving market 2 at price p_2^m is above the profit from the solution serving both markets described in section 4. If the market is dropped due to the minimum price rule, welfare (and consumer surplus) always decreases, as profit and demand in market 2 are the same as without the rule, while the gains from trade from market 1 are lost. The same results hold for the rebate-responsive minimum price rule with two markets as well.

Under the average price rule, even with two markets, and under the minimum price rule with three or more markets (with or without rebate-responsiveness) the analysis becomes significantly more complex when dropping markets is considered. The difficulty is that the problem with $n-1$ markets is, by itself, as complex as the problems we have analyzed where all markets are served, so that one ends up needing to compare the solution to multiple problems, each as complex as the problems we focus on in this paper. Characterizations and welfare results are thus hard to come by. We have, however, been able to find examples showing that it is possible for the minimum price rule with three markets (again with or without rebate-responsiveness) to increase welfare even when leading the monopolist to drop a market. Under the average price rule, while we suspect this can be true, we have been unable to find such an example.

7 Conclusion

Our analyses in sections 4 and 5 show how the minimum and average price rebate rules affect a monopolist's optimal pricing strategy as well as social welfare under third-degree price discrimination. In the context of the minimum price rule, we present our analysis with two markets. The minimum market price charged always rises compared to the no regulation case. In contrast, prices in markets where the minimum is not charged will fall. The welfare effect may be good or bad. A useful sufficient condition for the minimum price rule to be welfare improving is that it raise aggregate quantity. We also analyze a rebate-responsive version, in which MFC demand is affected by the post-rebate (i.e., minimum) price. We find that the minimum price rule has effects on prices and social welfare similar to those without rebate-responsiveness and suggest applications beyond the Medicaid context.

Under the average price rule, we find that all market prices move in the same direction in two different scenarios: when MFC demand is close enough to inelastic or when MFC demand is sufficiently similar to non-MFCs demand. When MFC demand is close to inelastic, all market prices decrease, resulting in an increase in aggregate quantity and social welfare. In contrast, when MFC demand is close to non-MFC demand, all market prices increase, resulting in a decrease in aggregate quantity and social welfare.

The analysis of these policies is surprisingly intricate, even in a relatively simple setting such as ours. This suggests that great care is needed when implementing such MFC rules and that making provisions for data collection to support follow-up empirical work measuring the pricing and demand response has high potential value in avoiding mistakes or helping fine-tune the policy. Some theoretical issues that we have not addressed here, such as incorporating competition, demand uncertainty, second-degree price discrimination or the effect on dynamic R&D incentives for the manufacturer are interesting topics for future work to explore. In related settings, these factors have been shown to modify conclusions found under monopoly in various ways (see e.g., Stole, 2007).

8 Appendix

Proof of Lemma 1. Suppose, if possible, $\widehat{p}_1 > \widehat{p}_2$. There are three possible cases to consider. First, let us suppose that $\widehat{p}_1 \geq p^u \geq \widehat{p}_2$. Compare this with charging $p_1 = p^u = p_2$. The rebate will be weakly lower in the latter case. Profits in the second market will be weakly higher, as price is moving closer to p_2^m (by Assumption 3). Similarly, in the first market profits will be weakly higher, as price is moving closer to p_1^m from above (by Assumption 3). Thus, $p_1 = p^u = p_2$ dominates $\widehat{p}_1 \geq p^u \geq \widehat{p}_2$. Next suppose $\widehat{p}_1 > \widehat{p}_2 \geq p^u$. Compare this with $p_1 = \widehat{p}_2 = p_2$. Again the rebate is lower in the latter case, profits from the second market are the same, while profits from the first market increase since p_1 is getting closer to p_1^m from above (by Assumption 3). Thus $p_1 = \widehat{p}_2 = p_2$ dominates $\widehat{p}_1 > \widehat{p}_2 \geq p^m$. Finally, suppose $p^u \geq \widehat{p}_1 > \widehat{p}_2$. By similar arguments this is dominated by $p_1 = \widehat{p}_1 = p_2$. ■

Proof of Proposition 1. Given Lemma 1, there are two possibilities to consider: $\widehat{p}_1 = \widehat{p}_2$ and $\widehat{p}_1 < \widehat{p}_2$. In the first scenario (i.e., when $\widehat{p}_1 = \widehat{p}_2$), the optimal solution will be to charge the uniform monopoly price p^u. Since both prices are the same, the effective rebate equals zero, which is the lowest possible effective rebate. Therefore, the solution of (4.1) is also the solution of the maximization problem when the monopolist maximizes profits (the first sum in equation 4.1), under the constraint of uniform pricing.
A remaining question is thus, when is the uniform monopoly price optimal? When it is not, we know that $\widehat{p}_1 < \widehat{p}_2$. We claim that it is sufficient to prove optimality of the uniform monopoly price (under Assumption 2) by checking whether starting from the uniform monopoly price it does not give a local improvement to either lower p_1 or raise p_2. When will these moves not give a local improvement? When the partial derivative of (4.1) with respect to p_1 when taken from below and evaluated at uniform monopoly prices is positive and the partial derivative of (4.1) with respect to p_2 when taken from above and evaluated at uniform monopoly prices is negative. Formally, these one-sided partial derivatives are, from below and above respectively:

$$\begin{aligned}
&(p-c)\,(1-\beta\gamma)\,q_1'\,(p) + ((1-\beta\gamma)\,q_1\,(p) + \beta\gamma z_1) \\
&-\gamma\,(p_q\,(p,p_2) - p)\,(1-\beta)\,q_1'\,(p) \\
&-\gamma\left(\frac{dp_q\,(p,p_2)}{dp} - 1\right)((1-\beta)\,(q_1\,(p) + q_2\,(p_2)) + \beta\,(z_1+z_2))
\end{aligned} \tag{8.1}$$

where

$$\frac{dp_q(p,p_2)}{dp} = \frac{(p - p_q(p,p_2))(1-\beta\gamma)\,q_1'(p) + ((1-\beta\gamma)\,q_1(p) + \beta\gamma z_1)}{((1-\beta\gamma)\,q_1(p) + \beta\gamma z_1) + ((1-\beta\gamma)\,q_2(p_2) + \beta\gamma z_2)}$$

and

$$\begin{aligned} &(p-c)(1-\beta\gamma)\,q_2'(p) + ((1-\beta\gamma)\,q_2(p) + \beta\gamma z_2) \\ &-\gamma\,(p_q(p_1,p) - p_1)(1-\beta)\,q_2'(p) \\ &-\gamma\frac{dp_q(p_1,p)}{dp}\,((1-\beta)(q_1(p) + q_2(p_2)) + \beta(z_1+z_2)) \end{aligned} \tag{8.2}$$

where

$$\frac{dp_q(p_1,p)}{dp} = \frac{(p - p_q(p_1,p))(1-\beta\gamma)\,q_2'(p) + ((1-\beta\gamma)\,q_2(p) + \beta\gamma z_2)}{((1-\beta\gamma)\,q_1(p) + \beta\gamma z_1) + ((1-\beta\gamma)\,q_2(p_2) + \beta\gamma z_2)}.$$

When calculated at $(p_1 = p^u, p_2 = p^u)$, (8.1) and (8.2) simplify to:

$$\begin{aligned} &(p^u - c)(1-\beta\gamma)\,q_1'(p^u) + ((1-\beta\gamma)\,q_1(p^u) + \beta\gamma z_1) \\ &+\gamma\,((1-\beta\gamma)\,q_2(p^u) + \beta\gamma z_2)\,\frac{\sum_{i=1}^2((1-\beta)\,q_i(p^u) + \beta z_i)}{\sum_{i=1}^2((1-\beta\gamma)\,q_i(p^u) + \beta\gamma z_i)} \end{aligned} \tag{8.3}$$

and

$$\begin{aligned} &(p^u - c)(1-\beta\gamma)\,q_2'(p^u) + ((1-\beta\gamma)\,q_2(p^u) + \beta\gamma z_2) \\ &\times\left(1 - \gamma\frac{\sum_{i=1}^2((1-\beta)\,q_i(p^u) + \beta z_i)}{\sum_{i=1}^2((1-\beta\gamma)\,q_i(p^u) + \beta\gamma z_i)}\right) \end{aligned} \tag{8.4}$$

respectively.

Recall that the uniform monopoly price is defined by the condition

$$(p^u - c)(1-\beta\gamma)(q_1'(p^u) + q_2'(p^u)) + \textstyle\sum_{i=1}^2((1-\beta\gamma)\,q_i(p^u) + \beta\gamma z_i) = 0.$$

Using this to substitute into (8.3) gives:

$$\begin{aligned} &-(p^u - c)(1-\beta\gamma)\,q_2'(p^u) - ((1-\beta\gamma)\,q_2(p^u) + \beta\gamma z_1) \\ &+\gamma\,((1-\beta\gamma)\,q_2(p^u) + \beta\gamma z_2)\,\frac{\sum_{i=1}^2((1-\beta)\,q_i(p^u) + \beta z_i)}{\sum_{i=1}^2((1-\beta\gamma)\,q_i(p^u) + \beta\gamma z_i)}, \end{aligned}$$

which is positive (so there is no gain from lowering p_1) exactly when

$$\begin{aligned}&(p^u - c)(1-\beta\gamma)\, q_2'(p^u) + ((1-\beta\gamma)\, q_2(p^u) + \beta\gamma z_2)\\ &\qquad \times \left(1 - \gamma \frac{\sum_{i=1}^{2}((1-\beta)\, q_i(p^u) + \beta z_i)}{\sum_{i=1}^{2}((1-\beta\gamma)\, q_i(p^u) + \beta\gamma z_i)}\right) \;\; \leq \;\; 0.\end{aligned}$$

The other partial (8.4) is negative (so there is no gain from raising p_2) exactly at the same condition. Thus, whenever (4.2) holds true, the uniform monopoly price is optimal, and otherwise the optimum will have $\widehat{p}_1 < p^u < \widehat{p}_2$.

We further claim that $\widehat{p}_1 \geq p_1^m$ and $\widehat{p}_2 \leq p_2^m$. At the uniform monopoly price, this is trivially true. Consider the possibility when $\widehat{p}_1 < \widehat{p}_2$. Suppose, if possible, $\widehat{p}_1 < p_1^m$. By raising p_1 a bit, we raise profits in market 1, while, because $\widehat{p}_1$ is getting closer to $\widehat{p}_2$ the total rebate shrinks (formally as shown above the total rebate shrinks by $\gamma q_2(p_2)$ as $\widehat{p}_1$ increases) thus it cannot be optimal to have $\widehat{p}_1 < p_1^m$. To see the other inequality, suppose $\widehat{p}_2 > p_2^m$. By lowering $\widehat{p}_2$ a bit we raise profits in market 2. What happens to the rebate? As long as $\widehat{p}_2$ is weakly below the monopoly price for market 2 that would hold if cost were $\widehat{p}_1$, then lowering $\widehat{p}_2$ lowers the total rebate. Furthermore, that is the relevant region because, assuming concavity, we need to look at only whether it would be optimal to raise $\widehat{p}_2$ above p_2^m starting from $\widehat{p}_2$. Thus it is never optimal to have $\widehat{p}_2 > p_2^m$. ■

Proof of Proposition 2. To apply Fact 1, take

$$\mathbf{x}(\mathbf{P}) = ((1-\beta\gamma)\, q_1(p_1) + \beta\gamma z_1, (1-\beta\gamma)\, q_2(p_2) + \beta\gamma z_2).$$

From the right-hand-side inequality of Fact 1, we see that

$$\begin{aligned}\triangle W &\geq (1-\beta\gamma)\left[(\widehat{p}_1 - c)(q_1(\widehat{p}_1) - q_1(p_1^m)) + (\widehat{p}_2 - c)(q_2(\widehat{p}_2) - q_2(p_2^m))\right]\\ &= (1-\beta\gamma)(\widehat{p}_1 - c)\textstyle\sum_{i=1}^{2}(q_i(\widehat{p}_i) - q_i(p_i^m)) \qquad (8.5)\\ &\quad + (1-\beta\gamma)(\widehat{p}_2 - \widehat{p}_1)(q_2(\widehat{p}_2) - q_2(p_2^m)).\end{aligned}$$

Notice that the change in aggregate demand, $\triangle Q$, is given by

$$\triangle Q = \textstyle\sum_{i=1}^{2}(1-\beta\gamma)(q_i(\widehat{p}_i) - q_i(p_i^m)) \qquad (8.6)$$

The inequality in (8.5) therefore gives us

$$\triangle W \geq (\widehat{p}_1 - c)\triangle Q + (1-\beta\gamma)(\widehat{p}_2 - \widehat{p}_1)(q_2(\widehat{p}_2) - q_2(p_2^m)).$$

By Proposition 1, $\widehat{p}_1 \leq \widehat{p}_2 \leq p_2^m$ and therefore, we have

$$(1-\beta\gamma)\left(\widehat{p}_2-\widehat{p}_1\right)\left(q_2\left(\widehat{p}_2\right)-q_2\left(p_2^m\right)\right) \geq 0.$$

Hence, we get the following

$$\triangle W \geq \left(\widehat{p}_1-c\right)\triangle Q.$$

Next, assume that $q_i\left(p\right)$ is concave in $p \geq 0$ for $i = 1, 2$. We, therefore, have

$$q_i\left(\widehat{p}_i\right)-q_i\left(p_i^m\right) \leq \left(\widehat{p}_i-p_i^m\right)q_i'\left(p_i^m\right).$$

Or,

$$\left(p_i^m-c\right)(1-\beta\gamma)\left(q_i\left(\widehat{p}_i\right)-q_i\left(p_i^m\right)\right) \leq \left(\widehat{p}_i-p_i^m\right)\left(p_i^m-c\right)(1-\beta\gamma)\,q_i'\left(p_i^m\right).$$

Since p_i^m maximizes $(p-c)\left((1-\beta\gamma)\,q_i\left(p\right)+\beta\gamma z_i\right)$ by definition, the first-order condition gives

$$\left(p_i^m-c\right)(1-\beta\gamma)\,q_i'\left(p_i^m\right) = -\left((1-\beta\gamma)\,q_i\left(p_i^m\right)+\beta\gamma z_i\right).$$

Therefore, we have

$$\begin{aligned} & \left(p_i^m-c\right)(1-\beta\gamma)\left(q_i\left(\widehat{p}_i\right)-q_i\left(p_i^m\right)\right) \\ \leq\ & \left(p_i^m-\widehat{p}_i\right)\left((1-\beta\gamma)\,q_i\left(p_i^m\right)+\beta\gamma z_i\right). \end{aligned} \tag{8.7}$$

From the left-hand-side inequality of Fact 1, we see that

$$\triangle W \leq (1-\beta\gamma)\left[\left(p_1^m-c\right)\left(q_1\left(\widehat{p}_1\right)-q_1\left(p_1^m\right)\right)+\left(p_2^m-c\right)\left(q_2\left(\widehat{p}_2\right)-q_2\left(p_2^m\right)\right)\right]$$

Combining the above inequality with the inequality in (8.7), we get

$$\triangle W \leq \textstyle\sum_{i=1}^{2}\left(p_i^m-\widehat{p}_i\right)\left((1-\beta\gamma)\,q_i\left(p_i^m\right)+\beta\gamma z_i\right). \tag{8.8}$$

The change in the monopolist's profit (including rebate) can be written as

$$\begin{aligned} & \triangle\pi \\ = \ & \textstyle\sum_{i=1}^{2}\left(\widehat{p}_i-c\right)\left((1-\beta\gamma)\,q_i\left(\widehat{p}_{1i}\right)+\beta\gamma z_i\right) \\ & -\textstyle\sum_{i=1}^{2}\left(p_i^m-c\right)\left((1-\beta\gamma)\,q_i\left(p_i^m\right)+\beta\gamma z_i\right) \\ = \ & \textstyle\sum_{i=1}^{2}\widehat{p}_i\left((1-\beta\gamma)\,q_i\left(\widehat{p}_i\right)+\beta\gamma z_i\right) \\ & -\textstyle\sum_{i=1}^{2}p_i^m\left((1-\beta\gamma)\,q_i\left(p_i^m\right)+\beta\gamma z_i\right)-c\triangle Q. \end{aligned}$$

After adding and subtracting $\triangle\pi$ to the right-hand side expression in (8.8) and rearranging terms, we get

$$\begin{aligned}
& \triangle W \\
\leq\; & \textstyle\sum_{i=1}^{2} \widehat{p}_i \left((1-\beta\gamma)\, q_i(\widehat{p}_i) + \beta\gamma z_i \right) \\
& -\textstyle\sum_{i=1}^{2} \widehat{p}_i \left((1-\beta\gamma)\, q_i(p_i^m) + \beta\gamma z_i \right) - c\triangle Q - \triangle\pi \\
\leq\; & (1-\beta\gamma)\left[\widehat{p}_1 \left(q_1(\widehat{p}_1) - q_1(p_1^m) \right) + \widehat{p}_2 \left(q_2(\widehat{p}_2) - q_2(p_2^m) \right) \right] - c\triangle Q - \triangle\pi \\
=\; & (\widehat{p}_1 - c)\triangle Q - \triangle\pi + (1-\beta\gamma)(\widehat{p}_2 - \widehat{p}_1)\left(q_2(\widehat{p}_2) - q_2(p_2^m) \right).
\end{aligned}$$

■

Proof of Proposition 3. We first calculate the derivative of the quantity weighted average price with respect to individual prices. Define $m_i \equiv \frac{\beta\gamma}{1-\beta\gamma} z_i$.

$$\begin{aligned}
\frac{d}{dp_j} p_q &= \frac{d}{dp_j} \frac{\sum_{i=1}^{n} p_i \left(q_i(p) + m_i \right)}{\sum_{i=1}^{n} \left(q_i(p) + m_i \right)} \\
&= \frac{\begin{array}{c} \left(\sum_{i=1}^{n} \left(q_i(p) + m_i \right) \right) \left(p_j q_j'\left((p_j) + q_j(p) + m_j \right) \right) \\ - \left(\sum_{i=1}^{n} p_i \left(q_i(p) + m_i \right) \right) q_j'(p_j) \end{array}}{\left(\sum_{i=1}^{n} \left(q_i(p) + m_i \right) \right)^2} \\
&= \frac{(p_j - p_q)\, q_j'(p_i) + q_j(p_i) + m_j}{\sum_{i=1}^{n} \left(q_i(p) + m_i \right)}.
\end{aligned}$$

The first-order conditions of the average price rule problem are given by

$$\begin{aligned}
& \frac{d}{dp_j} \left\{ \textstyle\sum_{i=1}^{n} (p_i - c)\left((1-\beta\gamma)\, q_i(p_i) + \beta\gamma z_i \right) \right\} \qquad (8.9) \\
& \quad -\alpha\gamma \frac{d}{dp_j} \left\{ p_q \textstyle\sum_{i=1}^{n} \left((1-\beta)\, q_i(p_i) + \beta z_i \right) \right\} = 0, \text{ for } j = 1, 2, \ldots, n.
\end{aligned}$$

If computed at $(p_1^m, \ldots, p_n^m)$, the left-hand side of the first-order condition is

$$-\alpha\gamma \frac{d}{dp_j} \left\{ p_q \textstyle\sum_{i=1}^{n} \left((1-\beta)\, q_i(p_i) + \beta z_i \right) \right\} \Big|_{p=\left(p_1^m, \ldots, p_n^m\right)} \qquad (8.10)$$

This is because the first part of (8.9), when computed at $(p_1^m, \ldots, p_n^m)$, is zero as $(p_1^m, \ldots, p_n^m)$ is the solution of the unconstrained problem. Further, notice that

$$
\begin{aligned}
& \frac{d}{dp_j}\left\{p_q \textstyle\sum_{i=1}^n \left(\left(1-\beta\right) q_i\left(p_i\right)+\beta z_i\right)\right\} \\
= \; & \left(\textstyle\sum_{i=1}^n \left(\left(1-\beta\right) q_i\left(p_i\right)+\beta z_i\right)\right) \frac{dp_q}{dp_j} + p_q\left(1-\beta\right) q_j'\left(p_j\right) \\
= \; & \left(1-\beta\right) p_q q_j'\left(p_j\right) + \left(\textstyle\sum_{i=1}^n \left(\left(1-\beta\right) q_i\left(p_i\right)+\beta z_i\right)\right) \frac{\left(p_j-p_q\right) q_j'\left(p_j\right)+q_j\left(p_j\right)+m_j}{\sum_{i=1}^n \left(q_i\left(p_i\right)+m_i\right)} \\
= \; & \frac{1}{\sum_{i=1}^n \left(q_i\left(p_i\right)+m_i\right)}\left(\begin{array}{c} p_q q_j'\left(p_j\right)\left(\begin{array}{c}\left(\left(1-\beta\right)\sum_{i=1}^n \left(q_i\left(p_i\right)+m_i\right)\right) \\ -\sum_{i=1}^n \left(\left(1-\beta\right) q_i\left(p_i\right)+\beta z_i\right)\end{array}\right) \\ \times\left(p_j q_j'\left(p_j\right)+q_j\left(p_j\right)+m_j\right) \\ \times\sum_{i=1}^n \left(\left(1-\beta\right) q_i\left(p_i\right)+\beta z_i\right)\end{array}\right) \\
= \; & \frac{1}{\sum_{i=1}^n \left(q_i\left(p_i\right)+m_i\right)}\left(\begin{array}{c} p_q q_j'\left(p_j\right)\left(\left(\left(1-\beta\right)\sum_{i=1}^n \frac{\beta\gamma z_i}{1-\beta\gamma}\right)-\sum_{i=1}^n \beta z_i\right) \\ \times\left(p_j q_j'\left(p_j\right)+q_j\left(p_j\right)+m_j\right) \\ \times\sum_{i=1}^n \left(\left(1-\beta\right) q_i\left(p_i\right)+\beta z_i\right)\end{array}\right) \qquad (8.11)
\end{aligned}
$$

Note that when computed at $\left(p_1^m,\ldots,p_n^m\right)$, $\left(p_i q_i'\left(p_i\right)+q_i\left(p_i\right)+m_i\right)=cq_i'\left(p_i\right)$ (by (3.1)). Also, as $\frac{\beta\gamma(1-\beta)}{1-\beta\gamma}-\beta=-\frac{\beta(1-\gamma)}{1-\beta\gamma}$, we can simplify (8.11), when computed at $\left(p_1^m,\ldots,p_n^m\right)$, as

$$
\frac{1}{\sum_{i=1}^n \left(q_i\left(p\right)+m_i\right)}\left(\begin{array}{c} cq_j'\left(p_j\right)\left(\sum_{i=1}^n \left(\left(1-\beta\right) q_i\left(p_i\right)+\beta z_i\right)\right) \\ -\frac{\beta(1-\gamma)}{1-\beta\gamma} p_q q_j'\left(p_j\right)\sum_{i=1}^n z_i\end{array}\right). \qquad (8.12)
$$

Therefore, the first-order derivative of the average price rule objective function, when computed at $\left(p_1^m,\ldots,p_n^m\right)$, can be written (by (8.10)) as

$$
\frac{\alpha\gamma}{\sum_{i=1}^n \left(q_i\left(p_i\right)+m_i\right)}\left(\begin{array}{c} \frac{\beta(1-\gamma)}{1-\beta\gamma} p_q q_j'\left(p_j\right)\sum_{i=1}^n z_i \\ -cq_j'\left(p_j\right)\left(\sum_{i=1}^n \left(\left(1-\beta\right) q_i\left(p_i\right)+\beta z_i\right)\right)\end{array}\right). \qquad (8.13)
$$

Notice that

$$
\begin{aligned}
& \frac{\alpha\gamma}{\sum_{i=1}^n \left(q_i\left(p\right)+m_i\right)}>0, \\
& \frac{\beta\left(1-\gamma\right)}{1-\beta\gamma} p_q q_j'\left(p_j\right)\textstyle\sum_{i=1}^n z_i \leq 0, \\
\text{and} \qquad & cq_j'\left(p_j\right)\left(\textstyle\sum_{i=1}^n \left(\left(1-\beta\right) q_i\left(p_i\right)+\beta z_i\right)\right)<0.
\end{aligned}
$$

Therefore, the sign of (8.13) will be determined by relative values of the two terms, $\frac{\beta(1-\gamma)}{1-\beta\gamma} p_q q_j'\left(p_j\right)\sum_{i=1}^n z_i$ and $cq_j'\left(p_j\right)\left(\sum_{i=1}^n \left(\left(1-\beta\right) q_i\left(p_i\right)+\beta z_i\right)\right)$. In general, the first-order derivative can take either sign.

In order to prove the proposition, we calculate the partial derivatives at two extreme values of β. At $\beta = 0$, when MFC demand is as elastic as non-MFCs, the term in (8.13) can be written as

$$-\alpha\gamma c q_j'(p_j),$$

which is always positive, as $q_j'(p_j) < 0$ for all $j = 1, 2, \ldots, n$. On the other hand, at $\beta = 1$, when MFC demand is completely inelastic, the term in (8.13) can be written as

$$\frac{\alpha\gamma(1-\gamma)}{\sum_{i=1}^{n}((1-\gamma)q_i(p)+\gamma z_i)}\left[(p_q - c)\, q_j'(p_j) \textstyle\sum_{i=1}^{n} z_i\right],$$

which is always negative for all $j = 1, 2, \ldots, n$.

Since (8.13) evaluated at $(p_1^m, \ldots, p_n^m)$ (itself a continuous function of β) is continuous in $\beta \in [0,1]$, and takes a positive value at $\beta = 0$ and a negative value at $\beta = 1$, for each j we can find two numbers, $\underline{\beta}_j$ and $\overline{\beta}_j$, such that $0 \leq \underline{\beta}_j \leq \overline{\beta}_j \leq 1$ and (8.13) evaluated at $(p_1^m, \ldots, p_n^m)$ is always negative for $\beta \in \left[\overline{\beta}_j, 1\right]$ and always positive for $\beta \in \left[0, \underline{\beta}_j\right]$. Set $\underline{\beta} = \min_i \left\{\underline{\beta}_i\right\}$ and $\overline{\beta} = \max_i \left\{\overline{\beta}_i\right\}$. Therefore, for $\beta \in \left[0, \underline{\beta}\right]$, the partial derivative with respect to p_j, computed at $(p_1^m, \ldots, p_n^m)$, is positive for all j. By global concavity, we see that price increases in every market, in comparison to $(p_1^m, \ldots, p_n^m)$, the solution of the unconstrained problem. Similarly, for $\beta \in \left[\overline{\beta}, 1\right]$ the partial derivative with respect to p_j, computed at $(p_1^m, \ldots, p_n^m)$, is negative for all j. By global concavity, we see that price decreases in every market, in comparison to $(p_1^m, \ldots, p_n^m)$, the solution of the unconstrained problem. ■

Proof of Proposition 5. Let a solution vector be $(p_1^*, \ldots, p_n^*)$ and $J = \{i \in \{1, 2, \ldots, n\} \mid p_i^* = \min\{p_1^*, \ldots, p_n^*\}\}$.

Claim 1: If $j \notin J$, then $p_j^* = \widetilde{p}_j$.

If $j \notin J$, then $p_j^* > \min\{p_1^*, \ldots, p_n^*\}$. p_j^* is also the solution of the optimization problem: $\max_p (p-c)\, q_j(p)$ such that $p \geq \min\{p_1^*, \ldots, p_n^*\}$.

If $\widetilde{p}_j > \min\{p_1^*, \ldots, p_n^*\}$, and as $\widetilde{p}_j$ maximizes $(p-c)\, q_j(p)$ globally, $p_j^* = \widetilde{p}_j$.

If $\widetilde{p}_j \leq \min\{p_1^*, \ldots, p_n^*\}$, then $(p-c)\, q_j(p)$ being concave in p, is maximized at $p = \min\{p_1^*, \ldots, p_n^*\}$ over the range $\{p : p \geq \min\{p_1^*, \ldots, p_n^*\}\}$. This implies that $p_j^* = \min\{p_1^*, \ldots, p_n^*\}$, or, $j \in J$. which is ruled out.

Claim 2: If $j \in J, l \notin J$, then $j < l$.

If not, let us suppose $\exists\, l \notin J$ and $j \in J$ such that $j > l$.

Then, Claim 1 suggests $p_l^* = \widetilde{p}_l$. Moreover, $\widetilde{p}_l > \min\{p_1^*, \ldots, p_n^*\}$ since $l \notin J$. As $j > l$, we have $\widetilde{p}_j > \widetilde{p}_l > \min\{p_1^*, \ldots, p_n^*\}$. Therefore, $j \notin J$. Contradiction.

Claim 3: $\min\{p_1^*, \ldots, p_n^*\} \in [\widetilde{p}_k, \widetilde{p}_{k+1})$ for $k = \max J$.

By Claim 1, $p_{k+1}^* = \widetilde{p}_{k+1} > \min\{p_1^*, \ldots, p_n^*\}$. Suppose $p_k^* < \widetilde{p}_k$. Then, the monopolist could strictly increase profits by setting $p_k^* = \widetilde{p}_k$. This increases profits from the non-MFC customers in market k, and leaves all other terms in the profit expression unchanged.

Claim 4: $k < n$.

Suppose $k = n$. Then $\min\{p_1^*, \ldots, p_n^*\} = p^u$, the uniform monopoly price. Since $\widetilde{p}_n > p^u$, this contradicts Claim 3.

Claims 1, 2, 3 and 4 together yield that the solution is of the desired form.

It remains to show that the solution is unique. Suppose $(\check{p}_1, \ldots, \check{p}_n)$ is a different solution. It can differ from $(p_1^*, \ldots, p_n^*)$ only in the choice of k and $\widehat{p}$. We now show that there is a unique profit maximizing choice of k and $\widehat{p}$ so that the existence of such different solutions is not possible. For any fixed k, it follows from Assumption 2 that there is a unique profit maximizing price which satisfies $\max_p \sum_{i=1}^{k}(p-c)q_i(p) + \gamma \sum_{i=k+1}^{n}(p-c)q_i(p)$. Call this $\widehat{p}(k)$. Suppose that there exist $k_1 < k_2$ such that $\widehat{p}(k_1) \in [\widetilde{p}_{k_1}, \widetilde{p}_{k_1+1})$ and $\widehat{p}(k_2) \in [\widetilde{p}_{k_2}, \widetilde{p}_{k_2+1})$ as was shown to be required for profit maximization by the first part of this proof. By revealed preference, profits from the first k_1 markets and the MFCs from the remaining markets are strictly higher when charging $\widehat{p}(k_1)$ rather than $\widehat{p}(k_2)$. Since $\widehat{p}(k_2) \in [\widetilde{p}_{k_2}, \widetilde{p}_{k_2+1})$, profits from the non-MFCs in markets $k_1+1, \ldots, k_2$ would be higher by charging the monopoly prices in those markets. Combining these facts implies that profits are higher with $k = k_1$ and $\widehat{p} = \widehat{p}(k_1)$ than with $k = k_2$ and $\widehat{p} = \widehat{p}(k_2)$. This shows that a profit maximizing solution of the required form must be unique. ■

Proof of Proposition 6. First, we show that the minimum price increases after the rebate-responsive minimum price rule is imposed. To study properties of the minimum market price, we construct an alternative optimization problem and show that its optimal solution coincides with the optimal solution of the original problem (6.1). We then derive properties of the optimal minimum market price by studying the first order condition of this modified problem.

Given Proposition 5, the maximization problem (6.1) may be rewritten as the following problem of maximizing with respect to k and $p_{\min}$, where only an upper bound on $p_{\min}$ is imposed:

$$\begin{aligned} \max_{p<\widetilde{p}_{k+1}, k\in\{1,2,\ldots,n\}} \quad & \sum_{i=1}^{k}(p-c)\left((1-\beta\gamma)q_i(p)+\beta\gamma z_i\right) \qquad (8.14) \\ +(1-\gamma)\sum_{i=k+1}^{n}(\widetilde{p}_i-c)q_i(\widetilde{p}_i) \quad & +\gamma\sum_{i=k+1}^{n}(p-c)\left((1-\beta)q_i(p)+\beta z_i\right) \end{aligned}$$

where $\widetilde{p}_{n+1}$ defined as ∞.[17]

Let $\widehat{p}$ and $\widehat{k}$ solve (8.14). We now show that the unique solution to the following unconstrained optimization problem is $p=\widehat{p}$:

$$\begin{aligned} \max_{p} & \sum_{i=1}^{\widehat{k}}(p-c)\left((1-\beta\gamma)q_i(p)+\beta\gamma z_i\right) \qquad (8.15) \\ & +\gamma\sum_{i=\widehat{k}+1}^{n}(p-c)\left((1-\beta)q_i(p)+\beta z_i\right). \end{aligned}$$

By strict concavity, this problem has a unique solution – call it p'. By inspection, if $p'<\widetilde{p}_{\widehat{k}+1}$ then $p'=\widehat{p}$. Otherwise, the monopolist could strictly increase profits by setting $p=p'$ (instead of $\widehat{p}$) in (8.14). Can it be that $p'\geq\widetilde{p}_{\widehat{k}+1}$ for some $\widehat{k}\in\{1,2,\ldots,n-1\}$? Then $\widehat{p}<p'$. Since p' optimizes a strictly concave function, any increase in p above $\widehat{p}$, no matter how small, will increase the value of the objective function in (8.15). But some increase is always feasible in problem (8.14), as $\widehat{p}<\widetilde{p}_{\widehat{k}+1}$ and so could be increased at least some amount and still remain the minimum. This would contradict the optimality of $\widehat{p}$ in (8.14) and so it cannot be that $p'\geq\widetilde{p}_{\widehat{k}+1}$. Therefore, $p'<\widetilde{p}_{\widehat{k}+1}$ and $p'=\widehat{p}$.

Therefore, $\widehat{k}$ and $\widehat{p}$ solve the first order condition (in price) of (8.15):

$$\begin{aligned} (1-\beta\gamma)\sum_{i=1}^{\widehat{k}}\left[(\widehat{p}-c)q_i'(\widehat{p})+q_i(\widehat{p})\right] \qquad (8.16) \\ +\gamma(1-\beta)\sum_{i=\widehat{k}+1}^{n}\left[(\widehat{p}-c)q_i'(\widehat{p})+q_i(\widehat{p})\right]+\beta\gamma\sum_{i=1}^{n}z_i \quad = \quad 0. \end{aligned}$$

The above condition characterizes the minimum market price under the rebate-responsive minimum price rule.

Let $j\in\{1,2,\ldots,n\}$ be the market in which the unconstrained monopoly price was lowest. Denoting that monopoly price by p_j^m, it is the unique solution

[17] In this scenario, prices in all markets could even be greater than $\widetilde{p}_n$. To accommodate such a possibility, we set the upper limit as infinity (by setting $\widetilde{p}_{n+1}=\infty$).

of

$$(1-\beta\gamma)\left[(p-c)q_j'(p)+q_j(p)\right]+\beta\gamma z_j=0. \tag{8.17}$$

If we can show that $p_j^m \leq \widehat{p}$, this will complete the first part of the proof. To see that $p_j^m \leq \widehat{p}$, consider the function

$$\begin{aligned} S(p) \equiv{}& (1-\beta\gamma)\sum_{i=1}^{\widehat{k}}\left[(p-c)q_i'(p)+q_i(p)\right] \\ &+\gamma(1-\beta)\sum_{i=\widehat{k}+1}^{n}\left[(p-c)q_i'(p)+q_i(p)\right]+\beta\gamma\sum_{i=1}^{n}z_i. \end{aligned}$$

By Assumption 2, $S(p)$ is decreasing in p. Furthermore, for every $i=1,2,3,...,n$, $(1-\beta\gamma)\left[(p_j^m-c)q_i'\left(p_j^m\right)+q_i(p_j^m)\right]+\beta\gamma z_i \geq 0$.
Notice that

$$\begin{aligned} S(p) &= \sum_{i=1}^{\widehat{k}}\left[(1-\beta\gamma)\left[(p-c)q_i'(p)+q_i(p)\right]+\beta\gamma z_i\right] \\ &\quad+\sum_{i=\widehat{k}+1}^{n}\left[\gamma(1-\beta)\left[(p-c)q_i'(p)+q_i(p)\right]+\beta\gamma z_i\right] \\ &\geq \sum_{i=1}^{\widehat{k}}\left[(1-\beta\gamma)\left[(p-c)q_i'(p)+q_i(p)\right]+\beta\gamma z_i\right] \\ &\quad+\sum_{i=\widehat{k}+1}^{n}\left[\gamma(1-\beta)\left[(p-c)q_i'(p)+q_i(p)\right]+\beta\gamma\frac{\gamma(1-\beta)}{1-\beta\gamma}z_i\right] \\ &= \sum_{i=1}^{\widehat{k}}\left[(1-\beta\gamma)\left[(p-c)q_i'(p)+q_i(p)\right]+\beta\gamma z_i\right] \\ &\quad+\frac{\gamma(1-\beta)}{1-\beta\gamma}\sum_{i=\widehat{k}+1}^{n}\left[(1-\beta\gamma)\left[(p-c)q_i'(p)+q_i(p)\right]+\beta\gamma z_i\right]. \end{aligned}$$

Hence, $S\left(p_j^m\right) \geq 0$ because term-by-term the final expression is non-negative. From (8.16), we know that $S(\widehat{p})=0$. Therefore, we have $p_j^m \leq \widehat{p}$ since $S(p)$ is decreasing in p.

The second part of the proposition, stating that prices decrease under the rebate-responsive minimum price rule in those markets where the minimum price is not charged, follows directly from Proposition 5. ■

Proof of Proposition 7. To make it easier to apply Fact 1, we make the following adjustment in our notation: when writing the price vector we will treat the MFC and non-MFC sections of each market as two different markets. The generic price vector is thus $\mathbf{P} = (p_1, \ldots, p_{2n}) \in \mathbb{R}_+^{2n}$ where p_i and p_{n+i} denote the prices faced by the non-MFC and the MFC sections of market i respectively. Without MPP, the monopolist's optimal price vector is given by $(p_1^m, \ldots, p_n^m, p_1^m, \ldots, p_n^m)$. Under the rebate-responsive minimum price rule, applying Proposition 5, the optimal price vector is $(\hat{p}, \ldots, \hat{p}, \widetilde{p}_{k+1}, \ldots \widetilde{p}_n, \hat{p}, \ldots, \hat{p})$ where k is endogenously determined such that the minimum price is charged in the first k markets, if Condition U is violated or $\left(\underbrace{\widehat{p}, \ldots, \widehat{p}}_{2n\ times}\right)$ if Condition U holds. With the split-market representation of prices, note that the corresponding market demands will be

$$\mathbf{x}(\mathbf{P}) = \left(\begin{array}{c} (1-\gamma) q_1(p_1), \ldots, (1-\gamma) q_n(p_n), \gamma((1-\beta) q_1(p_{n+1}) + \beta z_1), \\ \ldots, \gamma((1-\beta) q_n(p_{2n}) + \beta z_n) \end{array} \right).$$

Let us first consider the case when Condition U is violated. Applying Fact 1 yields

$$\begin{array}{rl} & (1-\beta\gamma)\sum_{i=1}^{k}(p_i^m - c)(q_i(\widehat{p}) - q_i(p_i^m)) \\ & +(1-\gamma)\sum_{i=k+1}^{n}(p_i^m - c)(q_i(\tilde{p}_i) - q_i(p_i^m)) \\ & +\gamma(1-\beta)\sum_{i=k+1}^{n}(p_i^m - c)(q_i(\widehat{p}) - q_i(p_i^m)) \\ \geq & \triangle W \\ \geq & (1-\beta\gamma)\sum_{i=1}^{k}(\widehat{p} - c)(q_i(\widehat{p}) - q_i(p_i^m)) \\ & +(1-\gamma)\sum_{i=k+1}^{n}(\tilde{p}_i - c)(q_i(\tilde{p}_i) - q_i(p_i^m)) \\ & +\gamma(1-\beta)\sum_{i=k+1}^{n}(\widehat{p} - c)(q_i(\widehat{p}) - q_i(p_i^m)). \end{array} \tag{8.18}$$

The change in aggregate demand, $\triangle Q$, is given by

$$\begin{array}{rcl} \triangle Q & = & (1-\beta\gamma)\sum_{i=1}^{k}(q_i(\widehat{p}) - q_i(p_i^m)) \\ & & +(1-\gamma)\sum_{i=k+1}^{n}(q_i(\tilde{p}_i) - q_i(p_i^m)) \\ & & +\gamma(1-\beta)\sum_{i=k+1}^{n}(q_i(\widehat{p}) - q_i(p_i^m)). \end{array} \tag{8.19}$$

The right-hand side inequality in (8.18) gives us

$$\begin{array}{rcl} \triangle W & \geq & (1-\beta\gamma)\sum_{i=1}^{k}(\widehat{p} - c)(q_i(\widehat{p}) - q_i(p_i^m)) \\ & & +(1-\gamma)\sum_{i=k+1}^{n}(\tilde{p}_i - \widehat{p} + \widehat{p} - c)(q_i(\tilde{p}_i) - q_i(p_i^m)) \\ & & +\gamma(1-\beta)\sum_{i=k+1}^{n}(\widehat{p} - c)(q_i(\widehat{p}) - q_i(p_i^m)) \\ & = & (\widehat{p} - c)\triangle Q + (1-\gamma)\sum_{i=k+1}^{n}(\tilde{p}_i - \widehat{p})(q_i(\tilde{p}_i) - q_i(p_i^m)) \\ & \geq & (\widehat{p} - c)\triangle Q, \end{array}$$

where the last inequality follows from $\widehat{p} \leq \tilde{p}_i \leq p_i^m$ for $i = k+1, \ldots, n$ by Proposition 6.

Next consider the possibility when Condition U holds. The optimal price vector under MPP is $\left(\underbrace{\widehat{p}, \ldots, \widehat{p}}_{2n\ times}\right)$ and the change in demand, $\triangle Q$, is given by,

$$\triangle Q = (1-\beta\gamma) \textstyle\sum_{i=1}^{n} \left(q_i\left(\widehat{p}\right) - q_i\left(p_i^m\right)\right).$$

Applying Fact 1 yields

$$\begin{aligned} & (1-\beta\gamma)\left(\textstyle\sum_{i=1}^{n}\left(p_i^m - c\right)\left(q_i\left(\widehat{p}\right) - q_i\left(p_i^m\right)\right)\right) \geq \triangle W \\ \geq \; & (1-\beta\gamma)\left(\textstyle\sum_{i=1}^{n}\left(\widehat{p} - c\right)\left(q_i\left(\widehat{p}\right) - q_i\left(p_i^m\right)\right)\right). \end{aligned} \tag{8.20}$$

The right-hand side inequality in (8.20) therefore gives us

$$\triangle W \geq \left(\widehat{p} - c\right) \triangle Q.$$

Next, assume that $q_i(p)$ is concave in $p \geq 0$ for all $i = 1, 2, \ldots, n$. We, therefore, have

$$q_i\left(\widehat{p}\right) - q_i\left(p_i^m\right) \leq \left(\widehat{p} - p_i^m\right) q_i'\left(p_i^m\right),$$

or, equivalently,

$$\left(p_i^m - c\right)\left(q_i\left(\widehat{p}\right) - q_i\left(p_i^m\right)\right) \leq \left(p_i^m - c\right)\left(\widehat{p} - p_i^m\right) q_i'\left(p_i^m\right).$$

Since p_i^m maximizes $(p-c)\left((1-\beta\gamma) q_i(p) + \beta\gamma z_i\right)$, using the first-order condition we get

$$\left(p_i^m - c\right) q_i'\left(p_i^m\right) = -q_i\left(p_i^m\right) - \frac{\beta\gamma z_i}{(1-\beta\gamma)}. \tag{8.21}$$

Therefore, we have

$$\left(p_i^m - c\right)\left(q_i\left(\widehat{p}\right) - q_i\left(p_i^m\right)\right) \leq \left(p_i^m - \widehat{p}\right)\left(q_i\left(p_i^m\right) + \frac{\beta\gamma z_i}{(1-\beta\gamma)}\right). \tag{8.22}$$

Similarly, we get

$$q_i\left(\tilde{p}_i\right) - q_i\left(p_i^m\right) \leq \left(\tilde{p}_i - p_i^m\right) q_i'\left(p_i^m\right),$$

or,

$$\left(p_i^m - c\right)\left(q_i\left(\tilde{p}_i\right) - q_i\left(p_i^m\right)\right) \leq \left(p_i^m - c\right)\left(\tilde{p}_i - p_i^m\right) q_i'\left(p_i^m\right). \tag{8.23}$$

Applying (8.21), inequality (8.23) becomes

$$\left(p_i^m - c\right)\left(q_i\left(\tilde{p}_i\right) - q_i\left(p_i^m\right)\right) \leq \left(p_i^m - \tilde{p}_i\right)\left(q_i\left(p_i^m\right) + \frac{\beta\gamma z_i}{(1-\beta\gamma)}\right). \qquad (8.24)$$

When Condition U is violated, combining (8.22) and (8.24) with the left-hand side inequality in (8.18), we get

$$\begin{aligned} \triangle W \leq\ & (1-\beta\gamma)\textstyle\sum_{i=1}^{k}\left(p_i^m - \widehat{p}\right)\left(q_i\left(p_i^m\right) + \dfrac{\beta\gamma z_i}{(1-\beta\gamma)}\right) \\ & + (1-\gamma)\textstyle\sum_{i=k+1}^{n}\left(p_i^m - \tilde{p}_i\right)\left(q_i\left(p_i^m\right) + \dfrac{\beta\gamma z_i}{(1-\beta\gamma)}\right) \qquad (8.25) \\ & + \gamma(1-\beta)\textstyle\sum_{i=k+1}^{n}\left(p_i^m - \widehat{p}\right)\left(q_i\left(p_i^m\right) + \dfrac{\beta\gamma z_i}{(1-\beta\gamma)}\right). \end{aligned}$$

Notice that the change in the monopolist's profit can be written as

$$\begin{aligned} \triangle\pi =\ & (1-\beta\gamma)\textstyle\sum_{i=1}^{k}\left(\widehat{p} - c\right)\left(q_i\left(\widehat{p}\right) + \dfrac{\beta\gamma z_i}{(1-\beta\gamma)}\right) \\ & + (1-\gamma)\textstyle\sum_{i=k+1}^{n}\left(\tilde{p}_i - c\right)q_i\left(\tilde{p}_i\right) \\ & + \gamma(1-\beta)\textstyle\sum_{i=k+1}^{n}\left(\widehat{p} - c\right)\left(q_i\left(\widehat{p}\right) + \dfrac{\beta z_i}{(1-\beta)}\right) \\ & - (1-\beta\gamma)\textstyle\sum_{i=1}^{n}\left(p_i^m - c\right)\left(q_i\left(p_i^m\right) + \dfrac{\beta\gamma z_i}{(1-\beta\gamma)}\right). \end{aligned}$$

After adding and subtracting $\triangle\pi$ to the right-hand side expression in (8.25) and rearranging terms, we get

$$\begin{aligned} \triangle W \leq\ & \left(\widehat{p} - c\right)\triangle Q - \triangle\pi + \frac{\beta\gamma(1-\gamma)}{1-\beta\gamma}\textstyle\sum_{i=k+1}^{n}\left(\widehat{p} - \widetilde{p}_i\right)z_i \\ & + (1-\gamma)\textstyle\sum_{i=k+1}^{n}\left(\widetilde{p}_i - \widehat{p}\right)\left(q_i\left(\widetilde{p}_i\right) - q_i\left(p_i^m\right)\right). \end{aligned}$$

Next consider the possibility when Condition U holds. The optimal price vector under MPP is $\Bigg(\underbrace{\widehat{p}, \ldots, \widehat{p}}_{2n\ times}\Bigg)$, and as shown above, we have

$$(1-\beta\gamma)\left(\textstyle\sum_{i=1}^{n}\left(p_i^m - c\right)\left(q_i\left(\widehat{p}\right) - q_i\left(p_i^m\right)\right)\right) \geq \triangle W.$$

Combining this inequality with (8.22), we get

$$\triangle W \leq (1-\beta\gamma)\left(\sum_{i=1}^{n}\left(p_i^m - \widehat{p}\right)\left(q_i\left(p_i^m\right) + \frac{\beta\gamma z_i}{(1-\beta\gamma)}\right)\right). \tag{8.26}$$

Further, when Condition U holds, the change in profit is given by

$$\triangle\pi = (1-\beta\gamma)\left(\begin{array}{c} \sum_{i=1}^{n}\left(\widehat{p}-c\right)\left(q_i\left(\widehat{p}\right) + \frac{\beta\gamma z_i}{(1-\beta\gamma)}\right) \\ -\sum_{i=1}^{n}\left(p_i^m - c\right)\left(q_i\left(p_i^m\right) + \frac{\beta\gamma z_i}{(1-\beta\gamma)}\right) \end{array}\right).$$

After adding and subtracting $\triangle\pi$ to the right-hand side expression in (8.26) and rearranging terms, we get

$$\triangle W \leq \left(\widehat{p}-c\right)\triangle Q - \triangle\pi.$$

■

References

Armstrong, M. and Vickers, J. 1991. "Welfare Effects of Price Discrimination by a Regulated Monopolist." *Rand Journal of Economics*, 22:4, 571-580.

Besanko, D and Lyon, T.P. 1993. "Equilibrium Incentives for Most-Favored-Customer Clauses in an Oligopolistic Industry." *International Journal of Industrial Organization*, 15, 347-367.

Butz, D.A. 1990. "Durable-Good Monopoly and Best-Price Provisions." *American Economic Review*, 80, 1062-1076.

Congressional Budget Office. 1996. "How the Medicaid Rebate on Prescription Drugs Affects Pricing in the Pharmaceutical Industry." Washington D.C.: Congressional Budget Office.

Cooper, T.E. 1986. "Most-Favored-Customer Pricing and Tacit Collusion." *RAND Journal of Economics*, 17, 377-388.

Cooper, T.E. and Fries, T.L. 1991. "The Most-Favored-Nation Pricing Policy and Negotiated Prices." *International Journal of Industrial Organization*. 9, 209-223.

Cowan, S. 2007. "The Welfare Effects of Third-Degree Price Discrimination with Nonlinear Demand Functions." *Rand Journal of Economics*. 38:2, 419-428.

Cowan, S. and Vickers, J. 2007. "Output and welfare effects in the classic monopoly price discrimination problem." Department of Economics Discussion Paper No. 355, University of Oxford.

Crocker, K.J. and Lyon, T.P. 1994. "What Do Facilitating Practices Facilitate? An Empirical Investigation of Most-Favored-Nation Clause in Natural Gas Contracts." *Journal of Law and Economics*, 37, 297-322.

Danzon, P. and Towse, A. 2003. "Differential Pricing for Pharmaceuticals: Reconciling Access, R&D and Patents." *International Journal of Health Care Finance and Economics*, 3, 183–205.

Duggan, M. and Scott Morton, F. 2006. "The Distortionary Effects of Government Procurement: Evidence for Medicaid Prescription Drug Purchasing." *Quarterly Journal of Economics*, 121:1, 1-30.

Hearne, J. 2006. "Prescription Drug Coverage Under Medicaid." CRS Report for Congress, Order Code RL30726. *http://opencrs.cdt.org/document/RL30726*

Heuer, A., Mejer, M. and Neuhaus, J. 2007. "The National Regulation of Pharmaceutical Markets and the Timing of New Drug Launches in Europe." Kiel Institute for the World Economy. *Working Paper.* No. 437.

Holahan, J. and Headen, I. May 2010. "Medicaid Coverage and Spending in Health Reform: National and State-by-State Results for Adults at or Below 133% FPL." Washington, DC: Kaiser Commission on Medicaid and the Uninsured.

Jacobson, G. A., Panangala, S.V. and Hearne, J. 2007. "Pharmaceutical Costs: A Comparison of Department of Veterans Affairs (VA), Medicaid, and Medicare Policies." CRS Report for Congress, Order Code RL33802. *http://opencrs.cdt.org/rpts/RL33802_20070117.pdf*

Jacobzone, S. 2000. "Pharmaceutical policies in OECD countries: Reconciling social and industrial goals." OECD Labour Market and Social Policy Occasional Papers No. 40, OECD Publishing. *http://dx.doi.org/10.1787/323807375536*

Jelovac, I., and Bordoy, C. 2005. "Pricing and Welfare Implications of Parallel Imports in the Pharmaceutical Industry." *International Journal of Health Care Finance and Economics. 5, 5-21.*

Garcia Mariñoso, B., Jelovac, I. and Olivella, P. 2010. "External Referencing and Pharmaceutical price Negotiation." *Health Economics, forthcoming.*

Martin, J. A. 2000. "Antitrust Analysis of 'Most Favored Nation' Clauses in Health Care Contracts." *Private Antitrust Litigation News*, Fall 2000.

O'Sullivan, J. and Tritz, K. 2006. "Medicare Prescription Drug Benefit: An Overview of Implementation for Dual Eligibles." CRS Report for Congress, Order Code RL33268. *http://opencrs.cdt.org/rpts/RL33268_20060206.pdf*

Neilson, W. S. and Winter, H. 1992. "Unilateral Most-Favored-Customer Pricing: A Comparison with Stackelberg." *Economics Letters.* 38:2, 229-232.

Neilson, W. S. and Winter, H. 1993. "Bilateral Most-Favored-Customer Pricing and Collusion." *Rand Journal of Economics*. 24:1, 147-155.

Neilson, W. S. and Winter, H. 1994. "Enhancing Bargaining Power with Most-Favored-Customer Pricing." *Economics Letters.* 44, 61-66.

Pecorino, P. 2002. "Should the US allow prescription drug reimports from Canada?" *Journal of Health Economics*, 21, 699-708.

Png, I.P.L. 1991. "Most-Favored-Customer Protection Versus price Discrimination over Time." *Journal of Political Economy*, 99, 1010-1028.

Png, I.P.L. and Hershleifer, D. 1987. "Price Discrimination trough Offers to Match Price." Journal of Business, 60, 365-383.

Robinson, J. 1933. *The Economics of Imperfect Competition.* London: Macmillan.

Salop, S.C. 1986. "Practices That (Credibly) Facilitate Oligopoly Coordination." In J.E. Stiglitz and G.F. Matthewson, eds., *New Developments in the Analysis of Market Structure*. Cambridge, Mass.: MIT Press.

Schmalensee, R. 1981. "Output and Welfare Implications of Monopolistic Third-Degree Price Discrimination." *American Economic Review*, 71, 242-247.

Schwartz, M. 1990. "Third-Degree Price Discrimination and Output: Generalizing a Welfare Result." *American Economic Review*, 80, 1259-1262.

Scott Morton, F. 1997a. "The Strategic Response by Pharmaceutical Firms to the Medicaid Most-Favored-Customer Rules." *The RAND Journal of Economics*, 28:2, 269-290.

Scott Morton, F. 1997b. "The Interaction between a MFC Clause and Price Dispersion: An Empirical Examination of the Medicaid Rebate Rules of 1990." *Journal of Economics and Management Strategy*, 6:1, 151-174.

Spier, K. 2003a. "The Use of 'Most-Favored-Nation' Clauses in Settlement of Litigation." *The RAND Journal of Economics*, 34:1, 78-95.

Spier, K. 2003b. "Tied to the Mast: Most-Favored-Nation Clauses in Settlement Contracts." *The Journal of Legal Studies*, 32:1. 91-120.

Stole, L. 2007. "Price Discrimination and Competition." In M. Armstrong and R. Porter, eds., *Handbook of Industrial Organization*. Elsevier, Volume 3, 2221-2299.

US Congress, Office of Technology Assessment. 1993. "Pharmaceutical R&D: Cost, risks and rewards." OTA-H-522. Washington D.C., USA: Government Printing Office.

United States General Accounting Office. 1991. "Changes in Drug Prices Paid by VA and DOD Since Enactment of Rebate Provisions." GAO report to Congressional Committees, GAO/HRD-91-139. Washington, D.C.

United States General Accounting Office. 1993. "Changes in Drug Prices Paid by HMOs and Hospitals Since Enactment of Rebate Provisions." GAO report to Congressional Committees, GAO/HRD-93-43. Washington, D.C.

Varian, H. R. 1985. "Price Discrimination and Social Welfare." *American Economic Review*, 75:4, 870-875.