

Risk sharing example: Pooling Investments

A friend and I have initial wealth W_a and W_b ; r_a and r_b are our coefficients of absolute risk aversion.

We own portfolios which have a expected returns of I_a and I_b , and variances $\text{Var}(I_a)$ and $\text{Var}(I_b)$, respectively..

Let τ be a cash transfer from me (indexed as a) to my friend (indexed as b).

Q: How do we efficiently split these streams of income?

To be determined:

How do we split the returns from each investment?

Will there be side payments between us? What will they be?

Let α be my share of I_a , let β be my share of I_b .

Let τ be a cash transfer from me (indexed as a) to my friend (indexed as b).

$$\begin{aligned} \text{My utility/CEQ: } & W_a - \tau + \alpha I_a + \beta I_b + \frac{1}{2} r_a \text{var}(\alpha I_a) + \frac{1}{2} r_a \text{var}(\beta I_b) \\ & = W_a - \tau + \alpha I_a + \beta I_b + \frac{1}{2} r_a [\alpha^2 \text{var}(I_a) + \beta^2 \text{var}(I_b)] \end{aligned}$$

$$\text{Friend's utility/CEQ: } W_b + \tau + (1-\alpha) I_a + (1-\beta) I_b + \frac{1}{2} r_b [(1-\alpha)^2 \text{var}(I_a) + (1-\beta)^2 \text{var}(I_b)]$$

Total value: $W_a + W_b + I_a + I_b + \text{Prem}_a + \text{Prem}_b$, where

$$\begin{aligned} \text{Prem}_a & = \frac{1}{2} \text{var}(I_a) [r_a \alpha^2 + r_b (1-\alpha)^2] \\ \text{Prem}_b & = \frac{1}{2} \text{var}(I_b) [r_a \beta^2 + r_b (1-\beta)^2] \end{aligned}$$

Note:

C τ drops out: transfer does not affect efficiency

C α, β only enter through risk premia

Choose α , β to maximize total value:

$$\frac{MTV}{M\alpha} = r_a \alpha \text{ var}(I_a) + r_b (1-\alpha) \text{ var}(I_a) = 0$$

$$\frac{\alpha}{1-\alpha} = \frac{r_b}{r_a}$$

$$\frac{MTV}{M\beta} = 0 \Leftrightarrow \frac{\beta}{1-\beta} = \frac{r_b}{r_a}$$

Therefore,

$$\alpha = \beta = \frac{r_b/r_a}{1 + r_b/r_a}$$

Efficiency dictates:

- C Higher r_b/r_a (more risk-tolerant I am relative to my friend), more of the share I should take.
- C Share does not depend on the riskiness of the stream.
- C Does not depend on who initially owns which stream.
- C Risk-neutral person bears entire risk.
- C *Efficiency gains arise because pooling allows us to allocate risk so that it imposes least costs.*

Individual rationality dictates:

- C ? must be chosen such that each of us does at least as well as we would if we did not pool our investments.
- C range of possible values that satisfy individual rationality.