Portfolio Theory 3

Using the Model

Our goal now is to:

- Implement the model, that is, determine how to calculate the portfolio measures that we discussed last week.
- Use these measures to:
  1. Figure out whether or not an asset is over-priced, under-priced or correctly priced. This helps us identify investments opportunities
  2. Evaluate portfolio performance -- is a successful portfolio manager smart (dumb) or lucky (unlucky)?

Implementing the Model -- Index Models

Suppose we take a more statistical approach and we model returns the following way:

Economy-wide Component of the Surprise → $\beta_i F$; Idiosyncratic Component of the Surprise → ε_i

$$\underbrace{r_i}_{\text{Actual Outcome}} = \underbrace{E(r_i)}_{\text{Expected Outcome}} + \underbrace{\beta_i F + \varepsilon_i}_{\text{Surprise}}$$

where
r_i = actual return for firm i
$E(r_i)$ = expected return for firm i
F = index which measures unanticipated economy-wide events
β_i = sensitivity of firm i to unanticipated economy-wide events
e_i = impact of unanticipated firm specific events on i

This is a **factor model** for stock returns. One can easily imagine a model with more factors but to keep things simple we=ll assume that firms are affected by only two factors, a macro factor and a firm specific factor.

Index Models

The obvious question that arises is “What is F?”.

One reasonable way to go is to assume that the excess return (i.e., the return minus the risk free rate) on a broad index of securities such as the S&P 500 is a good proxy for the common macro factor. This approach leads to a particular kind of factor model called the **single-index model**

Index Models

We can rewrite our factor model as:

$$R_i = \alpha_i + \beta_i R_M + e_i$$

where	R_i	=	r_i-r_f
	α_i	=	the expected return on security i
	$\beta_i R_M$	=	the component of the return due to economy-wide events
	R_M	=	$(r_{mkt} - r_f)$
	e_i	=	the component of the return due to firm specific events

The riskfree rate is subtracted off from the actual returns because we interested in explaining movements in returns having to do with risk rather than time value.

Index Models

Using this model we can write the variance (total risk) of a security's return as:

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma^2(e_i)$$

This decomposes a security's risk into two components:

market risk = $\beta_i^2 \sigma^2_M$

firm specific risk = $\sigma^2(e_i)$

The covariance between two assets is:

$Cov(R_i,R_j)$ $= Cov(\beta_i R_M + e_i, \beta_j R_M + e_j)$

$= \beta_i \beta_j \sigma^2_M$

$Correlation(R_i,R_j)$ $= (\beta_i \beta_j \sigma^2_M)/\sigma_i \sigma_j$

$= \rho_{iM} \rho_{jM}$

Index Models

To estimate the single index model we run a regression.

$$R_i = \alpha_i + \beta_i R_M + e_i$$

where $R_i = r_i - r_f$ and $R_M = r_M - r_f$.

Linear regression has the feature that the formula for the coefficient β_i is

$$\beta_i = \frac{Cov(R_i, R_M)}{Var(R_M)}$$

This exactly corresponds to the formula for β given by the CAPM.

Index Models

Recall from our discussion of portfolio theory that the model suggests that

$$E(r_i) = r_f + \beta_i(E(r_{mkt}) - r_f)$$

Rewriting this we get:

$$E(r_i) - r_f = \beta_i(E(r_{mkt}) - r_f)$$

Which suggest that on average

$$r_i - r_f = R_i = \beta_i(r_{mkt} - r_f) = \beta_i R_M$$

This means that if assets are "correctly priced" according to the model, the estimated alpha should be approximately equal to zero which in turn implies that when we run our regression,

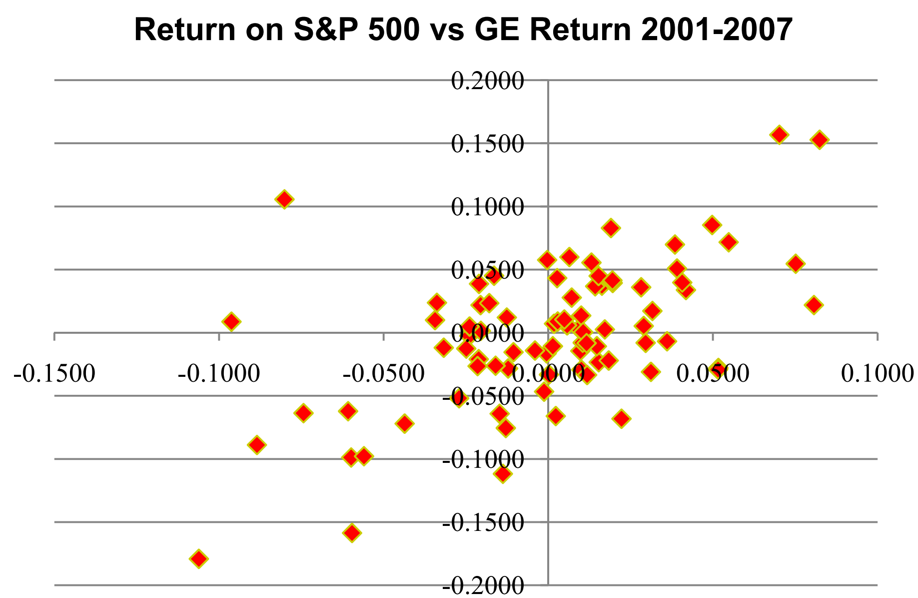
$$R_i = \alpha_i + \beta_i R_M + e_i$$

α should be approximately equal to zero.

Index Models

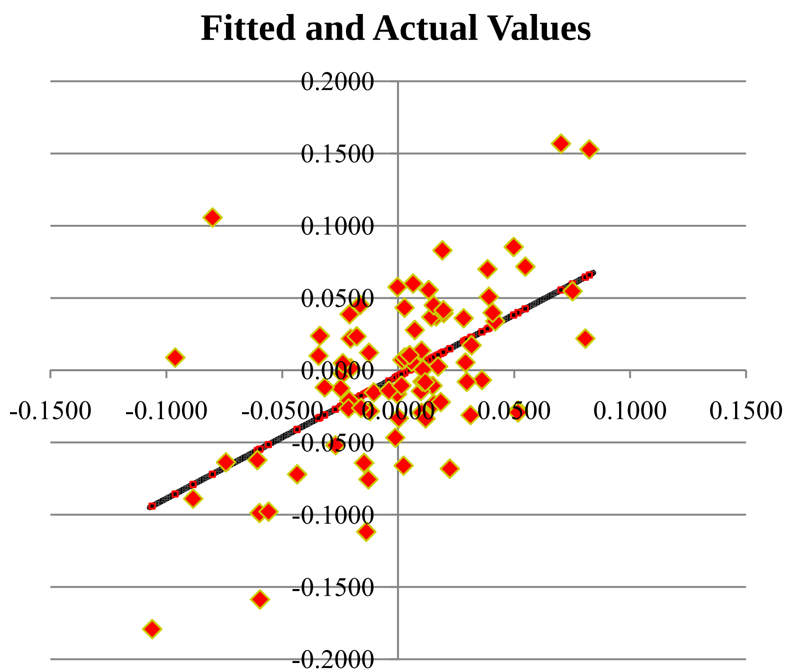
- Example: To estimate the beta of GE we would run a regression using the returns on the S&P 500 as the independent variable and the returns on GE as the dependent variable.

Date	Return on S&P	Return on GE
2/1/2001	-0.0962	0.0087
3/1/2001	-0.0598	-0.0988
4/2/2001	0.0702	0.1568
5/1/2001	0.0018	0.0071
6/1/2001	-0.0240	-0.0025
7/2/2001	-0.0138	-0.1118
8/1/2001	-0.0608	-0.0622
9/4/2001	-0.0885	-0.0889
10/1/2001	0.0153	-0.0234
11/1/2001	0.0752	0.0546
12/3/2001	0.0027	0.0433
1/2/2002	-0.0129	-0.0756
2/1/2002	-0.0210	0.0387
3/1/2002	0.0312	-0.0311
4/1/2002	-0.0596	-0.1587
5/1/2002	-0.0105	-0.0155



Index Models

- The results from the regression are:



	Coefficients	Standard Error
alpha	-0.0015	0.0052
beta	0.8482	0.1377
R^2	0.3191	
Standard Error for Y	0.0472	
Regression Sum of the squares	0.0846	
Residuals Sum of the squares	0.1805	
Observations	81	

Performance Evaluation

- There are two types of active portfolio management:
  - Security Selection
  - Market Timing

Performance Evaluation

- **Security Selection**
  - This involves locating mispriced securities and then taking a position which deviates from the market portfolio.
  - The benefit of stock selection is that the return may be higher.
  - The cost is the loss of diversification since not all the firm specific risk has been eliminated.
  - The size of the deviation entails weighting these two effects off against one another.

Alpha

Alpha is the key variable that tells us whether an asset is a good investment or a bad investment. Assets which are **correctly priced** (that is, the return is neither too high or too low relative to its market risk) will have a **zero alpha**.

An asset with a **positive α** has a return which is higher than the return it should be earning given its market risk. This makes it a good investment so it should receive **more than the market weight** in a optimal portfolio.

An asset with a **negative α** has a return which is lower than the return it should be earning given its market risk. This makes it a bad investment so it should receive **less than the market weight** in a optimal portfolio.

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Alpha

- A portfolio manager's job is often to identify securities with alpha's which are not equal to zero. The market component of the return , $\beta_i E(R_M)$, is measured statistically and it's estimate won't differ much across different portfolio managers.

Performance Evaluation

Market Timing

Consider two different investment strategies:

- On 1/1/26 invest $1000 in 30 day commercial paper and rolled the proceeds over every thirty days until 12/31/78 (52 years later). At the end of the invest period the investor would have $3,600.
- On 1/1/26 invert $1000 in the NYSE index. Reinvest all dividends and hold the portfolio until 12/31/78. At the end of the invest period the investor would have $67,500.

Define perfect market timing to be the ability to tell at the beginning of each month whether the NYSE index will outperform 30-day commercial paper.

- If a perfect market timer had invested $1000 on 1/1/26 how much would he have on 12/31/78?

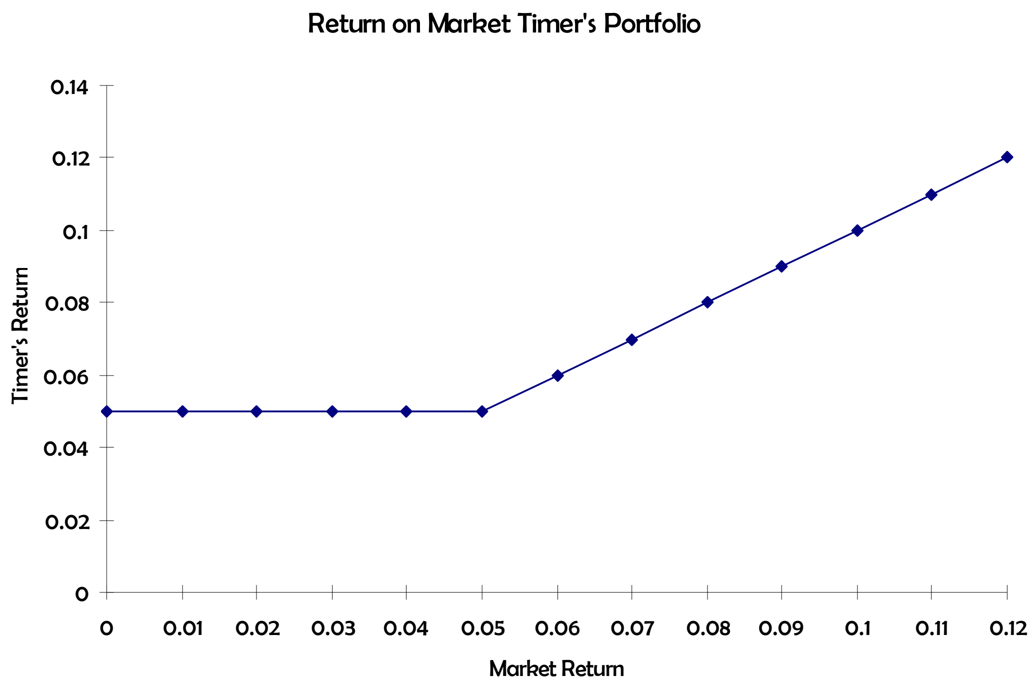
Performance Evaluation

- A market timer shifts funds between the market portfolio and the safe asset. He is either 100% in the market or 100% in the safe asset. A perfect market timer will never earn less than rf but he will earn more when the market return exceeds the risk free rate.

Performance Evaluation

The return on the market timer portfolio can be depicted in the following graph.

What does this payoff remind you of?



Performance Evaluation

There are three standard measures. They differ with respect to how they measure risk.

The Sharpe Measure is appropriate for the overall portfolio.

$$\frac{E(r_p) - r_f}{Var(r_p)}$$

The Treynor Measure is appropriate for measuring the risk of a piece of a larger portfolio

$$\frac{E(r_p) - r_f}{\beta_p}$$

Performance Evaluation

- Another measure is the Appraisal Ratio

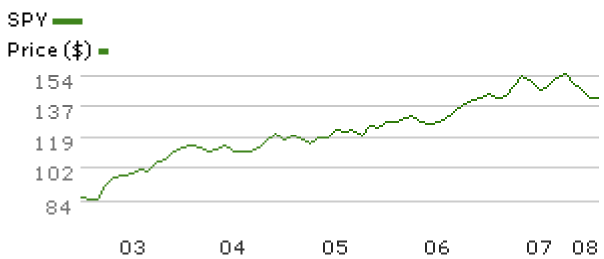
$$\frac{\alpha_p}{\mathrm{Var}(e_p)}$$

Performance Evaluation

SPDRs

Volatility Measurements	Trailing 3-Yr through 12-31-07		
Standard Deviation	7.77	Sharpe Ratio	0.54
Mean	8.86		

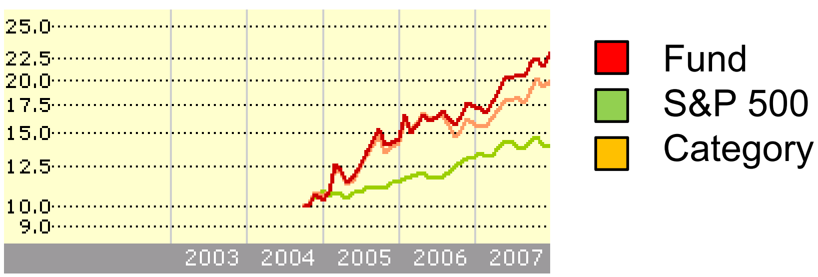
Modern Portfolio Theory Statistics	Trailing 3-Yr through 12-31-07	
	Standard Index S&P 500 TR	Best Fit Index S&P 500 TR
R-Squared	100	100
Beta	1	1
Alpha	-0.09	-0.09



Performance Evaluation

Vanguard Energy ETF VDE

Volatility Measurements	Trailing 3-Yr through 12-31-07		
Standard Deviation	20.07	Sharpe Ratio	1.23
Mean	33.08		
Modern Portfolio Theory Statistics	Trailing 3-Yr through 12-31-07		
	Standard Index S&P 500 TR	Best Fit Index Morningstar Energy Sec TR	
R-Squared	22	99	
Beta	1.21	1	
Alpha	19.48	-0.55	



Performance Evaluation

iShares MSCI Japan Index **EWJ**

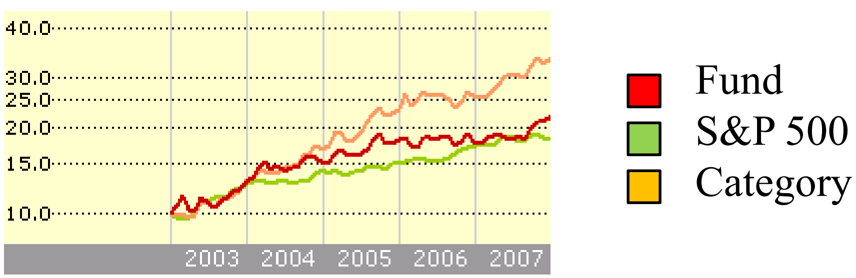
Volatility Measurements	Trailing 3-Yr through 12-31-07		
Standard Deviation	11.75	Sharpe Ratio	0.34
Mean	8.6		

Modern Portfolio Theory Statistics	Trailing 3-Yr through 12-31-07	
	Standard Index	Best Fit Index
	MSCI EAFE NDTR_D	MSCI Japan NDTR_D
R-Squared	50	100
Beta	0.87	1
Alpha	-6.23	-0.49

Performance Evaluation

PIMCO CommodityRealRet Strat Instl PCRIX

Volatility Measurements	Trailing 3-Yr through 12-31-07 |		*Trailing 5-Yr through 12-31-07	
Standard Deviation		15.46	Sharpe Ratio	0.6
Mean		14.44	Bear Market Decile Rank*	1
Modern Portfolio Theory Statistics	Trailing 3-Yr through 12-31-07			
	Standard Index		Best Fit Index	
	S&P 500 TR		Goldman Sachs Nat Res	
R-Squared	0		45	
Beta	0.12		0.54	
Alpha	8.71		-3.2	



Performance Evaluation

Fidelity Advisor Small Cap Growth I FCIGX

Volatility Measurements	Trailing 3-Yr through 12-31-07 |		*Trailing 5-Yr through 12-31-07
Standard Deviation	13.42	Sharpe Ratio	0.78
Mean	15.8	Bear Market Decile Rank*	---

Modern Portfolio Theory Statistics	Trailing 3-Yr through 12-31-07	
	Standard Index	Best Fit Index
	S&P 500 TR	Russell 2000 Growth
R-Squared	55	87
Beta	1.29	0.87
Alpha	4.92	6.49

Fund
S&P 500
Category