Options

Options

A European call option is a contract which gives its owner (the buyer) the right, but not the obligation, to **buy** a pre-specified piece of property (underlying asset) at a pre-specified price (the exercise price or strike price) on a pre-specified day (expiration day).

A European put option is a contract which gives its owner (the buyer) the right, but not the obligation, to **sell** a pre-specified piece of property (underlying asset) at a pre-specified price (the exercise price or strike price) on a pre-specified day (expiration day).

Options

An American Option is like a European option except it can be exercised any time prior to expiration.

Notation:

C	=	Call Price or Premium
P	=	Put Price or Premium
S	=	Price of the Underlying Asset
T	=	Expiration Date
K	=	Strike Price or Exercise Price

The creator of the option is referred to as the seller or the writer.

When Should You Exercise An Option?

The easiest case to consider is what to do at expiration.

Suppose on June 1 you buy a European call on IBM stock which expires on September 24 with K=40.

If the price of IBM is $45 on September 24, what should you do?

What is the profit <u>at expiration</u>?

When Should You Exercise An Option?

If the price of IBM is $35 on September 24, what should you do?

What is the profit <u>at expiration</u>?

When Should You Exercise An Option?

In general:

- Exercise (i.e., buy) when $K \leq S_T$

- Do Nothing when $K > S_T$

This implies that the value of the call at expiration is

$$C = S_T - K \quad \text{if } K \leq S_T$$
$$= 0 \quad \text{otherwise}$$

This can be rewritten as

$$C = \max[0, S_T - K]$$

When Should You Exercise An Option?

Note: when $K<S_T$, we say the call is **in-the-money**
when $K=S_T$, we say the call is **at-the-money**
when $K>S_T$, we say the call is **out-of-the-money**

When Should You Exercise An Option?

Suppose on June 1 you buy a European put on IBM stock which expires on September 24 with K=40.

If the price of IBM is $45 on September 24, what should you do?

What is the profit at expiration?

When Should You Exercise An Option?

If the price of IBM is $35 on September 24, what should you do?

What is the profit at expiration?

When Should You Exercise An Option?

In general:

- Exercise (i.e., sell) when $K \geq S_T$ (in-the-money)
- Do Nothing when $K < S_T$ (out-of-the-money)

This implies that the value of the put at expiration is

$$P = K - S_T \quad \text{if } K \geq S_T$$
$$= 0 \quad \text{otherwise}$$

This can be rewritten as

$$P = \max[0, K - S_T]$$

When Should You Exercise An Option?

Note: when $K>S_T$, we say the put is **in-the-money**
when $K=S_T$, we say the put is **at-the-money**
when $K<S_T$, we say the put is **out-of-the-money**

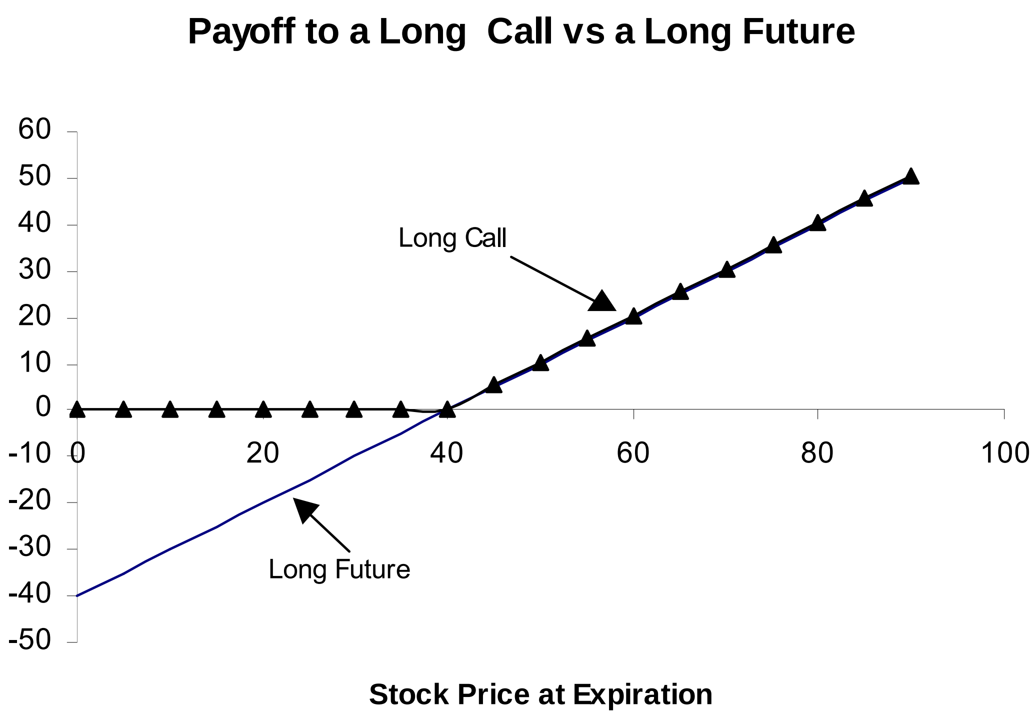
Comparison Between Options and Futures

Consider a call with K = \$40 and initial cost C = \$2.78

- The **payoff** at expiration on a long position in one call is

C	= 0			if $S_T < K = \$40$
	$= S_T - K$	=	$S_T - 40.00$	if $S_T \geq K = \$40$

Comparison Between Options and Futures

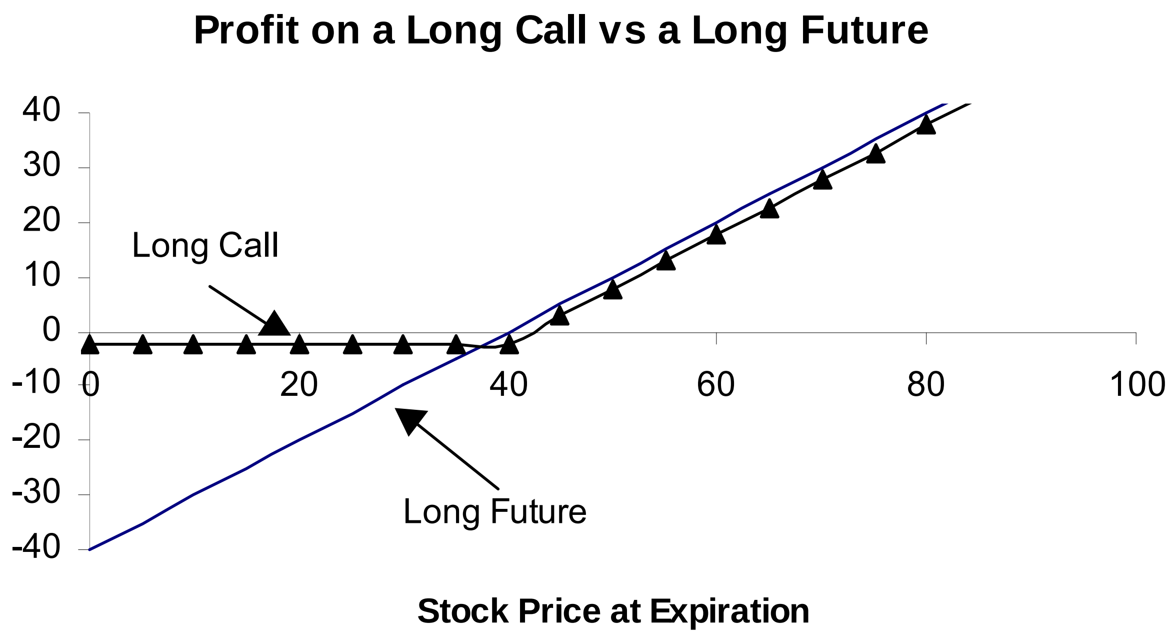


Comparison Between Options and Futures

The **profit** (payoff plus the initial cost) at expiration on a long position in one call is

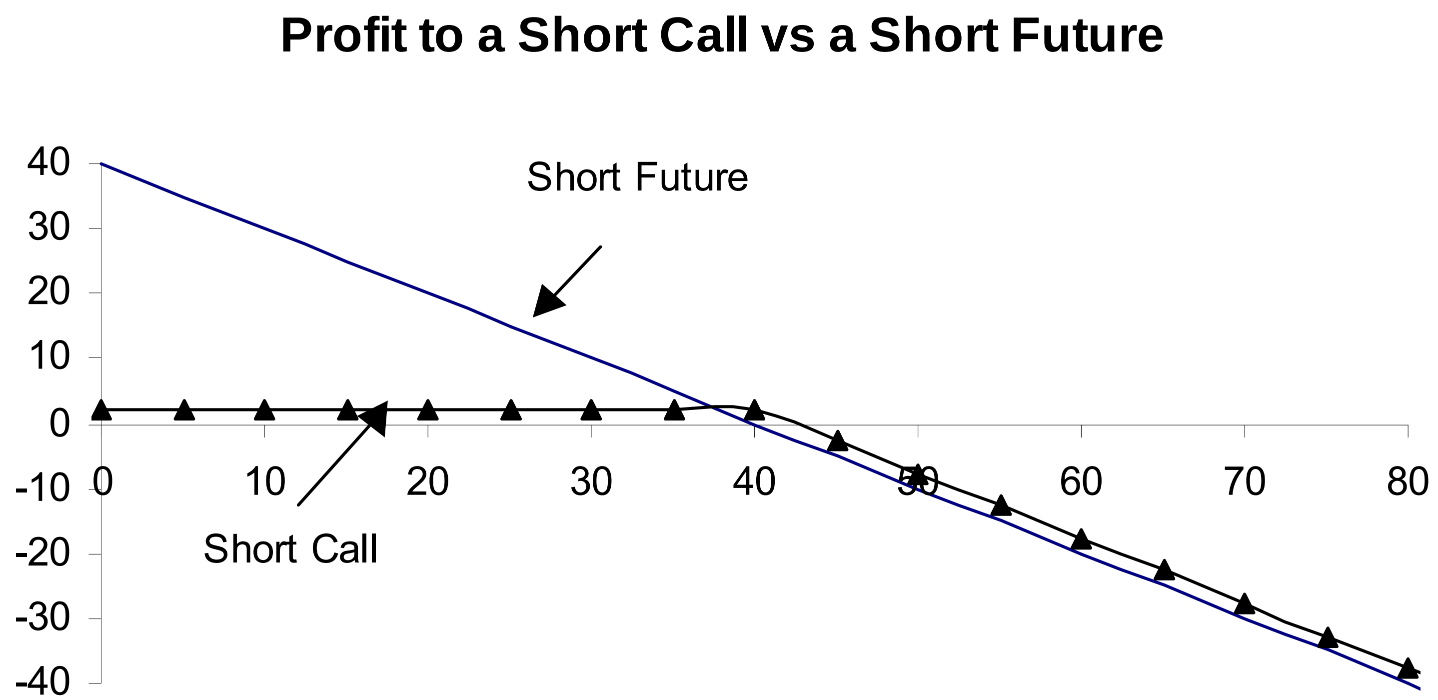
profit =	$-C$	$= -2.78$	if $S_T < K = \$40$
=	$-C+S_T-K$	$= S_T - 42.78$	if $S_T \geq K = \$40$

Comparison Between Options and Futures



Note that we are temporarily ignoring the time value associated with the initial cost of the option.

Comparison Between Options and Futures



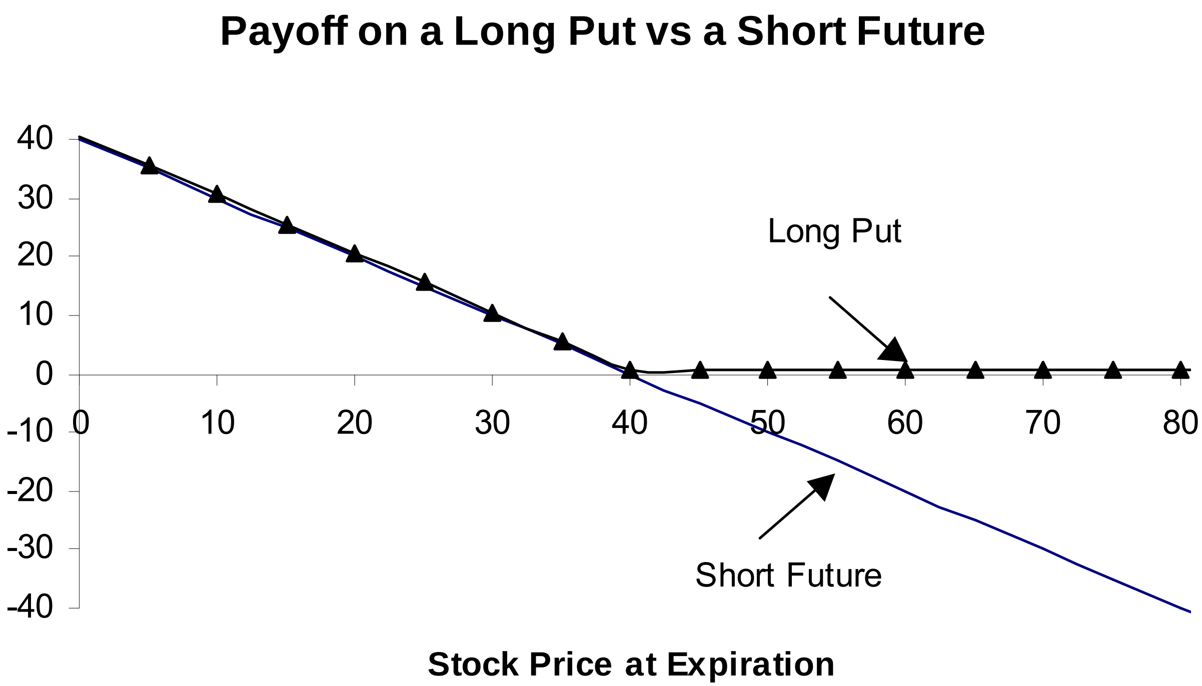
Comparison Between Options and Futures

Consider a put with K = \$40 and initial cost P = \$1.99

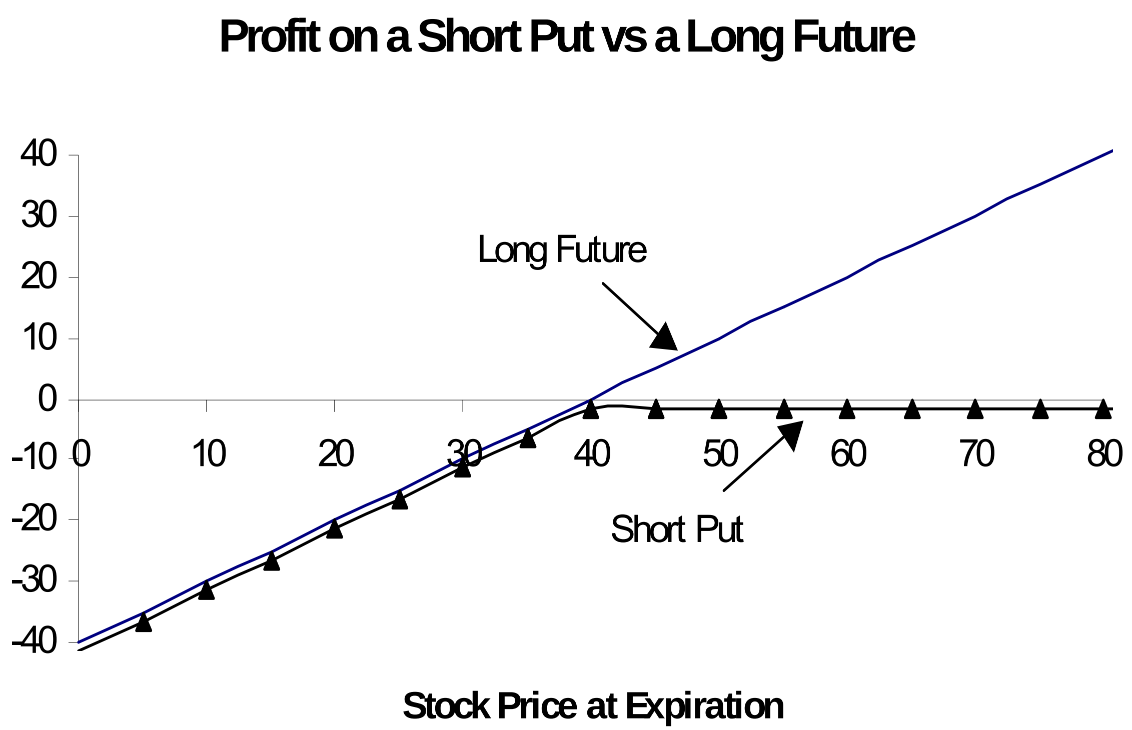
The **payoff** at expiration on a long position in one put is

$$
\begin{aligned}
P &= 0 && \text{if } K < S_T \\
&= K - S_T = 40 - S_T && \text{if } K \geq S_T
\end{aligned}
$$

Comparison Between Options and Futures



Comparison Between Options and Futures



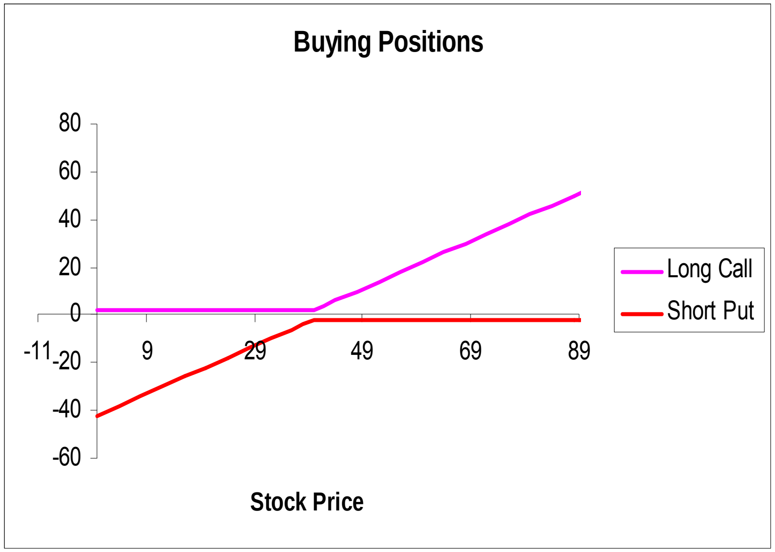
Comparison Between Options and Futures

Summary:

- Positions which involve buying:

    - Long Future
    - Long Call
    - Short Put

Comparison Between Options and Futures

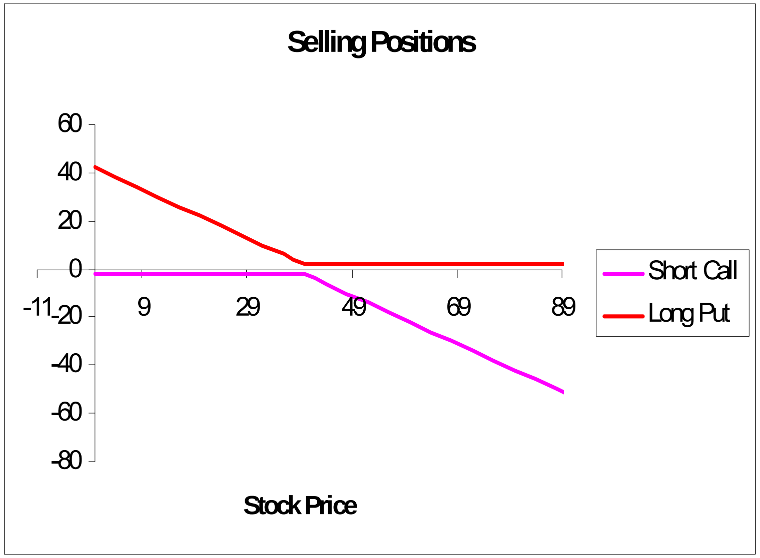


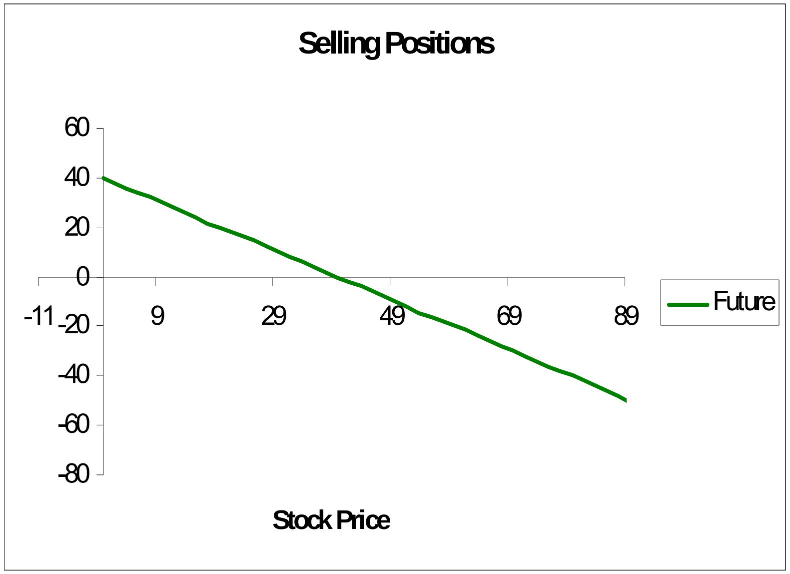
Comparison Between Options and Futures

Positions which involve selling:

- Short Future
- Short Call
- Long Put

Comparison Between Options and Futures



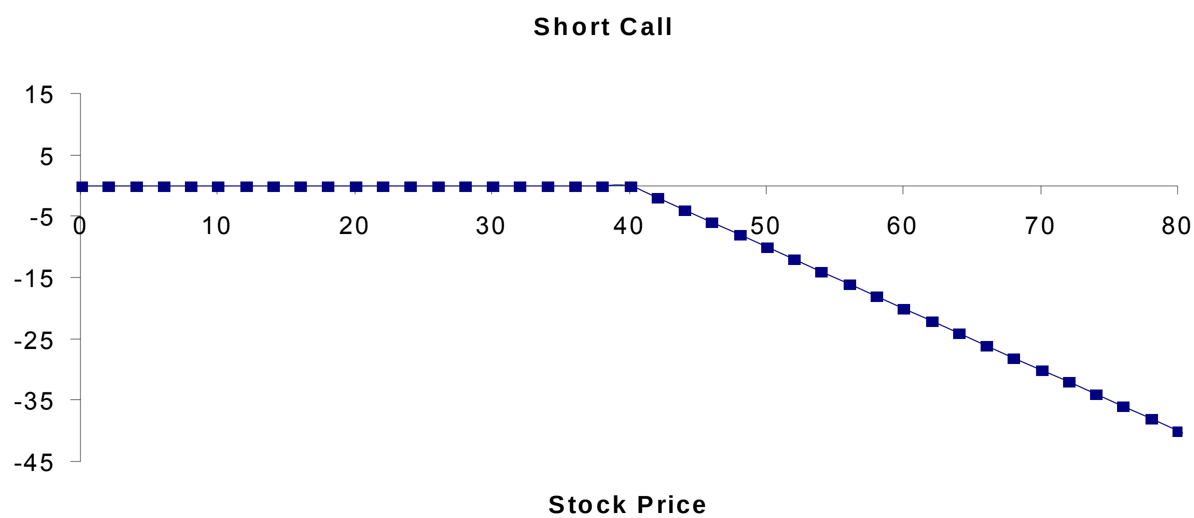


Hedging

- A **hedge** is a position where gains on one part of the position partially or wholly offset loses on another part of the position. For example, gains on an option position can offset loses on a position in the underlying asset and gains on the underlying asset can offset loses on an option position.

Hedging

- Covered Call -- Suppose you have written 1 call and you would like to hedge your position. What should you do?



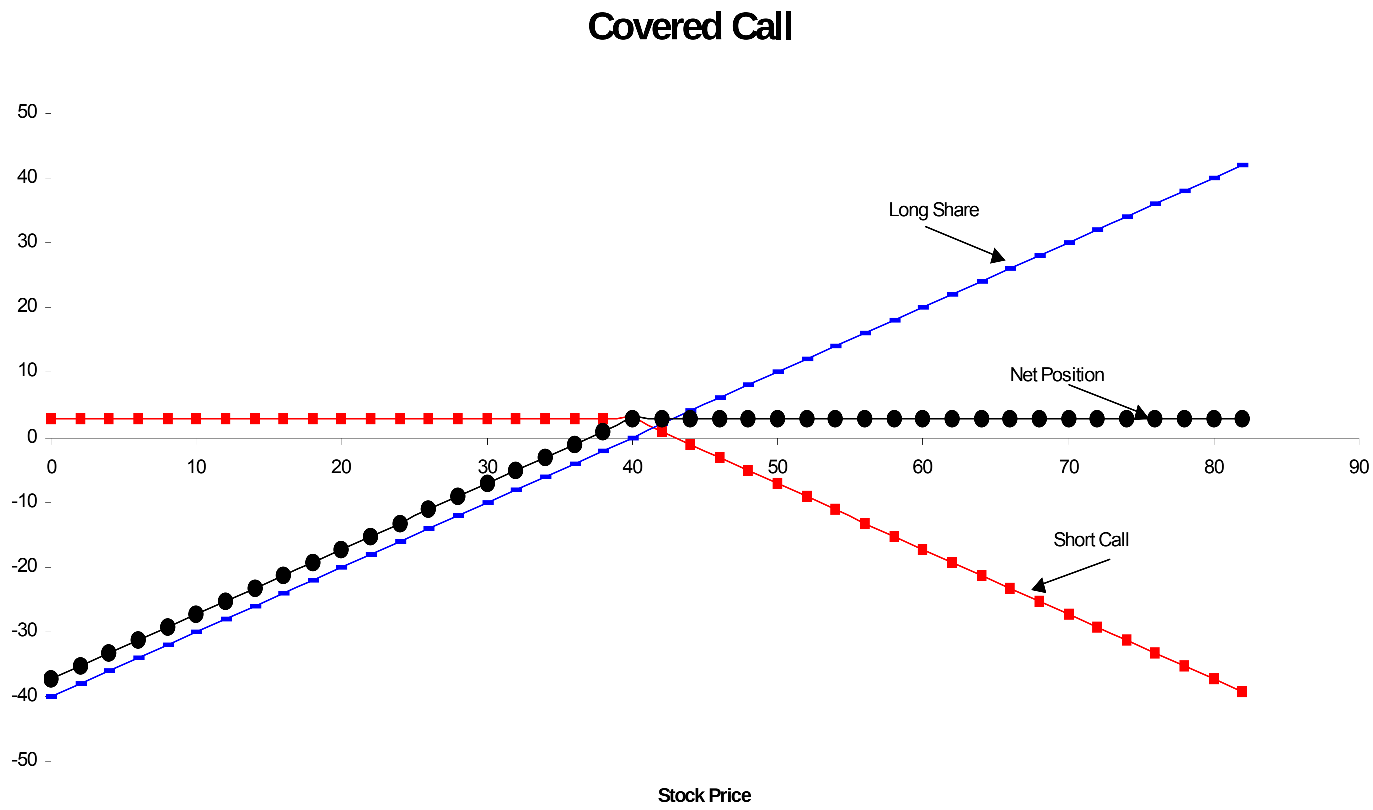
Hedging

Suppose that you buy one share in addition to shorting a call.

- Assume C=$2.78, K=40 and S_t =40.
- The initial cost and the payoff at expiration are given below:

	Today	$S_T < K$	$S_T \geq K$
WRITE 1 CALL	C=2.78	0	$K-S_T = 40-S_T$
BUY 1 SHARE	$-S_t=-40$	S_T	S_T
TOTAL	-37.22	S_T	40

Hedging

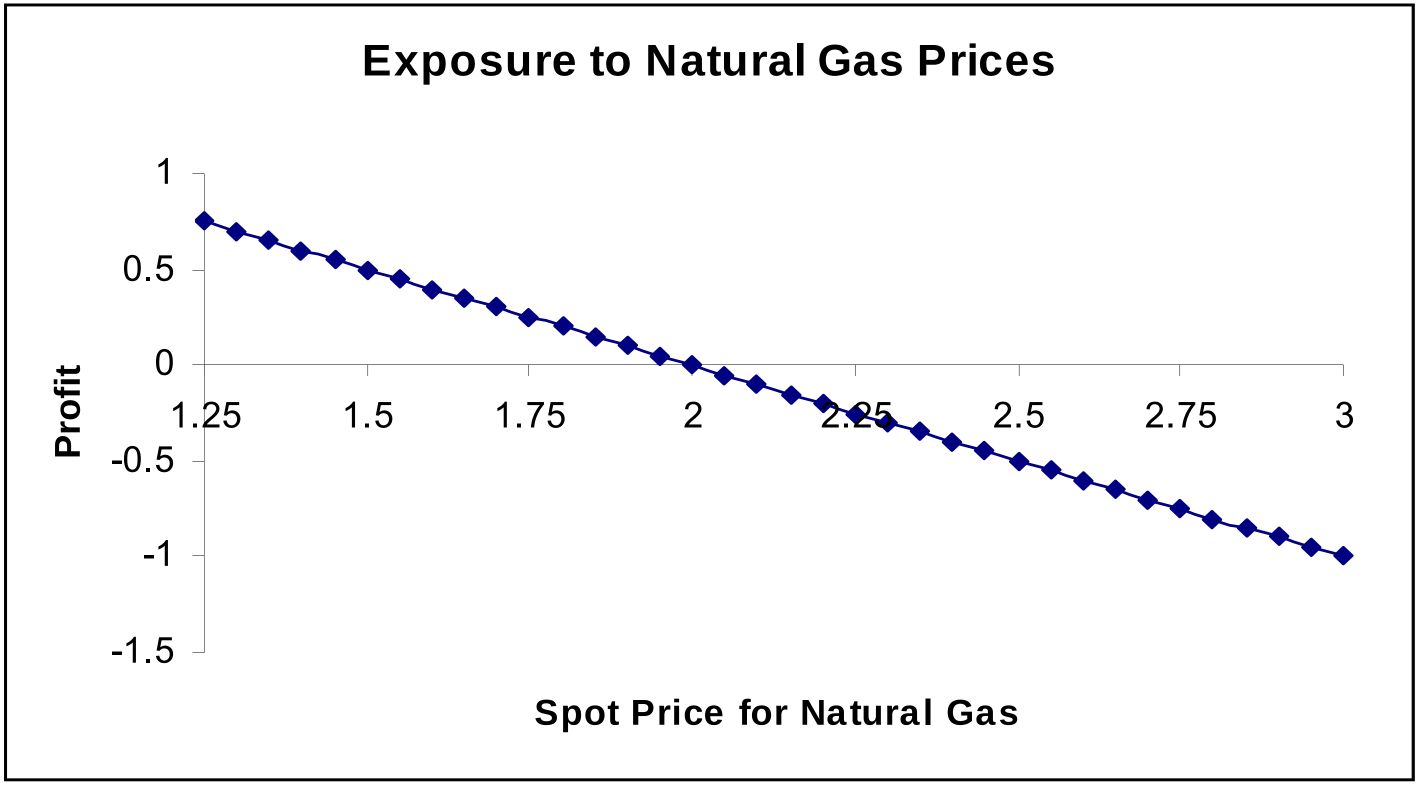

Covered Call
50
40
30
20
10
0
-10
-20
-30
-40
-50
0
10
20
30
40
50
60
70
80
90
Long Share
Net Position
Short Call
Stock Price

Hedging with Futures, Caps, Floors and Collars

- Suppose I am a producer of a chemical which uses natural gas as an input and my "break-even" cost for natural gas is $2.00 per MMBtu (i.e., if the cost of gas is above $2.00 per MMBtu, I lose money and if it is below $2.00, I make money.)

Note that my underlying exposure is **short**. That is, I like low prices and dislike high prices.

Hedging with Futures, Caps, Floors and Collars

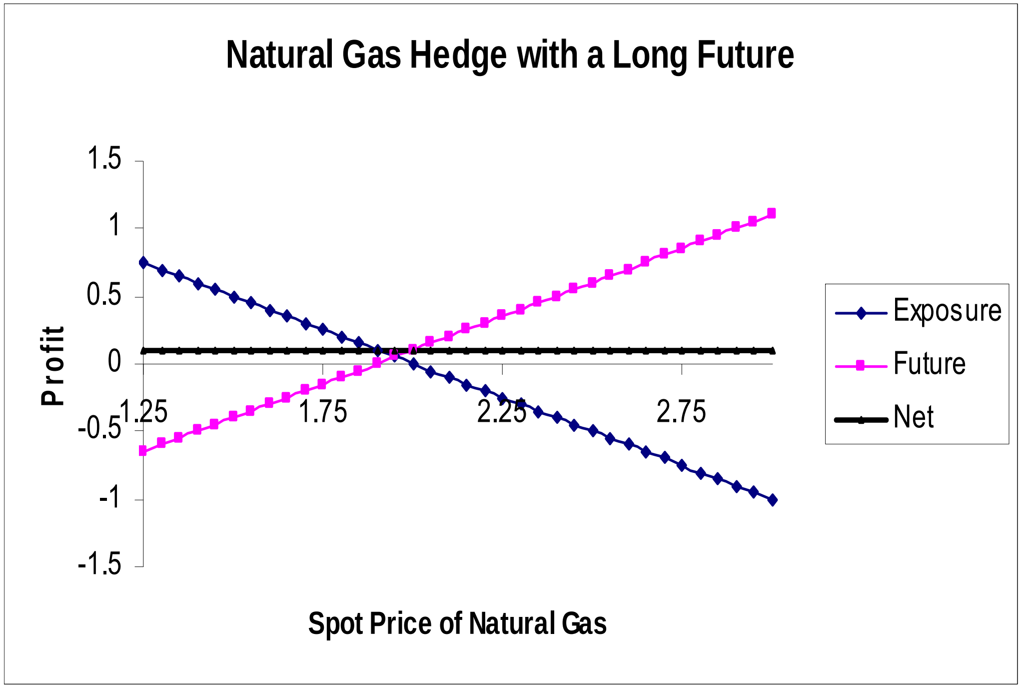


What are the ways that I can hedge my exposure?

Hedging with Futures, Caps, Floors and Collars

Long Future

Suppose the futures price is $1.90 per MMBtu. I could go **long** a natural gas future and lock-in a buying price of $1.90 per MMBtu. My hedged position looks like



Hedging with Futures, Caps, Floors and Collars

- One of the disadvantages of this position is that in order to eliminate the exposure to prices above $1.90, I also eliminate the ability to benefit from prices which are below $1.90.

- Is there a way to get the protection from price increases and still benefit from price declines?

Hedging with Futures, Caps, Floors and Collars

Buy a **cap**. A cap is just a call option. When gas prices increase, the gain on the option position offsets the loss due to higher gas prices.

Suppose natural gas ranges in price from about $1.25 to $3.00 per MMBtu. Suppose I buy a call option on natural gas with a strike price of $2.10. Each option contract is for 10,000 MMbtu's. The call price per MMBtu is .12.

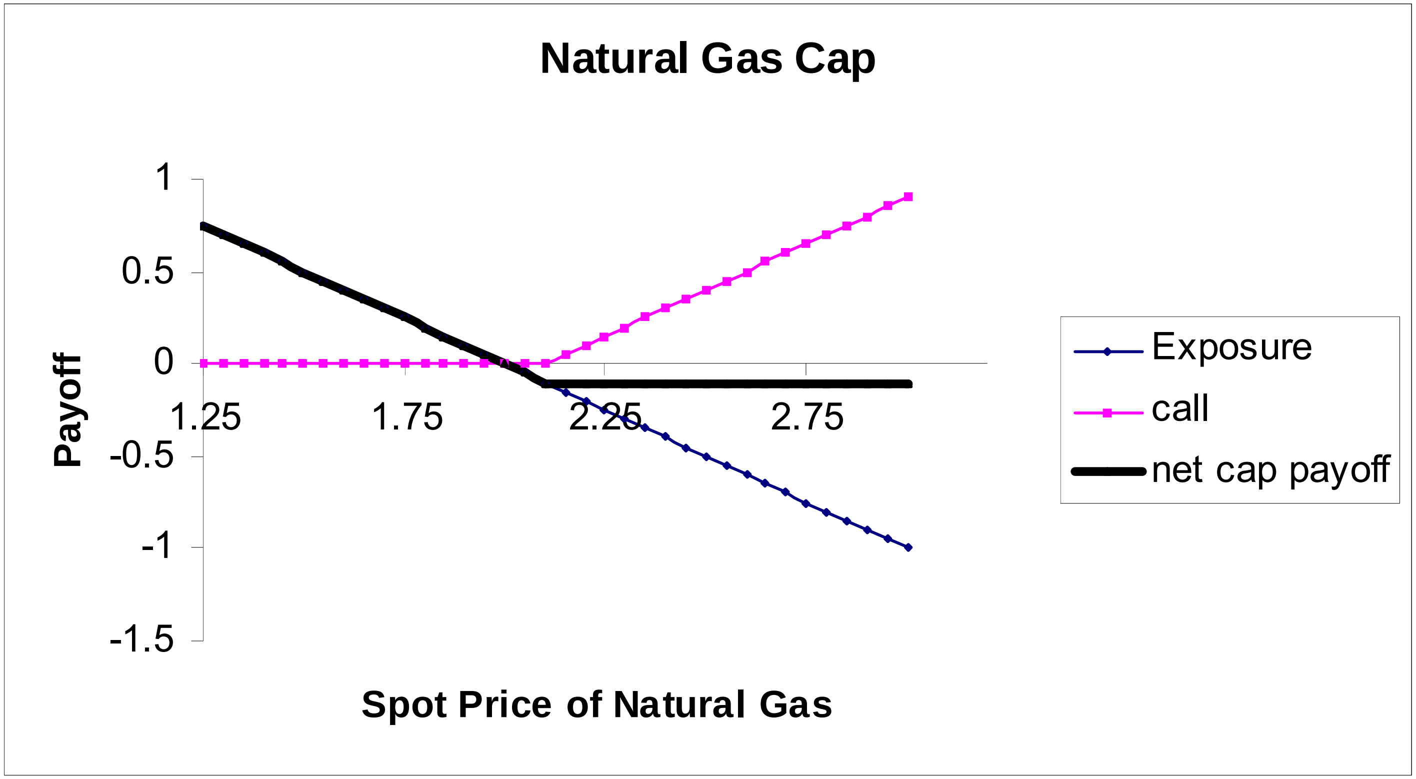
Hedging with Futures, Caps, Floors and Collars

My cash flow per MMBtu is given below.

	Today	G < K=2.10	G ∃ K=2.10
Buy I Call	-C=-.12	0	G-K=G-2.10
Buy 1 MMBtu of Gas at the Expiration Date		-G	-G
TOTAL	-C=-.12	-G	-2.10

Notice that the cost of gas is capped at $2.10/MMBtu and the "all in" cost of gas is capped at $2.22.

Hedging with Futures, Caps, Floors and Collars



Hedging with Futures, Caps, Floors and Collars

Suppose that I like the downside protection that I get from the cap but I find that the cap expensive. What can I do?

There are several possibilities:

1. Raise the strike price on the call. This lowers the cost of the call but provides less downside insurance (you get what you pay for).

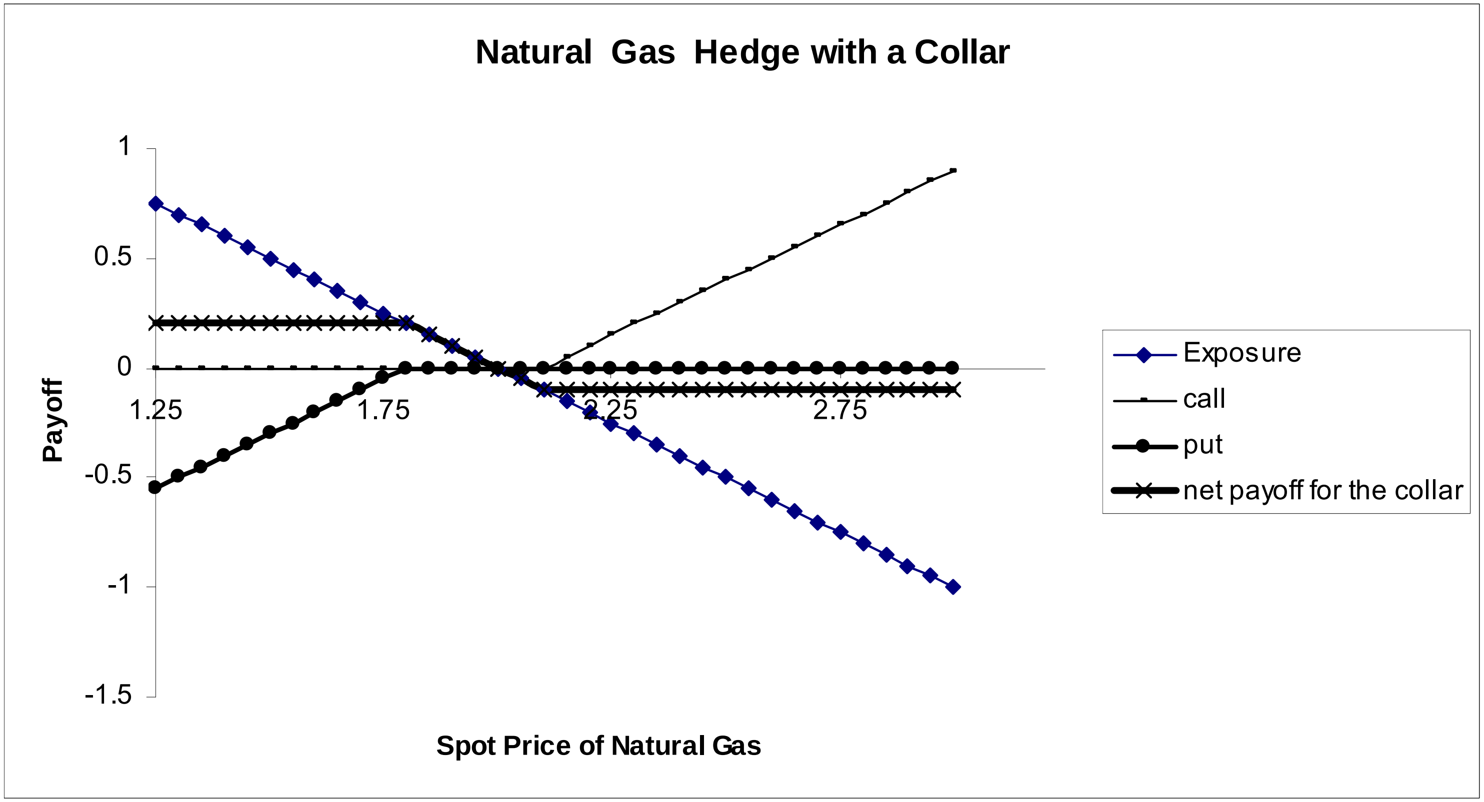
2. Sell off some of the potential gains which would occur if the price of gas was low. To do this, you sell a natural gas put. Buying a call and simultaneously selling a put is referred to as a **collar**. The revenue from selling the put offsets some of the cost of buying the call. Now there is both a cap and a floor on the cost of natural gas.

Hedging with Futures, Caps, Floors and Collars

Example: Suppose you buy a 2.10 call and sell a 1.80 put.

	Today	G < 1.80	1.80< G ≤2.10	G > 2.10
Sell 1 Put with K= 1.60	P= .065	**-(K-G) = -(1.80-G)**	0	0
Buy I Call with K= 2.10	-C= -.12		0	G-K= G-2.10
Buy 1 MMBtu of Gas at the Expiration Date		-G	-G	-G
TOTAL	P-C= -.055	-1.80	-G	-2.10

Hedging with Futures, Caps, Floors and Collars



Hedging with Futures, Caps, Floors and Collars

There are several things to note about this position:

- Your position has a 2.10 cap and a 1.80 floor. The cost this position is .055.
- The highest all-in cost of natural gas is 2.10+.055 = 2.155. This will be the cost when the spot price of natural gas is 2.10 or higher. Your profit in this case will be -.155.
- The lowest all-in cost of natural gas is 1.80+.055 = 1.855. You will pay this price if the spot price of natural gas is 1.80 or lower. Your profit in this case will be .145.
- You have some exposure to market prices when the spot price is between 1.80 and 2.10.
- If you wanted to make the collar less expensive you could raise the floor (i.e., sell-off more of the gains) or raise the cap (i.e., buy less insurance).

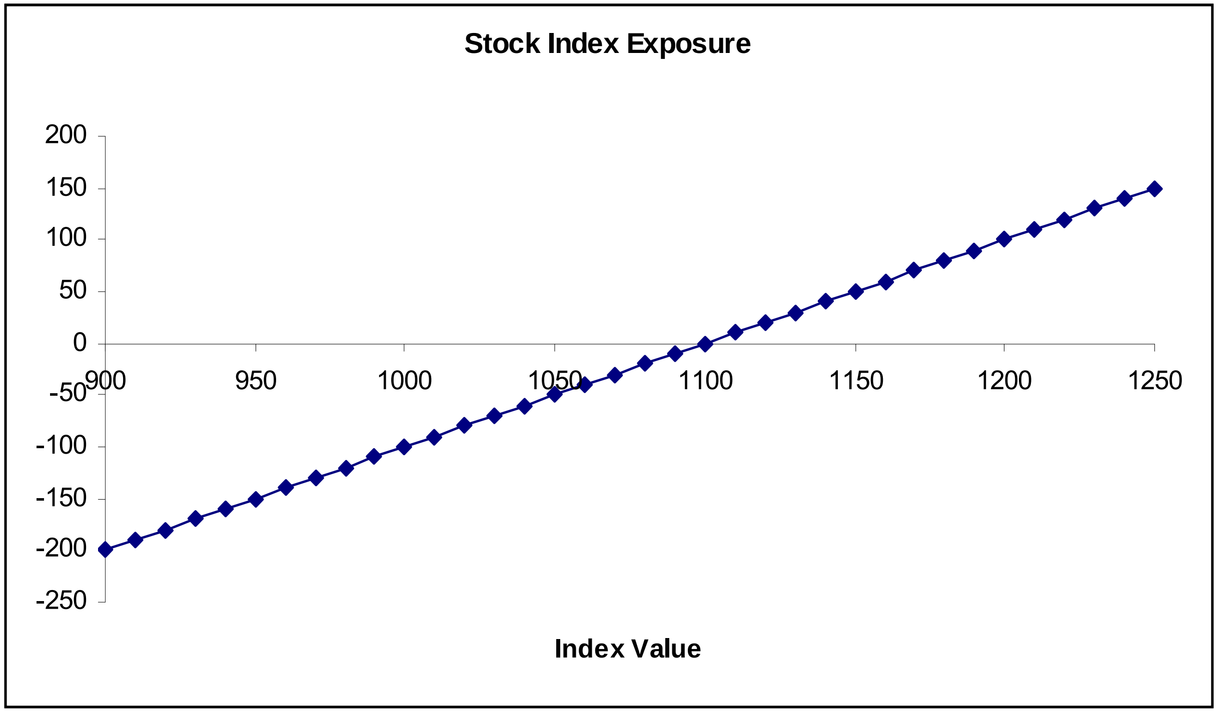
Hedging with Futures, Caps, Floors and Collars

Hedging Long Exposures

Suppose I am a manager of an index fund.

Suppose my “break-even” value for the index is 1100 (i.e., for whatever reason I feel that if the value of the index is above 1100, I’m ahead and if it is below 1100, I’m behind.)

Hedging with Futures, Caps, Floors and Collars



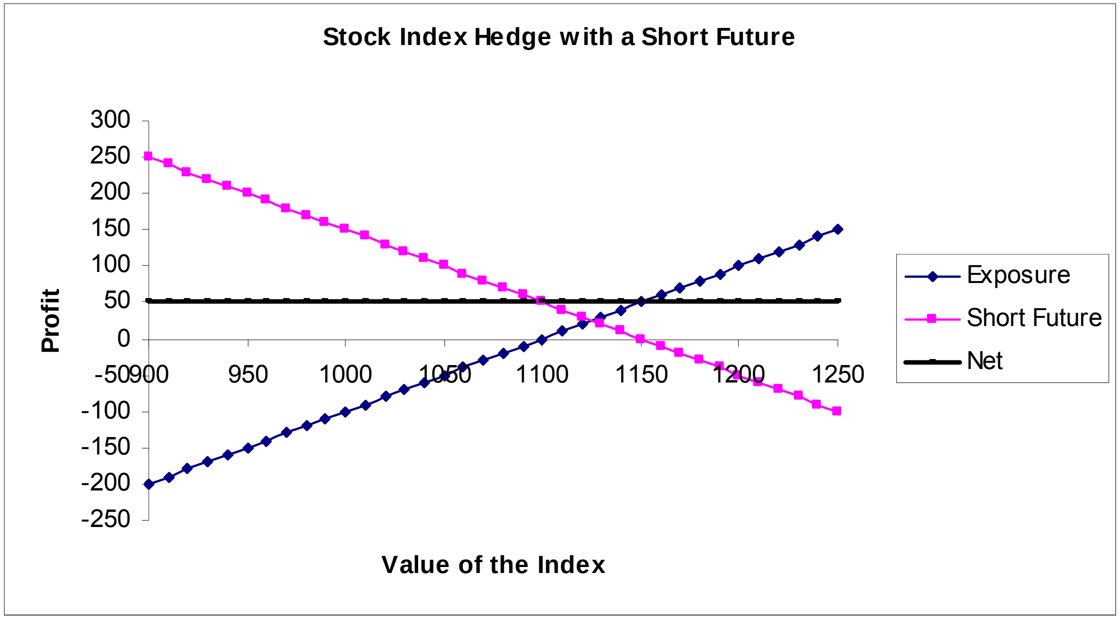
Note that my underlying exposure is **long**. That is, I like high values and dislike low values.

What are the ways that I can hedge my exposure?

Hedging with Futures, Caps, Floors and Collars

Short Future

Suppose the futures price is 1150. I could go **short** a stock index future and lock-in a selling price of 1150. My hedged position looks like:



Hedging with Futures, Caps, Floors and Collars

- One of the disadvantages of this position is that in order to eliminate the exposure to values below 1150, I also eliminate the ability to benefit from values which are above 1150. I have “synthetically” turned my stock index position into a T-bill and hence the rate of return on my position should be the risk-free rate.

- Is there a way to shield myself from drops in the value of my portfolio but receive some of the benefits if the index should increase in value?

Hedging with Futures, Caps, Floors and Collars

- Buy a **floor** (this is also called portfolio insurance). A floor is just a put option. When the portfolio drops in value, the gain on the option position offsets the loss on the index portfolio.

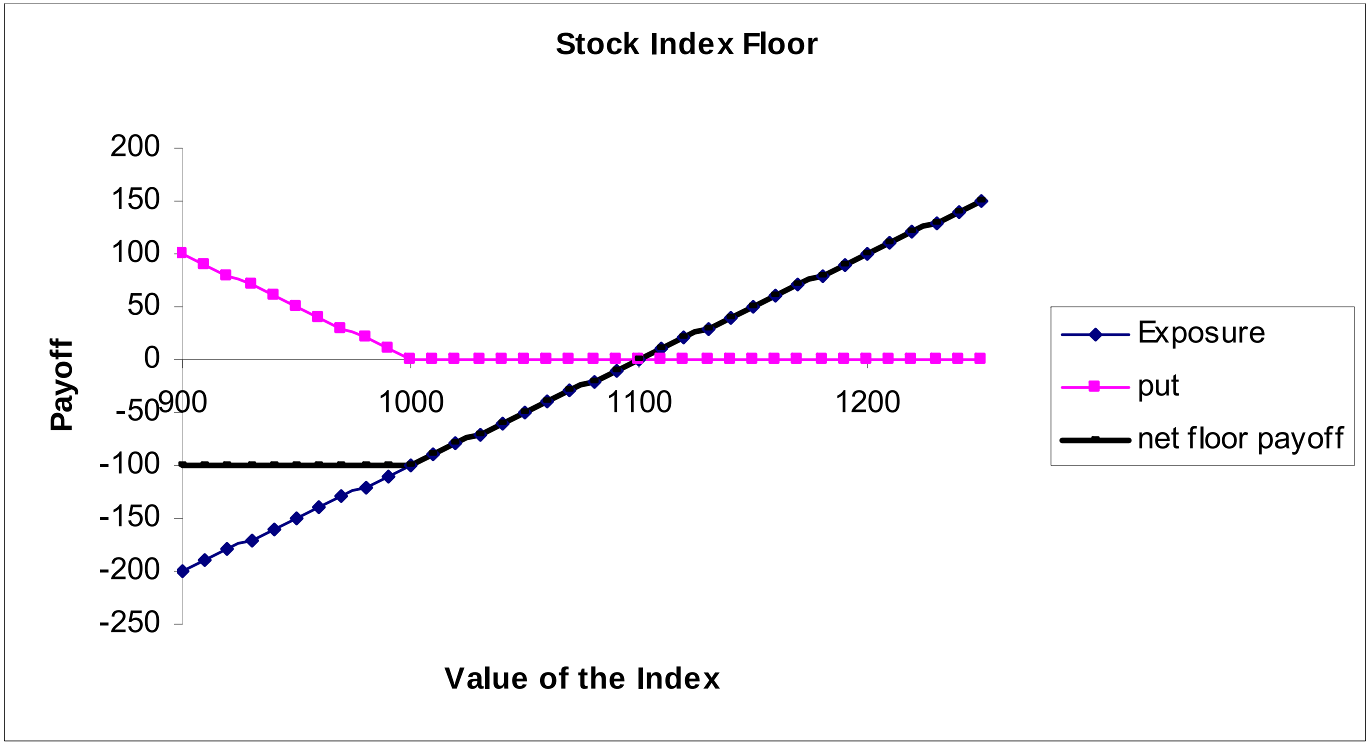
Hedging with Futures, Caps, Floors and Collars

The current value of the index is 1100. On March 24 I buy a S&P 500 put option with a strike price of 1000 which expires three months later. If the value of the S&P 500 index is below 1000 at expiration, the put is worth (1000-I_T) and if the S&P 500 Index is above 1000 at expiration, the put expires worthless. Suppose the put costs $2.63. My cash flow per unit of the index is given below.

	Today	I_T < K=1000	$I_T \geq$ K=1000
Buy I Put with K=450	-P = -2.63	K -I_T =1000-I_T	0
Buy 1 Unit of the Index Portfolio	-I_t = -1100	I_T	I_T
TOTAL	-1102.63	1000	I_T

Notice the manager pays $2.63 to insure that the portfolio has a 1000 floor over the next 3 months. On an annualized basis, the manager is giving up 1% of the return to pay for the insurance.

Hedging with Futures, Caps, Floors and Collars



Hedging with Futures, Caps, Floors and Collars

Suppose that I like the downside protection that I get from the floor but I find the floor expensive. What can I do? There are several possibilities:

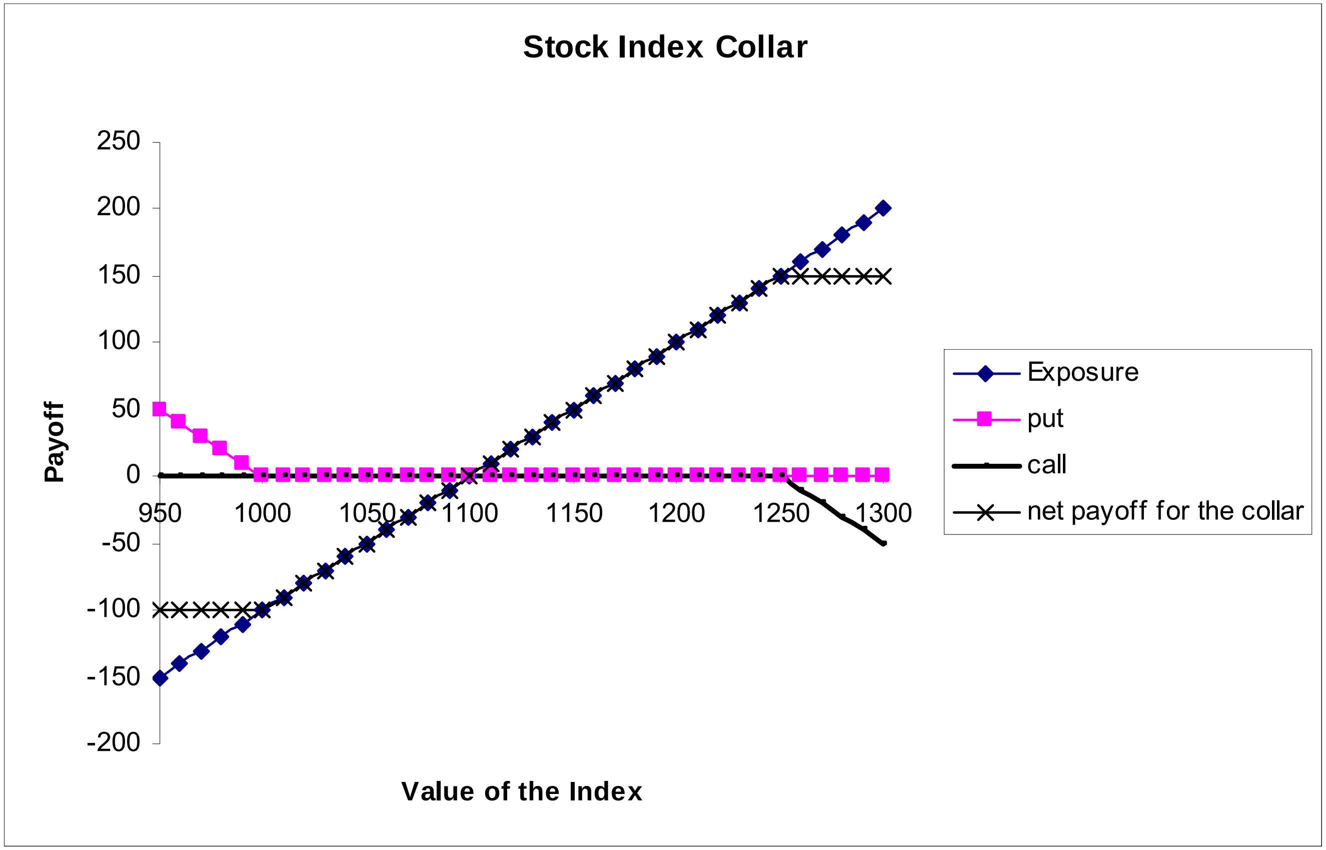
- Lower the strike price on the put. This lowers the cost of the put but provides less insurance (you get what you pay for). For example, an identical put with K= 950 instead of 1000 costs $.46.
- Sell off some of the potential gains which would occur if the price of the index increases. To do this you sell a stock index call. Buying a put and simultaneously selling a call is referred to as a **collar**. The revenue from selling the call offsets some of the cost of buying the put. Now the manager has both a cap and a floor on the value of his portfolio.

Hedging with Futures, Caps, Floors and Collars

Example: Suppose you buy a 1000 put and sell a 1250 call.

	Today	$I_T < 1000$	$1000<I_T<1250$	$I_T \geq 1250$
Buy I Put with K=1000	-P = -2.63	$1000-I_T$	0	0
Sell 1 Call with K=1250	C = 2.35	0	0	$1250-I_T$
Buy 1 Unit of the Index Portfolio	-It = -1100	I_T	I_T	I_T
TOTAL	-1100.28	1000	I_T	1250

Hedging with Futures, Caps, Floors and Collars



Hedging with Futures, Caps, Floors and Collars

There are several things to note about this position:

1. Your position has a 1250 cap and a 1000 floor. The cost of this position is .28.

2. The lowest all-in payoff on the portfolio is 1000-.28 = 999.72. This will be the payoff when the value of the index is 1000 or lower. The highest all-in payoff is 1250-.28 = 1249.72. This will be the payoff when the value of the index is 1250 or higher.

3. You have some exposure to market prices when the value of the index is between 1000 and 1250.

4. If you wanted to make the collar less expensive you could lower the cap (i.e., sell-off more of the gains) or lower the floor (i.e., buy less insurance).

Hedging with Futures, Caps, Floors and Collars

Summary:

	Short Exposure	Long Exposure
Hedge	Long Future	Short Future
Cost	$0	$0
Insurance	Buy a Call	Buy a Put
Cost	$C	$P
To Lower the Cost	Raise the Strike Price	Lower the Strike Price
Sell-Off Gains	Sell a Put	Sell a Call
Revenue	$P	$C
To Increase Revenue	Raise the Strike Price	Lower the Strike Price

Speculative Positions

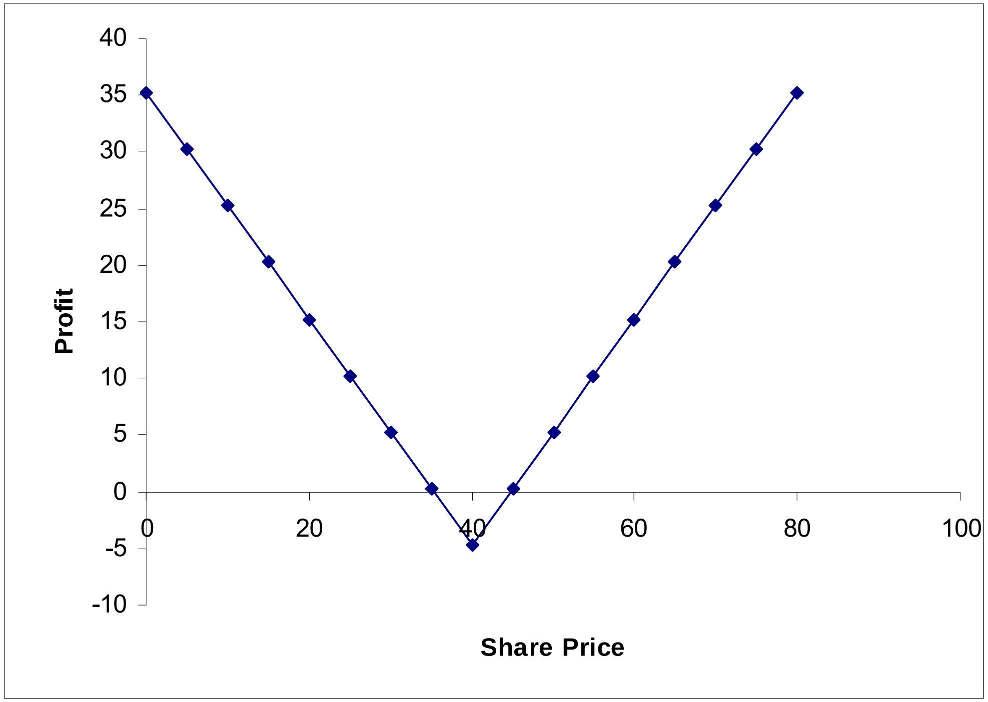
- There are many ways that options can allow an investors to tailor their positions to fit a particular view.
- Suppose an investor thinks that a stock will be more volatile in the near future and she would like to create a position where she could profit from that view if she is correct.

Speculative Positions

- Suppose the current stock price is \$40/share and she buys a call with K= 40 for \$2.78 and buys a put with K=40 for \$1.99. The cash flows for this position are:

		At Expiration	
	Today	$S_T \leq K$	$S_T > K$
Long Call K=40	-2.78	0	S_T - K
Long Put K=40	-1.99	K - S_T	0
Total	-4.77	K - S_T	S_T > K

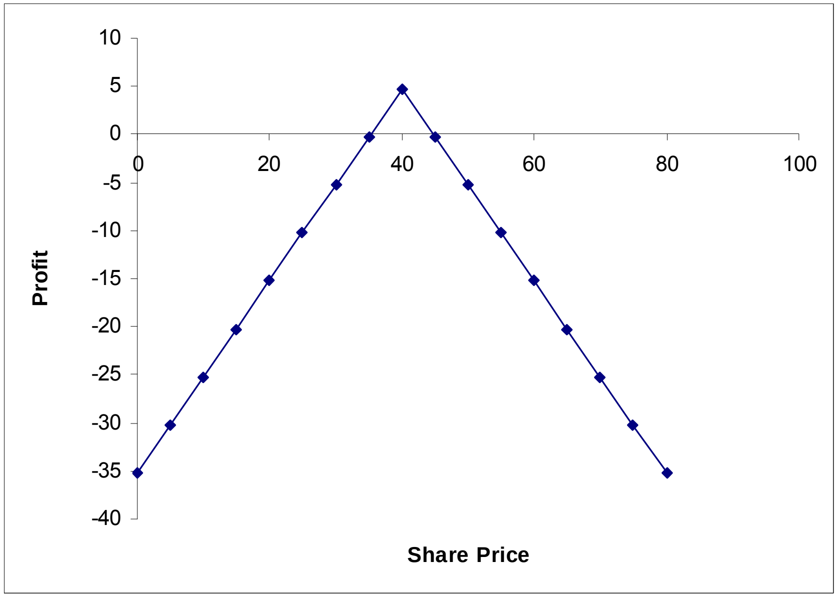
Speculative Positions



This position has a positive profit when there is a large price movement in either direction and loses money when there is a small change in the price.

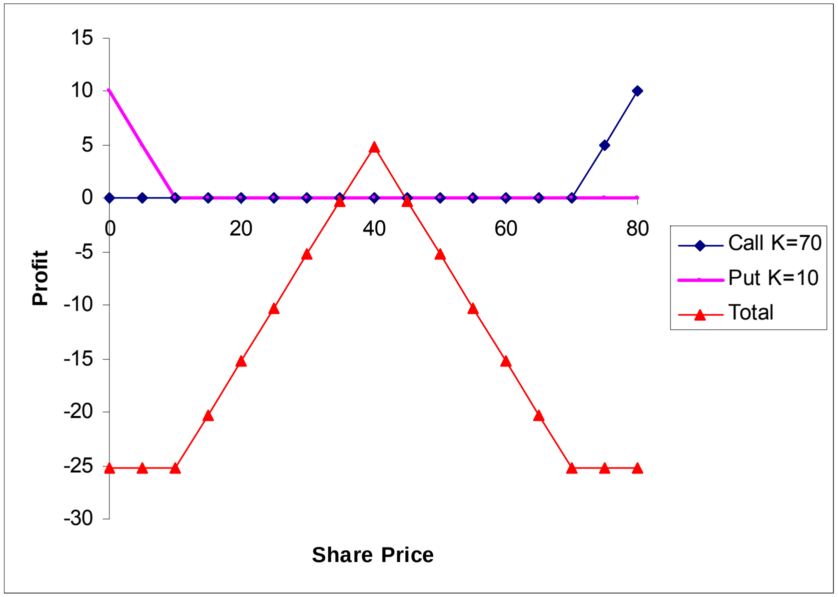
Speculative Positions

If an investor thinks that a stock will be less volatile in the near future and she could do the reverse, that is sell a call and sell a put.



Speculative Positions

If the investor finds this position too risky (due to the large negative tails), the risk can be reduce by buying a call with a high strike price and selling a put with a low strike price.



PUT-CALL PARITY

The relationship between the values of European puts and calls with the same expiration date, same exercise price and same underlying asset.

Previously we saw that:

looks like

- 1 written call ==============> 1 written put
- 1 long share

and

looks like

- 1 long put ===============> 1 long call
- 1 long share

PUT-CALL PARITY

This suggests that we can make:

- a "synthetic" put from a call and the stock

and

- a "synthetic" call from a put and the stock.

By synthetic, I mean exactly replicate the cash flows.

PUT-CALL PARITY

The cash flow from a long call is:

		At Expiration	
	Today	$S_T \leq K$	$S_T > K$
Long 1 Call	-C	0	S_T - K

PUT-CALL PARITY

Suppose we try to recreate the cash flows at expiration with a put and the stock:

		At Expiration	
	Today	$S_T \leq K$	$S_T > K$
Long Put	-P	K- S_T	0
Long Share	-S_t	S_T	S_T
Total	-P- S_t	K	S_T

The cash flows don't match! We have K too much at expiration.

PUT-CALL PARITY

Now try borrowing PV(K) to get the cash flows to match:

		At Expiration	
	Today	$S_T \leq K$	$S_T > K$
Long Put	-P	$K - S_T$	0
Long Share	$-S_t$	S_T	S_T
Borrow Present Value of K	Ke^{-rT}	-K	-K
Total	$-P- S_t-K$	0	$S_T - K$

Now the cash flows are the same!

PUT-CALL PARITY

Now I have two portfolios:

- 1 long call
- 1 long put
  1 long share ==========> SYNTHETIC CALL
  KR^{-T} borrowed

Both give exactly the same payoff regardless of the value of S_T.

PUT-CALL PARITY

Therefore, they must sell for the same price.

$$C = P + S - Ke^{-rT}$$

This is called an arbitrage relationship because if this relationship doesn't hold there is an arbitrage opportunity.

PUT-CALL PARITY

$$C = P + S - Ke^{-rT}$$

This relationship says that buying a call is like

1. Buying 1 share
2. Buying a long put
3. Borrowing (at the risk free rate!) the present value of K to partially finance the purchase of the share and the put.

Note that given a particular share price, the higher the K the greater the implicit leverage.