Introduction to Futures and Forwards

Forwards

- A **forward contract** is an agreement between two parties for the deferred delivery of an asset. The contract has the following characteristics:
    - The contract specifies:
        1. the quantity and type of asset
        2. delivery time and place
        3. price

- Both the buyer and the seller are **obligated** to the carry out the transaction.

Forwards

- Terminology:

  If you have agreed to be the buyer, you are said to be **long**
  If you have agreed to be the seller, your are said to be **short**

- Notation:

$f_{t,T}$ = the forward price for a contract initiated at time t for delivery at time T
P_t = the spot price at time t (i.e., the price for immediate delivery)

Note that entering into a forward contract is "free", that is, no money changes hands when the contract is initiated

Forwards

The gain per contract to the long position is

= (# of units in the contract)*(P_T - $f_{t,T}$)

The gain per contract to the short position is

= (# of units in the contract)*($f_{t,T}$ - P_T)

Payoff Diagram

Example:

On September 10, two parties enter into a forward contract for the delivery of 100 ounces of gold on December 1 at $450 per ounce.

	Price Increase	Price Decrease
Dec 1 Spot Price	$460	$430
Long Profit	$10	-$20
Short Profit	-$10	$20

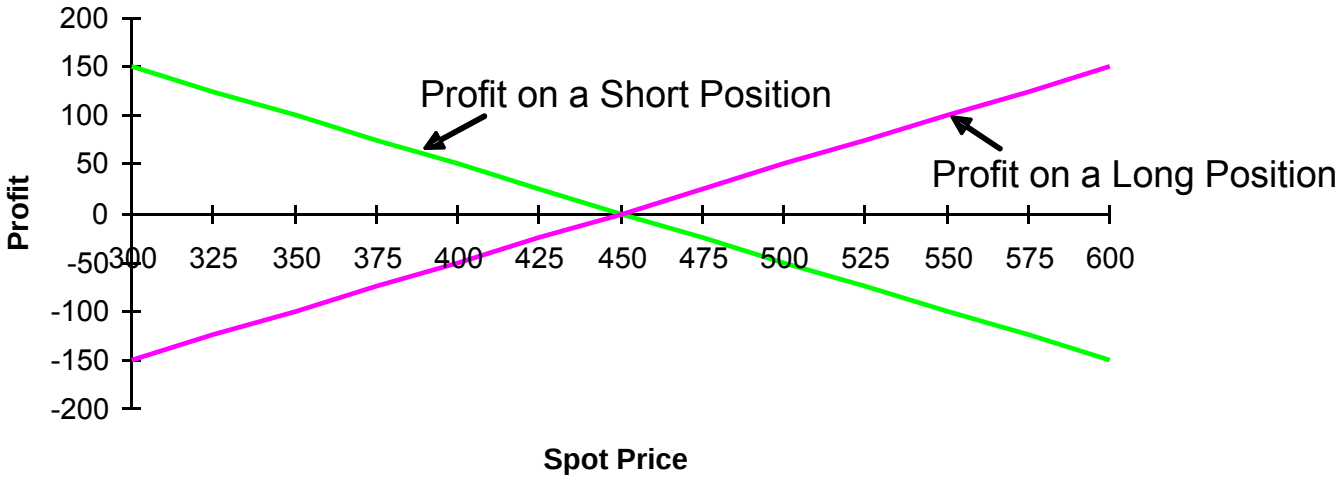
The long:

locks in a purchase price of $450
makes a profit if the spot price is above $450
takes a loss if the spot price is below $450

The short:

locks in a selling price of $450
makes a profit if the spot price is below $450
takes a loss if the spot price is above $450

Payoff Diagram



Futures

A future is an exchange traded forward contract. Exchange trading affects a contract in three ways:

1. The contract is standardized. This increases liquidity but the contract is not tailored to individual needs.

Futures

2. Margins are required

When a trader enters into a futures contract, the Clearing Corporation requires that the trader put up an initial margin. This amount depends upon the contract and whether the trader is a hedger or a speculator. If the balance in the margin account falls below a pre-specified amount referred to as the maintenance margin, the trader receives a margin call. The margin account balance must be brought back to the initial level or the position will be closed out by the Clearing Corporation

Futures

3. The contracts are marked-to-market. This means that gains and loses are realized every day.

Futures

Example: The following is an example of how marking-to-market and margin requirements help to minimize the problems associated with credit risk. Suppose a buyer and a seller enters into a futures contract on 7-1 for 5000 bushels of wheat to be delivered on 7-5 at $1/bushel. Suppose the initial margin is $1500 and the maintenance margin is $1200.

Futures

			Margin Account			
			Change		Balance	
Date	Transaction	$F_{t,T}$	Buyer	Seller	Buyer	Seller
7-1	Contract initiated. The buyer and the seller deposit the money in their margin accounts.	1.00	**1500**	**1500**	1500	1500
7-2	a. Price rises by .10, buyer gains (5000)(.10) and seller loses (5000)(.10). $500 is transferred from the sellers margin account to the buyer's margin account.	1.10	+500	-500	2000	1000
	b. Seller's account balance falls below $1200, so $500 must be deposited into the margin account. If not the Clearing Corporation liquidates the seller's position by buying an offsetting contract with $F_{t,T}$=1.10.			**+500**		1500
7-3	Price falls by .05, and the buyer loses (.05)(5000) and seller gains (.05)(5000). $250 is transferred from the buyers account to the seller's account.	1.05	-250	+250	1750	1750
7-4	a. Price falls by .15, and the buyer loses (.15)(5000) and seller gains (.15)(5000). $750 is transferred from the buyer's account to the seller's account.	.90	-750	+750	1000	2500
	b. Buyer's margin account falls below the maintenance margin, so $500 must be deposited into the margin account or the buyer's position will be liquidated.		**+500**		1500	
7-5	Price rises by .05, buyer gains by (.05)(5000) and the seller loses (.05)(5000). $250 is transferred from the seller's account to the buyer's account.	.95	+250	-250	1750	2250
7-5	The contract is settled at $.95. The buyer pays $4750 to the seller and receives5000 bushels of wheat.		-4750	+4750		
Net cash flow excluding margin payments			-5000	5000		
Net margin payments			**2000**	**2000**		

Futures

Even though the buyer paid $4750 at delivery, the net payment (excluding margin payments) made by the buyer over the life of the contract was $5000, the amount that was initially agreed upon. That is,

Date	Buyer	Seller
7-2	+500	-500
7-3	-250	-250
7-4	-750	-750
7-5	+250 -4750	-250 +4750
Total	-5000	+5000

Is a futures contract equivalent to a forward contract?

Futures Pricing

To price a future, we use the following procedure:

1. We create a riskless position using a futures contract and the spot asset.

2. The futures price is found by noting that all riskfree transactions should earn the riskfree rate.

Futures Pricing

To create a **riskless position**, you do what is called a cash and carry, that is, you buy the spot commodity and sell the future. The spot commodity is then carried to the delivery date of the futures contract and delivered against the short futures position. The cash flows associated with this position are given below.

	t	T
buy the spot	$-P_t$	*
sell the future		$F_{t,T}$
Total	$-P_t$	$F_{t,T}$

*The spot commodity which was purchased at t is used at time T to cover the short futures position.

Futures Pricing

The cash flows resemble the cash flows for a T-Bill because you pay a fixed amount today and receive a known amount at a pre-specified day in the future. The gross rate of return on a cash and carry is equal to:

$$\frac{F_{t,T}}{P_t}$$

Futures Pricing

- The gross rate of return on a real T-Bill over the period t to T is $e^{r(T-t)}$. Note that $e^{r(T-t)}$ is approximately equal to $(1+r)^{T-t}$

- To rule out an arbitrage opportunity, the rate of return on a riskless position must earn the riskfree rate. Therefore the fundamental no arbitrage relationship is:

$$\frac{F_{t,T}}{P_t} = e^{r(T-t)} \quad \rightarrow \quad F_{t,T} = e^{r(T-t)} P_t$$

Futures Pricing

Example: Suppose the spot price of gold is $325/ounce and the six month continuously compounded interest rate is 5.92%. The futures price for gold with delivery in 3 months is:

$$\$325 * e^{.25*(.0592)} = \$329.85$$

Futures Pricing

Example:
Suppose on t=4-1-06 a futures contract is created where 1 share of IBM will be traded on T=4-1-07 at $F_{t,T}$=$156/share.

Suppose the current spot price is $140/share and the continuously compounded T-Bill rate is 10%. Assume there are no dividends.

Is there an arbitrage opportunity?

The theoretical value for the future is

$$F_{t,T} = 140e^{.10*1} = 154.72$$

which is different from the market price.

Futures Pricing

To take advantage of this you should:

- Sell the future since it is overvalued.
- Buy the spot to hedge the risk associated with the short future.
- Borrow (short a T-bill) to finance the purchase of the spot.

Futures Pricing

The cash flows associated with this position are given below:

	t	T
sell a future		156
buy the spot	-140	*
short a T-Bill	140	$-140*e^{.10*1}$ =-154.72
Total	0	1.28

*The spot is used to cover the short position in the future.

Futures Pricing

Gold Comp. - comex

Data retrieved at Oct 30 21:40:38 GMT • All quotes are in Greenwich Mean Time • Data provided by eSignal

Contract	Month	Last	Chg	Open	High	Low	Volume	OpenInt	Exchange	Date	Time
GOLD	Nov '06	**604.7**s	**6.4**	608.5	608.9	608.0	5	3	COMX	10/30/06	18:42:08
GOLD	Dec '06	**606.2**	**-1.2**	606.5	607.5	606.0	634	183492	COMX	10/30/06	21:10:28
GOLD	Feb '07	**612.6**	**-0.9**	613.5	613.6	612.6	22	37501	COMX	10/30/06	20:12:20
GOLD	Apr '07	**619.4**s	**6.6**	613.6	623.5	613.6	84	10287	COMX	10/30/06	18:42:08
GOLD	Jun '07	**625.0**	**-0.2**	624.6	625.0	624.6	6	18863	COMX	10/30/06	19:17:21
GOLD	Aug '07	**631.0**s	**6.8**	628.8	635.4	628.8	5	1830	COMX	10/30/06	18:42:08
GOLD	Oct '07	**636.7**s	**6.9**	636.7	636.7	636.7	100	12662	COMX	10/30/06	18:42:08
GOLD	Dec '07	**642.3**s	**7.0**	642.3	642.3	642.3	1	28263	COMX	10/30/06	18:42:08
GOLD	Feb '08	**647.8**s	**7.1**	647.8	647.8	647.8	25	2301	COMX	10/30/06	18:42:08
GOLD	Apr '08	**653.4**s	**7.2**	653.4	653.4	653.4	77	1214	COMX	10/30/06	18:42:08
GOLD	Jun '08	**659.0**s	**7.3**	659.0	659.0	659.0	3000	3285	COMX	10/30/06	18:42:08
GOLD	Aug '08	**664.5**s	**7.4**	664.5	664.5	664.5	0	0	COMX	10/30/06	18:42:08
GOLD	Dec '08	**675.7**s	**7.6**	675.7	675.7	675.7	75	10597	COMX	10/30/06	18:42:08
GOLD	Jun '09	**692.6**s	**7.9**	692.6	692.6	692.6	3000	4846	COMX	10/30/06	18:42:08

Futures Pricing

Leverage and Futures Contracts

- A long futures position is equivalent to buying the underlying asset and financing 100% of the cost of the position by borrowing at the risk free rate.

Futures Pricing

A. Buy a share of stock and borrow 100% of the cost at the risk free rate. Assume the position is closed out at time T.

	Today	T
Buy a share of stock	$- S_t$	S_T
Borrow	$+S_t$	$S_t e^{r(T-t)}$
Total	0	$S_T - S_t e^{r(T-t)}$

B. Buy a future on the stock

	Today	T
Buy a future	0	$S_T – F_{t,T} =$ $S_T - S_t e^{r(T-t)}$