SWAPS

Commodity Swaps

Suppose you are a producer of methanol who purchases natural gas as an input. You plan to purchase 10,000 MMBtu's of natural gas every three months for one year starting on October 1 (assume Oct 1 is 3 months from now). Natural gas prices can be highly volatile so you would like to lock in the prices for the coming year. Assume the risk free rate is 3.35%

There are several ways to hedge the uncertainty.

1. Buy a strip of futures contracts

The producer could enter into a series of futures contracts, one for every month he plans to purchase natural gas. A series of futures is referred to as a strip. Natural Gas futures prices for the coming year are given below:

DATE	Futures Price
Oct	3.142
Jan	3.809
Apr	3.482
June	3.581

If the producer buys the four futures contracts he will have locked in a purchase price for gas for the coming year. The present value of the cost of gas is

DATE	Futures Price	Discount Factor	Present Value of Cost
Oct	3.142	0.992	3.116
Jan	3.809	0.984	3.747
Apr	3.482	0.976	3.397
June	3.581	0.968	3.465
		Total	13.725

2. Buy a swap

The producer could enter into an arrangement where he pays a fixed price for gas every three months over the next year. This is referred to as a swap. What should be the fixed price be?

Whether the producer goes with the swap or the strip of futures, he has a known set of cash flows for the natural gas. Since both these sets of cash flows are risk free, the present value of the swap payments and the present value of the futures' contract payments should be the same. Therefore, to find the swap price we need to find the fixed payment which has the same present value as the strip of futures.

DATE	Futures Price	Swap Payment	Discount Factor	Present Value of Cost	PV of Swap Payment
Oct	3.142	C	0.992	3.116	C*.992
Jan	3.809	C	0.984	3.747	C*.984
Apr	3.482	C	0.976	3.397	C*.976
June	3.581	C	0.968	3.465	C*.968
			Total	13.725	C*3.919

Therefore

$$C = \frac{13.725}{3.919} = 3.5022$$

Ignoring credit risk, any set of cash flows which has the same present value ($13.725 in this case), would be viewed as equivalent.

The swap could be settled through physical delivery of the natural gas or the contract could be cash settled.   If the contract is cash settled, a cash payment equal to the difference between the spot price and the swap price is made. When the spot price of natural gas is higher than the swap price, the producer receives a payment from the counter party. If the spot price of natural gas is lower than the swap price, the producer makes a payment to the counter party.

<u>Example</u>

If the spot price in October is $3.20, the producer pays the counter party $3.5024 - $3.20 = $.3024. The net cost of the natural gas to the producer is:

[$3.20 (for the spot gas) + .3024 (for the swap] = $3.5024.

If the spot price in October is $3.80, the producer receives from the counter party $3.80 - $3. 5024 = $.2976. The net cost of the natural gas to the producer is:

[$3.80 (for the spot gas) - .2976 (for the swap] = $3.5024.

Hedging Swap Positions

The risk of variable natural gas prices has been transferred from the producer to the counter party. How does the counter party hedge the risk associated with the swap?

The natural way to hedge would be for the counter party to buy a strip of futures to lock in a price for the natural gas he committed to deliver.

	Today	Oct	Jan	Apr	Jul	
Short the Swap	0.000	3.502	3.502	3.502	3.502	
Buy Oct Future	0.000	-3.142				
Buy Jan Future	0.000		-3.809			
Buy Apr Future	0.000			-3.482		
Buy June Future	0.000				-3.581	
Borrow(+)/Lend(-) Difference in the Cash Flows		-0.360	0.307	-0.020	0.079	NET
Future Value Borrowing and Lending		-0.369	0.312	-0.020	0.079	0.00

Note that this example assumed that the borrowing and lending were all done at a constant rate. In "real life" there may also be some interest rate risk associated with the borrowing and lending which must be hedged separately.

Market Value of the Swap

When the producer and the counter party enter into the swap, the swap is priced so that the market value of the swap is zero. Suppose that on December 1 the producer decides that he would like to get out of the swap. A common way to do this is for the producer to sell an offsetting swap. The swap price on December 1 can be computed from the futures prices. Suppose we have the following prices on Dec 1 for the futures:

Delivery Month	Futures Price
Jan	3.550
Apr	3.300
Jul	3.410

The swap price is found the same way as before, that is, find the quarterly payment which makes the present value of the cash flows the same as the present value of the strip of futures:

Delivery Month	Futures Price	Discount Factor	Present Value of Cost	Present Value of Swap Payment
Jan	3.550	$0.995	$3.531	C*.995
Apr	3.300	$0.986	$3.255	C*.986
Jul	3.410	$0.978	$3.336	C*.978
		Total	$10.121	C*2.958

Therefore the swap price is:

$$C = \frac{10.121}{2.958} = 3.422$$

The producer is currently a buyer of natural gas so in the offsetting swap he will need to be a seller. In January, April and July the producer will buy natural gas at $3.5024 and sell natural gas at $3.422. This generates a loss of $3.5024 - $3.422 = $.080 every time there is a transaction.

Delivery Month	Dollars Paid Out (Old Swap)	Dollars Paid In (New Swap)	Present Value of Difference
Jan	3.502	3.422	0.0800
Apr	3.502	3.422	0.0793
Jul	3.502	3.422	0.0787
			0.2380

The present value of the loss is .2380

This implies that the producer could pay a counter party $.2380 to take the swap off his hands. So a long position in the swap has a market value of -$.2380 on December 1.

Currency Swaps

Suppose a European firm has decided to issue debt in dollars because the CFO thinks the dollar denominated debt market is more liquid. The bond is a \$10 million dollar 4 year 3.35% coupon bond. The firm's revenues are in euros so the firm faces some exchange rate risk. Assume the continuously compounded dollar denominated interest rate is ln(1.0335)=3.3%, the euro denominated interest rate is ln(1.043)=4.21% and the current exchange rate is \$.92/euro (which could be restated as 1.087euros/\$). Because the firm's revenues are in euros and the coupon payments are in dollars, the firm faces some exchange rate risk, i.e., the firm doesn't know the euro cost of covering its coupon and principle payments.

This is exactly the same problem as the as the commodity swap problem except here the firm wants to buy dollars at fixed intervals instead of natural gas at fixed intervals.

There are several possible ways to handle this uncertainty.

1. The firm can enter into a series of currency forwards, one for each of the four years. This will lock in a euro cost of covering the payments on the bond.

   * Note that the forward price of a euro is:

$$F_{t,T} = 1.087e^{(r_{euro} - r_{US})(T-t)} = 1.087e^{(.0421-.03295)(T-t)}$$

Year	Dollar Debt Obligations	Forward Rate euro/$	Euro Discount Factor	Euro Cost of Debt Obligations	PV of Euro Cost of Debt Obligations (Discounted at Euro rate)
1	335,000	1.0969	0.9588	367,477.55	352,327.46
2	335,000	1.1070	0.9192	370,855.42	340,907.08
3	335,000	1.1172	0.8813	374,264.35	329,856.87
4	10,335,000	1.1275	0.8450	11,652,469.19	9,846,473.80
			3.6044		10,869,565.22

Note that hedging didn't not add any value in the sense that discounted value of the hedged euro cash flows is just the discounted value of the dollar cash flows multiplied by the current exchange rate, that is,

$$\$10,000,000 * (1/.92) = 10,869,565.22$$

2. The firm could enter into a swap where the firm makes a fixed set of euro payments. Let C denote the euro coupon payments and P be the principal payment made by the European firm.

What euro coupon and principle payments should the firm be required to make?

Whether the firm goes with the swap or the strip of forwards, he has a known cost of covering the dollar debt obligation. Since both these sets of cash flows are risk free, the present value of the swap payments and the present value of the forward payments should be the same. Therefore, to find the swap price we need to find a C and P which has the same present value as the forward position.

The cash flows are given below:

Year	Dollar Debt Obligations	Forward Rate euro/$	Euro Discount Factor	Euro Cost of Debt Obligations	PV of Euro Cost of Debt Obligations (Discounted at Euro rate)	Swap Cost of Debt Obligation (in Euros)	PV of Swap Cost of Debt Obligation (in Euros Dicounted at the Euro rate)
1	335,000	1.0969	0.9588	367,477.55	352,327.46	C	C*0.9588
2	335,000	1.1070	0.9192	370,855.42	340,907.08	C	C*0.9192
3	335,000	1.1172	0.8813	374,264.35	329,856.87	C	C*0.8813
4	10,335,000	1.1275	0.8450	11,652,469.19	9,846,473.80	C+P	(C+P)*0.8450
			3.6044		10,869,565.22		C*3.6044+P*0.8450

We want to find C and P such that the present value of the swap cost is the same as the present future of the cost using forwards.

$$10{,}869{,}565.22 = 3.6044*C + .08450*P$$

There is one equation and two unknowns, so there are an infinite number of solutions. How do we choose a solution?

It is often the case that the principal amounts are chosen to be approximately equivalent using the exchange rate at the time the swap is initiated. In this case, that means we would set

$$P = 10{,}000{,}000*(1/.92) = 10{,}869{,}565.22$$

With this value of P, we get C = 467,391.30 euros. The following table is a check to show that this value of C works.

Year	Dollar Debt Obligations	Forward Rate euro/$	Euro Discount Factor	Euro Cost of Debt Obligations	PV of Euro Cost of Debt Obligations (Discounted at Euro rate)	Swap Cost of Debt Obligation (in Euros)	PV of Swap Cost of Debt Obligation (in Euros Dicounted at the Euro rate)
1	335,000	1.0969	0.9588	367,477.55	352,327.46	467,391.30	448,122.06
2	335,000	1.1070	0.9192	370,855.42	340,907.08	467,391.30	429,647.23
3	335,000	1.1172	0.8813	374,264.35	329,856.87	467,391.30	411,934.06
4	10,335,000	1.1275	0.8450	11,652,469.19	9,846,473.80	11,336,956.52	9,579,861.88
			3.6044		10,869,565.22		10,869,565.22

Interest Rate Futures and Swaps

Zero Coupon Bonds

Define: P_{ij} = the price in period i for \$100 received in period j

Coupon paying bonds can be thought of as portfolios of zero coupon bonds. For example a 2 year bond with a 6% annual coupon and face value equal to 100 can be thought of as a portfolio containing

.06 of a 1 year zero
1.06 of a 2 year zero

The price of the 2 year coupon paying bond should be

$$\text{Bond Price} = .06\ P_{01} + 1.06\ P_{02}.$$

The price of the zero coupon bonds can be determined from the market prices of coupon paying bonds.

Example:
Consider the following bond prices:

	Price
1 year 5% coupon bond	100
2 year 6% coupon bond	100
3 year 4% coupon bond	98

This implies that

$$\begin{aligned} 1.05P_{01} &= 100 \\ .06P_{01} + 1.06P_{02} &= 100 \\ .04P_{01} + .04P_{02} + 1.04P_{03} &= 98 \end{aligned}$$

Solving we get:

$P_{01} = 95.23$ $P_{02}=88.95$ $P_{03}=87.15$

If you invest in a zero you pay P_{0i} today and receive $100 in period i. The rate of return on that position should be the riskfree rate over that period, that is,

$$\frac{100}{P_{0t}} = e^{r(0,t)*t}$$

Note that r(0,t) is the annualized continuously compounded risk free rate over the period 0 to t.

Therefore the price of the zero is

$$P_{0t} = 100e^{-r(0,t)*t}$$

Forward Prices

The forward prices can be computed from the zeros using a cash and carry.

Let $F_{0,i,j}$ = forward price in period 0 for a future on a zero which matures in period j with delivery in period i.

Suppose I want to price a forward on a 3 year zero with delivery in two years (i.e I want $F_{0,2,5}$).

	today	delivery(in two years)
buy spot which is a 5 year zero	$-P_{05}$	*
sell a forward		$F_{0,2,5}$
total	$-P_{05}$	$F_{0,2,5}$

* use the spot which is now a three year zero to cover the short forward position.

This is a riskless transaction so the rate of return should be equal to the two year T-Bill rate

$$\frac{F_{0,2,5}}{P_{05}} = e^{r(0,2)*2} \quad \Rightarrow \quad F_{0,2,5} = P_{05}e^{r(0,2)*2} \quad \Rightarrow \quad F_{0,2,5} = 100\frac{P_{05}}{P_{02}}$$

In general:

$$F_{0,i,j} = 100\frac{P_{0j}}{P_{0i}}$$

Swaps

Once we have the forward prices we can price a swap

Suppose A and B enter into a plain vanilla swap. Let A be the floating rate payer and let B be the fixed rate payer. B pays A cash flows equal to the interest at a pre-specified fixed rate on a notational principal for a pre-specified period. A pays B cash flows equal to the interest at a floating rate on the same notational principal for the same number of periods.

Define:
N = notational principal
r_{fixed} = fixed rate
r_{float} = floating rate
n = number of payment periods per year

Each period:

Fixed rate payer receives a net payment equal to:

$$N \times (1/n) \times (r_{float} - r_{fixed})$$

Floating rate payer receives a net payment equal to:

$$N \times (1/n) \times (r_{fixed} - r_{float})$$

Example: Suppose A and B enter into a swap on 9/22/03 with payments made twice a year (that is, n=2) beginning in 6 months. The notational principal is $10 million. The fixed rate is 10.5%, the floating rate is the 6 month LIBOR rate and the payments are exchanged every six months for 3 years. (Interest payments are typically made in arrears but we will ignore that for now.)

The cash flow for a possible set of LIBOR rates is given below.

Date	LIBOR	Fixed Payment (B Pays)	Floating Payment (A Pays)	Net Payment to the Fixed Rate Payer	Net Payment to the Floating Rate Payer
3/22/04	.094	525,000	470,000	-55,000	55,000
9/22/04	.097	525,000	485,000	-40,000	40,000
3/22/05	.10	525,000	500,000	-25,000	25,000
9/22/05	.108	525,000	540,000	15,000	-15,000
3/22/06	.11	525,000	550,000	25,000	-25,000
9/22/06	.10	525,000	525,000	0	0

Using swaps to switch floating rate loans to fixed rate loans

B might be interested in this type of swap if he wanted to hedge an existing floating rate loan. Suppose B has a floating rate loan of $N. Every six months B makes a floating rate payment equal to

$$N \times \frac{1}{2} \times r_{float}$$

What swap position should he take to eliminate the exposure to the floating rate?

If he is the fixed rate payer in the swap, he will receive

$$N \times \frac{1}{2} \times (r_{float} - r_{fixed})$$

Therefore his net payment (loan payment less swap revenue) is:

$$(N \times \frac{1}{2} \times r_{float}) - N \times \frac{1}{2} \times (r_{float} - r_{fixed}) = N \times \frac{1}{2} \times r_{fixed}$$

Switching a fixed rate loan to a floating rate loan is done in a analogous way.

ASIDE ON EURODOLLAR FUTURES CONTRACTS

The LIBOR rate is quoted on an add in yield basis using a 360 day year. For example, if the 3 month LIBOR rate is 8%, then the interest on $1,000,000 is

$$(.08)*(90/360)*(\$1M) = \$20,000$$

Eurodollar futures contracts:

Quantity: $1million/contract

Contracts are cash settled based on the 3 month LIBOR rate (this is the fixed rate on US dollar deposits in banks not subject to US banking regulations.

The expiration futures price (i.e. the settlement price) = 100*(1 - r_{LIBOR})

At expiration the payoff to the long is:

$$[(100(1 - r_{LIBOR}) - F] \times 2,500$$

and the payoff to the short is:

$$[F - 100(1 - r_{LIBOR})] \times 2,500$$

Euro dollar futures can be used to lock in borrowing and lending rates.

<u>Example</u>: Suppose in February,a firm would like to lock in a 3 month **lending** rate starting in June on $1 M. The Eurodollar futures price for June delivery was 97.74. If they **buy a the long Euro 3 month future**, they can lock in a 3 month lending rate of 100-97.74 =2.26%. To see this consider the following possible outcomes:

• Suppose the spot LIBOR rate in June is 2.10%

If the firm enters into m long futures contracts, the firm receives

$$[100(1-.021)-97.74]\mathrm{x}2500=400$$

From the 3 month lending they receive interest equal to

$$(.021)\times\frac{1}{4}\times 1{,}000{,}000 = 5{,}250$$

So the net payment is $5650 which results in a 2.26% return.

• Suppose the spot LIBOR rate is June is 2.35%

From the long futures position, the firm receives

$$[100(1-.0235)-97.74]\times 2500=-225$$

From the 3 month lending they receive interest equal to

$$(.0235)\times\frac{1}{4}\times 1{,}000{,}000 = 5{,}875$$

So the net payment is $5650 which results in a 2.26% return.

Example: Suppose in February, a firm would like to lock in a 3 month **borrowing** rate starting in June. The Eurodollar futures price for June delivery was 97.74. If they **sell a the long Euro 3 month future**, they can lock in a 3 month borrowing rate of 100-97.74 =2.26%. To see this consider the following possible outcomes:

• Suppose the spot LIBOR rate is June is 2.10%

From the short futures position, the firm receives

$$[97.74 - 100(1 - .021)] \times 2500 = -400$$

From the 3 month borrowing they pay interest equal to

$$(.021) \times \frac{1}{4} \times 1{,}000{,}000 = 5{,}250$$

So the net payment is $5650 which results in a 2.26% borrowing rate.

• Suppose the spot LIBOR rate is June is 2.35%

From the short futures position, the firm receives

$$[97.74 - (100 - 2.35)] \times 2500 = 225$$

From the 3 month borrowing they pay interest equal to

$$(.0235) \times \frac{1}{4} \times 1{,}000{,}000 = 5{,}875$$

So the net payment is $5650 which results in a 2.26% borrowing rate.

Swap Pricing

The swap can be priced (i.e the appropriate fixed rate for the swap can be determined) by using the futures prices.

Suppose there is a Euro dollar swap (i.e., floating rate is LIBOR) which makes payments every 3 months.

The floating rate payer's net revenue from the swap every 3 months is:

$$(1M)x(90/360)x(r_{fixed} - r_{LIBOR})$$

To hedge this position (that is, remove the exposure to the floating rate) the floating rate payer can take a short position in a strip of 3 month Euro dollar futures.

The net revenue from the strip of short futures every 3 months is:

$$2500x[F_{0,t,t+1} - 100(1 - r_{LIBOR})]$$

Every 3 months the total revenue from the two positions is:

$$(1M)x(90/360)x(r_{fixed} - r_{LIBOR}) + 2500x[F_{o,t,t+1} - 100(1 - r_{LIBOR})]$$

$$= 2500x[F_{0,t,t+1} - 100(1 - r_{fixed})]$$

This position produces a certain cash flow every 3 months. The initial cost of the position is zero. Since there is no risk and no capital required for the position (the strip and futures and the swap are both "free"), the rate of return on the position should be zero. This implies that the present value of the net cash flows should be zero or else there is an arbitrage opportunity. This implies that the fixed rate on the swap must be such that:

$$\sum_{t=1}^{n} 2500x[F_{0,t,t+1} - 100(1 - r_{fixed})]e^{-r(0,t)*t} = 0$$

Rearranging terms, we get:

$$r_{fixed} = \frac{\sum_t (100 - F_{0,t,t+1})e^{-r(0,t)*t}}{100\sum_t e^{-r(0,t)*t}}$$

This expression can be rewritten using the price of the zeros. Recall that

$$F_{0,i,j} = 100\frac{P_{0j}}{P_{0i}}$$

and

$$P_{0t} = 100e^{-r(0,t)*t}$$

Substituting, we get:

$$r_{fixed} = \frac{\sum_t (P_{0,t} - P_{0,t+1})}{\sum_t P_{0,t}}$$