Introduction to Derivatives

Forwards and Futures

• A <u>forward contract</u> is an agreement between two parties for the deferred delivery of an asset. The contract has the following characteristics:

- The contract specifies:
 - 1. the quantity and type of asset
 - 2. delivery time and place
 - 3. price

• Both the buyer and the seller are **obligated** to the carry out the transaction.

• Terminology:

If you have agreed to be the buyer, you are said to be **long** If you have agreed to be the seller, your are said to be **short**

• Notation:

 $f_{t,T}$ = the forward price for a contract initiated at time t for delivery at time T P_t = the spot price at time t (i.e., the price for immediate delivery)

- The gain per contract to the long position is
- = (number of units in the contract)*($P_T f_{t,T}$)
- The gain per contract to the **short position** is
- = (number of units in the contract)*($f_{t,T} P_T$)

Note that entering into a forward contract is "free", that is, no money changes hands when the contract is initiated.

Payoff Diagram:

Example: On September 10, two parties enter into a forward contract for the delivery of 100 ounces of gold on December 1 at \$450 per ounce.

	Price Increase	Price Decrease
Dec 1 Spot Price	\$460	\$430
Long Profit	\$10	-\$20
Short Profit	-\$10	\$20

The long:

- locks in a purchase price of \$450
- makes a profit if the spot price is above \$450
- takes a loss if the spot price is below \$450

The short:

- locks in a selling price of \$450
- makes a profit if the spot price is below \$450
- takes a loss if the spot price is above \$450



A <u>future</u> is an exchange traded forward contract. Exchange trading affects a contract in three ways:

1. The contract is standardized. This increases liquidity but the contract is not tailored to individual needs.

2. Margins are required.

When a trader enters into a futures contract, the Clearing Corporation requires that the trader put up an <u>initial margin</u>. This amount depends upon the contract and whether the trader is a hedger or a speculator. If the balance in the margin account falls below a pre-specified amount referred to as the <u>maintenance margin</u>, the trader receives a margin call. The margin account balance must be brought back to the initial level or the position will be closed out by the Clearing Corporation.

3. The contracts are <u>marked-to-market</u>. This means that gains and loses are realized every day.

The futures price for a contract which was initiated on date t for delivery on date T will be denoted $F_{t,T}$.

Example: The following is an example of how marking-to-market and margin requirements help to minimize the problems associated with credit risk. Suppose a buyer and a seller enter into a futures contract on 7-1 for 5000 bushels of wheat to be delivered on 7-5 at \$1/bushel. Suppose the initial margin is \$1500 and the maintenance margin is \$1200.

	Margin Account			Account		
	-		Cha	ange	Bala	ince
Date	Transaction	F _{t,T}	Buyer	Seller	Buyer	Seller
7-1	Contract initiated. The buyer and the seller deposit the money in their margin accounts.	1.00	1500	1500	1500	1500
7-2	a. Price rises by .10, buyer gains (5000)(.10) and seller loses (5000)(.10). \$500 is transferred from the sellers margin account to the buyer's margin account. b. Seller's account balance falls below \$1200, so \$500 must be deposited into the margin account. If not the Clearing Corporation liquidates the seller's position by buying an offsetting contract with $F_{t,T}$ =1.10.	1.10	+500	-500 +500	2000	1000
7-3	Price falls by .05, and the buyer loses (.05)(5000) and seller gains (.05)(5000). \$250 is transferred from the buyers account to the seller's account.	1.05	-250	+250	1750	1750
7-4	 a. Price falls by .15, and the buyer loses (.15)(5000) and seller gains (.15)(5000). \$750 is transferred from the buyer's account to the seller's account. b. Buyer's margin account falls below the maintenance margin, so \$500 must be deposited into the margin account or the buyer's position will be liquidated. 	.90	-750 +500	+750	1000 1500	2500
7-5	Price rises by .05, buyer gains by (.05)(5000) and the seller loses (.05)(5000). \$250 is transferred from the seller's account to the buyer's account.	.95	+250	-250	1750	2250
7-5	The contract is settled at \$.95. The buyer pays \$4750 to the seller and receives5000 bushels of wheat.		-4750	+4750		
Net ca	ash flow excluding margin payments		-5000	5000		
Net m	argin payments		2000	2000		

Even though the buyer paid \$4750 at delivery, the net payment (excluding margin payments) made by the buyer over the life of the contract was \$5000, the amount that was initially agreed upon. That is,

Date	Buyer	Seller
7-2	+500	-500
7-3	-250	-250
7-4	-750	-750
7-5	+250 -4750	-250 +4750
Total	-5000	+5000

Is a futures contract equivalent to a forward contract?

No. This is because of time value considerations. If prices are rising the buyer benefits from marking-to-market because he has access to the gains early on. If prices are falling, the buyer is hurt by marking-to-market because he has to pay some of the cost prior to the delivery date.

Options

A <u>European call option</u> is a contract which gives its owner (the buyer) the right, but <u>not</u> the obligation, to <u>buy</u> a pre-specified piece of property (underlying asset) at a pre-specified price (the exercise price or strike price) on a pre-specified day (expiration day).

A <u>European put option</u> is a contract which gives its owner (the buyer) the right, but <u>not</u> the obligation, to <u>sell</u> a pre-specified piece of property (underlying asset) at a pre-specified price (the exercise price or strike price) on a pre-specified day (expiration day).

An <u>American Option</u> is like a European option except it can be exercised any time prior to expiration.

Notation:

С	=	Call Price or Premium
Ρ	=	Put Price or Premium
S	=	Price of the Underlying Asset
Т	=	Expiration Date
Κ	=	Strike Price or Exercise Price

The creator of the option is referred to as the <u>seller</u> or the <u>writer</u>.

Transaction Summary for an American Option:

At t=0

\$C or \$P Buyer -----> Writer <-----Option

At t>0 the buyer of the option has three choices:

1. Exercise

	\$K	
For a Call:	Buyer	> Writer
	<	
	Underlying Asse	et

	Underlying Asset	:
For a Put:	Buyer	> Writer
	<	
	\$K	

2. Do Nothing

If t < T, position remains open

If t = T, option expires

3. Close Out Position by Reversing the Trade

If you bought (wrote) a call --> write (buy) of call

If you bought (wrote) a put ---> write (buy) a put

When Should You Exercise An Option?

The easiest case to consider is what to do at expiration.

• Suppose on June 1 you buy a European call on IBM stock which expires on September 24 with K=40.

If the price of IBM is \$45 on September 24, what should you do?

What is the profit <u>at expiration</u>?

If the price of IBM is \$35 on September 24, what should you do?

What is the profit at expiration?

In general:

- Exercise (i.e., buy) when $K \leq S_T$
- Do Nothing when $K > S_T$

This implies that the value of the call at expiration is

 $\begin{array}{rrrr} C & = & S_T - K & \mbox{if } K \leq S_T \\ & = & 0 & \mbox{otherwise} \end{array}$

This can be rewritten as

Note: when $K < S_T$, we say the call is <u>in-the-money</u> when $K = S_T$, we say the call is <u>at-the-money</u> when $K > S_T$, we say the call is <u>out-of-the-money</u> • Suppose on June 1 you buy a European put on IBM stock which expires on September 24 with K=40.

If the price of IBM is \$45 on September 24, what should you do?

What is the profit <u>at expiration</u>?

If the price of IBM is \$35 on September 24, what should you do?

What is the profit <u>at expiration</u>?

In general:

- Exercise (i.e., sell) when $K \ge S_T$ (in-the-money)
- Do Nothing when $K < S_T$ (out-of-the-money)

This implies that the value of the put at expiration is

 $\begin{array}{rcl} \mathsf{P} & = & \mathsf{K} - \mathsf{S}_\mathsf{T} & \text{ if } \mathsf{K} \geq \mathsf{S}_\mathsf{T} \\ = & \mathsf{0} & \text{ otherwise} \end{array}$

This can be rewritten as

$$P = max[0, K - S_T]$$

Note: when $K>S_T$, we say the put is <u>in-the-money</u> when $K=S_T$, we say the put is <u>at-the-money</u> when $K<S_T$, we say the put is <u>out-of-the-money</u>

Comparison Between Options and Futures.

Consider a futures contract on a stock with a futures price, $F_{t,T}$ = \$40/share

 \bullet The **profit** (and the payoff since futures are free) on a long future at time T is S_T - $F_{t,T}$

 \bullet The **profit** (and the payoff since futures are free) on a short future at time T is $\ensuremath{\,F_{t,T}}$ - $\ensuremath{S_T}$

Consider a call with K = \$40 and initial cost C = \$2.78

• The payoff at expiration on a long position in one call is

$$\begin{array}{ll} = 0 & \mbox{if } S_T < K = \$40 \\ = S_T \text{-} K & = & S_T \text{-} 40.00 & \mbox{if } S_T \geq K = \$40 \end{array}$$



Payoff to a Long Call vs a Long Future

• The **profit** (payoff plus the initial cost) at expiration on a long position in one call is

=

 $\begin{array}{rl} -C &= -2.78 & \mbox{if } S_T < K = \$40 \\ -C + S_T - K &= S_T - 42.78 & \mbox{if } S_T \ge K = \$40 \end{array}$



Note that we are <u>temporarily</u> ignoring the time value associated with the initial cost of the option.

• The **payoff** at expiration on a short position in one call is

• The profit at expiration on a short position in one call is





Consider a put with K = \$40 and initial cost P = \$1.99

• The payoff at expiration on a long position in one put is

$$\begin{array}{lll} = & 0 & \quad \mbox{if } \mathsf{K} < \mathsf{S}_{\mathsf{T}} \\ = & \mathsf{K} {-} \mathsf{S}_{\mathsf{T}} = & 40 - \mathsf{S}_{\mathsf{T}} & \quad \mbox{if } \mathsf{K} \geq \mathsf{S}_{\mathsf{T}} \end{array}$$



• The profit at expiration on a long position in one put is

• The payoff at expiration on a short position in one put is

• The profit at expiration on a short position in one put is





Summary

Positions which involve buying:

- Long Future
- Long Call
- Short Put





Positions which involve selling:

- Short Future
- Short Call
- Long Put





So far we have looked at uncovered positions in the option or the underlying asset. Now we want to look at positions which involve combinations of different securities.

• A **hedge** is a position where gains on one part of the position partially or wholly offset loses on another part of the position. For example, gains on an option position can offset loses on a position in the underlying asset and gains on the underlying asset can offset loses on an option position.

• Covered Call -- Suppose you have written 1 call and you would like to hedge your position. What should you do?



Suppose that you buy one share in addition to shorting a call.

Assume C=\$2.78, K=40 and S_t=40.

The initial cost and the payoff at expiration are given below:

	Today	S⊤ < K	$S_T \ge K$
WRITE 1 CALL	C=2.78	0	$K-S_T = 40-S_T$
BUY 1 SHARE	-S _t =-40	ST	S _T
TOTAL	-37.22	ST	40

Does adding a share to the position create a hedge?



Covered Call

Hedging with Futures, Caps, Floors and Collars

Hedging Short Exposures

• Suppose I am a producer of a chemical which uses natural gas as an input and my "break-even" cost for natural gas is \$2.00 per MMBtu (i.e., if the cost of gas is above \$2.00 per MMBtu, I lose money and if it is below \$2.00, I make money.) Graphically my exposure looks like this:

Note that my underlying exposure is **short**. That is, I like low prices and dislike high prices.



What are the ways that I can hedge my exposure?

Long Future

Suppose the futures price is \$1.90 per MMBtu. I could go **long** a natural gas future and lock-in a buying price of \$1.90 per MMBtu. My hedged position looks like:



One of the disadvantages of this position is that in order to eliminate the exposure to prices above \$1.90, I also eliminate the ability to benefit from prices which are below \$1.90.

Is there a way to get the protection from price increases and still benefit from price declines?

Buy a **cap**. A cap is just a call option. When gas prices increase, the gain on the option position offsets the loss due to higher gas prices.

Suppose natural gas ranges in price from about \$1.25 to \$3.00 per MMBtu. Suppose I buy a call option on natural gas with a strike price of \$2.10. Each option contract is for 10,000 MMbtu's. The call price per MMBtu is .12. My cash flow per MMBtu is given below.

	Today	G < K=2.10	G ≥ K=2.10
Buy I Call	-C=12	0	G-K=G-2.10
Buy 1 MMBtu of Gas at the Expiration Date		-G	-G
TOTAL	-C=12	-G	-2.10

Notice that the cost of gas is capped at \$2.10/MMBtu and the "all in" cost of gas is capped at \$2.22.



Suppose that I like the downside protection that I get from the cap but I find that the cap expensive. What can I do?

There are several possibilities:

1. Raise the strike price on the call. This lowers the cost of the call but provides less downside insurance (you get what you pay for).

2. Sell off some of the potential gains which would occur if the price of gas was low. To do this, you sell a natural gas put. Buying a call and simultaneously selling a put is referred to as a **collar**. The revenue from selling the put offsets some of the cost of buying the call. Now there is both a cap and a floor on the cost of natural gas.

	Today	G < 1.80	1.80< G ≤ 2.10	$G \ge 2.10$
Sell 1 Put with K= 1.60	P= .065	-(K-G) = -(1.80-G)	0	0
Buy I Call with K= 2.10	-C=12		0	G-K= G-2.10
Buy 1 MMBtu of Gas at the Expiration Date		-G	-G	-G
TOTAL	P-C=055	-1.80	-G	-2.10

Example: Suppose you buy a 2.10 call and sell a 1.80 put.



There are several things to note about this position:

1. Your position has a 2.10 cap and a 1.80 floor. The cost this position is .055.

2. The highest all-in cost of natural gas is 2.10+.055 = 2.155. This will be the cost when the spot price of natural gas is 2.10 or higher. Your profit in this case will be -.155. The lowest all-in cost of natural gas is 1.80+.055 = 1.855. You will pay this price if the spot price of natural gas is 1.80 or lower. Your profit in this case will be .145.

3. You have some exposure to market prices when the spot price is between 1.80 and 2.10.

4. If you wanted to make the collar less expensive you could raise the floor (i.e., sell-off more of the gains) or raise the cap (i.e., buy less insurance).

Hedging Long Exposures

• Suppose I am a manager of an index fund. Suppose my "break-even" value for the index is 1100 (i.e., for whatever reason I feel that if the value of the index is above 1100, I'm ahead and if it is below 1100, I'm behind.) Graphically my exposure looks like this:



Note that my underlying exposure is **long**. That is, I like high values and dislike low values.

What are the ways that I can hedge my exposure?

Short Future

Suppose the futures price is 1150. I could go **short** a stock index future and lock-in a selling price of 1150. My hedged position looks like:



One of the disadvantages of this position is that in order to eliminate the exposure to values below 1150, I also eliminate the ability to benefit from values which are above 1150. I have "synthetically" turned my stock index position into a T-bill and hence the rate of return on my position should be the risk-free rate.

Is there a way to shield myself from drops in the value of my portfolio but receive some of the benefits if the index should increase in value?

Buy a **floor** (this is also called portfolio insurance). A floor is just a put option. When the portfolio drops in value, the gain on the option position offsets the loss on the index portfolio.

The current value of the index is 1100. On March 24 I buy a S&P 500 put option with a strike price of 1000 which expires three months later. If the value of the S&P 500 index is below 1000 at expiration, the put is worth $(1000-I_T)$ and if the S&P 500 Index is above 1000 at expiration, the put expires worthless. Suppose the put costs \$2.63. My cash flow per unit of the index is given below.

	Today	I _T < K=1000	$I_T \ge K$ =1000
Buy I Put with K=450	-P = -2.63	K -I _T =1000-I _T	0
Buy 1 Unit of the Index Portfolio	$-I_t = -1100$	Ι _Τ	Ι _Τ
TOTAL	-1102.63	1000	Ι _Τ

Notice the manager pays \$2.63 to insure that the portfolio has a 1000 floor over the next 3 months. On an annualized basis, the manager is giving up 1% of the return to pay for the insurance.



Suppose that I like the downside protection that I get from the floor but I find the floor expensive. What can I do?

There are several possibilities:

1. Lower the strike price on the put. This lowers the cost of the put but provides less insurance (you get what you pay for). For example, an identical put with K=950 instead of 1000 costs \$.46.

2. Sell off some of the potential gains which would occur if the price of the index increases. To do this you sell a stock index call. Buying a put and simultaneously selling a call is referred to as a **<u>collar</u>**. The revenue from selling the call offsets some of the cost of buying the put. Now the manager has both a cap and a floor on the value of his portfolio.

Example: Suppose you buy a 1000 put and sell a 1250 call.

	Today	I _T < 1000	1000 <i<sub>T<1250</i<sub>	$I_T \geq 1250$
Buy I Put with K=1000	-P = -2.63	1000-I _T	0	0
Sell 1 Call with K=1250	C = 2.35	0	0	1250-I _⊺
Buy 1 Unit of the Index Portfolio	-It = -1100	Ι _Τ	Ι _Τ	Ι _Τ
TOTAL	-1100.28	1000	Ι _Τ	1250



There are several things to note about this position:

1. Your position has a 1250 cap and a 1000 floor. The cost of this position is .28.

2. The lowest all-in payoff on the portfolio is 1000-.28 = 999.72. This will be the payoff when the value of the index is 1000 or lower. The highest all-in payoff is 1250-.28 = 1249.72. This will be the payoff when the value of the index is 1250 or higher.

3. You have some exposure to market prices when the value of the index is between 1000 and 1250.

4. If you wanted to make the collar less expensive you could lower the cap (i.e., sell-off more of the gains) or lower the floor (i.e., buy less insurance).

Aside on collar pricing:

Suppose an executive owns many shares in her firm. In order to avoid the losses which would occur if the stock price drops, the executive decides to put a zero cost collar on her position. The current stock price is \$50/share, the volatility of the stock is 40%, the risk-free rate is 5% and the stock does not pay any dividends. The collar consists of a 3 year 50 put and a 3 year 74.25 call. Both the put and the call cost \$9.40.

Why is it that the collar seems so asymmetrical (i.e., a call which is out-ofthe-money by almost \$25 is financing the purchase of an at-the-money put)?

The put isn't really at-the-money. If the stock was worth \$50 in 3 years the executive wouldn't have broken even. The stock must appreciate by at least the risk-free rate. The minimum break-even stock price is $50^{*}e^{3^{*.05}} = 58.09$. If the strike price on the put was set at \$58.09, the cost of the put would be \$13.54 and the strike price on the call which has the same cost would be also be \$58.09.

What do you think the futures price is for one share with delivery in 3 years?

Summary:

	Short Exposure	Long Exposure
Hedge	Long Future	Short Future
Cost	\$0	\$0
Insurance	Buy a Call	Buy a Put
Cost	\$C	\$P
To Lower the Cost	Raise the Strike Price	Lower the Strike Price
Sell-Off Gains	Sell a Put	Sell a Call
Revenue	\$P	\$C
To Increase Revenue	Raise the Strike Price	Lower the Strike Price

Speculative Positions

• **Vertical Spread** This is a position involving options which have the same expiration date but different exercise prices

Example: Bullish Vertical Spread

Buy 1 call with K=40	 refer to this as C(40)
Write 1 call with K=60	 refer to this option as C(60)

Suppose C(40) = \$2.78 and C(60) = \$.011.

	Today	S _T # 40	$40 < S_T \le 60$	S _T > 60
BUY C(40)	-2.78	0	S⊤-40	S _⊤ -40
WRITE C(60)	.011	0	0	60-S _T
TOTAL	-2.769	0	S⊤-40	20

Example: Butterfly Vertical Spread

Buy 1 call with K=40 Write 2 calls with K=50 Buy 1 call with K=60

Suppose C(40)=\$2.78, C(50)=\$.267 and C(60)=.011.

	Today	$S_T \!\!\leq\! 40$	40 <s<sub>T≤50</s<sub>	50 <s⊤≤60< th=""><th>S⊤ >60</th></s⊤≤60<>	S⊤ >60
BUY C(40)	-2.78	0	S _T -40	S _T -40	S _T -40
WRITE 2 C(50)	.534	0	0	100 - 2S _T	100 - 2S _T
BUY C(60)	011	0	0	0	S _T -60
TOTAL	-2.257	0	S⊤-40	60-S⊤	0

• Horizontal Spread - a combination of options with the same exercise price but different expiration dates.

• Diagonal Spread - different exercise prices and different expiration dates.

Corporate Finance Applications

• **PERCS** (Preferred Equity Redemption Cumulative Stock)

Like cumulative preferred stock because:

There is a preset dividend rate and the holders of the PERCS must be paid any promised but unpaid dividends before any common shareholders receive dividends.

Not like cumulative preferred stock because:

There is a mandatory redemption with common shares at a predetermined time and rate. In particular the PERCS share is capped at some predetermined price Simple Example:

Suppose neither the common stock nor the PERCS pays a dividend. Suppose current share price is \$40/share. The PERCS expires in three years and is capped at \$60/share. The possible payoffs to the PERCS are given below:

Share Price at Expiration	Number of Common Shares Received in Exchange for the PERCS	Value of the PERCS at Expiration
20	1	20
40	1	40
60	1	60
80	.75	60
100	.60	60
120	.50	60



A payoff which is exactly the same can be generated by buying one share of common stock and selling a call with a strike price equal to \$60 (i.e., a covered call)

Share Price at Expiration	Value of Common Share	Value of Short Call	Total Value
20	20	0	20
40	40	0	40
60	60	0	60
80	80	-20	60
100	100	-40	60
120	120	-60	60

Since the payoff at expiration of the covered call and the PERCS is the same, the value of the PERCS must be equal to the value of the covered call. If the current stock price is \$40 and the price of the call is 5.34, then the price of the covered call is 40-5.34 = 34.66 and the PERCS must also sell for 34.66.

Slightly more complicated example:

Suppose the stock pays no dividend but the PERCS will pay a dividend. The annual interest rate is 6%. If the firm wants to sell the PERCS at par (i.e., 40/share) what does the dividend have to be?

The value of the security must be increased by (\$40-\$34.66) = \$5.34. Therefore, the present value of the dividend must equal \$5.34. The value of the annual dividend, denoted D, that does the job is found by solving

$$\frac{D}{(1.06)} + \frac{D}{(1.06)^2} + \frac{D}{(1.06)^3} = 5.34$$

The solution is D = \$2.00

WorldCom's Offer for MCI

WorldCom's offer for MCI had the following form:

The shareholders of MCI would receive shares of WorldCom. The number of shares would be determined in the following way:

- 1. The 20-day average of the high and low prices of WorldCom stock prior to the closing of the deal would be computed.
- 2a. If the average price of WorldCom stock was between \$29 and \$41:

MCI shareholders would receive shares worth \$51 for each MCI share. The actual number of shares received is :

\$51/ AVERAGE PRICE If the average share price was \$39, an MCI shareholder would receive \$51/39 = 1.3077 shares of WorldCom stock for every MCI share.

2b. If the average price was less than \$29/share:

MCI shareholders would receive 1.7586 (= 51/29) shares for each share of MCI. The value of this would be (1.7586 * share price). The value is always lower than \$51 and the lower the stock price the lower the value.

If the value of WorldCom stock was \$25, the MCI shareholders would receive shares worth 1.7586*\$25 = \$43.965

2c. If the average price is greater than \$41/share:

MCI shareholders would receive 1.2439 (= 51/41) shares for each share of MCI. The value of this would be (1.2439 * share price). The value is always greater than \$51 and the higher the stock price the higher the value.

If the value of WorldCom stock was \$55, the MCI shareholders would receive shares worth 1.2439*\$55 = \$68.415





There are a number of ways to create an equivalent position using puts, calls and shares of WorldCom.

I. This offer is equivalent to WorldCom giving the MCI shareholders the following portfolio:

- 1.2439 long puts with K=41 1.7586 short puts with K=29
- 1.2439 shares of WorldCom Stock

To see this, compare the cash flows:

WorldCom Offer

S < \$29	$29 < S \le 41$	S > \$41
1.7586 * S	\$51	1.2439*S

Option Portfolio:

	S < \$29	$\$29 \le S \le \41	S > \$41
1.2439 shares	1.2439 * S	1.2439 * S	1.2439 * S
1.2439 long puts with K = 41	1.2439*(41-S)	1.2439*(41-S)	0
1.7589 short puts with K=29	1.7589*(S-29)	0	0
Total	1.7589*S	\$51	1.2439 * S

This means that the value of the offer can be determined by valuing the option portfolio.

Using the Black-Scholes option pricing formula we can compute the value of the deal.

If WorldCom stock was trading:

at \$35.00/share, the offer was worth \$39.58;/share, at \$38.00/share, the offer was worth \$42.01/share, at \$48.50/share, the offer was worth \$51.00/share.

II. How would you create an equivalent position using calls on WorldCom and WorldCom stock?