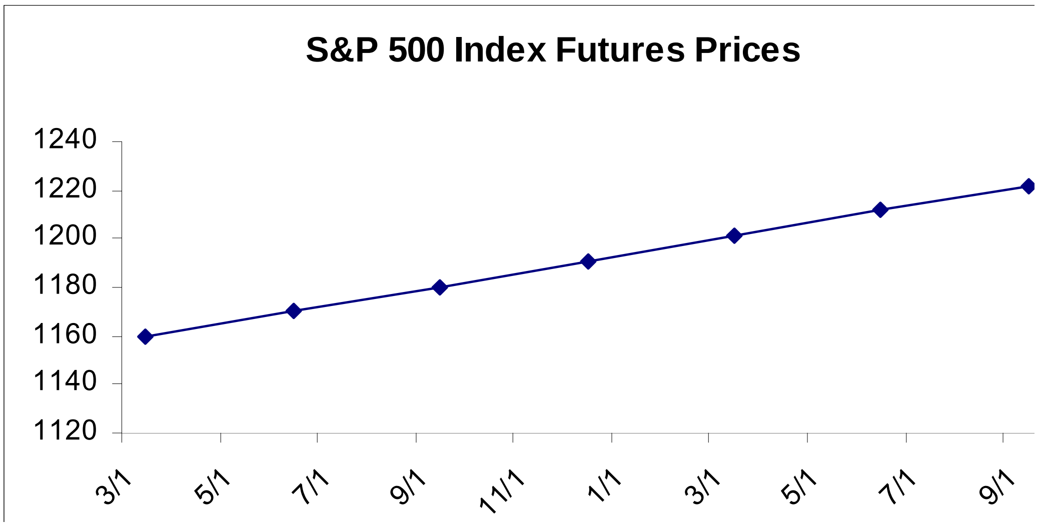
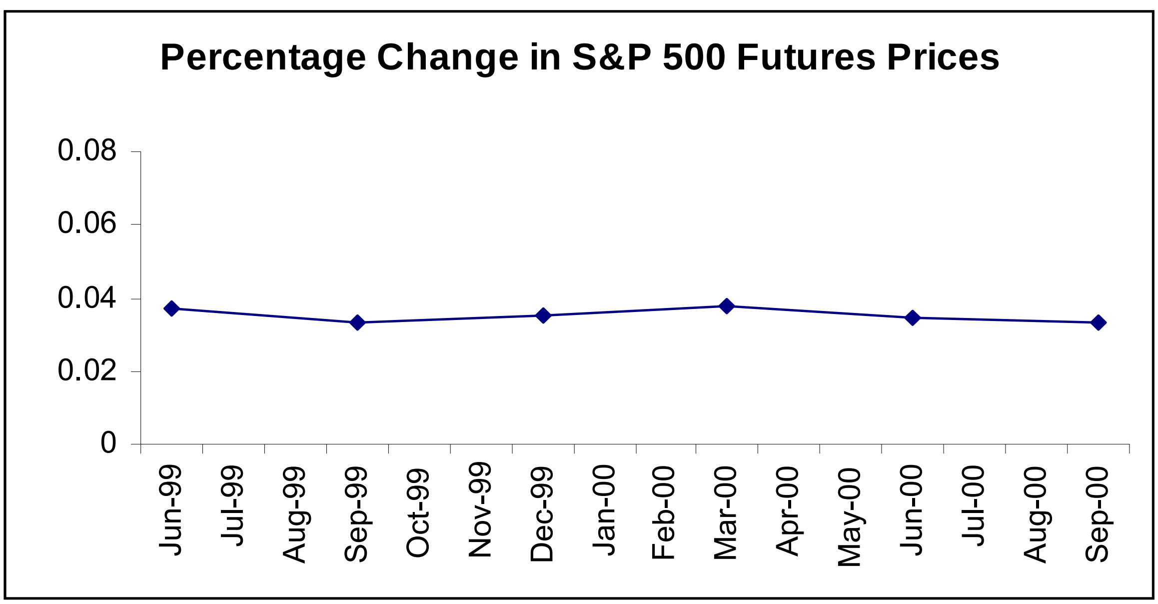
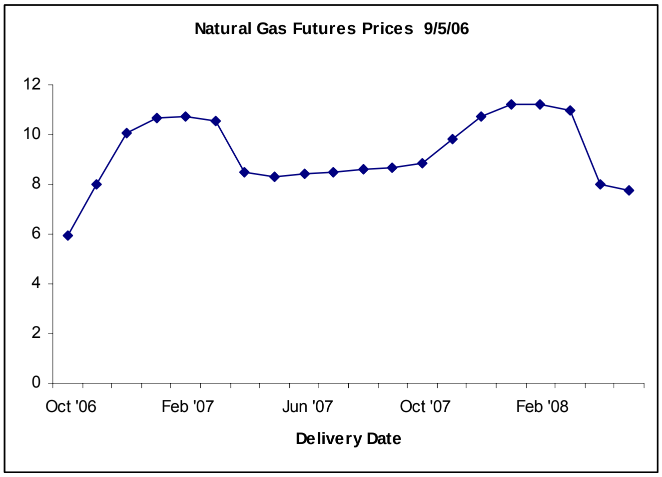
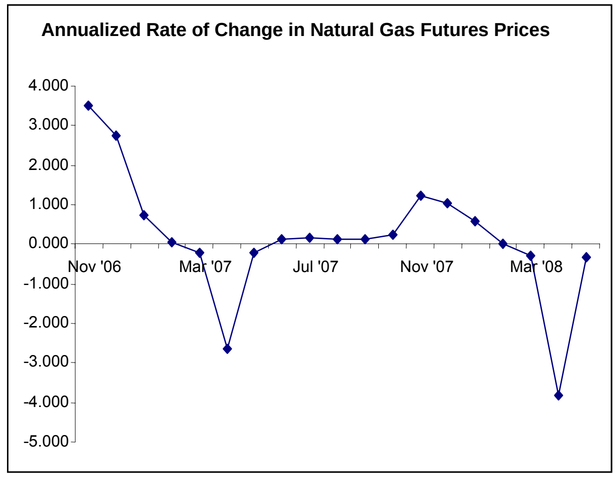
Commodity Futures

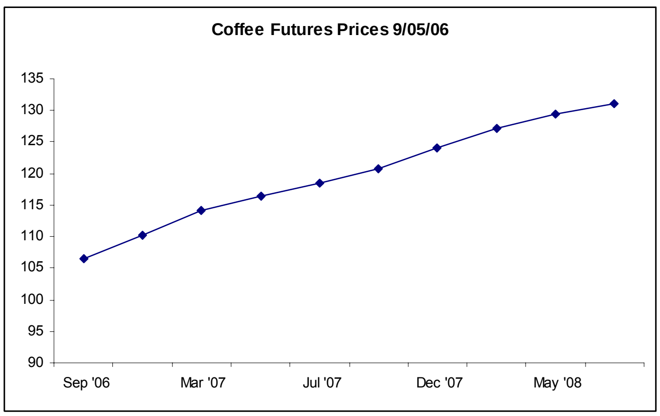
Commodity futures appear to behave very differently from stock index futures. Consider the following data:

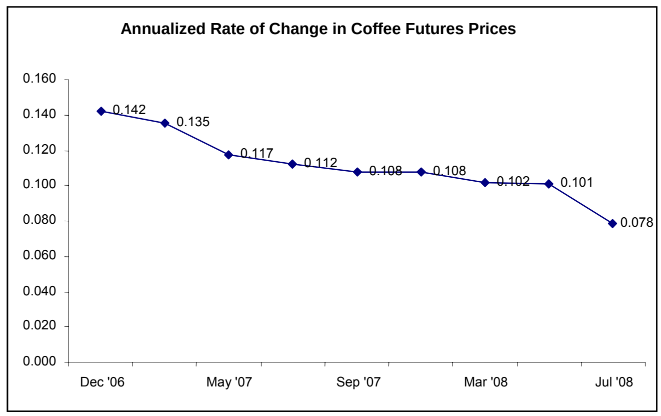


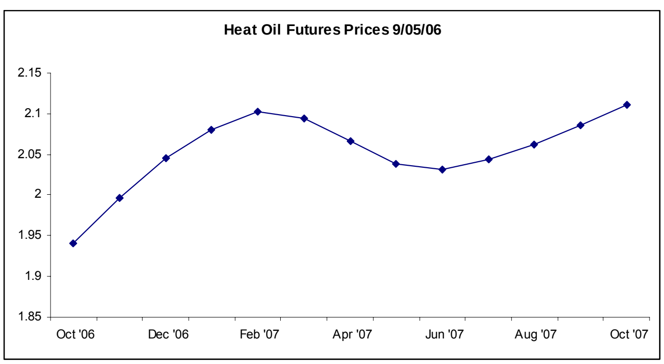


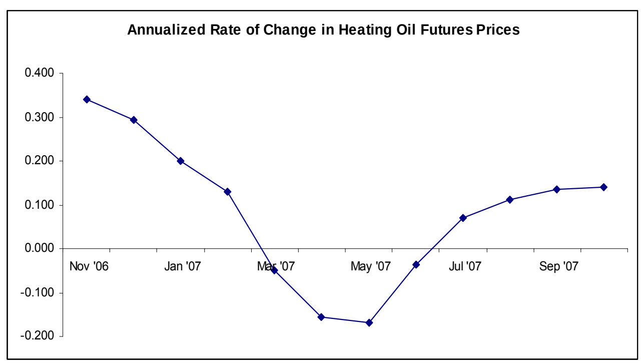

Natural Gas Futures Prices 9/5/06
12
10
8
6
4
2
0
Oct '06
Feb '07
Jun '07
Oct '07
Feb '08
Delivery Date


Annualized Rate of Change in Natural Gas Futures Prices
4.000
3.000
2.000
1.000
0.000
-1.000
-2.000
-3.000
-4.000
-5.000
Nov '06
Mar '07
Jul '07
Nov '07
Mar '08


Coffee Futures Prices 9/05/06
135
130
125
120
115
110
105
100
95
90
Sep '06
Mar '07
Jul '07
Dec '07
May '08


Annualized Rate of Change in Coffee Futures Prices
0.160
0.140
0.120
0.100
0.080
0.060
0.040
0.020
0.000
0.142
0.135
0.117
0.112
0.108
0.108
0.102
0.101
0.078
Dec '06
May '07
Sep '07
Mar '08
Jul '08


Heat Oil Futures Prices 9/05/06
2.15
2.1
2.05
2
1.95
1.9
1.85
Oct '06
Dec '06
Feb '07
Apr '07
Jun '07
Aug '07
Oct '07


Annualized Rate of Change in Heating Oil Futures Prices
0.400
0.300
0.200
0.100
0.000
-0.100
-0.200
Nov '06
Jan '07
Mar '07
May '07
Jul '07
Sep '07

What is different about commodities?

Carrying Costs

Many commodities have carrying costs associated with them. For example, carrying a commodity such as wheat involves warehousing, spoilage and insurance costs. These costs can vary across time due to things such as the variation in the level of inventories. To see how these costs affect futures pricing consider the following modified cash and carry:

t	t'	T
sell the future buy the spot	borrow and pay carrying costs	future expires and delivery occurs

The cash flows associated with this position are given below:

	t	t'	T
buy spot	$-P_t$		
sell future			$F_{t,T}$
pay carrying costs		-C	
borrow		C	$-e^{r(T-t')}C$
Total	$-P_t$	0	$F_{t,T}-e^{r(T-t')}C$

*The spot is used to cover the short futures position.

This position resembles a T-Bill. In order for there not to be an arbitrage opportunity, the rate of return on the cash and carry must be the same as the rate of return on a T-Bill. That is,

$$\frac{F_{t,T} - Ce^{r(T-t')}}{P_t} = e^{r(T-t)}$$

or

$$F_{t,T} = P_t e^{r(T-t)} + Ce^{r(T-t')}$$

Note that in this case the futures price is always greater than the spot price.

The cash and carry with a continuous carrying cost is done slightly differently. We assume that when you purchase the spot, you also pay the carrying costs which are proportional to the cost of the asset.
The cash flows are:

	t	T
Buy the spot	$-P_t e^{k(T-t)}$	
Sell the future		$F_{t,T}$
Total	$-P_t\, e^{k(T-t)}$	$F_{t,T}$

Since this is a risk-free transaction, it should earn the risk-free rate.

$$\frac{F_{t,T}}{P_t e^{k(T-t)}} = e^{r(T-t)}$$

Rewriting this we get

$$F_{t,T} = e^{r(T-t)} P_t e^{k(T-t)} = P_t e^{(r+k)(T-t)}$$

Convenience Yield

The cash and carry with carrying costs explains some of the behavior of commodity futures prices, but not all. This approach to pricing requires that an arbitrageur be able to short the spot commodity. With many types of assets this may be difficult. Why are some assets difficult to short?

In the type of short sales that we have discussed thus far, as long as the lender didn't need the asset for the duration of the short sale, lending the asset out was equivalent to holding the asset. This type of asset is called a **pure asset**. It is held for capital gains and explicit payout such as dividends and coupon payments. Physical possession isn't needed to earn a return. Financial assets such as stocks and bonds are pure assets.

Assets which are hard to short are assets which are held for their physical services as well as their investment returns. This type of asset is known as a **convenience asset**. Commodities such as corn or copper are convenience assets. These assets are held to avoid production shortages. The value of these services is referred to as the **convenience value**. This value will differ across individuals and across times of the year.

When pricing a future on a convenience asset, the convenience value can be treated like a dividend. Just as a shareholder is willing to lend out his share if he is compensated for any dividends paid, a corn processor is willing to lend out the corn he has on hand if he is compensated for the potential losses associated with shortages.

Therefore the futures price is

$$F_{t,T} = P_t e^{(r+k-\delta)(T-t)}$$

where r is the continuously compounded risk free rate, k is the continuously compounded carry cost and δ is the continuously compounded convenience value (or yield).

The calendar spread is given by:

$$F_{t,T2} = F_{t,T1} e^{(r+k-\delta)(T2-T1)}$$

A **full carry market** is a market where the convenience value is zero.

A **non-full carry market** is a market where the convenience value is greater than zero.

Oil has acted as a pure asset at some times and a convenience asset at other times. Consider the following data:

Crude Oil Futures Prices			
	PRICE	MONTH TO MONTH PERCENTAGE DIFFERENCE	ANNUALIZED DIFFERENCE
NOV	14.11		
DEC	13.74	-2.62%	-31.47%
JAN	13.61	-0.95%	-11.35%
FEB	13.55	-0.44%	-5.29%
MAR	13.56	0.07%	0.89%
APR	13.59	0.22%	2.65%
MAY	13.63	0.29%	3.53%
JUN	13.67	0.29%	3.52%

In this time period, the market expected crude oil to be a convenience asset. In order for there to be full carry the futures price must rise enough to cover both the interest and storage costs. This was clearly not the case since T-Bill rates at this time were above 7%. The futures data suggest that there was a temporary oil shortage which was going to ease over the coming months. Only those whose convenience value was very high would be willing to hold crude oil between November and December.

The situation in a different time period was quite different.

Crude Oil Futures Prices			
MAY	12.25		
JUNE	12.62	3.02%	36.24%

The T-Bill rate at this time was below 7%. The futures price was a full carry. This type of situation occurs if there is an oil glut. The price of oil dropped earlier in the year since production exceeded consumption. All those whose wanted oil for its convenience value were fully stocked and some of the oil was being held by people with a zero convenience value. Before taking the oil into storage, these individuals required a return which would cover both the interest and carrying costs.

Exchange of Futures for Physicals (EFPs)

Suppose a refiner wishes to lock in the price of North Sea oil in March and at the same time a producer of North Sea oil would like to lock in a sales price for North Sea oil in March.

Both parties could use an exchange-traded contract on West Texas crude oil with a March delivery but they would be exposed to some basis risk.

Alternatively they could enter into a forward agreement with each other for North Sea oil but there would be some credit risk

A third possibility is to enter into an EFP agreement. This entails the two parties entering into an exchange-traded contract with a March delivery and entering into an additional agreement that they will trade North Sea oil in March at a price equal to the exchange futures price plus some differential. When delivery occurs the exchange pairs the two parties up and they trade North Sea oil instead of West Texas oil at the West Texas crude oil price plus the differential. This mechanism allows the two parties to reduce credit risk since the exchange position is guaranteed and reduce some of the basis risk.

Heating Oil Futures

Like crude oil, the most important issue in pricing heating oil is whether the asset is a convenience asset or a pure asset. This is determined primarily by seasonal factors.

Demand for heating oil is high in the winter and low in the summer. Producers adjust their output to account for the change in demand. However due to limitations on refining capacity it is inefficient to refine large quantities of oil in the winter and have idle capacity during the summer. So during the fall and summer, refiners build up a stock of heating oil to be consumed in the winter. Inventories tend to peak in the late fall and are at their lowest levels in late spring.

Heating oil is a pure asset in the summer and fall and a convenience asset in the late winter and early spring when stocks are low.

Consider the following prices:

Crude Oil Futures Prices

	PRICE	MONTH TO MONTH PERCENTAGE DIFFERENCE	ANNUALIZED DIFFERENCE
NOV	0.4084		
DEC	0.4167	2.03%	24.36%
JAN	0.4224	1.37%	16.44%
FEB	0.4214	-0.24%	-2.88%
MAR	0.4044	-4.03%	-48.36%
APR	0.3884	-3.96%	-47.52%
MAY	0.3784	-2.57%	-30.84%
JUN	0.3724	-1.59%	-19.08%

The T-Bill rate at this time was about 7.3%.

In November the inventories were at their highest levels, so heating oil was being held as a pure asset. Note that the futures price must rise enough to cover the interest expense and the increasing cost of storage. The differential between January and December is high but lower than the differential between November and December. This is because the cost of storage drops as the inventory levels begin to fall.

The price of oil falls in the late winter and spring. The convenience value is extremely high since only those with an extreme potential need would store oil over this period.

In the summer the market returns to full carry as inventories start to build up again.

Contango and Backwardation

If the forward curve (that is, the curve which plots futures prices as a function of time to delivery) is upward-sloping, the market is said to be in contango. This implies that the convenience yield is less than the risk free rate.

If the forward is downward-sloping, the market is said to be in backwardation. This implies that the convenience yield is greater than the risk free rate.

A perfect hedge requires that there is futures contract on an asset whose price is perfectly correlated with the price of the asset whose value one wishes to hedge. If this is not the case, we have an asset mismatch. For example, if a bank wants to hedge against increases in mortgage rates but the only futures are T-Bond futures, there is an asset mismatch and a perfect hedge is not possible.

Example:

Today is t=12-1-04 and I plan to buy heating oil on 2-1-05. The futures contract which is most highly correlated with heating oil is crude oil. The correlation is positive but it is less than 1.

In order to hedge against a rise in the cost of heating oil, I should buy a crude oil future since a long position in a future earns a profit when prices rise. How many contracts should I buy?

Define the following:

P_2^{HO} = the price of heating oil on 2-1-05
F_1^{CO} = the futures price of crude oil on 12-1-04
F_2^{CO} = the futures price of crude oil on 2-1-05

Note that F_2^{CO} equals the spot price since 2-1-05 is the delivery date.

h = hedge ratio
= number of futures contracts per unit of the asset being hedged

The cost on 2-1-05 when I purchase heating oil and the futures position is closed out equals

$$P_2^{HO} - h(F_2^{CO} - F_1^{CO}).$$

The second term in the expression above is the gain on the futures position which is an offset to the cost of the heating oil. F_1^{CO} is known on 12-1-04 and if we knew (P_2^{HO} - h F_2^{CO}) then there would be no uncertainty about our revenue on 2-1-05. This difference in price, known as the basis, is not usually known. The uncertainty which arises from this is known as basis risk.

In order to minimize the basis risk how many contracts should I sell? To answer this we need to define some notation.

ΔP = the per unit change in cost of heating oil during the life of the hedge

ΔF = the per unit change in crude oil futures price during the life of the hedge

σ_P = standard deviation of ΔP

σ_F = standard deviation of ΔF

ρ = correlation between ΔP and ΔF

The change in the value of my position between 12-1-04 and 2-1-05 is

$$\Delta P + h\,\Delta F$$

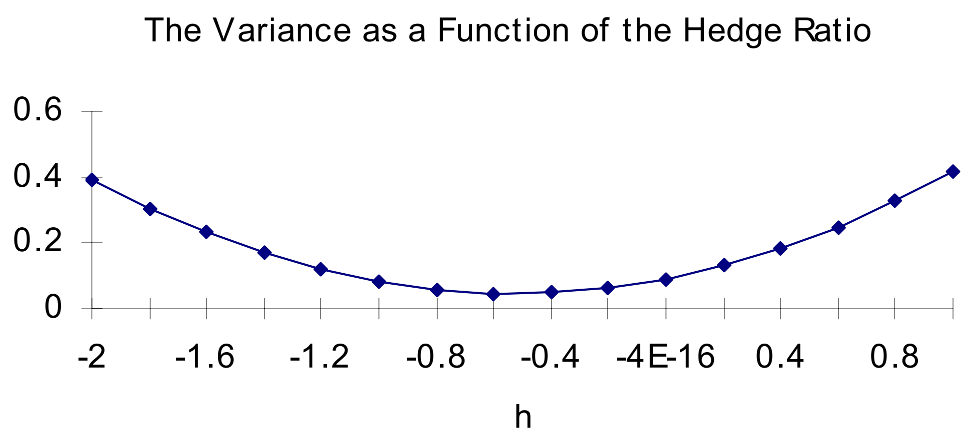
and the variance of my position is*

$$\sigma_P^2 + h^2\,\sigma_F^2 + 2\,h\,\rho\,\sigma_P\,\sigma_F$$

*Recall the formula for the variance of the sum (aX + bY) where a and b are constants and X and Y are random variables is:

$$Var(aX+bY) = a^2 Var(X) + b^2 Var(Y) + 2abCov(X,Y)$$

The graph below shows how the variance changes with the hedge ratio when $\sigma_P = .3$ $\sigma_F = .4$ and $\rho = .7$.



When hedging we would like as little change in the value of the position as possible. This means we would like to minimize the variance of the change in the value of the position.

The variance is minimized at the point where the slope is zero. This holds when

$$h = -\rho \frac{\sigma_P}{\sigma_F}$$

If h is negative, the hedge should be the reverse of the underlying position, i.e., if the underlying position is short, the hedge should be a long position and if the underlying position is long, the hedge should be a short position.

Examples:

- Suppose ρ =1. Then $h=-\sigma_P/\sigma_F$ and the variance of the change in the value of the position is zero. This means that you have a perfect hedge. This would also be the case when ρ = -1.

- If ρ = 0, then h = 0. If the futures contract is uncorrelated with the asset whose value you would like to hedge, it doesn't do you any good so the hedge ratio is zero

- Suppose the standard deviation of heating oil prices over the life of the hedge is 20%, the standard deviation of crude oil futures prices over the life of the hedge is 10%, and correlation is .99. Then

$$h = -.99*.20/.10 =-1.98$$

The larger the variance of the change in the spot relative to the variance of the change in the futures contract, the bigger the hedge ratio.

Note that it is possible that the variance could be reduced further by hedging with more than one type of contract.