NORTHWESTERN UNIVERSITY
KELLOGG GRADUATE SCHOOL OF MANAGEMENT

Professor Kathleen Hagerty Fall 2006
Finance 465

Practice Problems

1. Do problems 13.1, 13.3, 13.9, 13.12

2. A stock is priced at $30/share. The interest rate is 7%/year. A three-month European call option with a strike price of $35 has a Black-Scholes price of $.28

a. What is the value of a European put with the same underlying asset, same strike price and same time to expiration?

b. The delta of the call is .1515. How would you make a synthetic call (i.e., how many shares and how many dollars in T-bills)?

c. What is the delta of an otherwise identical put? How would you make a synthetic put?

d. If a trader sold 100 calls, what share position would he/she need to take to be delta neutral?

e. Suppose the gamma of the call is .0625. If the stock price increases by $1, how does the delta change? If the stock price increases by $3, does the hedged position make money or lose money?

3. A stock is priced at $48/share. The volatility is 36%/year and the interest rate is 7%/year. Consider the following information on one-year options with a strike price of $45.

| | Call | Put |
|---|---|---|
| Price | $9.88 | $3.84 |
| Delta | .71 | -.29 |
| Gamma | .0198 | .0198 |
| Theta* | -.013 | -.0047 |
| Vega* | .1643 | .1643 |
| Rho* | .2420 | -.1775 |

* The theta is the change in option price per day. The rho is the change in the option price per 1% change in the interest rate (i.e., if the interest rate changes from 7% to 8% the call price will change by ($.2520 *1) = $.2520). The vega is the change in the option per 1% change in the volatility (i.e., if the volatility increases from 36% to 37%, the price of the call would increase by .1643*1 = .1643)

a. Suppose a trader sold 50 calls. Assuming the stock price remained unchanged for one week, estimate the dollar profit on the position.
b. Suppose a trader buys 50 put options. Estimate the change in the value of the position if the volatility increased from 36% to 40%.
c. What is the elasticity of the call?
d. If the beta of the stock is 1.30, what is the beta of the call?

4. A stock is priced at $20/share. The volatility is 30%/year and the interest rate is 7%/year. Consider the following information on three month at-the-money options.

| | Call | Put |
|---|---|---|
| Price | $1.37 | $1.02 |
| Delta | .576 | -.424 |
| Gamma | .1306 | .1306 |
| Theta | -.0084 | -.0046 |
| Vega | .0392 | .0392 |
| Rho | .0254 | -.0237 |

A trader wants to establish a position which will make money if the volatility increases.

a. Using the call and put given above, establish a position which is delta neutral and has a vega equal to .0404. What is the gamma of this position?
b. Compare this position to a straddle. In particular compute the delta and the vega of the straddle.
c. What is the gamma of a straddle?
d. If an investor buys one call and sells one put, what is the delta?

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Answers to Practice Problems

13.1

The delta of one long call at S=40 is .2815. Therefore the delta of short 100 calls is (-100)(.28)=-28. To hedge this position you should go long 28 shares.

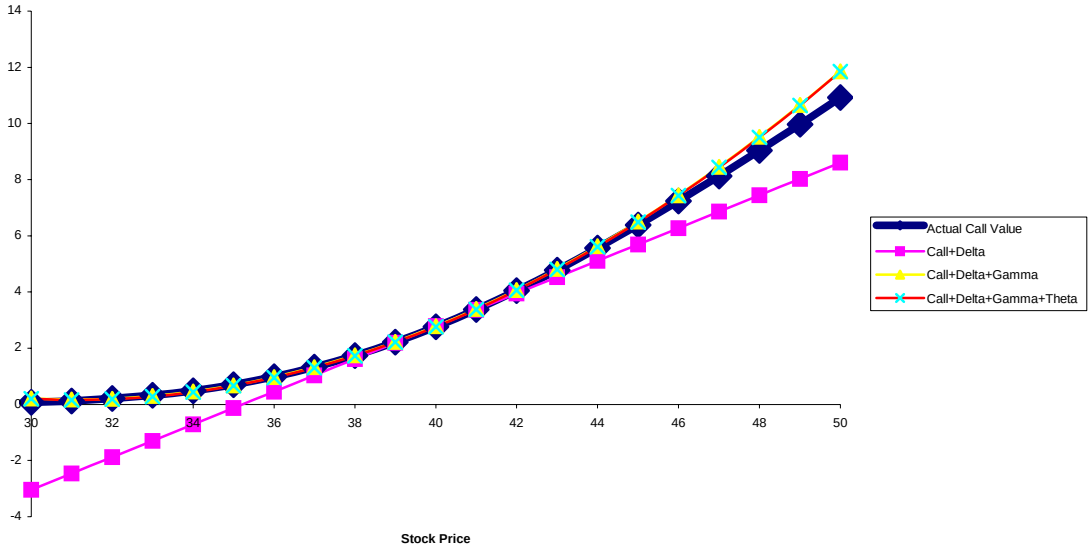
| | S=40 T=91/365 | S=39 T=90/360 | S=40.50 T=90/360 |
|---|---|---|---|
| Delta | 0.2815 | 0.225513 | 0.30841 |
| Call | **0.9710*100= 97.10** | **0.7054*(-100) =-70.54** | **1.1046*(-100)=-110.46** |
| Stock | **-28*(40)= -1120** | **28*39= 1092** | **28*40.50=1134** |
| Initial Investment | **-1022.90** | | |
| Fv of Initial Investment | | **1022.90*($e^{.08*1/365}$) =1023.04** | **1022.90*($e^{.08*1/365}$) =1023.04** |
| Profit | | **-1.58** | **.50** |

13.3
In a bull spread you buy a call at 40 and sell an otherwise identical call at 45. The delta of the position is .5824(1)+.2815(-1)=.301

| | S=40 T=91/365 | S=39 T=90/360 | S=40.50 T=90/360 |
|---|---|---|---|
| Delta of 40 Call | 0.5824 | 0.514737 | 0.614203 |
| Delta of 45 Call | 0.2815 | 0.225513 | 0.30841 |
| Buy 40 Call | **-2.780398** | **2.214415** | **3.062096** |
| Sell 45 Call | **0.9710** | **0.7054*(-1) =.7054** | **1.1046*(-1)=1.1046** |
| Stock Position | **.301*(40)= 12.036** | **.301*39= -11.74** | **.301*40.50=12.19** |
| Initial Investment | **10.2266** | | |
| FV of Initial Investment | | **10.2266e*(e.08*1/365) = 10.22884** | **10.2266e*(e.08*1/365) = 10.22884** |
| Profit | | **-.0022** | **-.0038** |

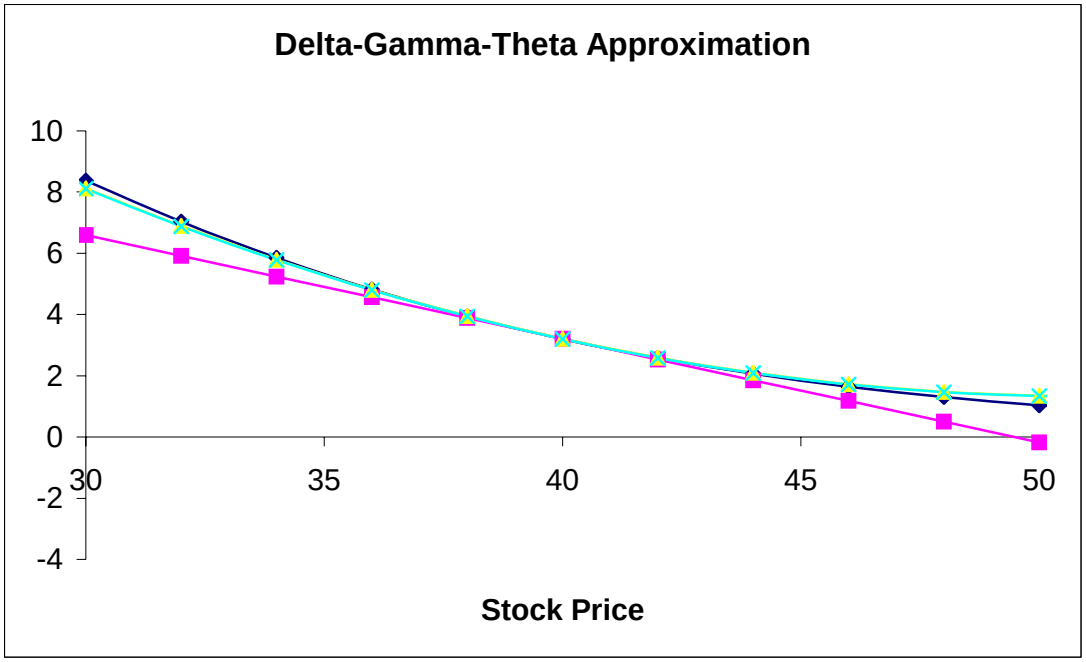
13.9

| | Actual Call Value | Call+Delta | Call+Delta+Gamma | Call+Delta+Gamma+Theta |
|---|---|---|---|---|
| 30 | 0.0730 | -3.0436 | 0.2142 | 0.1968 |
| 31 | 0.1267 | -2.4612 | 0.1776 | 0.1602 |
| 32 | 0.2083 | -1.8788 | 0.2062 | 0.1888 |
| 33 | 0.3263 | -1.2964 | 0.2999 | 0.2825 |
| 34 | 0.4893 | -0.7140 | 0.4588 | 0.4414 |
| 35 | 0.7055 | -0.1316 | 0.6828 | 0.6655 |
| 36 | 0.9819 | 0.4508 | 0.9720 | 0.9547 |
| 37 | 1.3238 | 1.0332 | 1.3264 | 1.3090 |
| 38 | 1.7343 | 1.6156 | 1.7459 | 1.7286 |
| 39 | 2.2144 | 2.1980 | 2.2306 | 2.2132 |
| 40 | 2.7630 | 2.7804 | 2.7804 | 2.7631 |
| 41 | 3.3770 | 3.3628 | 3.3954 | 3.3780 |
| 42 | 4.0519 | 3.9452 | 4.0755 | 4.0582 |
| 43 | 4.7820 | 4.5276 | 4.8208 | 4.8035 |
| 44 | 5.5610 | 5.1100 | 5.6313 | 5.6139 |
| 45 | 6.3827 | 5.6924 | 6.5069 | 6.4895 |
| 46 | 7.2405 | 6.2748 | 7.4476 | 7.4303 |
| 47 | 8.1286 | 6.8572 | 8.4536 | 8.4362 |
| 48 | 9.0416 | 7.4396 | 9.5246 | 9.5073 |
| 49 | 9.9746 | 8.0220 | 10.6609 | 10.6435 |
| 50 | 10.9237 | 8.6044 | 11.8623 | 11.8449 |



13.12

| Stock Price | Long Put | Delta Approximation | Delta-Gamma Approximation | Delta-Gamma-Theta Approximation |
|---|---|---|---|---|
| 30 | 8.372107 | 6.593787212 | 8.117839 | 8.106128815 |
| 32 | 7.031004 | 5.916864898 | 6.892258 | 6.880547951 |
| 34 | 5.847552 | 5.239942583 | 5.788601 | 5.776891209 |
| 36 | 4.819803 | 4.563020269 | 4.806869 | 4.795158589 |
| 38 | 3.940235 | 3.886097955 | 3.94706 | 3.935350092 |
| 40 | 3.197364 | 3.20917564 | 3.209176 | 3.197465717 |
| 42 | 2.577325 | 2.532253326 | 2.593215 | 2.581505463 |
| 44 | 2.065225 | 1.855331012 | 2.099179 | 2.087469332 |
| 46 | 1.646194 | 1.178408698 | 1.727067 | 1.715357323 |
| 48 | 1.306121 | 0.501486383 | 1.476879 | 1.465169437 |
| 50 | 1.032107 | -0.175435931 | 1.348616 | 1.336905672 |
| 52 | 0.812707 | -0.852358245 | 1.342276 | 1.33056603 |
| 54 | 0.637999 | -1.529280559 | 1.45786 | 1.446150509 |
| 56 | 0.499541 | -2.206202874 | 1.695369 | 1.683659111 |
| 58 | 0.390263 | -2.883125188 | 2.054802 | 2.043091835 |
| 60 | 0.304322 | -3.560047502 | 2.536159 | 2.524448681 |
| 62 | 0.236939 | -4.236969816 | 3.13944 | 3.127729649 |
| 64 | 0.184244 | -4.913892131 | 3.864645 | 3.852934739 |



2. a. Using put-call parity the value of the put is

$$P = C - S + Ke^{-rT} = \$.28 - 30 + 35\, e^{-.07*.25} = \$4.67$$

b.You would buy .1515 shares and borrow )S - C = (.1515)*(30)- .28 = $4.265

c. The delta of a put is $)_{call}$ -1 = .1515 - 1 =-.8485. To make a synthetic put, you would short .8485 shares and lend .8485*30 + 4.67 = 30.125.

d.The delta of the position is .1515*100 = 15.15, so you buy 15.15 shares

e.The delta is approximately .1515 + .0625 = .2140. Since the gamma is positive, the position makes money.

3. a. The initial revenue from the position is 50*$9.88 = $494. The theta is -.0127 so in one week the change in the value of the position is -.0127*7 = -.0894. So the call can be repurchased for $9.88 - .0894 = $9.79/call. So the profit on the position is 50*(.0894) = $4.47.

b. If the volatility increases by .04, vega tells us that the value of the option will increase by 4*.1643 = .6572. So the change in the value of the position is 50*.6572 = $32.86.

c. The elasticity is equal to )*S/C = .071*48/9.88 =.345

d. The beta of the call is Σ*∃ = 3.45*1.30 = 4.485

4. a. To find the right combinations of calls and puts you have to solve two equations with two unknowns. Let $n_c$ be the number of calls and let $n_p$ be the number of puts.

$n_c\Delta_c + n_p\Delta_p$ = $n_c(.576) + n_p(-.424)$ = 0 0 (Delta neutral)

$n_c v_c + n_p v_p$ = $n_c(.0392) + n_p(.0392)$ = .0404 (Vega of .0404)

Solving we get:

$n_c$ = .437 $n_p$ = .5936

The gamma of the position is .437*.1306 + .5936*.1306 = .1346

b. A straddleconsists of long one call and long one put. The delta of a straddle is $_{.c} +_{.p}$ = .576 - .424 = .1520. The vega of a straddle is $v_c + v_p = 2*.0392 = .0784$

c. The gamma is equal to 2 * .1306 = .2612

d. The delta is .576 + .424 = 1