

Northwestern University
Kellogg Graduate School of Management

Kathleen Hagerty
Finance 465

Fall 2003

Midterm Exam

You are allowed one 8 ½" by 11" page of notes (both sides). Answer the questions in the space provided. **SHOW ALL YOUR WORK!** Assume all the interest rates are annualized continuously compounded rates. You can spend up to 90 minutes on the exam.

Section 71

The exam is due in class on Wednesday, November 5

Sections 61 & 62

The exam is due Monday, November 3 by 5 pm in the Finance Dept.

I. Answer the following questions true or false and explain why.

a. (5 points) A commodity futures forward curve must rise at least the risk free rate.

b. (5 points) A gold mining firm might hedge their gold inventory by selling puts and buying calls.

c. (5 points) The expected return on a short S&P 500 futures position is negative.

II. An investor has a \$100 million portfolio with a beta equal to .85. She would like to hedge the risk of her portfolio over the next 6 months using a S&P 500 futures contract. The current value of the S&P 500 Index is 1040, the dividend yield is 2% and the risk free rate is 1%.

a. (10 points) How many futures contracts should she buy or sell to minimize the risk of the portfolio.

b. (10 points) The initial margin and the maintenance margin on an S&P 500 index futures contract are \$17,000 and \$14,000 respectively. If the value of the **index** rises to 1060, what is the change in the value of the investor's margin account? Will she get a margin call?

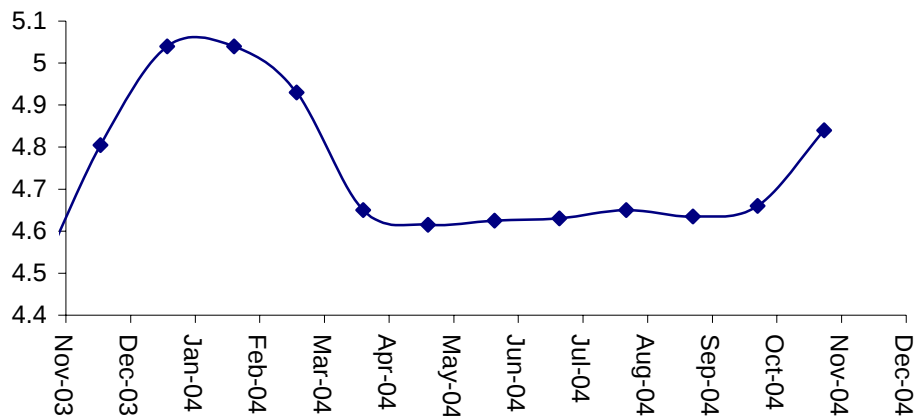
II. Consider the following information. Assume the risk free rate is 1.3% and that November delivery date is 1 month from now.

Natural Gas Comp. - nymex

ta retrieved at 10/28/03 10:35:08 • All quotes are in exchange local

	Contract	Month	Last	OpenInt	Date
	Natural Gas	Nov '03	4.470	24566	10/28/2003
	Natural Gas	Dec '03	4.805	54052	10/28/2003
	Natural Gas	Jan '04	5.040	35332	10/28/2003
	Natural Gas	Feb '04	5.040	24848	10/28/2003
	Natural Gas	Mar '04	4.930	21058	10/28/2003
	Natural Gas	Apr '04	4.650	21885	10/28/2003
	Natural Gas	May '04	4.615	15601	10/28/2003
	Natural Gas	Jun '04	4.625	14526	10/28/2003
	Natural Gas	Jul '04	4.630	10985	10/28/2003
	Natural Gas	Aug '04	4.650	10918	10/28/2003
	Natural Gas	Sep '04	4.635	8905	10/28/2003
	Natural Gas	Oct '04	4.660	8831	10/28/2003
	Natural Gas	Nov '04	4.840	7684	10/28/2003

Natural Gas Forward Curve



- a. (10 points) What is the annualized net convenience value, $\delta - k$, between December 03 and January 04 and between March 04 and April 04. Given an intuition for your results.
- b. (15 points) Suppose someone would like to lock in a price for natural gas for the months January, February, March and April using a swap. In addition suppose they want to swap payments to increase by 10% each month, i.e., the first payment would be C , the second payment would be $C \cdot (1.10)$, the third payment would be $C \cdot (1.20)$ etc. What should C be?

III. Suppose a firm in England sells widgets in the Mexico. The firm will be receiving 2 million pesos in eight months which it will then convert into pounds. The firm would like to hedge the exchange rate risk associated with this transaction. The break even exchange rate for the firm is .06 £/peso. The current exchange rate is .0532 £/peso, the risk free rate in England is 3% and the risk free rate in Mexico is 12%.

- a. (10 points) Draw the firm's exposure to the exchange rate. What futures position should the firm take to hedge its currency exposure?

- b. (10 points) If the firm hedged its exposure with a future what profit or loss would it lock in?

c. (15 points) Suppose you observe the following prices.

	Bid (£/peso)	Ask (£/peso)
Future (delivery in 8 months)	.0490	.0495
Spot Exchange Rate	.0530	.0538

Assume that you can borrow and lend at 3% in England and you can borrow and lend at 12% in Mexico. Show how you can earn an arbitrage profit. (Make sure you do your calculations out to the 4th decimal place.)

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Answers to the Midterm Exam

I. Answer the following questions true or false and explain why.

a. (5 points) A commodity futures forward curve must rise at least the risk free rate.

False. The rate of increase in the futures price is measured by $(r+k-\delta)$ which can be less than r if $k < \delta$.

b. (5 points) A gold mining firm might hedge their gold inventory by selling puts and buying calls.

False. A gold mining firm has a long exposure so buying calls and selling puts would only increase their exposure to gold prices.

c. (5 points) The expected return on a short S&P 500 futures position is negative.

True. The expected return on a long position in an S&P 500 index future is $r_{S\&P500} - r_{riskfree} > 0$, so a short position would have a negative expected return.

II. An investor has a \$100 million portfolio with a beta equal to .85. She would like to hedge the risk of her portfolio over the next 6 months using a S&P 500 futures contract. The current value of the S&P 500 Index is 1040, the dividend yield is 2% and the risk free rate is 1%.

a. (10 points) How many futures contracts should she buy or sell to minimize the risk of the portfolio.

$$\text{The number of contracts} = H = \frac{I_p}{N} \beta$$

where I = the value of the investors portfolio

N = the notional value of the future contract
= units per contract x futures price

The futures price is $F_{t,T} = 1040e^{(.01-.02)(.5)} = 1034.813$.

$$H = -\frac{\$100M}{250 * 1034.813} (.85) = -328.56 \rightarrow \text{The investor should short 328 contracts}$$

b. (10 points) The initial margin and the maintenance margin on an S&P 500 index futures contract are \$17,000 and \$14,000 respectively. If the value of the **index** rises to 1060, what is the change in the value of the investor's margin account? Will she get a margin call?

If the index rises to 1060 then the futures price will become

$$F_{t,T} = 1060e^{(.01-.02)(.5)} = 1054.713$$

Since the investor is short futures, she incurs a loss per contract equal to

$$250 * (1034.813 - 1054.713) = -\$4,975$$

Since the initial margin and \$17,000 and the maintenance margin is \$14,000, a loss of \$3,000 or more will result in a margin call. Since her loss is greater than \$3000, she does get a margin call.

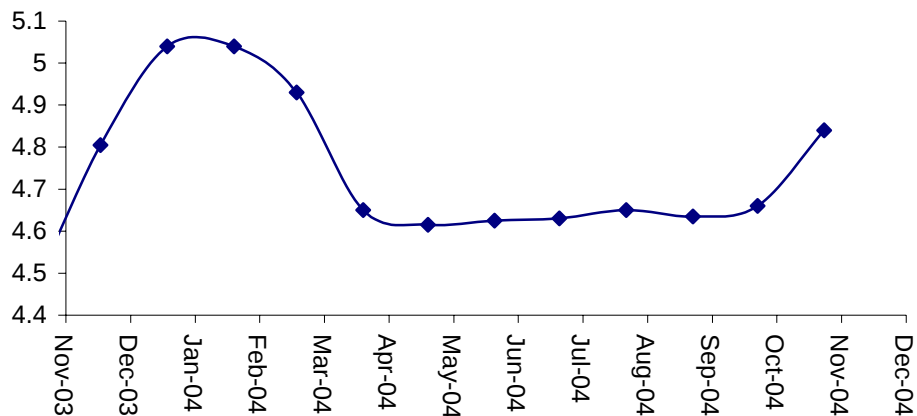
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Natural Gas Forward Curve



- c. (10 points) What is the annualized net convenience value, $\delta - k$, between December 03 and January 04 and between March 04 and April 04. Given an intuition for your results.

For December 03 to January 04

$$\delta - k = r - \frac{\ln\left(\frac{F_{t,T2}}{F_{t,T1}}\right)}{(T2 - T1)} = .013 - \frac{\ln\left(\frac{5.040}{4.805}\right)}{1/12} = -56\%$$

For March 04 to April 04

$$\delta - k = r - \frac{\ln\left(\frac{F_{t,T2}}{F_{t,T1}}\right)}{(T2 - T1)} = .013 - \frac{\ln\left(\frac{4.65}{4.93}\right)}{1/12} = 71.47\%$$

Between December and January, $\delta - k \ll 0$. Carrying costs must be very high suggesting that natural gas inventories are extremely high.

Between March and April, $\delta - k \gg 0$. The convenience value is clearly very high relative to carrying costs, suggesting that natural gas inventories will be extremely low.

- d. (15 points) Suppose someone would like to lock in a price for natural gas for the months January, February, March and April using a swap. In addition suppose they want to swap payments to increase by 10% each month, i.e., the first payment would be C, the second payment would be $C*(1.10)$, the third payment would be $C*(1.20)$ etc. What should C be?

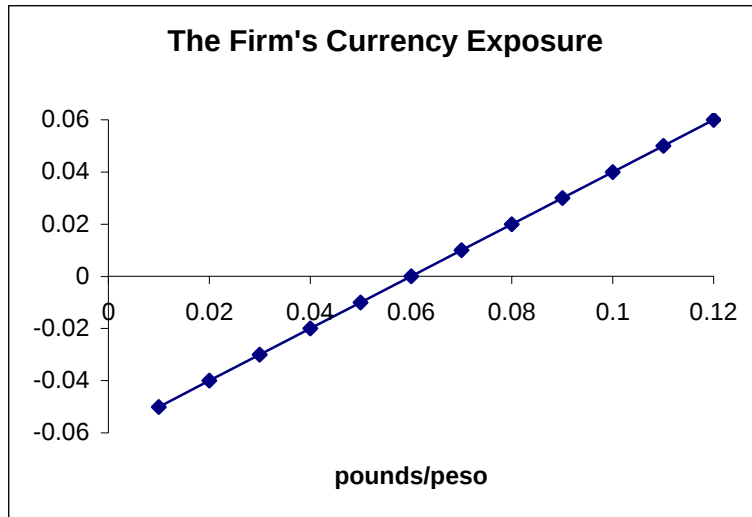
	futures price	discount factor	PV of futures price	Swap Payments	PV of Swap Payments
Jan-04	5.04	0.996755	5.023647	1*C	0.996755*1.0*C
Feb-04	5.04	0.995676	5.018207	1.1*C	0.995676*1.1*C
Mar-04	4.93	0.994598	4.903368	1.2*C	0.994598*1.2*C
Apr-04	4.65	0.993521	4.619873	1.3*C	0.993521*1.3*C
			19.56509		

$$C = 4.274567$$

CHECK	
	PV of Swap Payments
Jan-04	4.274567
Feb-04	4.702024
Mar-04	5.12948
Apr-04	5.556937
	19.56509

III. Suppose a firm in England sells widgets in the Mexico. The firm will be receiving 2 million pesos in eight months which it will then convert into pounds. The firm would like to hedge the exchange rate risk associated with this transaction. The break even exchange rate for the firm is .06 £/peso. The current exchange rate is .0532 £/peso, the risk free rate in England is 3% and the risk free rate in Mexico is 12%.

- c. (10 points) Draw the firm's exposure to the exchange rate. What futures position should the firm take to hedge its currency exposure?



Given that the firm has a long exposure, the firm should go short futures (i.e. sell peso for pounds).

- b. (10 points) If the firm hedged its exposure with a future what profit or loss would it lock in?

The futures price is $F_{t,T} = (.0532)e^{(.03-.12)(8/12)} = .0501$ £/peso.

The firm's profit is:

$(.0501 - .06) \text{ £/peso} \times (2 \text{ million pesos}) = -19,796.3 \text{ £}.$

c. (15 points) Suppose you observe the following prices.

	Bid (£/peso)	Ask (£/peso)
Future (delivery in 8 months)	.0490	.0495
Spot Exchange Rate	.0530	.0538

Assume that you can borrow and lend at 3% in England and you can borrow and lend at 12% in Mexico. Show how you can earn an arbitrage profit. (Make sure you do your calculations out to the 4th decimal place.)

The two no arbitrage restrictions are:

$$P_t^{bid} e^{(r-\delta)T} - F_{t,T}^{ask} \leq 0 \quad \text{Since } (.053)e^{(.03-.12)(8/12)} - .0495 = .0499 - .0495 > 0$$

➔ Arbitrage Opportunity

$$F_{t,T}^{bid} - P_t^{ask} e^{(r-\delta)T} \leq 0 \quad \text{Since } .049 - (.0538)e^{(.03-.12)(8/12)} = .0490 - .0507 < 0$$

➔ No Arbitrage Opportunity

To take advantage of the arbitrage opportunity you should buy the future, short $e^{-.12*8/12}$ pesos and lend the proceeds from the short sales.

	Today	At Delivery
Buy the future		-.0495
Short $e^{-.12*8/12}$ pesos	$e^{-.12*8/12} .053$	
Lend	$-e^{-.12*8/12} .053$	$e^{(.03-.12)*8/12} .053 = .0499$
Total	0	.0004