Final Exam

You are allowed two 8 ½" by 11" page of notes (both sides). Answer the questions in the space provided. SHOW ALL YOUR WORK! Assume all the interest rates are annualized and continuously compounded unless it is explicitly stated otherwise

I. Answer the following questions true or false and explain why.

a. (5 points) Calls will always sell for more than the value of immediate exercise.

b. (5 points) The implicit leverage in a call option is greatest when the call in deep in-the-money.

c. (5 points) Suppose you are planning to buy gold in six months and you would like to put a zero cost collar your exposure to gold prices. If the strike price on the call in the collar is equal to the current futures price, then the strike price on the put in the collar must also be equal to the futures price.

d. (5 points) The spot price of silver is $420/ounce and the continuously compounded annual risk free rate is 3%. Consider the following forward curve for silver:

Year	0	1	2	3
Forward Price	420	428.48	437.14	445.97

The forward curve given above implies that silver has a carrying cost.

II. On the next page is a binomial tree which gives the yen price of 1 British pound, that is, the units are yen/£. The length of each binomial period is .25. The annual continuously compounded Japanese risk free rate is 2% and the risk neutral probability of an up move is .4316

g. (5 points) What are u and d?

h. (5 points) What is σ?

i. (5 points) What is the British risk free rate?

d. (15 points) At each node in the tree on the following page, fill in the European and American call prices for calls with a strike equal to 140 yen.

208.38
Eur:
Amer:

170.80
Eur:
Amer:

S = 140
Eur:
Amer:

140.00
Eur:
Amer:

114.75
Eur:
Amer:

94.06
Eur:
Amer:

e. (5 points) What is the price of an otherwise identical European put 6 months from expiration?

f. (5 points) What are the delta and the elasticity two periods from expiration of the American call?

g. (5 points) Suppose that two periods from expiration you observe that the American call price is 15 yen. Using only pounds and Japanese T-bills, show how would you take advantage of this. (Show the exact positions and the cash flows).

h. (10 points) Suppose you observed that the price of a pound-yen futures contract (you buy pounds with yen) with delivery in six months is 133 yen/**£**. Explain in detail (give positions and cash flows) how you can earn an arbitrage profit.

III. Use the following information to answer the questions in this section. Assume the current stock price is $30, the annual continuously compounded risk free rate is 5%, the continuously compounded dividend yield is 2%, the volatility is 40% and there is 6 months to expiration. All the options are European.

a. (10 points) Fill in as much as possible on the following table

	Price	Delta	Gamma	1-day Theta	Vega
Call K=30	3.54			-0.01	
Put K=30		-0.42	0.045		0.082
Call K=50		0.053	0.013	-0.002	0.023
Put K=50	19.23			0.002	

b. (5 points) Suppose the stock price doesn't change but the price of the 30 strike price call rises to $3.70. Approximately what is the implied volatility at this new price?

c. (5 points) Draw the payoff diagram for the following position.

Long 1 Share
Short 1 30 Call
Short 1 50 Call

d. (5 points) If the expected return on the stock is 15%, what is the expected return on the position in c.?

d. (5 points) Assuming the stock price is \$30, how would you delta hedge the position in c.?

e. (5 points) Suppose the stock price increases by \$.75 over the course of **one day**. Use the delta-gamma-theta approximation to compute the change in the value of the **hedged position**. At what stock price will the hedged position break even?

IV. Consider the following call prices. Assume all the options have the same time to expiration and the same underlying asset.

Strike Price	100	110	140
Call Price	24.74	22.82	9.81

(10 points) Find an arbitrage opportunity and show how you can take advantage of it.

Answers to the Final Exam

I. Answer the following questions true or false and explain why.

a. (5 points) Calls will always sell for more than the value of immediate exercise.

False. European calls on dividend paying stocks can sell for less than S-K.

b. (5 points) The implicit leverage in a call option is greatest when the call in deep in-the-money.

False. The implicit leverage is highest when the call is deep out-of-the-money.

c. (5 points) Suppose you are planning to buy gold in six months and you would like to put a zero cost collar your exposure to gold prices. If the strike price on the call in the collar is equal to the current futures price, then the strike price on the put in the collar must also be equal to the futures price.

True. If you have a call whose strike is equal to the futures price, the only strike on the put that will produce a zero cost collar is the futures price.

d. (5 points) The spot price of silver is $420/ounce and the continuously compounded annual risk free rate is 3%. Consider the following forward curve for silver:

Year	0	1	2	3
Forward Price	420	428.48	437.14	445.97

The forward curve given above implies that silver has a carrying cost.

Ambiguous. To see this note:

$$\frac{\ln\left(\frac{F_{t,T2}}{F_{t,T1}}\right)}{(T2-T1)} = r - \delta + k = \ln\left(\frac{428.48}{420}\right) = \ln\left(\frac{437.40}{428.48}\right) = \ln\left(\frac{445.97}{437.14}\right) = .02$$

Since r=.03 ➔ (k-δ) = -.01 but you can't tell how the -.01 is divided between k and δ.

II. On the next page is a binomial tree which gives the yen price of 1 British pound, that is, the units are yen/£. The length of each binomial period is .25. The annual continuously compounded Japanese risk free rate is 2% and the risk neutral probability of an up move is .4316

a. (5 points) What are u and d?

u= $\frac{uS}{S} = \frac{170.80}{140} = 1.22$ d = $\frac{dS}{S} = \frac{114.75}{140} = .8196$

b. (5 points) What is σ?

$$u = e^{\sigma\sqrt{\frac{T}{n}}} \rightarrow \frac{\ln(u)}{\sqrt{\frac{T}{n}}} = \frac{\ln(1.22)}{\sqrt{.25}} = .397$$

c. (5 points) What is the British risk free rate?

$$p = \frac{e^{(r_d - r_f)(T/n)} - d}{u - d} \quad \rightarrow \quad r_f = 5\%$$

d. (15 points) At each node in the tree on the following page, fill in the European and American call prices for calls with a strike equal to 140 yen.

208.38
Eur: 68.38
Amer: 68.38

170.80

Eur: $= \frac{.4316 * 68.36}{e^{.02*.25}} = 29.35$

Amer: =max[29.35, 170.80-140 = 30.80] = 30.80

S = 140

Eur: $= \frac{.4316 * 29.35}{e^{.02*.25}} = 12.60$

Amer: $= \frac{.4316 * 30.80}{e^{.02*.25}} = 13.23$

140.00

Eur: 0

Amer: 0

114.75
Eur: 0
Amer: 0

94.06
Eur: 0
Amer: 0

e. (5 points) What is the price of an otherwise identical European put 6 months from expiration?

$$P = C - Se^{r_f T} + Ke^{r_d T} = 12.60 - 140e^{-.05*.5} + 140e^{-.02*.5} = 14.66$$

f. (5 points) What are the delta and the elasticity two periods from expiration of the American call?

$$\Delta = \frac{C_u - C_d}{R_f S(u-d)} = \frac{30.80}{e^{.05*.25}(170.80 - 114.75)} = .542$$

$$\Omega = \Delta * S / C = (.542)(140)/13.23 = 5.735$$

g.(5 points) Suppose that two periods from expiration you observe that the American call price is 15 yen. Using only pounds and Japanese T-bills, show how would you take advantage of this. (Show the exact positions and the cash flows).

The call is overpriced so you should sell the call and buy a synthetic call.

See the call	**15.00**
Buy Δ pounds	**-(.542)*(140)**
Borrow B yen	**62.65**
Total	**1.77**

h. (10 points) Suppose you observed that the price of a pound-yen futures contract (you buy pounds with yen) with delivery in six months is 133 yen/**£**. Explain in detail (give positions and cash flows) how you can earn an arbitrage profit.

$\mathbf{F_{t,T} = 140e^{(.02-.05)(.5)} = 137.91}$ ➔ The future is underpriced

	Today	**At Delivery**
Buy the Future		**-133**
Short $e^{-.05*.5}$ pounds	**+136.54**	**Cover short with pounds from the futures position**
Lend	**-136.54**	**$136.54e^{.02*.5} = 137.91$**
Total	**0**	**4.91**

III. Use the following information to answer the questions in this section. Assume the current stock price is $30, the annual continuously compounded risk free rate is 5%, the continuously compounded dividend yield is 2%, the volatility is 40% and there is 6 months to expiration. All the options are European.

b. (10 points) Fill in as much as possible on the following table

	Price	Delta	Gamma	1-day Theta	Vega
Call K=30	3.54	**.58**	**.045**	-0.01	**.082**
Put K=30	**3.099**	-0.42	0.045		0.082
Call K=50	**.166**	0.053	0.013	-0.002	0.023
Put K=50	19.23	**-.947**	**.013**	0.002	**.082**

$\mathbf{P_{30} = 3.54 - 30\, e^{-.02*.5} + 30\, e^{-.05*.5} = 3.099}$ $\mathbf{C_{50} = 19.23 + 30\, e^{-.02*.5} - 50\, e^{-.05*.5} = .166}$

b. (5 points) Suppose the stock price doesn't change but the price of the 30 strike price call rises to \$3.70. Approximately what is the implied volatility at this new price?

3.70 – 3.54 = .16

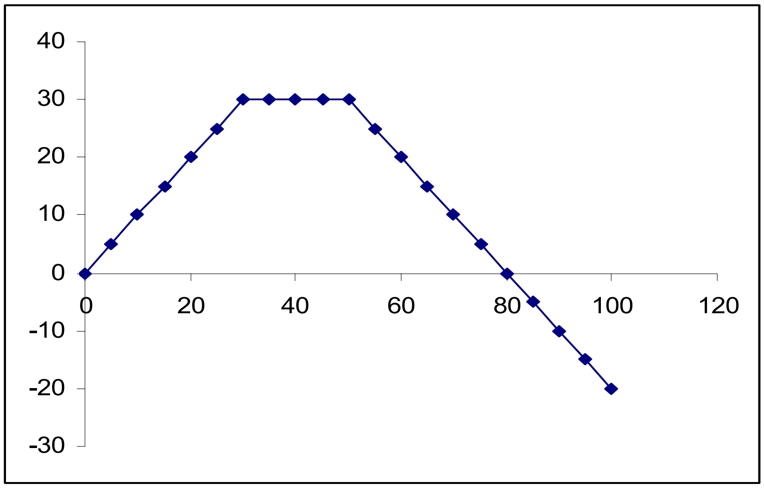
$\Rightarrow$ (change in volatility) * (vega) = .16

$\Rightarrow$ (change in volatility) = 1.95

$\Rightarrow$ new volatility = 41.95%

c. (5 points) Draw the payoff diagram for the following position.

Long 1 Share
Short 1 30 Call
Short 1 50 Call



d. (5 points) If the expected return on the stock is 15%, what is the expected return on the position in c.?

$ER_{position} - r = \Omega(ER_{stodk} - r)$

$\Delta_{position}$ = (1)(1) + (-1)(.58) + (-1)(.053) = .367

Value of the position = 30 + (-1)(3.54) + (-1)(.166) = 26.29

Ω = ΔS/[value of the position] = .367*30/26/29 = .418

$ER_{position}$ = .05 + (.418)(.15-.05) = .092

d. (5 points) Assuming the stock price is \$30, how would you delta hedge the position in c.?

The delta of the position is .367 so you should short .367 shares.

e. (5 points) Suppose the stock price increases by \$.75 over the course of **one day**. Use the delta-gamma-theta approximation to compute the change in the value of the **hedged position**. At what stock price will the hedged position break even?

The change in the value of the hedged position is

$$\frac{1}{2}\varepsilon^2\Gamma + \theta h = \frac{1}{2}(.75)^2(-.058) + (.012) = -.0043$$

Since Γ= (-1)(.045) + (-1)(.013) = -.058
and
θh =daily theta = (-1)(-.01)+(-1)(.002)=.012

Break-even stock price change = $\pm$ **one standard deviation =** $\pm\, 30\frac{.40}{\sqrt{365}} = \pm .628$

IV. Consider the following call prices. Assume all the options have the same time to expiration and the same underlying asset.

Strike Price	100	110	140
Call Price	24.74	22.82	9.81

(10 points) Find an arbitrage opportunity and show how you can take advantage of it.

22.82 > (3/4)(24.74) + (1/4)(9.81) = 21.0075 ➔ violated convexity

		At Expiration			
	Today	S<100	100<S<110	110<S<140	S>140
Buy 3 100 Calls	-74.22	0	3(S-100)	3(S-100)	3(S-100)
Buy 1 140 Call	-9.81	0	0	0	(S-140)
Sell 4 110 Calls	91.28	0	0	4(110-S)	4(110-S)
Total	7.25	0	3(S-100)	140-S	0