Name ________________________

Northwestern University
Kellogg Graduate School of Management

Kathleen Hagerty — Spring 2003
Finance 465 — Sections 61 & 62

Final Exam

You are allowed two 8 ½" by 11" page of notes (both sides). Answer the questions in the space provided. SHOW ALL YOUR WORK! Assume all the interest rates are annualized and continuously compounded unless it is explicitly stated otherwise. You can spend up to 2 hours on the exam.

I. Answer the following questions true or false and explain why.

a. (5 points) A deep in-the-money put has a negative delta, a positive gamma and positive theta.

b. (5 points) A commodity futures forward curve must rise at least the risk free rate.

c. (5 points)A gold mining firm might hedge their gold inventory by selling puts and buying calls.

d. (5 points) For a call to have a positive theta, the dividend yield must be greater than zero.

II. Use the following information to answer the questions in this section. The annual continuously compounded risk free rate is 3%, the current spot price for gold is $300/ounce and the annual volatility is 30%.

a. (10 points) Using a two period binomial model compute the price of a European call option which gives the holder the right to buy one ounce of gold in six months for $240/ounce. What is the price of an American call?

b. (5 points) What is the price of an otherwise identical European put?

c. (10 points) Using a two period binomial model compute the price of a European call option **on a gold future** with a strike price equal to $240/ounce. What is the price of an American call option on a gold future?

d. (5 points) What is the price of otherwise identical European put on a gold future?

e. (10 points) Now suppose the annual carrying cost of gold is 4% per year. Suppose you observe that a gold futures contract with delivery in 9 months is \$305/ounce. Show how you could earn an arbitrage profit.

III. Use the following information to answer the questions in this section. Assume the current stock price is $50, the annual continuously compounded risk free rate is 2%, the continuously compounded dividend yield is 1% , the annual standard deviation is 30% and there is 1 year to expiration. All the options are European.

a. (10 points) Fill in as much as possible on the following table

	Price	Delta	Gamma	1-day Theta	Vega
Call K=40	11.937	.815	.017	-0.01	.1285
Put K=40				-.005	
Call K=50	6.1226	.567	0.026	-0.008	0.1942
Put K=50					

b. (10 points) Suppose that over the day, the stock price increase by one standard deviation. What is the delta approximation for the value of the call with K=40 at the end of the day. What is the delta-gamma-theta approximation for the value of the call at the end of the day.

c. Consider the following position. Buy 1 put with K=40 and sell 1 call with K=50.

i. (5 points) How would you delta hedge this position?

ii. (10 points) Suppose the stock price moves by $2.00 over the course of **one week**. Use the delta-gamma-theta approximation to compute the change in the value of the **hedged position**.

IV. Consider the following call prices. Assume all the options have the same time to expiration and the same underlying asset.

Strike Price	40	42	50
Call Price	11.93	9.55	6.12

(10 points) Find an arbitrage opportunity and show how you can take advantage of it.

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Answers to the Final Exam

I. Answer the following questions true or false and explain why.

a. (5 points) A deep in-the-money put has a negative delta, a positive gamma and positive theta.

True.

b. (5 points) A commodity futures forward curve must rise at least the risk free rate.

False. If the convenience yield is very high then the slope could be negative. That is, $(r + k- \delta) < r$ if $k < \delta$.

c. (5 points)A gold mining firm might hedge their gold inventory by selling puts and buying calls.

False. A gold mining firm is long gold (i.e. likes high prices and dislikes low prices), so it would buy puts and sell calls.

d. (5 points) For a call to have a positive theta, the dividend yield must be greater than zero.

True. The theta is only positive if you are missing out on something by not being able to exercise the option. For a call, this will only by the case is if there is a dividend or some other cash flow associated with holding the asset.

II. Use the following information to answer the questions in this section. The annual continuously compounded risk free rate is 3%, the current spot price for gold is $300/ounce and the annual volatility is 30%.

a. (10 points) Using a two period binomial model compute the price of a European call option which gives the holder the right to buy one ounce of gold in six months for $240/ounce. What is the price of an American call?

u	d	R	p
1.161834	0.860708	1.007528	0.48757

		404.9576
	348.5503	
300		300
	258.2124	
		222.2455

		164.9576
	110.3435	
68.16579		60
	29.03563	
		0

		164.9576
	110.3435	
68.16579		60
	29.03563	
		0

Stock price tree **European call price tree** **American call price tree**

b. (5 points) What is the price of an otherwise identical European put?

$P = C - S + Ke^{-rT} = 68.16579 - 300 + 240e^{-.03*.5} = 4.5926$

c. (10 points) Using a two period binomial model compute the price of a European call option **on a gold future** with a strike price equal to $240/ounce. What is the price of an American call option on a gold future?

u	d	R	pf
1.161834	0.860708	1.007528	0.46257

Futures price tree

		411.0778
	353.8179	
304.5339		304.5339
	262.1148	
		225.6043

European call price tree

		171.0778
	112.9675	
67.66916		64.53392
	29.62842	
		0

American call price tree

		171.0778
	113.8179	
68.05961		64.53392
	29.62842	
		0

d. (5 points) What is the price of otherwise identical European put on a gold future?

$P = C - F + Ke^{-rT} = 67.669 - 304.5339e^{-.03*.5} + 240e^{-.03*.5} = 4.59$

e. (10 points) Now suppose the annual carrying cost of gold is 4% per year. Suppose you observe that a gold futures contract with delivery in 9 months is $305/ounce. Show how you could earn an arbitrage profit.

$F = 300\, e^{((.03+.04)*.75)} = 316.17$

Since the future is undervalued you should buy the future, short the spot and lend the proceeds.

	Today	**At Delivery**
Buy the future		**-305**
Short the $e^{.04*.5}$ units of the spot *	**$300e^{.04*.5}$**	**Use the gold from the long future to cover the short position**
Lend proceeds	**$-300e^{.04*.5}$**	**$300\, e^{.04*.5}\, e^{.03*.5} = 316.17$**
Total	**0**	**11.17**

*** The person you borrowed the gold from will lend you $e^{.04*.5}$ units and only require that you return 1 unit since you saved them the carrying costs by borrowing the gold.**

III. Use the following information to answer the questions in this section. Assume the current stock price is $50, the annual continuously compounded risk free rate is 2%, the continuously compounded dividend yield is 1% , the annual standard deviation is 30% and there is 1 year to expiration. All the options are European.

b. (10 points) Fill in as much as possible on the following table

	Price	Delta	Gamma	1-day Theta	Vega
Call K=40	11.937	.815	.017	-0.01	.1285
Put K=40	**1.643**	**-.185**	**.017**	-.005	**.1285**
Call K=50	6.1226	.567	.026	-0.008	.1942
Put K=50	**5.63**	**-.433**	**.026**		**.1942**

b. (10 points) Suppose that over the day, the stock price increase by one standard deviation. What is the delta approximation for the value of the call with K=40 at the end of the day. What is the delta-gamma-theta approximation for the value of the call at the end of the day.

$\sigma = \frac{.30}{\sqrt{365}} = .0157$ ===> Std deviation move = $50*.0157 = $.785

$C_{new} = C_{old} + \epsilon\Delta =$ 11.937 + (.785)(.815) = 12.58

$C_{new} = C_{old} + \epsilon\Delta + \frac{1}{2}\epsilon^2\Gamma + h\theta$ = 11.937 + (.785)(.815) + .5*$(.785)^2$(.017) + (-.01) =12.575

c. Consider the following position. Buy 1 put with K=40 and sell 1 call with K=50.

i. (5 points) How would you delta hedge this position?

Δ= (1)(-.185) + (-1)(.567) = -.752

You should buy .752 shares.

ii. (10 points) Suppose the stock price moves by \$2.00 over the course of **one week**. Use the delta-gamma-theta approximation to compute the change in the value of the **hedged position**.

Initial value = 1.643 - 6.1226 = - 4.48

$\Delta_{\text{hedged position}}$ **= 0**
$\Gamma_{\text{hedged position}}$ **= (1)(.017) + (-1)(.026) = -.009**
$\theta_{\text{hedged positino}}$ **= (1)(-.005) + (-1)(-.008) = .003 (this is the one day theta)**
(note the stock position doesn't affect the gamma or the theta)

delta-gamma-theta approximation $= \text{original value} + \epsilon\Delta + \frac{1}{2}\epsilon^2\Gamma + h\theta$
$= -4.48 + 0 + \frac{1}{2}(2)^2(-.009) + 5*.003$
$= -4.483$

IV. Consider the following call prices. Assume all the options have the same time to expiration and the same underlying asset.

Strike Price	40	42	50
Call Price	11.93	9.55	6.12

(10 points) Find an arbitrage opportunity and show how you can take advantage of it.

It should be the case that $K_2 - K_1 \geq C(K_1) - C(K_2)$ however this restriction is violated for the 40 and 42 strike price options. This means that the 40 strike price call is overpriced relative to the 42 strike price call. You should sell the 40 call and buy the 42 call.

	Today	**At Expiration**		
		$S_T < 40$	$40 \leq S_T < 42$	$42 \leq S_T$
sell 40 call	**11.93**	**0**	$40 - S_T$	$40 - S_T$
buy 42 call	**-9.55**	**0**	**0**	$S_T - 42$
lend \$2	**-2.00**	**2 + int(\$2)**	**2 + int(\$2)**	**2 + int(\$2)**
Total	**.38**	**2 + int(\$2)**	$42 - S_T$ **+ int(\$2)**	**int(\$2)**