Name ______________________

Northwestern University
Kellogg Graduate School of Management

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Finance 465

Fall 2001
Sections 61, 62 &71

Final Exam

You are allowed two 8 ½" by 11" page of notes (both sides). Answer the questions in the space provided. SHOW ALL YOUR WORK! Assume all the interest rates are annualized and continuously compounded unless it is explicitly stated otherwise. You can spend up to 2 hours on the exam.

I. Answer the following questions true or false and explain why.

a. (5 points) Calls will always sell for more than the value of immediate exercise.

b. (5 points) The implicit leverage in a call option is greatest when the call in deep in-the-money.

II. Use the following information to answer the questions in this section. The annual continuously compounded Japanese risk free rate is 2% and the annual continuously compounded British risk free rate is 5%. The current exchange rate is 140 yen/£ and the annual volatility is 40%.

a. (10 points) Using a two period binomial model compute the price of a European call option which gives the holder the right to buy 1 British pound for 140 yen in six months.

b. (5 points) What is the price of an otherwise identical European put?

c. (10 points) Using a two period binomial model find the price of an otherwise identical American call. What are the delta and the elasticity two periods from expiration?

d. (5 points) Suppose you observe that the American call price is 15 yen. Using only pounds and Japanese T-bills, show how would you take advantage of this. (Show the exact positions and the cash flows).

e. (10 points) Suppose you observed that the price of a pound-yen futures contract (you buy pounds with yen) with delivery in six months is 133 yen/£. Explain in detail (give positions and cash flows) how you can earn an arbitrage profit.

III. Use the following information to answer the questions in this section. Assume the current stock price is \$30, the annual continuously compounded risk free rate is 5%, the continuously compounded dividend yield is 2% and there is 6 months to expiration. All the options are European.

a. (10 points) Fill in as much as possible on the following table

	Price	Delta	Gamma	Theta	Vega
Call K=30	3.54			-0.01	
Put K=30		-0.42	0.045		0.082
Call K=50		0.053	0.013	-0.002	0.023
Put K=50	19.23			0.002	

b. (5 points) Draw the payoff diagram for the following position.

Short 1 Share
Long 1 30 Call
Long 1 50 Call

c. (5 points) To hedge the position in part b., what stock position should you take?

d. (5 points) If the stock price makes a small move, will the hedge make or loss money? What happens if there is a big stock price move?

e. (5 points) If the stock price increases by $1, by approximately how much will the stock position have to change in order to continue to be delta hedged?

f. (5 points) Holding the stock price constant, will the value of the position increase or decrease as the time to expiration gets closer by one day?

IV. Consider the following put prices. Assume all the options have the same time to expiration and the same underlying asset.

Strike Price	50	53	55	58
Put Price	6.2	7.2	8.8	10

(10 points) Find an arbitrage opportunity and show how you can take advantage of it.

V. The spot price of silver is $420/ounce and the continuously compounded annual risk free rate is 3%.

Consider the following forward curve for silver :

Year	0	1	2	3
Forward Price	420	428.48	437.14	445.97

a. (5 points) Does the prices in the forward curve given above imply that silver has a carrying cost?

b. (5 points) Suppose a silver producer wants to borrow $1500. At the end of three years the borrower will deliver two ounces of silver. In addition, the borrower will pay x ounces of silver at the end of years 1,2 and 3. What is x?

Answers to the Final Exam

I. Answer the following questions true or false and explain why.

a. (5 points) Calls will always sell for more than the value of immediate exercise.

If the stock doesn't pay any dividends this is true. However if the stock pays a large dividend, a European call may be worth less than S-K.

b. (5 points) The implicit leverage in a call option is greatest when the call in deep in-the-money.

False. The leverage is highest when the call is deep out-of-the-money.

II. Use the following information to answer the questions in this section. The annual continuously compounded Japanese risk free rate is 2% and the annual continuously compounded British risk free rate is 5%. The current exchange rate is 140 yen/£ and the annual volatility is 40%.

a. (10 points) Using a two period binomial model compute the price of a European call option which gives the holder the right to buy 1 British pound for 140 yen in six months.

$$u = e^{.4\sqrt{\frac{.5}{2}}} = 1.22 \qquad d = 1/u = .82 \qquad p = \frac{\frac{e^{.02*.25}}{e^{.05*.25}} - .82}{1.22 - .82} = .43$$

$$R_d = e^{.02*.25} = 1.005 \qquad R_f = e^{.05*.25} = 1.0126$$

		208.376
	170.80	
S=140		**140**
	114.80	
		94.136

C_{uu} = 68.376 C_{ud} = 0 C_{dd} = 0

C_u = (68.376)(.43)/(1.005) = 29.34
C_d = 0

C = (29.34)(.43)/(1.005) = 12.55

b. (5 points) What is the price of an otherwise identical European put?

$$P = C - Se^{-r_f T} + Ke^{-r_d T} = 12.55 - 140e^{-.05*.5} + 140e^{-.02*.5} = 14.61$$

c. (10 points) Using a two period binomial model find the price of an otherwise identical American call. What are the delta and the elasticity two periods from expiration?

		208.376
	170.80	
S=140		**140**
	114.80	
		94.136

C_{uu} = 68.376 C_{ud} = 0 C_{dd} = 0

C_u = max(170.80-140 = 30.80, 29.34) = 30.80
C_d = 0

C = (30.80)(.43)/(1.005) = 131.78

$$\Delta = \frac{30.80 - 0}{(1.0126)(140)(1.22 - .82)} = .543 \qquad \Omega = \frac{\Delta}{C/S} = \frac{.543}{13.178/140} = 5.768$$

d. (5 points) Suppose you observe that the American call price is 15 yen. Using only pounds and Japanese T-bills, show how would you take advantage of this. (Show the exact positions and the cash flows).

Since the call is overvalued you should sell the call and buy the synthetic.

	Today
Sell Call	**15**
Buy Delta Pounds	**(.543)(140)=76.02**
Borrow B yen	**62.842**
Total	**1.822**

e. (10 points) Suppose you observed that the price of a pound-yen futures contract (you buy pounds with yen) with delivery in six months is 133 yen/**£**. Explain in detail (give positions and cash flows) how you can earn an arbitrage profit.

$$F = (140)e^{(.02-.05)*.5} = 137.91$$

Since the future is undervalued you should buy the future and short the spot.

	Today	**Delivery**
Buy the Future		**-133**
Short $e^{-.05*.5}$ pounds	**(.9753)(140)**	**
Lend the proceeds	**-(.9753)(140)**	**140 $e^{-.05*.5}$ $e^{.02*.5}$=137.91**
Total	**0**	**4.91**

**** Use the pounds from the futures position to cover the short pound position.**

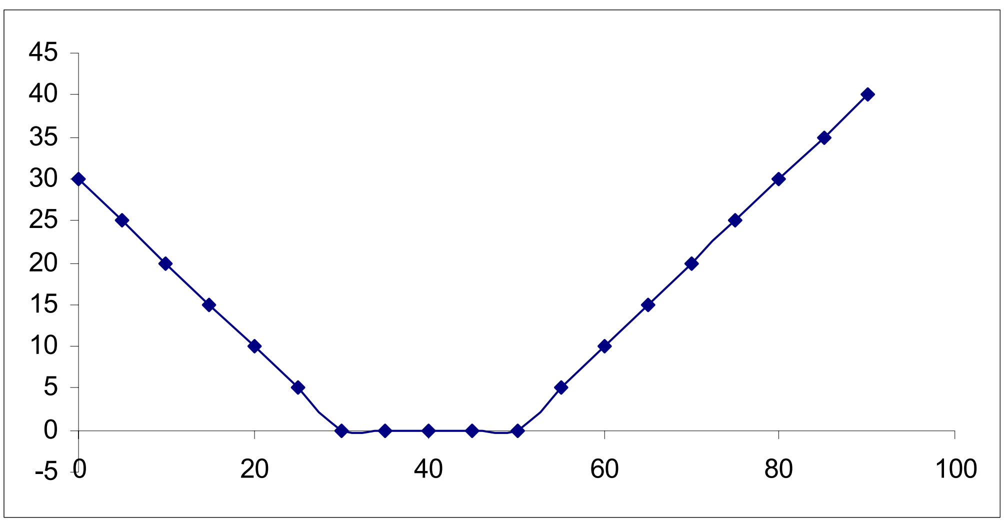
III. Use the following information to answer the questions in this section. Assume the current stock price is $30, the annual continuously compounded risk free rate is 5%, the continuously compounded dividend yield is 2% and there is 6 months to expiration. All the options are European.

a. (10 points) Fill in as much as possible on the following table

	Price	Delta	Gamma	Theta	Vega
Call K=30	3.54	**.58**	**.045**	-0.01	**.082**
Put K=30	**3.10**	-0.42	0.045		0.082
Call K=50	**.17**	0.053	0.013	-0.002	0.023
Put K=50	19.23	**.947**	**.013**	0.002	**.023**

b. (5 points) Draw the payoff diagram for the following position.

Short 1 Share
Long 1 30 Call
Long 1 50 Call



c. (5 points) To hedge the position in part b., what stock position should you take?

Delta = (-1)(1) + (1)(.58) + (1)(.053) = -.367
You should go long .367 shares.

d. (5 points) If the stock price makes a small move, will the hedge make or loss money? What happens if there is a big stock price move?

Gamma = (-1)(0) + (1)(.045) + (1)(.013) = .058

Since the gamma is positive, the position will lose money for small moves and make money on big moves.

e. (5 points) If the stock price increases by $1, by approximately how much will the stock position have to change in order to continue to be delta hedged?

The new delta is approximately equal to the old delta plus gamma, which in this case is = -.367 + .058 = -.309
If the stock price increase by $1, the delta of the position will be less negative, so you will reduce your share position by .058.

f. (5 points) Holding the stock price constant, will the value of the position increase or decrease as the time to expiration gets closer by one day?

Theta = (1)(-.01)+(1)(-.002) = (-.012)

If one passes, the value of the position will drop by .012 due to time decay.

IV. Consider the following put prices. Assume all the options have the same time to expiration and the same underlying asset.

Strike Price	50	53	55	58
Put Price	6.2	7.2	8.8	10

(10 points) Find an arbitrage opportunity and show how you can take advantage of it.

There is a convexity violation with the 50,55 and 58 strike price puts.

8=3/8

8.8 > (3/8)(6.2)+(5/8)(10) = 8.57

	Today	**At Expiration**			
		S<50	**50<S<55**	**55<S<58**	**S>58**
Buy 3 50 puts	**-18.69**	**3(50-S)**	**0**	**0**	**0**
Buy 5 58 puts	**-50.00**	**5(58-S)**	**5(58-S)**	**5(58-S)**	**0**
Sell 8 55 puts	**70.40**	**-8(55-S)**	**-8(55-S)**	**0**	**0**
Total	**1.71**	**0**	**3S-150**	**290-5S**	**0**

V. The spot price of silver is $420/ounce and the continuously compounded annual risk free rate is 3%.

Consider the following forward curve for silver :

Year	0	1	2	3
Forward Price	420	428.48	437.14	445.97

a. (5 points) Does the prices in the forward curve given above imply that silver has a carrying cost?

No, the forward curve is rising by less than the risk free rate. If there was carrying cost it would rise by more than the risk free rate.

b. (5 points) Suppose a silver producer wants to borrow $1500. At the end of three years the borrower will deliver two ounces of silver. In addition, the borrower will pay x ounces of silver at the end of years 1,2 and 3. What is x?

The future value of the loan = $1500e^{.03*3}$=1641.26

The bank will sell the silver it receives forward, so the future value of the forward sales must equals 1641.26

$x(428.48)e^{.03*2} + x(437.14)e^{.03*1}+(x+2)(445.97) = 1641.26$

==> x = .5545