Name

Northwestern University
Kellogg Graduate School of Management

Kathleen Hagerty
Fall 2000
Finance D65
Sections 61 and 62

Final Exam

You are allowed two 8 ½" by 11" page of notes (both sides). There are 90 total points. Answer the questions in the space provided. SHOW ALL YOUR WORK! Assume all the interest rates are annualized unless it is explicitly stated otherwise. You can spend up to 2 hours on the exam.

I. Answer the following true or false and explain why.

a. (5 points) An investor owns $600,000 of a stock with a beta equal to 1.25. The current value of the S&P 500 is 1500. If the investor wants to hedge the risk of this position, she should sell 2 S&P 500 Index futures contracts.

b. (5 points) Suppose an investor who is long natural gas buys a collar. The break-even price of natural gas for the investor is $5.00/MMBtu. The collar consists of a long put with K= $4.50/MMBtu and a short call with K= $5.50/MMBtu. If six months from expiration the current spot price for gas is $4.50/MMBtu, then the delta of the net position (i.e., long position in gas, long put plus short call) has a positive delta and a negative gamma.

c. (5 points) A European call on a <u>yen future</u> should have the same price as an otherwise identical (same expiration date, same strike price) European call on spot yen.

d. (5 points) When replicating a call on a sugar future, the replicating portfolio involves buying sugar futures and shorting T-bills.

e. (5 points) If the slope of the forward curve for contracts where you can buy pesos with dollars is positive, then the Mexican T-bill rate is higher than the U.S. T-bill rate.

II. a. (10 points) Currently a European call option on the NASDAQ 100 with delivery in two months and K=$2500 is selling for $250 and an otherwise identical put is selling for $110. The current value of the index is 2600 and the dividend yield is 1%. Assume the risk-free rate is 6%. Show how you can earn an arbitrage profit.

III. a. (10 points) Using a two period binomial model, compute the price of 18 month American call on a GE stock with with K= \$45. Assume the annual risk-free rate is 6%, the annual dividend yield is 5%, the current stock price is \$48 per share , the annual volatility is 40%.

b. (5 points) What would be the price of otherwise identical European call?

c. (5 points) Suppose the beta of GE is 1.2. What is the beta of the American call 18 months prior to expiration?

d. Suppose you had a position which consisted of the a long European call like the one described above and three short European puts with the same underlying asset, same strike and same time to expiration as the call.

i. (5 points) Draw the payoff to this position at expiration

ii. (10 points) How would you hedge this position eighteen months prior to expiration (assume the stock price is 48/share)? (**Be specific**)   If there is a large change in the value of the future, will the hedge make money or lose money? Why?

IV. Suppose a trader enters into 2 long futures contracts for 5000 bushels of wheat with $F_{t,T}$ =2.50/bushel, an initial margin equal to $5000 and a maintenance margin equal to $3000.

a. (5 points) What price move will imply that $500 can be withdrawn from the trader's margin account.

b. (5 points) What price move will generate a margin call?

V. (10 points) Consider the following set of European call prices. Identify one mispricing and explain how you could earn an arbitrage profit. Assume the current stock price is $40 and the annual risk-free rate is 10%.

Strike Price \ Time to Expiration	30 days	60 days
30	10.00	11.63
45	5.20	5.36
50	.39	1.48

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Final Exam Answers

I. Answer the following true or false and explain why.

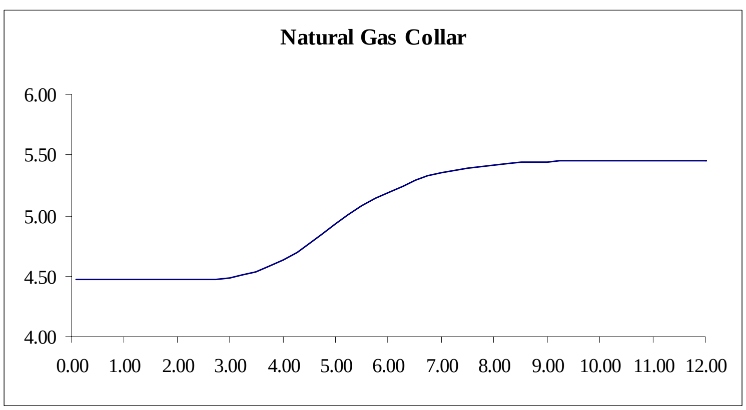
a. (5 points) An investor owns $600,000 of a stock with a beta equal to 1.25. The current value of the S&P 500 is 1500. If the investor wants to hedge the risk of this position, she should sell 2 S&P 500 Index futures contracts.

True.

h= -β = -1.25 ==> You should short 1.25 dollars of the S&P 500 Index future for every dollar in the stock.

$600,000*1.25 = $750,000 ==>
750,000/1500 = 500 units of the index ==>
500/250 = 2 contracts

b. (5 points) Suppose an investor who is long natural gas buys a collar. The break-even price of natural gas for the investor is $5.00/MMBtu. The collar consists of a long put with K= $4.50/MMBtu and a short call with K= $5.50/MMBtu. If six months from expiration the current spot price for gas is $4.50/MMBtu, then the delta of the net position (i.e., long position in gas, long put plus short call) has a positive delta and a negative gamma.



False. The delta and the gamma are both positive.

c. (5 points) A European call on a yen future should have the same price as an otherwise identical (same expiration date, same strike price) European call on spot yen.

True. The payoffs at expiration are the same so the prices are the same.

d. (5 points) When replicating a call on a sugar future, the replicating portfolio involves buying sugar futures and shorting T-bills.

False. When replicating a call option on a future, the T-bill position is always long.

e. (5 points) If the slope of the forward curve for contracts where you can buy pesos with dollars is positive, then the Mexican T-bill rate is higher than the U.S. T-bill rate.

False. The slope of the forward curve is equal to (r_d -r_f). If $r_d < r_f$, the slope is negative.

II. a. (10 points) Currently a European call option on the NASDAQ 100 with delivery in two months and K=$2500 is selling for $250 and an otherwise identical put is selling for $110. The current value of the index is 2600 and the dividend yield is 1%. Assume the risk-free rate is 6%. Show how you can earn an arbitrage profit.

Put-call parity says that $C = P + Se^{-\delta T} - Ke^{-rT}$ ==> However,

$$250 \neq 110 + 2600\ e^{-.01*1/6} - 2500\ e^{-.06*1/6} = 230.54$$

Since put-call parity is violated, you can earn an arbitrage profit.

	Today	**At Expiration**	
		$S_T < 2500$	$S_T \geq 2500$
Sell 1 Call	**250**	**0**	**2500 - S_T**
Buy 1 Put	**-110**	**2500 - S_T**	**0**
Buy $e^{-.01*1/6}$ Shares	**-2600 $e^{-.01*1/6}$**	S_T	S_T
Borrow PV of K	**2500 $e^{-.06*1/6}$**	**-2500**	**-2500**
Total	**19.46**	**0**	**0**

III. a. (10 points) Using a two period binomial model, compute the price of 18 month American call on a GE stock with with K= $45. Assume the annual risk-free rate is 6%, the annual dividend yield is 5%, the current stock price is $48 per share , the annual volatility is 40%.

$$\mathbf{u} = e^{.4*\sqrt{1.5/2}} = \mathbf{1.41} \quad \mathbf{d} = 1/\mathbf{u} = .707 \quad \mathbf{p} = \frac{\left(\frac{1.046}{1.038}\right) - .707}{1.41 - .707} = .427 \quad \mathbf{R} = e^{.06*1.5/2} = \mathbf{1.046}$$

$\theta = e^{.05*(1.5/2)} = 1.038$

		95.43
	67.68	
S = 48		**48**
	33.94	
		24

C_{uu} = 50.43
C_{ud} = 3
C_{dd} = 0
C_u = max{67.68 - 45, [.427*50.43 + (1-.427)*3]/(1.046)} = max{22.68, 22.23} = 22.68
C_d = [.427*3 + (1-.427)*0]/(1.046) = 1.22
C = max{48 - 45, [.427*22.68 + (1-.427)*1.22]/(1.046)} = max{3, 9.93} =9.93

b. (5 points) What would be the price of otherwise identical European call?

C_{uu} = 50.43
C_{ud} = 3
C_{dd} = 0
C_u = [.427*50.43 + (1-.427)*3]/(1.046) = 22.23
C_d = [.427*3 + (1-.427)*0]/(1.046) = 1.22
C = [.427*22.23 + (1-.427)*1.22]/(1.046)} = 9.74

c. (5 points) Suppose the beta of GE is 1.2. What is the beta of the American call 18 months prior to expiration?

To compute the value of beta, we need to compute the elasticity. To compute the elasticity we need to compute the delta of the call.

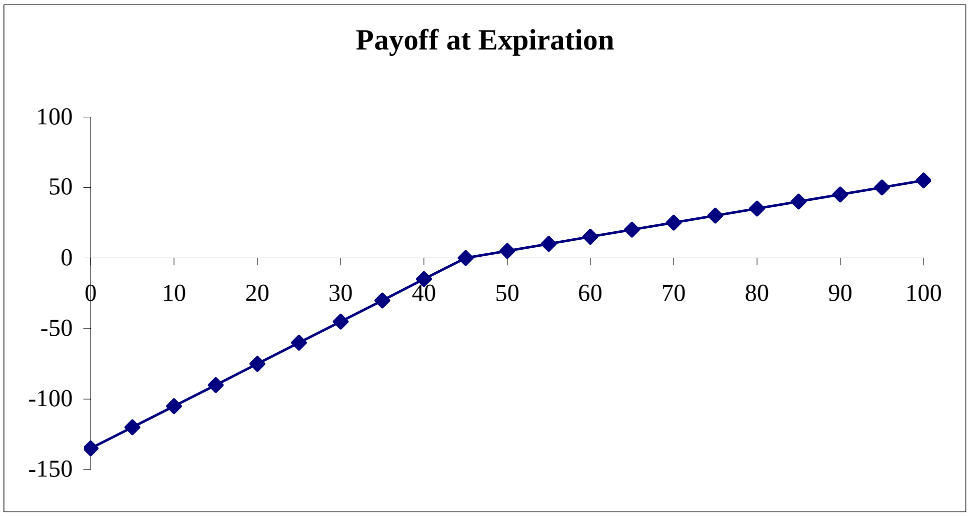
Δ = (22.68 - 1.22)/[(1.038)(67.68 - 33.94)] = .613

$\Omega_{call} = \Delta * S/C$ = .613 * (48/9.93) = 2.96

$\beta_{call} = \Omega_{call} * \beta_{stock}$ = 2.96 * 1.2 = 3.55

d. Suppose you had a position which consisted of the a long European call like the one described above and three short European puts with the same underlying asset, same strike and same time to expiration as the call.

i. (5 points) Draw the payoff to this position at expiration



ii. (10 points) How would you hedge this position eighteen months prior to expiration (assume the stock price is 48/share)? (**Be specific**) If there is a large change in the value of the future, will the hedge make money or lose money? Why?

Δ_{call} = (22.23 - 1.22)/[(1.038)(67.68 - 33.94)] = .60
Δ_{put} = Δ_{call} - 1 = .60 -1 = -.40
$\Delta_{position}$ = .60 + (-3)(-.4) = 1.8

To hedge this position you should short 1.8 shares of the stock. Since the gamma of the position is negative, the position would lose money on a big stock price move.

IV. Suppose a trader enters into 2 long futures contracts for 5000 bushels of wheat with $F_{t,T}$ =2.50/bushel, an initial margin equal to $5000 and a maintenance margin equal to $3000.

a. (5 points) What price move will imply that $500 can be withdrawn from the trader's margin account.

Since the trader is long the futures he will make money when the price rises. Each contract is for 5000 bushels. If the price rises by .05/bushel, the trader will have a gain of .05*2*5000 = $500 which can be withdrawn from the trader's margin account.

b. (5 points) What price move will generate a margin call?

Since the trader is long the futures he will lose money when the price falls. If the trader loses more than $2000, there will be a margin call. Each contract is for 5000 bushels. If the price falls by .20/bushel or more, the trader will have a loss of .20*2*5000 = $2000 or more.

V. (10 points) Consider the following set of European call prices. Identify one mispricing and explain how you could earn an arbitrage profit. Assume the current stock price is $40 and the annual risk-free rate is 10%.

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There is a violation of the convexity condition. That is, it should be the case that $C(45) < \lambda C(30) + (1-\lambda)C(50)$. However, 5.20 > (50-45)/(50-30)*10 + (45-30)/(50-30)*.39 = 2.79. The arbitrage is:

	Today	**At Expiration**			
		$S_T<30$	$30\le S_T<45$	$45\le S_T<50$	$50\le S_T$
Sell 4 C(45)	**20.80**	**0**	**0**	**$4*(45-S_T)$**	**$4*(45-S_T)$**
Buy 1 C(30)	**-10.00**	**0**	**S_T-30**	**S_T-30**	**S_T-30**
Buy 3 C(50)	**-1.17**	**0**	**0**	**0**	**$3*(S_T-50)$**
Total	**9.63**	**0**	**S_T-30**	**$150-3S_T$**	**0**