Merger Policy with Merger Choice

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- Analyze the optimal policy of an antitrust authority when merger proposals are endogenous and firms choose among alternative mergers
- The main result of the paper shows that the optimal policy of an antitrust authority that seeks to maximize expected consumer surplus imposes a tougher standard on "larger" mergers

The Model

- a homogeneous goods industry in which firms engaged in cournot competition
- **N**$= 0, 1, 2, ..., N$ denotes the initial set of firms.
- assumption
  (i) $P'(Q) + qP''(Q) < 0$ for all q $\in [0, Q]$;
  (ii) $\lim_{Q\to\infty} P(Q) = 0$
  these assumptions ensure the existence of a unique stable NE in cournot competition

Pre-merger equilibrium

- vector of output levels in equil. is denoted by $q^\circ \equiv (q_0^\circ, q_1^\circ, ..., q_N^\circ)$ $q_i^\circ > 0$ for all i
- first order condition $P(Q^\circ) + P'(Q^\circ)q_i^\circ = c_i$
- $CS \equiv \int_0^{Q^\circ} P(s)ds - P^\circ Q^\circ$
- $\pi_i^\circ \equiv [P^\circ - c_i]q_i^\circ$
- firm i's market share $s_i^\circ \equiv q_i^\circ / Q^\circ$

Merger

- suppose there is a set **K** of K potential mergers, each between firm 0(acquirer) and a single merger partner(target) k $\in$ **K**
- $\phi_k \in (0, 1)$ determines whether the merger between firm 0 and firm k is feasible
- $\theta_k \equiv Pr(\phi_k = 1) > 0$ probability that the merger is feasible
- a feasible merger is denoted by $M_k = (k, \bar{c_k})$
  k is the identity of the target and $\bar{c_k}$ is the realized post-merger marginal cost
- $\bar{c_k}$ is drawn from distribution function G_k with support $[l, h_k]$
- random draws of ϕ_k and $\bar{c_k}$ are independent across mergers
- realized set of feasible mergers is denoted $\mathbf{F} \equiv \{M_k : \phi_k = 1\}$

post-merger equilibrium

If merger M_k is implemented

- vector of outputs in equil. is denoted by
  $q(M_k) \equiv (q_1(M_k), ..., q_N(M_k))$
  we assume all nonmerging firms remains active after any merger
- market share of firm i $s_i(M_k) \equiv q_i(M_k)/Q(M_k)$
- first order condition
  $P(Q(M_k)) + P'(Q(M_k))q_i(M_k) = c_i$
  $P(Q(M_k)) + P'(Q(M_k))q_k(M_k) = \bar{c_k}$
- post-merger profit
  $\pi_i(M_k) \equiv [P(Q(M_k)) - c_i]q_i(M_k)$
  $\pi_k(M_k) \equiv [P(Q(M_k)) - \bar{c_k}]q_k(M_k)$

change in consumer surplus

the induced change in consumer surplus is
$\Delta CS(M_k) \equiv \int_0^{Q(M_k)} P(s)ds - P(Q(M_k))Q(M_k) - CS^\circ$

- M_k is CS-neutral if $\Delta CS(M_k) = 0$, CS-increasing if $\Delta CS(M_k) > 0$, and CS-decreasing if $\Delta CS(M_k) < 0$
- if no merger is implemented, the status quo obtains, which we denote by M°
  same outcomes as in the pre-merger equilibrium and M° is CS-neutral

antitrust authority

If merger M_k, k$\in$ **F**, is proposed, the antitrust authority can observe all aspects of that merger and knows as well the pre-merger cost levels of all firms.
What it doesn't observe are the characteristics of any feasible mergers that are not proposed.
Also, assume antitrust authority can commit ex ante to its policy.
In this paper, the authors confine attention to deterministic policies.
Assume only one of the mutually exclusive mergers can be proposed to, and evaluated by, the antitrust authority.

- The antitrust authority commits to a merger-specific approval policy by specifying an approval set A.
  $A \equiv \{M_k : \bar{c_k} \in A_k\}$ where $A_k \subseteq [l, h_k]$ for $k \in \mathbf{K}$

- The feasible mergers M_k that would be approved if proposed are given by the set $\mathbf{F} \cap A$
- A bargaining process among the firms determines which feasible merger is actually proposed.
  Firm 0 makes a take-it-or-leave-it offers of an acquisition price t_k to a single firm k of its choosing, where k is such that $M_k \in (\mathbf{F} \cap A)$, and firm 0 acquires the target in return for the payment t_k
- By choosing the payment t_k that makes firm k just indifferent from accepting and not, firm 0 can exact the entire bilateral profit gain $\Delta\Pi(M_k)$
- change in bilateral profit of the merging parties is denoted by $\Delta\Pi(M_k) \equiv \pi_k(M_k) - [\pi_0^\circ + \pi_k^\circ]$
  $\Delta\Pi(M^\circ)$

- Given $\mathbf{F} \cap A$, the merger outcome in the equilibrium of the offer game is $M^*(\mathbf{F}, A)$
  $M^*(\mathbf{F}, A) \equiv$
  $$\begin{cases} argmax_{M_k \in (\mathbf{F} \cap A)} \Delta\Pi(M_k) & if \ max_{M_k \in (\mathbf{F} \cap A)} \Delta\Pi(M_k) > 0 \\ M^\circ & otherwise \end{cases}$$
- antitrust authority's optimization problem:
  $max_A E_{\mathbf{F}}[\Delta CS(M^*(\mathbf{F}, A))]$
- assumption: For all $k \in \mathbf{K}$, the support of the post-merger cost distribution includes both CS increasing and CS decreasing mergers.
- Let $\mathbf{K} \equiv \{1, ...K\}$ and label firms 1 through K in decreasing order of their pre-merger marginal costs: $c_1 > c_2 > ... > c_K$.

Lemma1

Suppose merger M_k is CS neutral. Then

- (i) the merger causes no changes in the output of any nonmerging firm nor in the joint output of the merging firms
- (ii) $P(Q^\circ) - \bar{c}_k = [P(Q^\circ) - c_0] + [P(Q^\circ) - c_k]$
- (iii) the merger is profitable for merging firms
- (iv) the merger increases aggregate profit

Lemma2

Conditional on merger M_k being implemented, a reduction in the post-merger marginal cost $\bar{c}_k$ causes aggregate output, consumer surplus, and the merged firm's profit to increase.

Lemma3

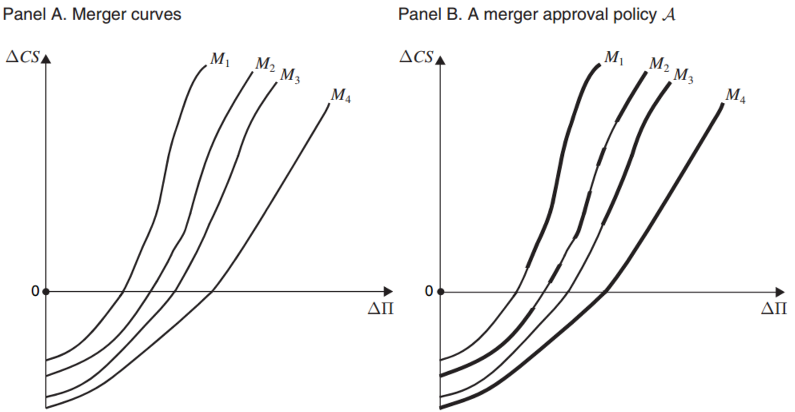
Suppose two mergers, M_j and M_k with $k > j$, induce the same nonnegative change in consumer surplus. Then, the larger merger M_k induces a greater increase in bilateral profit of the merger partners.

Corollary1

COROLLARY 1: *If two CS-nondecreasing mergers M_j and M_k with $k > j$ have $\Delta\Pi(M_k) \leq \Delta\Pi(M_j)$, then $\Delta CS(M_k) < \Delta CS(M_j)$.*

PROOF:

Suppose instead that $\Delta CS(M_k) \geq \Delta CS(M_j)$. Then there exists a $\overline{c}'_k > \overline{c}_k$ such that $\Delta CS(k, \overline{c}'_k) = \Delta CS(M_j)$. But this implies (using Lemma 2 for the first inequality and Lemma 3 for the second) that $\Delta\Pi(M_k) > \Delta\Pi(k, \overline{c}'_k) > \Delta\Pi(M_j)$, a contradiction.


Panel A. Merger curves
ΔCS
M_1
M_2
M_3
M_4
0
$\Delta\Pi$
Panel B. A merger approval policy $\mathcal{A}$
ΔCS
M_1
M_2
M_3
M_4
0
$\Delta\Pi$

Optimal Merger Policy

- Let largest allowable post-merger cost level be
  $\bar{a}_k \equiv max\{\bar{c}_k : \bar{c}_k \in A_k\}$

- $\underline{\Delta CS_k} \equiv \Delta CS(k, \bar{a}_k)$
  $\underline{\Delta \Pi_k} \equiv \Delta \Pi(k, \bar{a}_k)$

  These are the lowest levels of CS and bilateral profit in any allowable merger between firm 0 and firm k.(lemma2)
  such mergers are called marginal margers

Proposition 1

Any optimal approval policy A approves the smallest merger if and only if it is CS non decreasing, approves only mergers $k \in K^{+} \equiv \{1, ..., \hat{K}\}$ with positive probability and satisfies $0 = \underline{\Delta CS_1} < \underline{\Delta CS_2} < ... < \underline{\Delta CS_{\hat{K}}}$ *for all* $k \leq \hat{K}$.

That is, the lowest level of the consumer surplus change that is acceptable to the antitrust authority equals zero for the smallest merger M_1, is strictly positive for every other merger M_k, and is monotonically increasing in the size of the merger, while the largest merger may never be approved.

step1

Step 1
An optimal policy doesn't approve CS-decreasing mergers.

Proof
Suppose the approval set A includes CS-decreasing mergers, and consider the set A^+ that removes any mergers in A that reduce CS.

The change in expected CS from the change in the approval policy equals $Pr(M^*(F, A) \in A \backslash A^+)$ times
$E_F[\Delta CS(M^*(F, A^+)) - \Delta CS(M^*(F, A)) | M^*(F, A) \in A \backslash A^+]$.

The change is strictly positive.

step2

Step 2
Any smallest merger M_1 that is CS nondecreasing must be approved.

Proof
Suppose that the approval set is A but that
$A \subset A' \equiv (A \cup \{(1, \bar{c_1}) : \Delta CS(1, \bar{c_1}) \geq 0\})$

The change in expected CS by using A' rather than A equals
$Pr(M^*(F, A') \in A' \backslash A)$ times
$E_F[\Delta CS(M^*(F, A')) - \Delta CS(M^*(F, A)) | M^*(F, A') \in A' \backslash A]$
which is strictly positive by corollary 1 and the fact that $A' \backslash A$ contains only smallest mergers

step3

Step 3

in any optimal policy, $\underline{\Delta CS_k}$ must equal the expected change in CS from the next-most profitable merger

Proof

Defining the expected change in CS from the next-most profitable merger $M^*(F \backslash M_k, A)$, conditional on merger $M_k = (k, \bar{c_k})$ being the most profitable merger in $F \cap A$, to be

$$E_k^A(\bar{c_k})$$
$$\equiv E_F[\Delta CS(M^*(F \backslash M_k, A)) | M_k = (k, \bar{c_k}) \ and \ M_k = M^*(F, A)]$$
$$= E_F[\Delta CS(M^*(F \backslash M_k, A)) | M_k = (k, \bar{c_k}) \ and \ \Delta\Pi(M_k) \geq \Delta\Pi(M^*(F \backslash M_k, A))]$$

We need to show $\underline{\Delta CS_k} = E_k^A(\bar{a_k})$ holds for all k

Suppose first that $\underline{\Delta CS_{k'}} > E_{k'}^A(\bar{a_{k'}})$ for some k' and consider the alternative approval set $A \cup A_{k'}^{\epsilon}$, where
$A_{k'}^{\epsilon} \equiv \{M_k : M_k = (k', \bar{c_{k'}})\ with\ \bar{c_{k'}} \in (\bar{a_{k'}}, \bar{a_{k'}} + \epsilon)\}$

The change in expected CS from A to the alternative set equals
$Pr(M^*(F, A \cup A_{k'}^{\epsilon}) \in A_{k'}^{\epsilon})$ times
$E_F[\Delta CS(M^*(F, A \cup A_{k'}^{\epsilon})) - \Delta CS(M^*(F, A)) | M^*(F, A \cup A_{k'}^{\epsilon}) \in A_{k'}^{\epsilon}]$

$$= E_F[\Delta CS(M^*(F, A \cup A_{k'}^{\epsilon})) - E_{k'}^A(\bar{c_{k'}}) | M^*(F, A \cup A_{k'}^{\epsilon}) \in A_{k'}^{\epsilon}]$$

by continuity, the above conditional expectation is strictly positive for some ϵ
similar argument applies if $\underline{\Delta CS_{k'}} < E_{k'}^A(\bar{a_{k'}})$

step4

Step 4
for all $j < k$, $\underline{\Delta\Pi_j} \leq \underline{\Delta\Pi_k}$

Proof
let $k' \equiv argmin_{k>j}\underline{\Delta\Pi_k}$ and suppose that $\underline{\Delta\Pi_j} > \underline{\Delta\Pi_{k'}}$. By step 3, $\underline{\Delta CS_{k'}} = E^A_{k'}(\bar{a_{k'}})$ Let $\bar{c_j'}$ be the post-merger cost level satisfying $\Delta\Pi(j, \bar{c_j'}) = \underline{\Delta\Pi_{k'}}$ and consider a change in the approval set from A to $A \cup \bar{A}^\epsilon_j$ where $\bar{A}^\epsilon_j \equiv \{M_j : M_j = (j, \bar{c}_j)\ with\ \bar{c}_j \in (\bar{c_j'}, \bar{c_j'} + \epsilon)\}$

The change in expected CS from the change in the approval set equals $Pr(M^*(F, A \cup \bar{A}^\epsilon_j \in \bar{A}^\epsilon_j)$ times
$E_F[\Delta CS(M^*(F, A \cup \bar{A}^\epsilon_j)) - E^A_j(\bar{c}_j)|M^*(F, A \cup \bar{A}^\epsilon_j) \in \bar{A}^\epsilon_j]$

As ϵ goes to zero, the expected change converges to

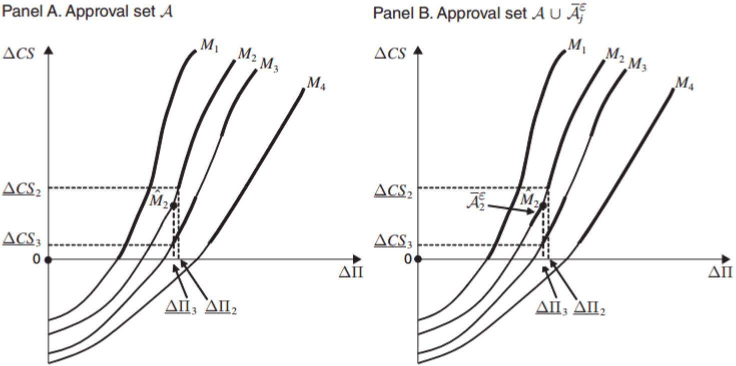
$$\Delta CS(j, \bar{c}'_j) - E^A_j(\bar{c}'_j)$$

$$= \Delta CS(j, \bar{c}'_j) - E^A_{k'}(\bar{a}'_k)$$

$$> \underline{\Delta CS_{k'}} - E^A_{k'}(\bar{a}_{k'}) = 0$$

where the inequality follows from corollary 1 since
$\Delta\Pi(j, \bar{c}'_j) = \underline{\Delta\Pi_{k'}}$

Proposition1



step5

Step 5
$\underline{\Delta CS_j} < \underline{\Delta CS_k}$ *for all* $j, k \in K^+$ *with* $j < k$

Proof
Suppose for some $j, h \in K^+$ *with with* $h > j$, we have
$\underline{\Delta CS_j} \geq \underline{\Delta CS_h}$
Define $k = argmin\{h \in K^+ : h > j \text{ and } \underline{\Delta CS_j} \geq \underline{\Delta CS_h}\}$
By step 3, $E_j^A(\bar{a}_j) = \underline{\Delta CS_j} \geq \underline{\Delta CS_k} = E_k^A(\bar{a}_k)$

$E_k^A(\bar{a}_k)$ is a weighted average of the following two conditional expectations.

$(1)E_F[\Delta CS(M^*(F\backslash M_k, A))|M_k = (k, \bar{a_k}), M_k = M^*(F, A),\ \Delta\Pi(M^*(F\backslash M_k, A)) < \underline{\Delta\Pi_j}]$

and

$(2)E_F[\Delta CS(M^*(F\backslash M_k, A))|M_k = (k, \bar{a_k}), M_k = M^*(F, A),\ \Delta\Pi(M^*(F\backslash M_k, A)) \in [\underline{\Delta\Pi_j}, \underline{\Delta\Pi_k}]]$

(1) no merger in A by either k or j can have such profit level(by step4), (1)=$E_j^A(\bar{a_j}) = \overline{\Delta CS_j}$
(2) case1 when $M^*(F\backslash M_k, A) = (j, \bar{c_j}) \geq \underline{\Delta CS_j}$
case2 when $M^*(F\backslash M_k, A) = (r, \bar{c_r})$ *for some* $r < j$ (by corollary 1) $> \underline{\Delta CS_j}$
case3 when $M^*(F\backslash M_k, A) = (r, \bar{c_r})$ *for some* $j < r < k$ (by definition of k) $> \underline{\Delta CS_j}$ lead to a contradiction.

step6

Step 6 If there is a merger M_j that will never be approved under the optimal policy A, then no larger merger M_k, $k > j$, will ever be approved.
observe that $\Delta CS(k, l)$ is decreasing in k
by argument similar to those showing monotonicity of $\underline{\Delta CS_k}$ in k for $k \in K^+$
this implies if merger M_k is never approved, the neither is any merger that is larger than M_k

Cutoff Policy

- Proposition 1 does not fully characterize the marginal mergers. Identifying the marginal merger for each target would be simpler if we knew the optimal policy had a "cutoff" structure.
- A cutoff policy $A^c = (\bar{a_1^c}, ...\bar{a_k^c})$ such that $M_k = (k, \bar{c}_k) \in A^c$ if only if $\bar{c}_k \leq \bar{a_k^c}$
  Proposition 1 implies that the marginal mergers can be found by a simple recursive procedure:
- accept all CS-nondecreasing M_1
- for k=2,...,K, recursively identify the largest post-merger cost level $\bar{a_k^c}$ for which $\Delta CS_k(k, \bar{a_k^c}) = E_k^{A^c}(\bar{a_k^c})$
- If $\Delta CS(k, \bar{c}_k) < E_k^{A^c}(\bar{c}_k)$ for all $\bar{c}_k \in [l, h_k]$, then no such cutoff exists for all $k' \geq k$

Conclusion

- in this environment, the antitrust authority optimally commits to a policy that imposes a tougher standard on mergers involving firms with a larger pre-merger market share.
- the optimal policy rejects some consumer surplus-enhancing larger mergers to induce firms to propose better smaller ones